Eletromagnetismo Avançado

3º ciclo
Aula de 29 outubro

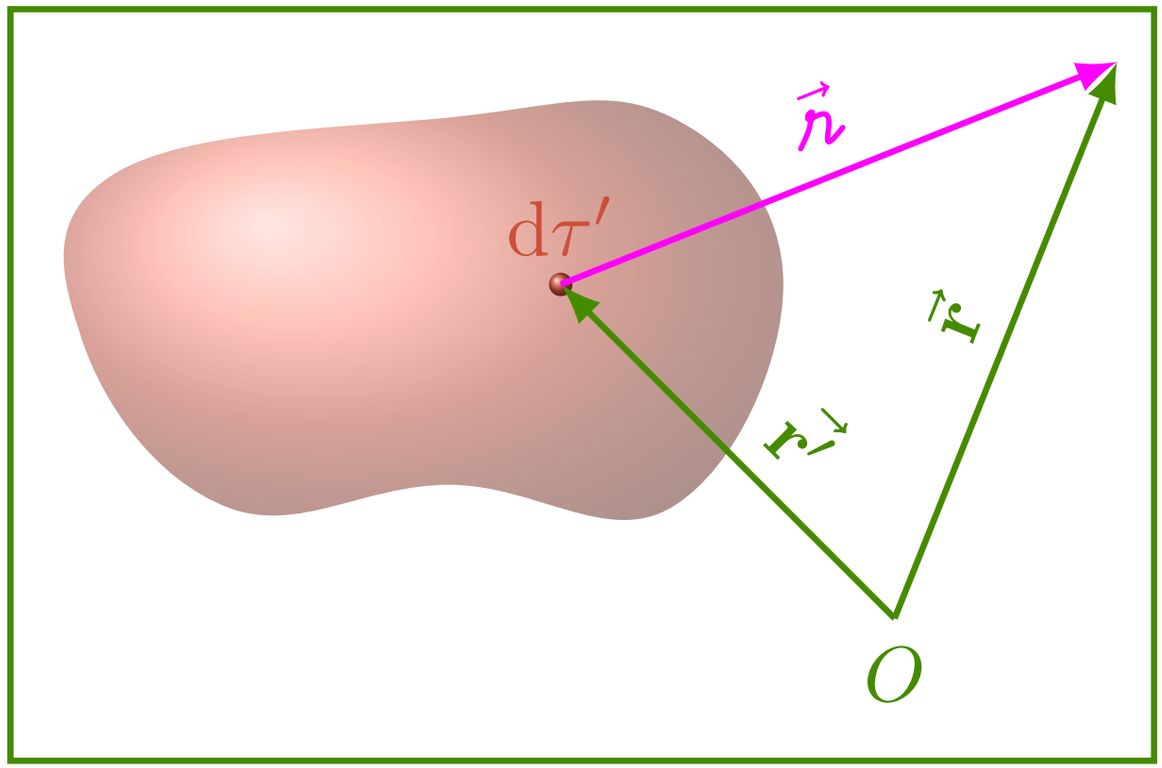
Potencial e potencial vetor

$$\Box^2 V = -\frac{\rho}{\epsilon_0}$$

$$V(\vec{\mathbf{r}},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{\mathbf{r}'},t_r)}{\mathcal{r}}\ \mathrm{d}\tau'$$

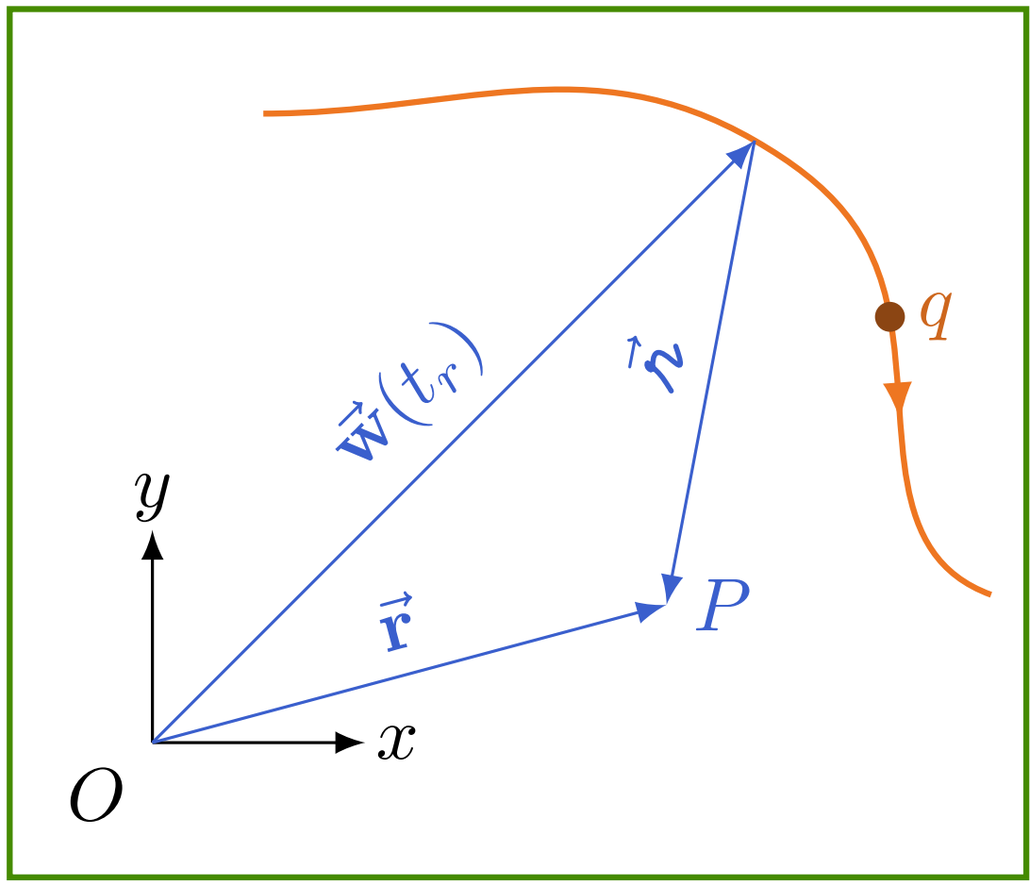
$$\Box^2 \vec{\mathbf{A}} = -\mu_0 \vec{\mathbf{J}}$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{\mathbf{J}}(\vec{\mathbf{r}'},t_r)}{\mathcal{r}}\ \mathrm{d}\tau'$$



$$t_r \equiv t - \frac{\mathcal{r}}{c}$$

Potenciais de Liénard e Wiechert



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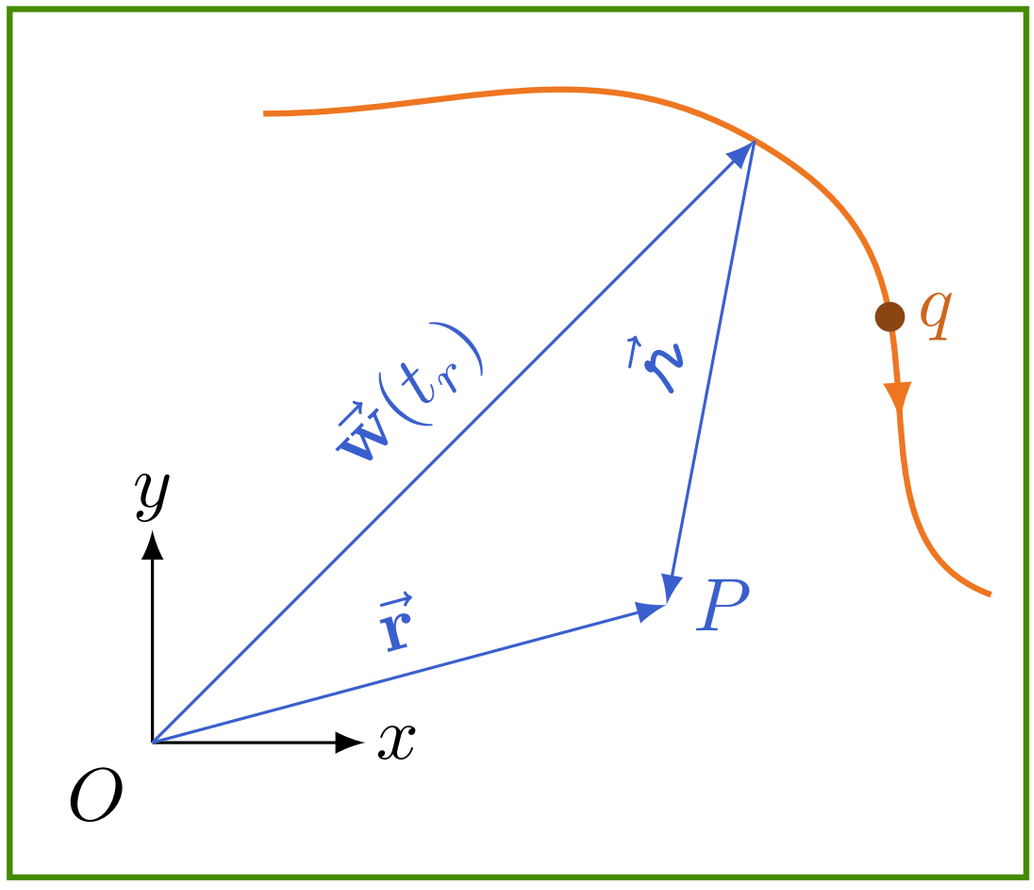
$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{\mathbf{J}}(\vec{\mathbf{r}'},t_r)}{\mathscr{r}}\,\mathrm{d}\tau'$$

$$\mathrm{d}\tau'_{\mathrm{ret}} = \mathrm{d}\tau'\,\frac{1}{1-\hat{\mathscr{r}}\cdot\frac{\vec{\mathbf{v}}}{c}}$$



$$V(\vec{\mathbf{r}},t) = \frac{1}{4\pi\epsilon_0}\,\frac{qc}{c\mathscr{r}-\vec{\mathbf{v}}\cdot\vec{\mathscr{r}}}$$

$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t) = \frac{\mu_0}{4\pi}\,\frac{qc\vec{\mathbf{v}}}{c\mathscr{r}-\vec{\mathbf{v}}\cdot\vec{\mathscr{r}}}$$

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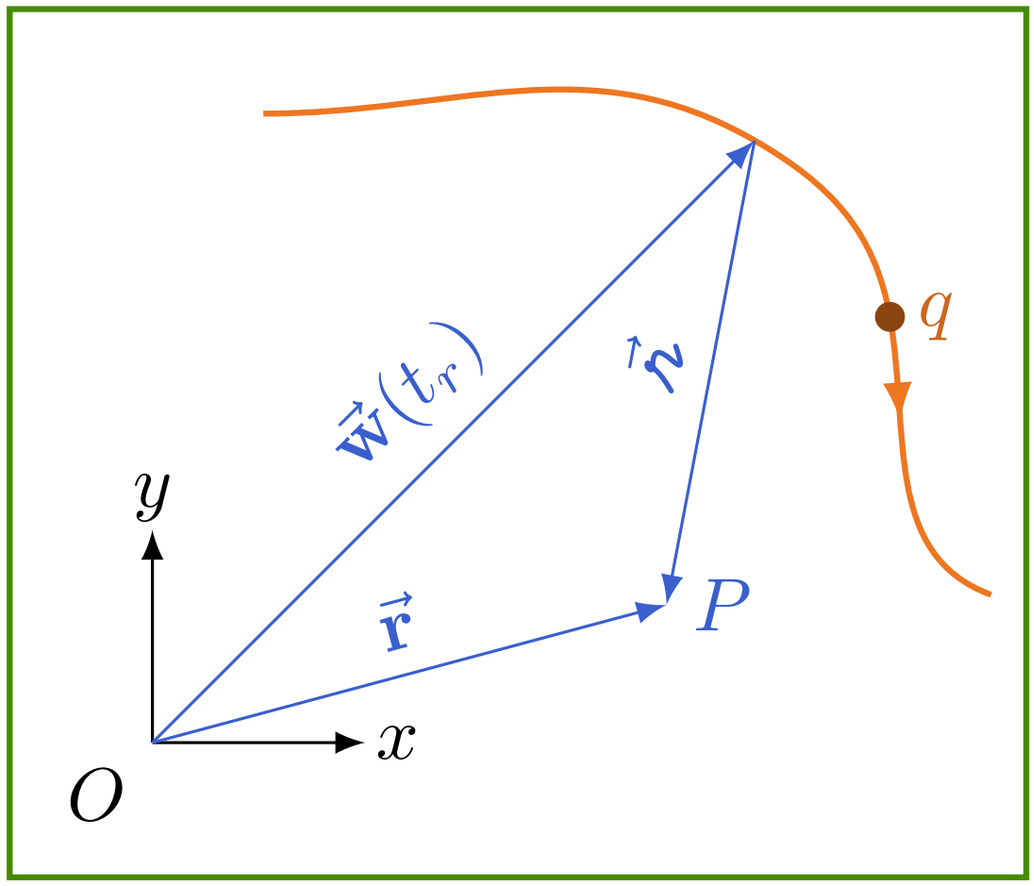

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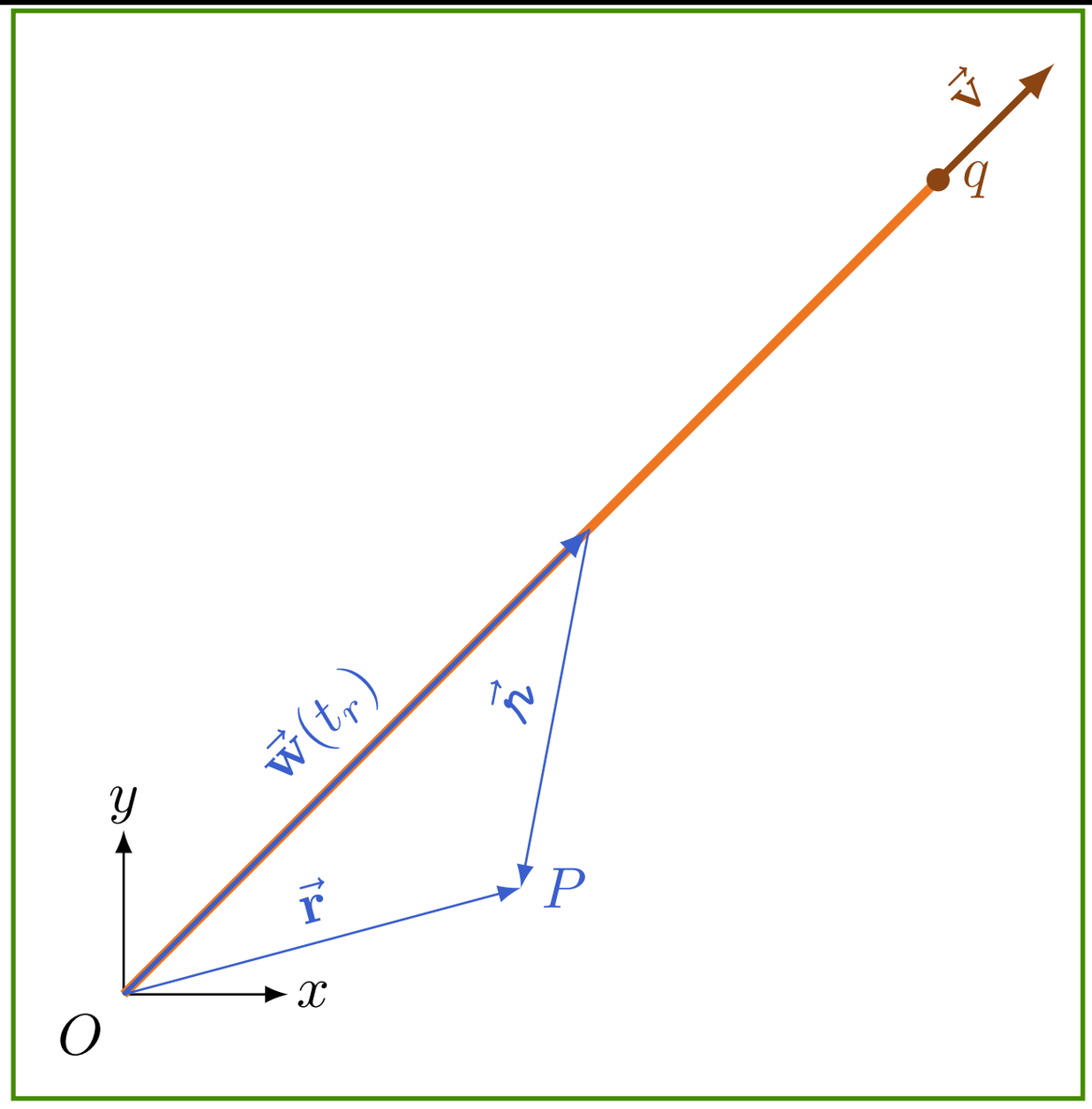

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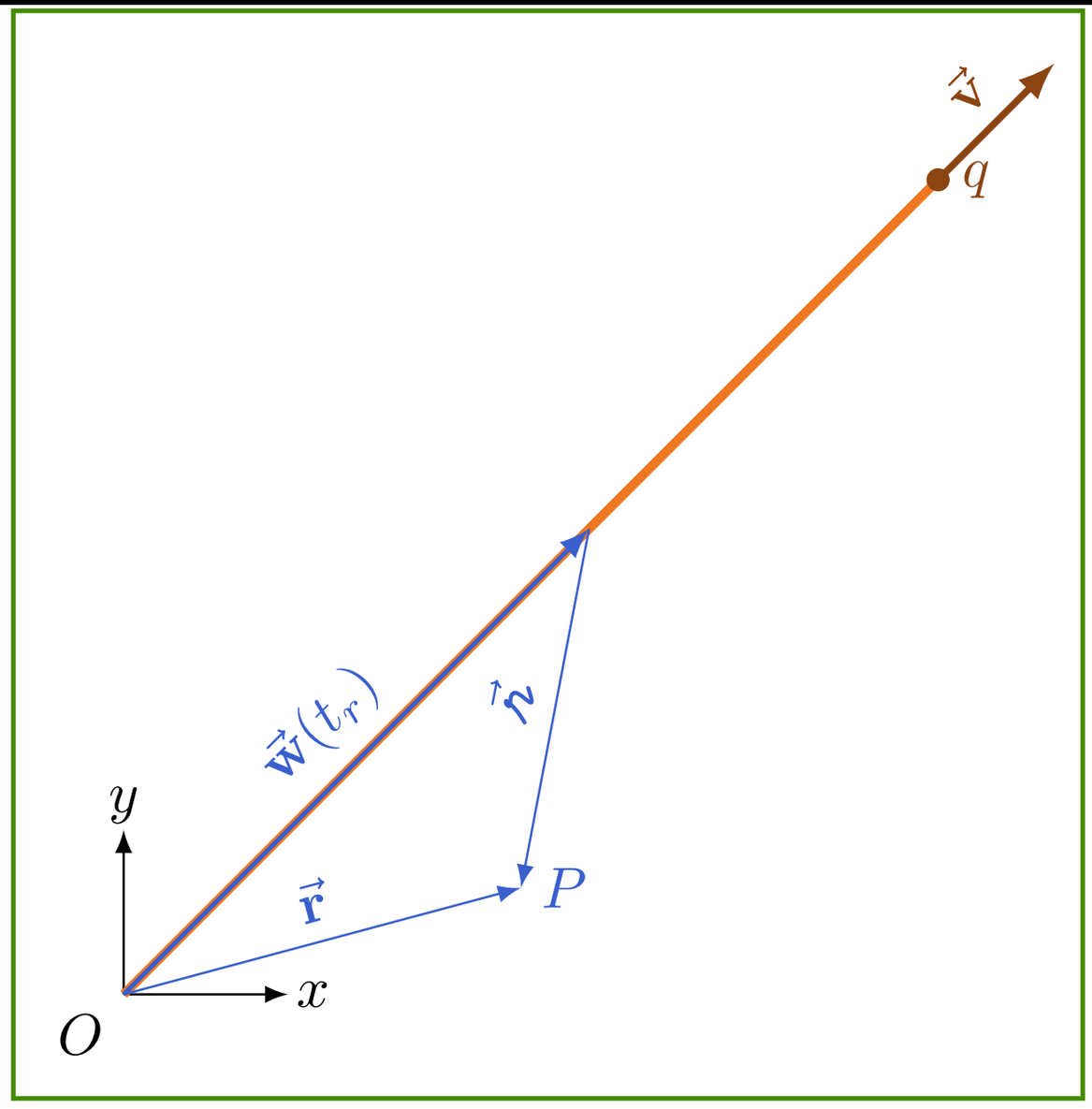

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$$t_r = \frac{c^2t - \vec{\mathbf{r}}\cdot\vec{\mathbf{v}} \pm \sqrt{(c^2t - \vec{\mathbf{r}}\cdot\vec{\mathbf{v}})^2 - (c^2t^2 - r^2)(c^2 - v^2)}}{c^2 - v^2}$$

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$$v = 0 \Rightarrow \quad t_r = \frac{c^2t \pm \sqrt{c^4t^2 - (c^2t^2 - r^2)c^2}}{c^2}$$

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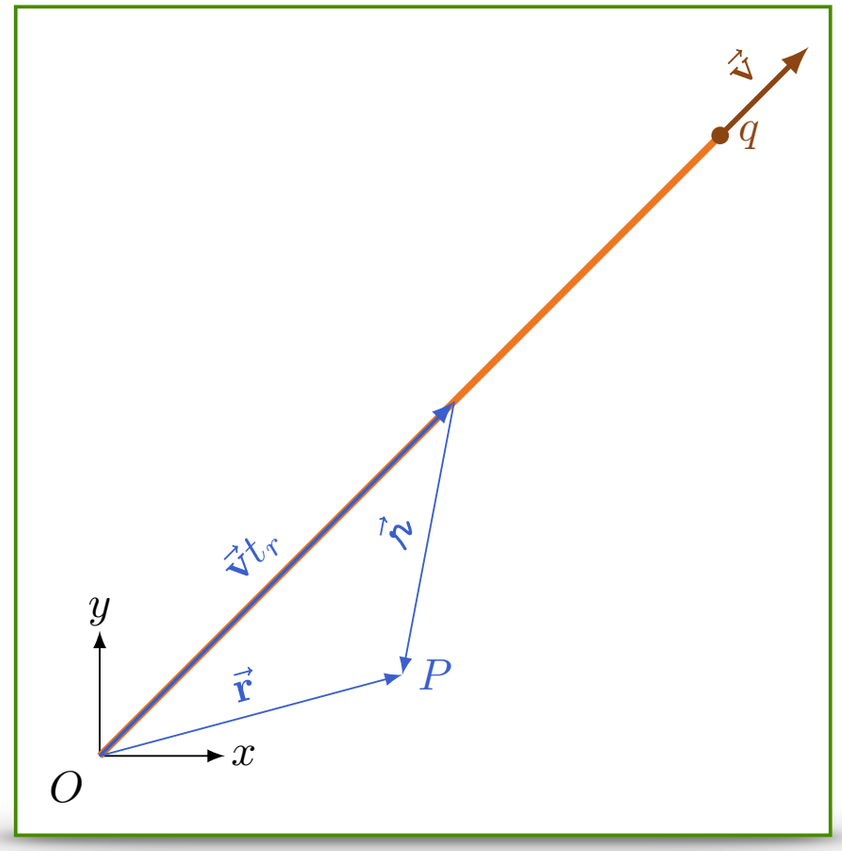

$$(c^2 - v^2)t_r = c^2t - \vec{\mathbf{r}}\cdot\vec{\mathbf{v}} - \sqrt{(c^2t - \vec{\mathbf{r}}\cdot\vec{\mathbf{v}})^2 - (c^2t^2 - r^2)(c^2 - v^2)}$$

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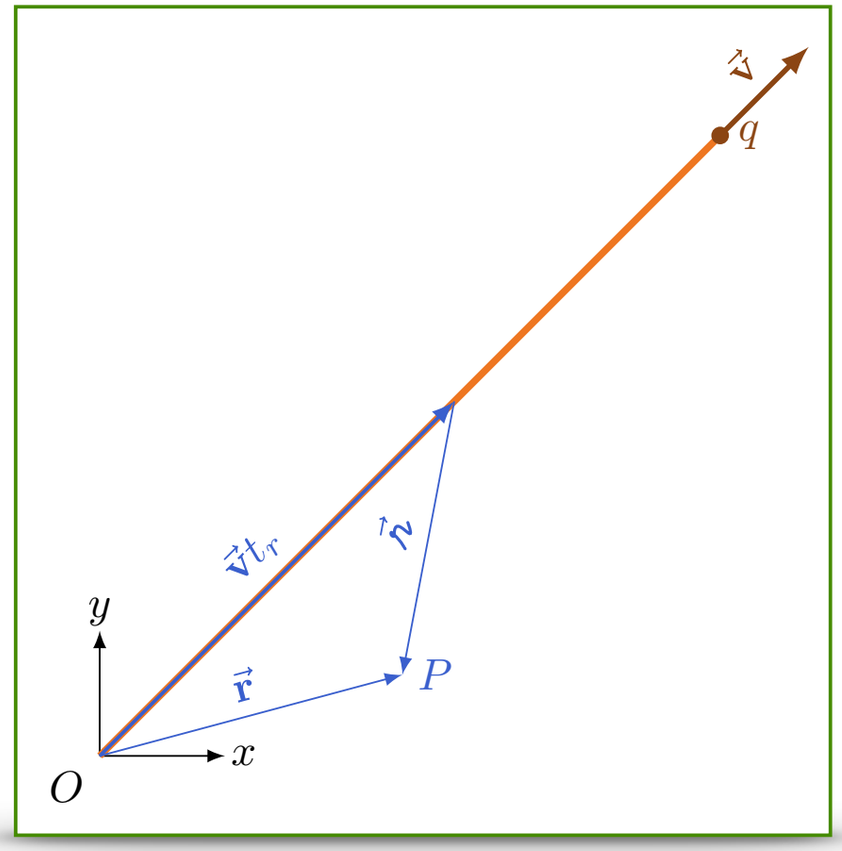
$$cr - \vec{\mathbf{v}}\cdot\vec{r} = c^2(t - t_r) - \vec{\mathbf{v}}\cdot\vec{\mathbf{r}} + v^2 t_r$$



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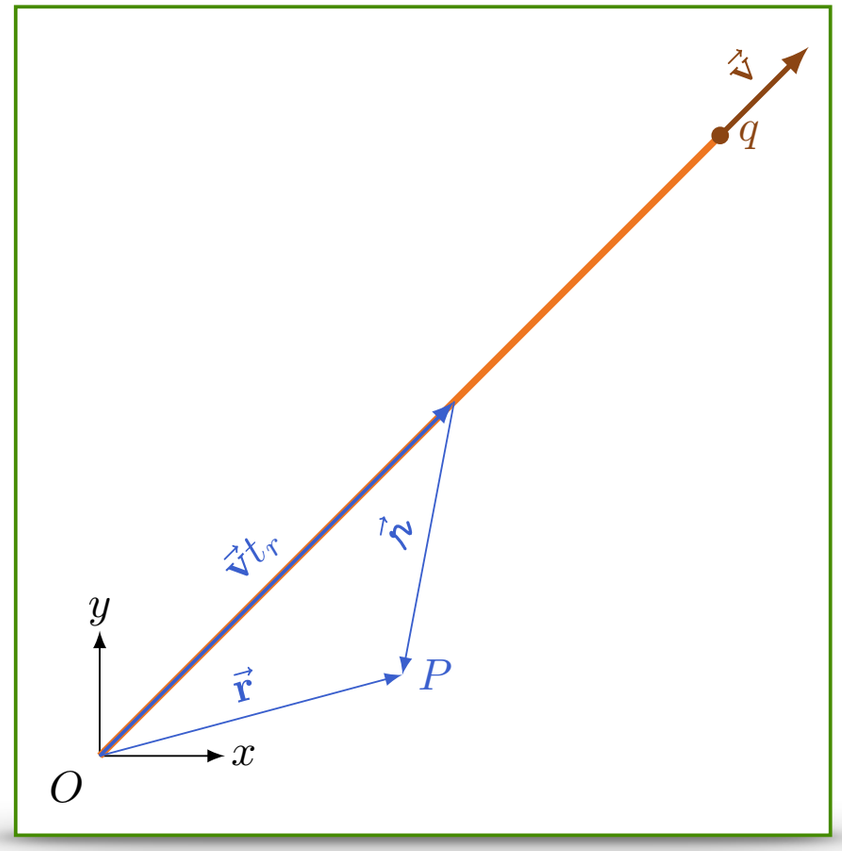

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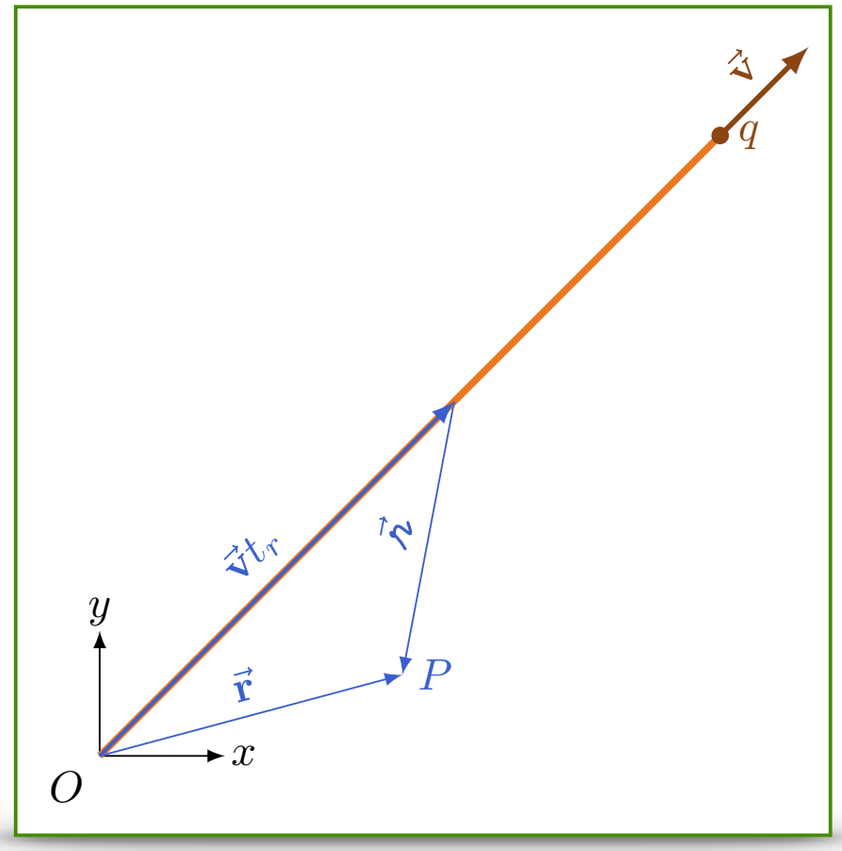

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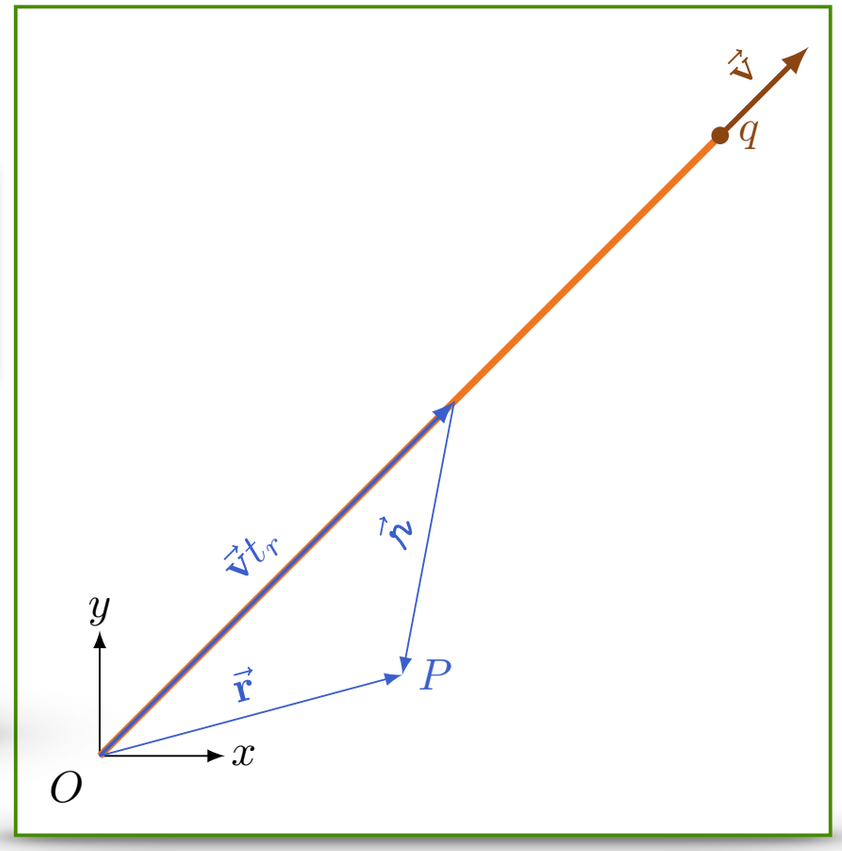

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