Home Work 06

Oct/28/2021

1. Let $S \subseteq \mathbb{R}^n$. Prove the following results
   a) $S^{\perp} \subseteq \mathbb{F}^n$;
   b) $(S^{\perp})^{\perp} = S$.

2. Determine bases for the four fundamental spaces of the LT below

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 2 & 2 & 1 \\ 2 & 5 & 5 & 3 \end{bmatrix}$$

3. Consider a matrix $A_{3\times 3}$ such that

$$R = \mathrm{Sp}\left\{\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}1\\-1\\2\end{bmatrix}\right\} \quad \text{and} \quad S = \mathrm{Sp}\left\{\begin{bmatrix}-2\\1\\0\end{bmatrix}\right\},$$

and consider a linear system $Ax = b$ in which $R(A) = R$ and $N(A) = S$, and $b^T = [1 \ -7 \ 0]$

   a) Explain why $Ax = b$ must be consistent;
   b) Explain why $Ax = b$ cannot have a unique solution.

4. Consider the matrices $A = \begin{bmatrix} 1 & 1 & 5 \\ 2 & 0 & 6 \\ -1 & 2 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -4 & 4 \\ 4 & -8 & 6 \\ 0 & -4 & 5 \end{bmatrix}$

   a) Do A and B have the same row space?
   b) Do A and B have the same col space?
   c) Do A and B have the same null space?
   d) Do A and B have the same left null space?