## PROBLEMS

**1.1.** If a charged particle is undeflected in passing through uniform crossed electric and magnetic fields **E** and **B** (mutually perpendicular, and both perpendicular to the direction of motion), what is its velocity? If we now turn off the electric field, and the particle moves in an arc of radius $R$, what is its charge-to-mass ratio?

**1.2.** The mass of Yukawa's meson can be estimated as follows. When two protons in a nucleus exchange a meson (mass $m$) they must temporarily violate the conservation of energy by an amount $mc^2$ (the rest energy of the meson). The Heisenberg uncertainty principle says that you may "borrow" an energy $\Delta E$, provided you "pay it back" in a time $\Delta t$ given by $\Delta E \, \Delta t = \hbar$ (where $\hbar \equiv h/2\pi$). In this case we need to borrow $\Delta E = mc^2$ long enough for the meson to make it from one proton to the other. It has to cross the nucleus (size $r_0$), and it travels, presumably, at some substantial fraction of the speed of light, so, roughly speaking, $\Delta t = r_0/c$. Putting this all together, we have

$$m = \frac{\hbar}{r_0 c}$$

Using $r_0 = 10^{-13}$ cm (the size of a typical nucleus), calculate the mass of Yukawa's meson. Express your answer as a multiple of the electron's mass, and compare the observed mass of the pion. [If you find that argument compelling, I can only say that you're pretty gullible. Try it for an *atom*, and you'll conclude that the mass of the photon is about $7 \times 10^{-30}$ g, which is nonsense. Nevertheless, it is a useful device for "back-of-the-envelope" calculations, and it does very well for the pi meson. Unfortunately, many books present it as though it were a rigorous derivation, which it certainly is *not*. The uncertainty principle does *not* license violation of conservation of energy (nor does any such violation occur in this process; we shall see later on how this comes about). Moreover, it's an *inequality*, $\Delta E \, \Delta t \geqslant \hbar$, which at most could give you a *lower bound* on $m$. It *is* typically true that the *range* of a force is inversely proportional to the mass of the mediator, but the size of a bound state is not always a good measure of the range (that's why the argument fails for the photon: The range of the electromagnetic force is infinite, but the size of an atom is not). In general, when you hear a physicist invoke the uncertainty principle, keep a hand on your wallet.]

**1.3.** In the period before the discovery of the neutron many people thought the nucleus consisted of protons and *electrons,* with the atomic number equal to the excess number of protons. Beta decay seemed to support this idea—after all, electrons come popping out; doesn't that imply that there were electrons inside? Use the position-momentum uncertainty relation, $\Delta x \, \Delta p \geqslant \hbar$, to estimate the minimum momentum of an electron confined to a nucleus (radius $10^{-13}$ cm). From the relativistic energy-momentum relation, $E^2 - \mathbf{p}^2c^2 = m^2c^4$, determine the corresponding energy, and compare it with that of an electron emitted in, say, the beta decay of tritium (Fig. 1.6). (This result convinced some people that the beta-decay electron could *not* have been rattling around inside the nucleus, but must be produced in the disintegration itself.)

**1.4.** The *Gell-Mann/Okubo mass formula* relates the masses of members of the baryon octet (ignoring small differences between $p$ and $n$; $\Sigma^+$, $\Sigma^0$, and $\Sigma^-$; and $\Xi^0$ and $\Xi^-$):

$$2(m_N + m_\Xi) = 3m_\Lambda + m_\Sigma$$

Using this formula, together with the known masses of the *nucleon* $N$ (use the average of $p$ and $n$), $\Sigma$ (again, use the average), and $\Xi$ (ditto), "predict" the mass of the $\Lambda$. How close do you come to the observed value?

**1.5.** The same formula applies to the mesons (with $\Sigma \to \pi$, $\Lambda \to \eta$, etc.); only, for reasons that remain something of a mystery, in this case you must use the *squares* of the masses. Use this to "predict" the mass of the $\eta$. How close do you come?

**1.6.** The mass formula for decuplets is much simpler—equal spacing between the rows:

$$M_\Delta - M_{\Sigma^*} = M_{\Sigma^*} - M_{\Xi^*} = M_{\Xi^*} - M_\Omega$$

Use this formula (as Gell-Mann did) to predict the mass of the $\Omega^-$. (Use the average of the first two spacings to estimate the third.) How close is your prediction to the observed value?

**1.7.** **(a)** Members of the baryon decuplet typically decay after $10^{-23}$ sec into a lighter baryon (from the baryon octet) and a meson (from the pseudo-scalar meson octet). Thus, for example, $\Delta^{++} \to p^+ + \pi^+$. List all decay modes of this form for the $\Delta^-$, $\Sigma^{*+}$, and $\Xi^{*-}$. Remember that these decays must conserve charge and strangeness (they are *strong* interactions).

**(b)** In any decay, there must be sufficient mass in the original particle to cover the masses of the decay products. (There may be *more* than enough; the extra will be "soaked up" in the form of kinetic energy in the final state.) Check each of the decays you proposed in part (a) to see which ones meet this criterion. The others are kinematically forbidden.

**1.8.** **(a)** Analyze the possible decay modes of the $\Omega^-$, just as you did in Problem 1.7 for the $\Delta$, $\Sigma^*$, and $\Xi^*$. See the problem? Gell-Mann predicted that the $\Omega^-$ would be "metastable" (i.e., much longer lived than the other members of the decuplet), for precisely this reason. (The $\Omega^-$ *does* in fact decay, but by the much slower *weak* interaction, which does not conserve strangeness.)

(b) From the bubble chamber photograph (Fig. 1.11, measure the length of the $\Omega^-$ track, and use this to estimate the lifetime of the $\Omega^-$. (Of course, you don't know how fast it was going, but it's a safe bet that the speed was less than the velocity of light; let's say it was going about $0.1c$. Also, you don't know if the reproduction has enlarged or shrunk the scale, but never mind: this is quibbling over factors of 2, or 5, or maybe even 10. The important point is that the lifetime is many orders of magnitude longer than the $10^{-23}$ sec characteristic of all other members of the decuplet).

**1.9.** Check the *Coleman-Glashow relation* [*Phys. Rev.* **B134**, 671 (1964)]:

$$\Sigma^+ - \Sigma^- = p - n + \Xi^0 - \Xi^-$$

(the particle names stand for their masses).

**1.10.** Look up the table of "known" mesons compiled by M. Roos in *Rev. Mod. Phys.* **35**, 314 (1963), and compare the current Particle Data Booklet[20] to determine which of the 1963 mesons have stood the test of time. (Some of the names have been changed, so you will have to work from other properties, such as mass, charge, strangeness, etc.)

**1.11.** Of the spurious particles you identified in Problem 1.10, which are "exotic" (i.e., inconsistent with the quark model)? How many of the surviving mesons are exotic?

**1.12.** How many different *meson* combinations can you make with 1, 2, 3, 4, 5, or 6 different quark flavors? What's the general formula for $n$ flavors?

**1.13.** How many different *baryon* combinations can you make with 1, 2, 3, 4, 5, or 6 different quark flavors? What's the general formula for $n$ flavors?

**1.14.** Using four quarks ($u$, $d$, $s$, and $c$), construct a table of all the possible baryon species. How many combinations carry a charm of +1? How many carry charm +2, and +3?

**1.15.** Same as Problem 1.14, but this time for *mesons*.

**1.16.** De Rujula, Georgi, and Glashow [*Phys. Rev.* **D12**, 147 (1975)] estimated the quark masses to be: $m_u = m_d = 336$ MeV/$c^2$, $m_s = 540$ MeV/$c^2$, and $m_c = 1500$ MeV/$c^2$ (the bottom quark is about 4500 MeV/$c^2$). If they are right, the average binding energy for members of the baryon octet is −62 MeV. If they all had *exactly* this binding energy, what would their masses be? Compare the *actual* values, and give the percent error. (Don't try this on the other supermultiplets, however. There really is no reason to suppose the binding energy is the same for all members of the group. The problem of hadron masses is a thorny issue, to which we shall return in Chapter 5.)

**1.17.** M. Shupe [*Phys. Lett.* **86B**, 87 (1979)] has proposed that all quarks and leptons are composed of two even more elementary constituents: $c$ (with charge −1/3) and $n$ (with charge zero)—and their respective antiparticles, $\bar{c}$ and $\bar{n}$. You're allowed to combine them in groups of three particles or three antiparticles ($ccn$, for example, or $\bar{n}\bar{n}\bar{n}$). Construct all of the eight quarks and leptons in the first generation in this manner. (The other generations are supposed to be excited states.) Notice that each of the *quark* states admits three possible permutations ($ccn$, $cnc$, $ncc$, for example)—these correspond to the three colors. Mediators can be constructed from three particles plus three antiparticles. $W^\pm$, $Z^0$, and $\gamma$ involve three *like* particles and three like antiparticles ($W^- = ccc\bar{n}\bar{n}\bar{n}$, for instance). Construct $W^+$, $Z^0$, and $\gamma$ in this way. Gluons involve mixed combinations ($ccn\bar{c}\bar{n}\bar{n}$, for instance). How many possibilities are there in all? Can you think of a way to reduce this down to eight?