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<u>Particle Physics</u> (Introduction to Elementary Particles - David Griffiths John Wiley (1987))

1897 - Discovery of <u>electron</u> by J. J. Thomson in cathode rays

~ 1900 - The <u>photon</u> (hard to say who discovered)

1911 - Rutherford's discovery of the <u>atomic nucleus</u>

1917 - " " " " <u>proton</u>

$$^{14}N + \alpha \to {}^{17}O + p$$

1932 - James Chadwick discovery of the <u>neutron</u>

α radiation at beryllium short produced neutral particles

<u>Mesons</u>

What holds the nucleus together? → strong force
short range

1934 - Yukawa's theory of strong force

range $\Delta r = 10^{-15}$ m

$\circ \quad m \quad \circ$

$$\Delta E = mc^2$$
$$\Delta E \Delta t \sim \hbar \quad \to \quad \Delta x = c\Delta t = \frac{\hbar}{mc}$$

$$m = \frac{\hbar}{\Delta x c}$$

$$\hbar = 6.582 \times 10^{-22} \text{ MeV s}$$
$$= 1.054 \times 10^{-34} \text{ J. s}$$

$$mc^2 = \frac{\hbar c}{\Delta x}$$

$$\boxed{\cancel{c^2 m = \frac{6.582 \times 10^{-22} \text{ MeV s}}{10^{-15} \text{ m} \; 3 \times 10^{8} \text{ m s}^{-1}}}}$$

$$mc^2 = \frac{6.582 \times 10^{-22}\ \mathrm{MeV\,s} \times 3 \times 10^{8}\ \mathrm{m/s}}{10^{-15}\ \mathrm{m}}$$

$$= 3 \times 6.582 \times 10^{1}\ \mathrm{MeV} = 197{,}5\ \mathrm{MeV}$$

1937 – meson discovered in cosmic rays. (wrong lifetime, lighter than Yukawa)

1946 discovered it interacts weakly with nuclei

That was the muon μ^-. (105,65 MeV)

1947 – Discovery of the π-meson in cosmic ray (desintegrates before reaching the ground)

Powell, ~~Occhialini~~ Occhialini, Lattes

$\pi^{\pm}$ 139,57 MeV

π^{0} 134,977 MeV

Antiparticles

1927 – Dirac predicts anti-electron (positron)

1932 – Anderson's discovery of the positron.

Crossing symmetry

$$A + B \to C + D$$

$$\downarrow$$

$$A \to \bar{B} + C + D$$

$$A + \bar{C} \to \bar{B} + D$$

$$\bar{C} + \bar{D} \to \bar{A} + \bar{B}$$

(restricted by energy conservation)

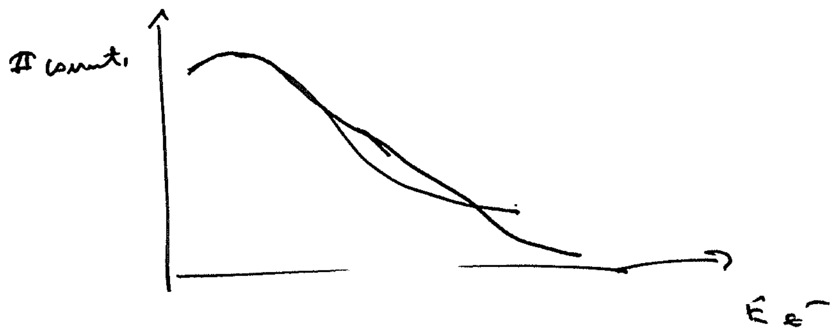
Asymmetry → matter/antimatter in the Universe.

Neutrinos

Beta decay $\quad A \to B + e^-$

$^{40}_{19}K \to {}^{40}_{20}Ca \qquad {}^{64}_{29}Cu \to {}^{64}_{30}Zn \qquad {}^{3}_{1}H \to {}^{3}_{2}He$

spectrum of e^-



Pauli suggested another particle was emitted and not detected.

Fermi called it neutrino. (in fact antineutrino)

$$n \to p^+ + e^- + \bar{\nu}_e$$

$$\pi^- \to \mu^- + \bar{\nu}_\mu \qquad \pi^+ \to \mu^+ + \nu_\mu$$

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

1959 Reines and Cowan discover neutrinos in large tank of water in the inverse beta decay

$$\bar{\nu} + p^+ \to n + e^+$$

Davis and Harmer proved neutrino is not its antiparticle. From crossing

$$\nu + n \to p^+ + e^-$$

and $\qquad \bar{\nu} + n \to p^+ + e^-$ did not occur.

1953 Konopinski and Mahmoud → conservation of lepton number

However the reaction

$\mu^- \to e^- + \gamma$ is not observed

→ conservation of lepton number in each family (e^-, μ^-)

Strange particles

1947 – Rochester and Butler – see the production of a neutral particle in cloud chamber that decays into 2 pions:

$$K^0 \to \pi^+ + \pi^-$$

1949 – Powell ~~detected a trace~~ detected the decay of a charged kaon:

$$K^+ \to \pi^+ + \pi^+ + \pi^-$$

Meanwhile the mesons η, ϕ, ω and ρ were discovered.

1950 – Anderson discovers the lambda (baryon)

$$\Lambda \to p^+ + \pi^-$$

(Stueckelberg proposed the conservation of baryon number to account for proton stability

$p^+ \to e^+ + \gamma$ is not seen)

More baryons were discovered Σ, Ξ, and Δ

These new particles were called "strange".

They are produced at a time scale of 10^{-23} s.

but they decay at a time scale of 10^{-10} s

Pairs: They are produced by strong force but decay by the weak force

So, strange particles should be produced in pairs.

Gell-Mann introduces "strangeness" that is conserved in strong interaction

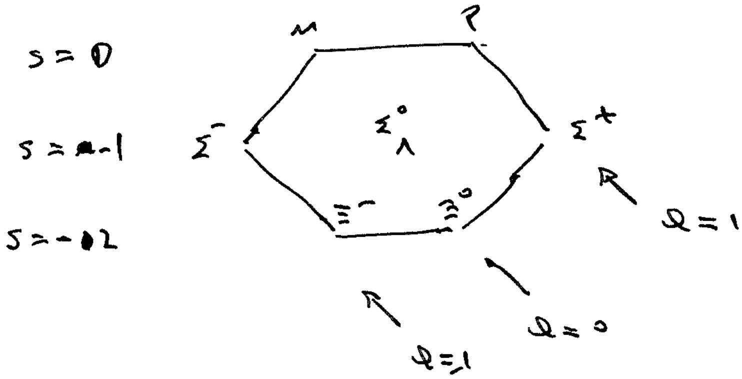
$$\pi^- + p^+ \to K^+ + \Sigma^- \qquad K \quad S=1$$
$$\to K^0 + \Sigma^0 \qquad \Lambda, \Sigma \quad S=-1$$
$$\to K^0 + \Lambda$$

Weak interaction does not conserve strangeness.

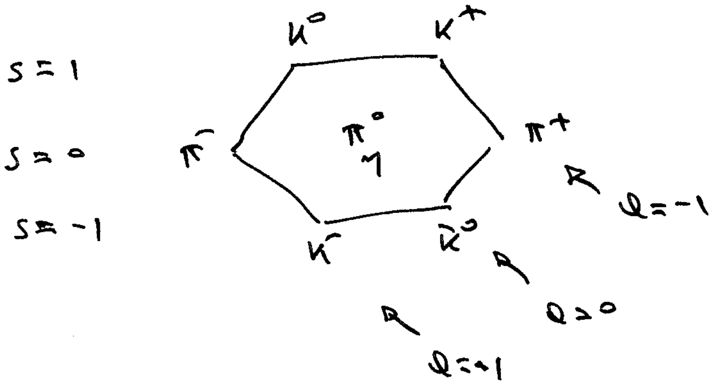
$$\Lambda \to p^+ + \pi^-$$
$$\Sigma^+ \to p^+ + \pi^0$$
$$\to n + \pi^+$$

The eightfold way (1961 - 1964)

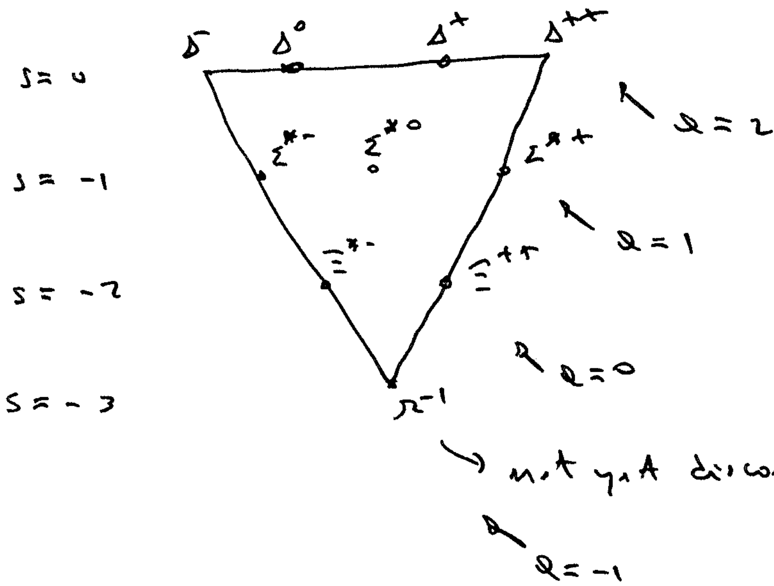
Gell-Mann and independently Ne'eman proposed a "period table" of particles in terms of representation of SU(3)



baryon octet



meson octet



Baryon Decuplet

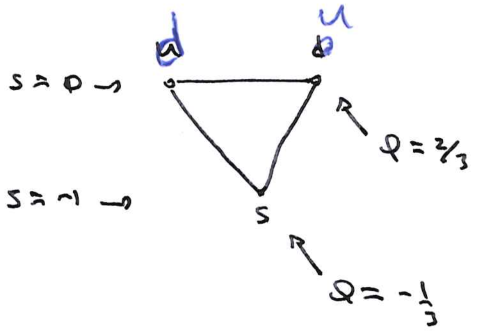
→ not yet discovered, discovered in 1964 as predicted by Gell-Mann hypercharge was introduced

$$Y = S + B$$

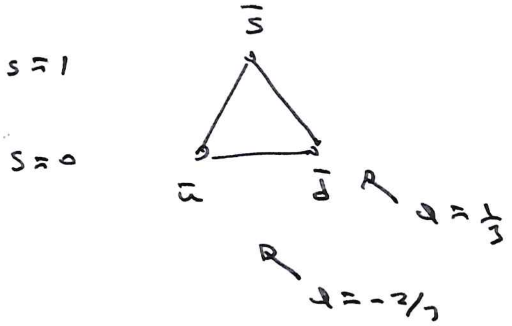
S: strangeness; B: baryon #

The quark model (1964)

Gell-Mann and Zweig (independently) proposed that hadrons are composed of "quarks" with 3 flavours u, d, s



quarks



anti-quarks

Baryon: composed of 3 quarks

Meson: " of a quark and anti-quark

Note that

$$3 \otimes \bar{3} = 8 + 1$$

So, there should be nine mesons and not 8. The nineth was the η' with $Q=0$, $S=0$. $\eta' \equiv 957{,}78$ MeV

In addition

$$3 \otimes 3 \otimes 3 = \underbrace{10}_{\text{symmetric}} + \underbrace{8}_{\text{antisymmetric in 1 and 2}} + \overbrace{8}^{\text{antisymmetric in 2 and 3}} + \underbrace{1}_{\text{completely antisymmetric}}$$

Note p and Δ^+ are made of uud $\quad p \equiv 938{,}27$ MeV

π^+ and ρ^+ " " of $u\bar{d}$ $\quad \pi^+ \equiv 139{,}57$ MeV, $\rho^+ \equiv 775{,}26$ MeV

→ different bound states of the same quarks.

Note: there can not be baryons with $S=0$, $Q=-2$

" " " " meson " $Q=+2$ $S=-3$

This could kill the quark model!!!

→ quarks never found directly → <u>confinement</u>

Can probe with "deep inelastic scattering" with electrons (SLAC 70's)

Color (O.W. Greenberg 1964)

We have

$$\Delta^{++} \equiv u\,u\,u$$

$$\Delta^{-} \equiv d\,d\,d$$

$$\Omega^{-} \equiv s\,s\,s$$

If quarks are fermions how can all be in the same state?

Quarks should exist with another quantum number taking 3 values, or 3 colors

Using that one can have antisymmetry in $u\,u\,u$.

hypothesis

"All naturally occurring particles are colorless"

So there can not be hadrons made of two or four quarks, but only $q\,\bar{q}$

Can have

$$q\,\bar{q}$$
$$q\,q\,q$$
$$\bar{q}\,\bar{q}\,\bar{q}$$

The ~~psi~~ J/ψ meson (1974)

- First observed at Brookhaven by C. C. Ting group in 1974 (J)
- Burton Richter's group at SLAC (ψ)
- 3 times the mass of the proton
- long life time 10^{-20} s, when typical life times of hadrons was 10^{-23} s (1000 times shorter)

It was agreed the J/ψ was a bound state of a new quark $J/\psi \equiv c\bar{c}$

c = the charm quark.

(Bjorken and Glashow wondered why 4 leptons and only 3 quarks)

So, there should exist lots of new hadrons!!

The first discovered was ~~(1975)~~

$$\left.\begin{array}{l} \Lambda_c^+ = udc \\ \Sigma_c^+ = uuc \end{array}\right) (1975)$$

$$\left.\begin{array}{l} D^0 = c\bar{u} \\ D^+ = c\bar{d} \end{array}\right| (1976)$$

$$F^+ = c\bar{s} \quad (1977)$$
$$\shortparallel$$
$$D_s$$

GIM - Mechanism (Glashow, Iliopoulos and Maiani) predicts a fourth quark in 1970

Forbids $\Delta S = 2$ transition in neutral current weak interaction

A new lepton ≡ the τ (1975)

Discovery of upsilon meson (1977) → the beauty or bottom quark

$$\Upsilon \equiv b\bar{b}$$

Search for new hadrons with b quark started.

$\Lambda_b \doteq udb$ (1981)

$$\left.\begin{array}{l} B^0 = b\bar{d} \\ B^- = b\bar{u} \end{array}\right) (1983)$$

The top quark (1984) → some evidence at CERN in 1984

Discovered in 1995 at CDF and D0 experiments at Fermilab.

$t \doteq 173$ GeV

Weak interactions

1934 - Fermi theory to explain β decay:

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} (\bar{\psi}_p \gamma_\mu \psi_n)(\bar{\psi}_e \gamma_\mu \psi_\nu)$$

G_F determined from μ decay $\Gamma(\mu^- \to e^- + \bar{\nu}_e + \nu_\mu) = \frac{G^2 m_\mu^2}{192\pi^3}$

$$G_F = 1.01 \times 10^{-5}\, m_p^{-2}$$

The β decay is

$$n \to p + e^- + \bar{\nu}_e$$

we also have

$$\pi^- \to \mu^- + \bar{\nu}_\mu$$

$$\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$$

μ lifetime:
$2{,}934 \times 10^{-6}$ s
$m_\mu = 105{,}659$ MeV

1936 - Gamow and Teller proposed an extension of Fermi's theory

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \sum_i C_i (\bar{\psi}_p \Gamma^i \psi_n)(\bar{\psi}_e \Gamma^i \psi_\nu)$$

$$\Gamma^S = 1, \quad \Gamma^P = \gamma_5, \quad \Gamma^V_\mu = \gamma_\mu, \quad \Gamma^A_\mu = \gamma_\mu \gamma_5, \quad \Gamma^T_{\mu\nu} = \sigma_{\mu\nu}$$

Parity violation in weak interactions

1956 - Lee and Yang propose to test parity conservation in weak interactions (β decay). Got Nobel Prize in 1957

1956 - Madame Chien-Shiung Wu made the experiment to test it, at the "Low Temperature Group of the US National Bureau of Standards".

Got the first Wolf Prize in 1978 !!

The experiment

Look for the decay

$${}^{60}_{27}Co \rightarrow {}^{60}_{28}Ni + e^- + \bar{\nu}_e + 2\gamma$$

↳ electromagnetic decay of excited state of Ni

↓

was known to respect parity

γ rays act as a control for the polarization of e^- and uniformity of alignment of ${}^{60}Co$ atoms.

Magnetic field aligned atoms of ${}^{60}Co$ in a temperature ~~0.01k~~ near absolute zero. $\underline{(0.003\,K)}$

Result: ~~the~~ electrons were emitted preferably in a direction opposite to the gamma rays.
Most of e^- were emitted opposed to the nuclear spin. Later was established that P-violation was maximal.

In 1957 it was established that the electrons emitted were mostly left-handed.

So the lagrangian had to be

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \sum_i c_i (\bar{\psi}_p \Gamma_i \psi_n)(\bar{\psi}_e \Gamma^i (1 \pm \gamma_5) \psi_\nu)$$

The intermediate vector boson IVB

1957 Schwinger
Lee and Yang developed the idea of an intermediate vector boson

The Fermi theory was not renormalizable theory
It violates unitarity for $P_{cm} \sim 300$ GeV

- IVB could cure it!
- It should be charged since weak process always involve charged currents.
- It should be very massive due to the short range of weak interactions.
- It should have the V-A structure to account for parity violation.

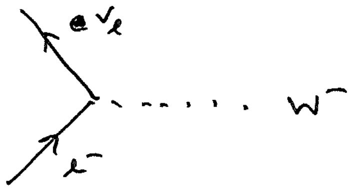
$$\mathcal{L} = \underset{\substack{\uparrow \\ \text{new coupling}}}{G_W} (J^\alpha W^+_\alpha + J^{+\alpha} W^-_\alpha)$$

In fact

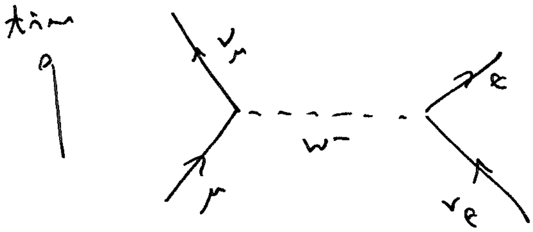
$$G_W^2 = \frac{M_W^2 G_F}{\sqrt{2}} \qquad M_W \simeq \text{mass of IVB}$$

G_W is dimensionless.

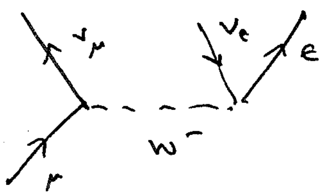
However, at high energies the IVB theory still violates unitarity.



vertex



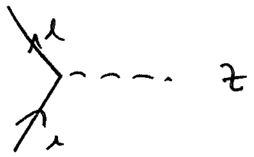
$\mu^- + \nu_e \to e^- + \nu_\mu$



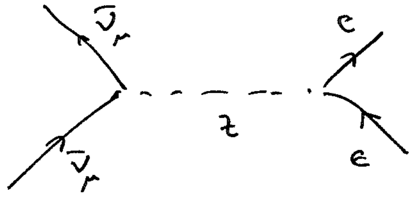
$\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$ (μ decay)

Neutral currents in weak interactions

1958 Leite Lopes Nucl Phys 8, 234 (1958)
Gamow, Teller Phys. Rev. 51 288 (1937)
N. Kemmer Phys. Rev. 52 906 (1937)
G. Wentzel Helv. Phys. Acta 10, 108 (1937)
S. Bludman Nuovo Cimento 9, 433 (1958



vertex

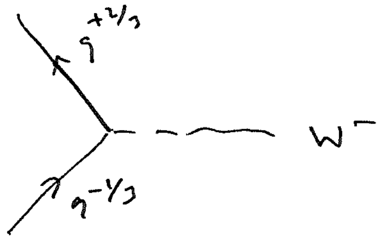


$\bar{\nu}_\mu + e^- \to \bar{\nu}_\mu + e^-$

First observed at CERN
F. J. Hasert et. al.
Phys. Lett <u>46B</u>, 121 (1973)
138 (?)

The weak interaction respects the lepton families, i.e., lepton number in each family is conserved.

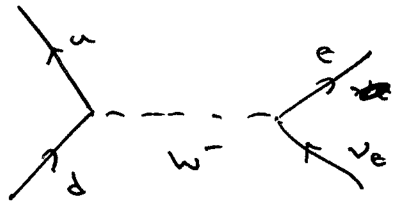
The same does not happen to quarks



$-\frac{1}{3}$ (d, s, b)

$\frac{2}{3}$ (u, c, t)

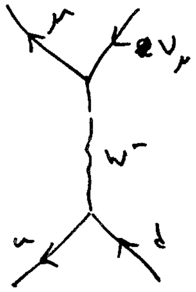
~~quark~~ outgoing quark carries the same color but different flavor. Weak interaction does not conserve flavor of quark.



~

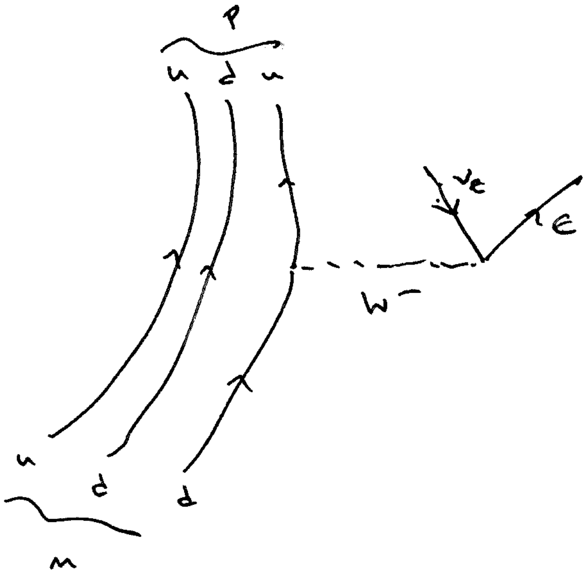
$\pi^- \equiv d\bar{u}$

$\pi^- \to e^- + \bar{\nu}_e$ (π^- decay)



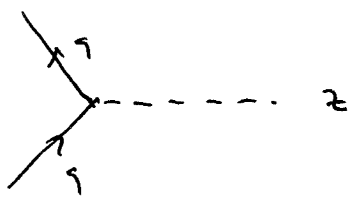
$\pi^- \to \mu^- + \bar{\nu}_\mu$ (much more often)

beta decay

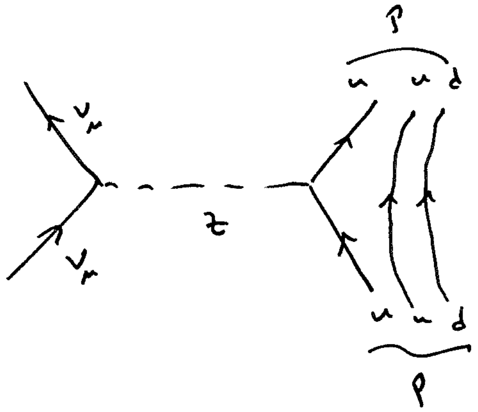


$n \to p^+ + e^- + \bar{\nu}_e$

The neutral current vertex



It leads to



$\nu_\mu + p \to \nu_\mu + p$

It is masked by electromagnetic interactions.

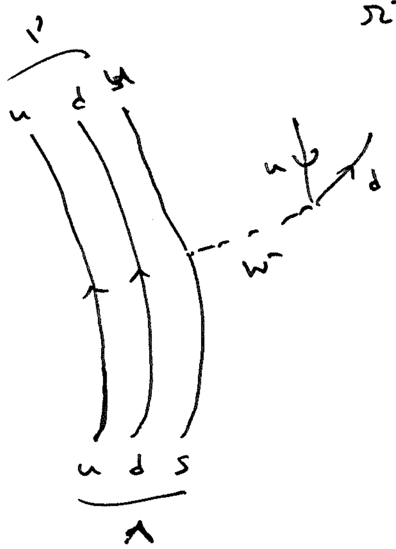
However that picture is too simple

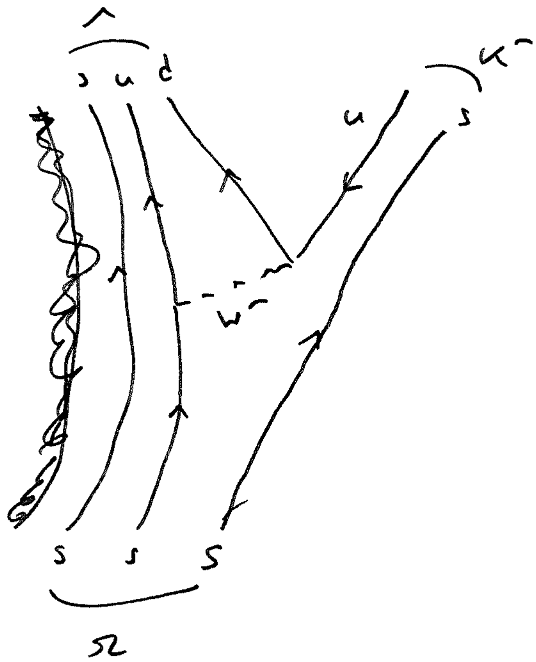
We have strangeness-changing weak interactions.

Such as

$\Lambda \to p^+ + \pi^+$

$\Omega^- \to \Lambda + K^-$





Solved by

Cabibbo (1963)

Glashow, Illiopoulos, Maiani (1970)

Kobayashi, Maskawa (1973)

The weak force does not couple

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

but

$$\begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix} \quad \begin{pmatrix} t \\ b' \end{pmatrix}$$

where

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} 3\times 3 \\ \text{KM matrix} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$ It is a unitary matrix

as for 2010 it's entries are (only the modulus)

$$|V_{ij}| = \begin{pmatrix} 0.97427 & 0.22534 & 0.00351 \\ 0.22520 & 0.97344 & 0.0412 \\ 0.00867 & 0.0404 & 0.999146 \end{pmatrix}$$

It specifies the mismatch of quantum states of quarks when they propagate freely and when they take part in weak interactions.

It is important in understanding CP violation

From Particle Data Group 2018 we have

$$|V_{ij}| = \begin{pmatrix} 0.97420 \pm 0.00021 & 0.2243 \pm 0.0005 & 0.00394 \pm 0.00036 \\ 0.218 \pm 0.004 & 0.997 \pm 0.017 & 0.0422 \pm 0.008 \\ 0.0081 \pm 0.0005 & 0.0394 \pm 0.0023 & 1.019 \pm 0.025 \end{pmatrix}$$

The coupling with the vector boson is given by

$$-\frac{g}{\sqrt{2}}(\bar{u}_L, \bar{c}_L, \bar{t}_L)\gamma^\mu W_\mu^+ V_{CKM} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + h.c.$$

with V_{CKM} the Cabibbo-Kobayashi-Maskawa matrix given above (the modulus only).

Conservation laws

- Energy and momentum: all 3 interactions conserve
  since comes from Special Relativity

- electric ~~field~~ charge: all 3 conserve

- color : all 3 conserve

- baryon number: all 3 conserve

- electron number
  muon " : all 3 conserve
  tau "

- quark flavor : strong and electromagnetic conserve
  weak does not due to
  the mixing of CKM matrix

Tuvi yoS Eq de Dirac

We have:

$$(\gamma^\mu p_\mu - mc)\psi = 0 \quad \rightsquigarrow \quad \left[i\gamma^\mu \partial_\mu - \frac{mc}{\hbar}\right]\psi = 0 \qquad p_\mu = i\hbar\partial_\mu$$

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

The spin is:

$$\vec{S} = \frac{\hbar}{2}\vec{\Sigma} = \frac{\hbar}{2}\begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$$

The helicity operator is

$$\vec{\Sigma}.\vec{P} = \begin{cases} 1 & \text{right handed} \\ -1 & \text{left} \quad " \end{cases}$$

We also have:

$$\gamma_5^* = \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

The solutions of the Dirac eq. are: $\left(\psi = u^{(s)} e^{-i p.x/\hbar}\right)$

$$u^{(1)} = N\begin{pmatrix} 1 \\ 0 \\ p_3 c/(E+mc^2) \\ (p_1 + i p_2)c/(E+mc^2) \end{pmatrix} \qquad u^{(2)} = N\begin{pmatrix} 0 \\ 1 \\ (p_1 - i p_2)c/(E+mc^2) \\ -p_3 c/(E+mc^2) \end{pmatrix}$$

$$u^{(3)} = N \begin{pmatrix} -p_3 c/(|E|+mc^2) \\ -(p_1+ip_2)c/(|E|+mc^2) \\ 1 \\ 0 \end{pmatrix} \qquad u^{(4)} = N \begin{pmatrix} -(p_1-ip_2)c/(|E|+mc^2) \\ p_3 c/(|E|+mc^2) \\ 0 \\ 1 \end{pmatrix}$$

It is common to work with

$$u_r = u^{(r)} \qquad r = 1,2$$

$$\left.\begin{aligned} v_1(\vec{p}) &= u^{(4)}(-\vec{p}) \\ v_2(\vec{p}) &= u^{(3)}(-\vec{p}) \end{aligned}\right\} \text{ \& } E \to -E$$

& so

$$v_1 = N \begin{pmatrix} (p_1-ip_2)c/(|E|+mc^2) \\ -p_3 c/(|E|+mc^2) \\ 0 \\ 1 \end{pmatrix} \qquad v_2 = N \begin{pmatrix} p_3 c/(|E|+mc^2) \\ (p_1+ip_2)c/(|E|+mc^2) \\ 1 \\ 0 \end{pmatrix}$$

If we take $\vec{p}$ to be along the 3-axis, i.e. $p_1 = p_2 = 0$ then $\vec{\Sigma}\cdot\vec{p} = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$

and so

$u^{(1)}$ has spin up

$u^{(2)}$ " " down

$u^{(3)}$ " " up

$u^{(4)}$ " " down.

v_1 has spin up

v_2 " " down

since momentum was reversed.

Helicity is not a Lorentz invariant quantity. So we define <u>chirality</u>. Consider the projectors.

$$P_L = \frac{1}{2}(1-\gamma_5) \qquad P_R = \frac{1}{2}(1+\gamma_5)$$

We define

$$u_{\substack{R\\L}} = P_{\substack{R\\L}}\, u$$

and

$$v_{\substack{R\\L}} = P_{\substack{L\\R}}\, v$$ since momentum was reversed.

In the case when $m=0$ and $p = p_z = \frac{E}{c}$ we have that in the ultrarelativistic limit $v \to c$ that

$$u_1 \Rightarrow \begin{pmatrix}1\\0\\1\\0\end{pmatrix} \qquad u_2 \Rightarrow \begin{pmatrix}0\\1\\0\\-1\end{pmatrix}$$

$$v_1 \Rightarrow \begin{pmatrix}0\\-1\\0\\1\end{pmatrix} \qquad v_2 \Rightarrow \begin{pmatrix}1\\0\\1\\0\end{pmatrix}$$

and so

$$\gamma_5 u_1 \to u_1 \qquad \gamma_5 u_2 \to -u_2$$

$$\gamma_5 v_1 \to -v_1 \qquad \gamma_5 v_2 \to v_2$$

So:

$$\left(\frac{1+\gamma_5}{2}\right)u_1 = u_1 \qquad \left(\frac{1-\gamma_5}{2}\right)u_1 = 0 \qquad \left(\frac{1+\gamma_5}{2}\right)u_2 = 0 \qquad \left(\frac{1-\gamma_5}{2}\right)u_2 = u_2$$

$$\left(\frac{1-\gamma_5}{2}\right)v_1 = v_1 \qquad \left(\frac{1+\gamma_5}{2}\right)v_1 = 0 \qquad \left(\frac{1+\gamma_5}{2}\right)v_2 = v_2 \qquad \left(\frac{1-\gamma_5}{2}\right)v_2 = 0$$

We now define:

$$\psi_{\substack{L\\R}} = P_{\substack{L\\R}}\,\psi \qquad\qquad \psi = \psi_L + \psi_R$$

Then:

$$\bar\psi_L = (P_L\psi)^\dagger\gamma_0 = \psi^\dagger P_L^\dagger\gamma_0 = \psi^\dagger P_L\gamma_0 = \psi^\dagger\gamma_0 P_R = \bar\psi P_R$$

$$\bar\psi_R = \bar\psi P_L$$

Note the mass term mixes left and right:

$$\bar\psi\psi = (\bar\psi_R + \bar\psi_L)(\psi_R + \psi_L) =$$

$$= \underbrace{\bar\psi_R\psi_R}_{=\,\bar\psi P_L P_R\psi\,=\,0} + \bar\psi_R\psi_L + \bar\psi_L\psi_R + \underbrace{\bar\psi_L\psi_L}_{=\,\bar\psi P_R P_L\psi\,=\,0} = \bar\psi_R\psi_L + \bar\psi_L\psi_R$$

The vector current does not mix

$$\bar\psi\gamma^\mu\psi = (\bar\psi_R + \bar\psi_L)\gamma^\mu(\psi_R + \psi_L) = \bar\psi_R\gamma^\mu\psi_R + \bar\psi_L\gamma^\mu\psi_L$$

since $\gamma^\mu P_L = P_R\gamma^\mu$, $\{\gamma^\mu, \gamma^5\} = 0$

Now:

$$\bar\psi_L\gamma^\mu\psi_L = \bar\psi P_R\gamma^\mu P_L\psi = \bar\psi\gamma^\mu P_L^2\psi = \bar\psi\gamma^\mu P_L\psi$$

$$= \frac{1}{2}\bar\psi\gamma^\mu(1-\gamma_5)\psi$$

$$\bar\psi_R\gamma^\mu\psi_R = \bar\psi P_L\gamma^\mu P_R\psi = \bar\psi\gamma^\mu P_R^2\psi = \bar\psi\gamma^\mu P_R\psi$$

$$= \frac{1}{2}\bar\psi\gamma^\mu(1+\gamma_5)\psi$$

In weak interaction only the current $\bar\psi_L\gamma^\mu\psi_L$ appears.

The electroweak theory

- the lepton numbers (e, μ, τ) are separately conserved

Then each family of leptons have to transform under ~~repr~~ separate representations.

- only the left handed spinors couple to weak interaction

So we shall denote $l = e, \mu, \tau$ and sum over them on the Lagrangian.

The current coupling to the charged vector boson is

$$J_\mu^+ = \bar{l}\,\gamma_\mu (1-\gamma_5)\nu = 2\,\bar{l}_L\,\gamma_\mu \nu_L$$

We introduce the left handed isospin doublet

$$L = \begin{pmatrix} \nu \\ l \end{pmatrix}_L = \begin{pmatrix} P_L \nu \\ P_L l \end{pmatrix} = \begin{pmatrix} \nu_L \\ l_L \end{pmatrix}$$

There is no right-handed neutrinos. So, right handed lepton is a isospin singlet.

$$R = P_R l = l_R$$

The isospin current is:

$$J_\mu^i = \bar{L}\gamma_\mu \frac{\sigma_i}{2} L$$

and so:

$$J_\mu^1 = \frac{1}{2}(\bar{\nu}_L \ \bar{l}_L)\gamma_\mu \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \nu_L \\ l_L \end{pmatrix} = \frac{1}{2}\left(\bar{l}_L\gamma_\mu\nu_L + \bar{\nu}_L\gamma_\mu l_L\right)$$

$$J_\mu^2 = \quad '' \quad \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad '' \quad = \frac{i}{2}\left(\bar{l}_L\gamma_\mu\nu_L - \bar{\nu}_L\gamma_\mu l_L\right)$$

$$J_\mu^3 = \quad '' \quad \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \quad '' \quad = \frac{1}{2}\left(\bar{\nu}_L\gamma_\mu\nu_L - \bar{l}_L\gamma_\mu l_L\right)$$

So $\quad J_\mu^+ = 2\left(J_\mu^1 - i J_\mu^2\right)$

The weak hypercharge current is: $\left(Y = -\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \to \text{doublets}\right.$

$Y = -2 \to$ singlets

$$J_\mu^Y = -\left(\bar{L}\gamma_\mu L + 2\bar{R}\gamma_\mu R\right) = -\left(\bar{\nu}_L\gamma_\mu\nu_L + \bar{l}_L\gamma_\mu l_L + 2\bar{l}_R\gamma_\mu l_R\right)$$

The electromagnetic current is:

$$J_\mu^{em} = -\bar{l}\gamma_\mu l = -\left(\bar{l}_L\gamma_\mu l + \bar{l}_R\gamma_\mu l_R\right)$$

$$= J_\mu^3 + \frac{1}{2}J_\mu^Y$$

So, we have the Gell-Mann-Nishijima relation

$$Q = T_3 + \frac{Y}{2} \qquad \left(\frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}\right)$$

$$\left(0 - \frac{2}{2} = -1\right)$$

So we take the gauge group as

$$G = SU(2)_L \otimes U(1)_Y$$

and introduce the gauge bosons

$$SU(2)_L \to W_\mu^1, W_\mu^2, W_\mu^3 \qquad [T_i, T_j] = i\varepsilon_{ijk} T_k$$

$$U(1)_Y \to B_\mu \qquad [T_i, Y] = 0$$

The field tensors are:

$$W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g\,\varepsilon^{ijk} W_\mu^j W_\nu^k$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

and Lagrangian:

$$\mathcal{L}_{gauge} = -\frac{1}{4} W_{\mu\nu}^i W^{\mu\nu i} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

and

$$\mathcal{L}_{leptons}^{free} = \bar{R} i\partial\!\!\!/ R + \bar{L} i\partial\!\!\!/ L$$

$$= \bar{l}_R i\partial\!\!\!/ l_R + \bar{l}_L i\partial\!\!\!/ l_L + \bar{\nu}_L i\partial\!\!\!/ \nu_L$$

$$= \bar{l} i\partial\!\!\!/ l + \bar{\nu} i\partial\!\!\!/ \nu$$

The covariant derivatives are:

$$D^L_\mu = \partial_\mu + i\frac{g}{2}\sigma_i W^i_\mu + i\frac{g'}{2} Y B_\mu$$

$$D^R_\mu = \partial_\mu + i\frac{g'}{2} Y B_\mu$$

We have $Y = -1$ for doublet

$Y = -2$ " singlet

Therefore the interacting terms (minimal coupling)

$$\mathcal{L}_{\text{lepton}} = \bar{R}\, i \not{D}_R R + \bar{L}\, i \not{D}_L L$$

$$= \bar{l}_R\, i \gamma^\mu \left(\partial_\mu + i\frac{g'}{2} Y B_\mu\right) l_R$$

$$+ \bar{L}\, i\gamma^\mu\left(\partial_\mu + i\frac{g}{2}\sigma_i W^i_\mu + i\frac{g'}{2} Y B_\mu\right) L$$

$$= \bar{l}_R i \not{\partial} l_R + \bar{l}_L i \not{\partial} l_L + \bar{\nu}_L i \not{\partial} \nu_L + g' \bar{l}_R \gamma^\mu l_R B_\mu$$

$$- g \bar{L}\gamma^\mu \underbrace{\left(\frac{\sigma_1}{2} W^1_\mu + \frac{\sigma_2}{2} W^2_\mu + \frac{\sigma_3}{2} W^3_\mu\right)}_{\frac{1}{2}\begin{pmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & -W^3_\mu \end{pmatrix}} L + \frac{g'}{2}\bar{L}\gamma^\mu L B_\mu$$

So we define

$$W^\pm_\mu = \frac{1}{\sqrt{2}}\left(W^1_\mu \mp i W^2_\mu\right)$$

Comparing with the isospin and hypercharge current on page (27) we have:

$$\mathcal{L}_{leptons} = \bar{\ell}\, i \not{\partial}\, \ell + \bar{\nu}_L\, i \not{\partial}\, \nu_L - \frac{g'}{2} J^Y_\mu B_\mu - g J^i_\mu W^i_\mu$$

Note that the diagonal part of the current is

$$-\frac{g'}{2} J^Y_\mu B_\mu - g J^3_\mu W^3_\mu = +\frac{g'}{2}\left(\bar{\nu}_L \gamma_\mu \nu_L + \bar{\ell}_L \gamma_\mu \ell_L + 2 \bar{\ell}_R \gamma_\mu \ell_R\right) B_\mu$$

$$- \frac{g}{2}\left(\bar{\nu}_L \gamma_\mu \nu_L - \bar{\ell}_L \gamma_\mu \ell_L\right) W^3_\mu$$

$$= \bar{\nu}_L \gamma_\mu \nu_L \left(\frac{g'}{2} B_\mu - \frac{g}{2} W^3_\mu\right) + \bar{\ell}_L \gamma_\mu \ell_L \left(\frac{g'}{2} B_\mu + \frac{g}{2} W_\mu\right)$$

$$+ \bar{\ell}_R \gamma_\mu \ell_R\, g' B_\mu$$

The off-diagonal part of the current is:

$$- g J^1_\mu W^1_\mu - g J^2_\mu W^2_\mu = - g\left[J^1_\mu \left(\frac{W^+_\mu + W^-_\mu}{\sqrt{2}}\right) + J^2_\mu \left(\frac{W^+_\mu - W^-_\mu}{i\sqrt{2}}\right)\right]$$

$$= -\frac{g}{\sqrt{2}}\left[W^+_\mu \left(J^1_\mu + i J^2_\mu\right) + W^-_\mu \left(J^1_\mu - i J^2_\mu\right)\right]$$

$$= -\frac{g}{2\sqrt{2}}\left[W^+_\mu J^-_\mu + W^-_\mu J^+_\mu\right] = -\frac{g}{\sqrt{2}}\left[W^-_\mu \bar{\ell}_L \gamma_\mu \nu_L + W^+_\mu \bar{\nu}_L \gamma_\mu \ell_L\right]$$

$$J^\pm_\mu = 2\left(J^1_\mu \mp i J^2_\mu\right)$$

and so!

pag 13 $\to G_W^2 = \frac{M_W^2 G_F}{\sqrt{2}} \to \boxed{\frac{g}{2\sqrt{2}} = \sqrt{\frac{M_W^2 G_F}{\sqrt{2}}}}$ (31)

$$-g\left(J_\mu^1 W_\mu^1 + J_\mu^2 W_\mu^2\right) = -\underset{\substack{\uparrow \\ G_W \text{ pag } 14}}{\frac{g}{2\sqrt{2}}}\left(W_\mu^+ \bar{\nu}\gamma_\mu(1-\gamma_5)\ell + W_\mu^- \bar{\ell}\gamma_\mu(1-\gamma_5)\nu\right)$$

We now introduce the photon A_μ and the neutral boson Z_μ

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\vartheta_W & \sin\vartheta_W \\ -\sin\vartheta_W & \cos\vartheta_W \end{pmatrix}\begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}$$

$\vartheta_W \equiv$ Weinberg angle

$$A_\mu = \cos\vartheta_W B_\mu + \sin\vartheta_W W_\mu^3$$

$$Z_\mu = -\sin\vartheta_W B_\mu + \cos\vartheta_W W_\mu^3$$

$$\sin\vartheta_W = \frac{g'}{\sqrt{g^2+g'^2}}$$

$$\cos\vartheta_W = \frac{g}{\sqrt{g^2+g'^2}}$$

or:

$$B_\mu = \cos\vartheta_W A_\mu - \sin\vartheta_W Z_\mu$$

$$W_\mu^3 = \sin\vartheta_W A_\mu + \cos\vartheta_W Z_\mu$$

Therefore

$$g' B_\mu - g W_\mu^3 = \left(-\frac{g'^2}{\sqrt{g^2+g'^2}} - \frac{g^2}{\sqrt{g^2+g'^2}}\right) Z_\mu = -\sqrt{g^2+g'^2}\, Z_\mu$$

$$g' B_\mu + g W_\mu^3 = \frac{2gg'}{\sqrt{g^2+g'^2}} A_\mu + \frac{(g^2-g'^2)}{\sqrt{g^2+g'^2}} Z_\mu$$

We define the ~~[illegible]~~ electromagnetic coupling

$$e = g\sin\vartheta_W = g'\cos\vartheta_W = \frac{gg'}{\sqrt{g^2+g'^2}}$$

Therefore the diagonal part of the current becomes

$$-\frac{g'}{2} J^Y_\mu B_\mu - g J^3_\mu W^3_\mu = \frac{\sqrt{g^2+g'^2}}{2}\, \bar\nu_L \gamma_\mu \nu_L Z_\mu$$

$$+ \bar l_L \gamma_\mu l_L \left(e A_\mu + \frac{1}{2} \frac{(g^2-g'^2)}{\sqrt{g^2+g'^2}} Z_\mu \right)$$

$$+ \bar l_R \gamma_\mu l_R \left(e A_\mu - \frac{g'^2}{\sqrt{g^2+g'^2}} Z_\mu \right)$$

$$= e A_\mu \bar l \gamma_\mu l + Z_\mu \left[\frac{\sqrt{g^2+g'^2}}{2}\, \bar\nu \gamma_\mu \frac{(1-\gamma_5)}{2} \nu \right.$$

$$\left. + \underbrace{\frac{\sqrt{g^2+g'^2}}{2}}_{= g/2\cos\theta_W} \left(\frac{g^2-g'^2}{g^2+g'^2}\, \bar l \gamma_\mu \frac{(1-\gamma_5)}{2} l - \frac{2g'^2}{g^2+g'^2}\, \bar l \gamma_\mu \frac{(1+\gamma_5)}{2} l \right) \right]$$

$$= e A_\mu \bar l \gamma_\mu l + \frac{g}{2\cos\theta_W} Z_\mu \left[\bar\nu \gamma_\mu \frac{(1-\gamma_5)}{2} \nu \right.$$

$$\left. + \underbrace{(\cos^2\theta_W - \sin^2\theta_W)}_{= 1 - 2\sin^2\theta_W} \bar l \gamma_\mu \frac{(1-\gamma_5)}{2} l - 2\sin^2\theta_W\, \bar l \gamma_\mu \frac{(1+\gamma_5)}{2} l \right]$$

$$= e A_\mu \bar l \gamma_\mu l + \frac{g}{2\cos\theta_W} Z_\mu \left[\bar\nu \gamma_\mu \frac{(1-\gamma_5)}{2} \nu \right.$$

$$\left. + \bar l \gamma_\mu \left(\frac{1 - 4\sin^2\theta_W - \gamma_5}{2} \right) l \right]$$

Note the neutrino coupling is V−A but the lepton is $(g_V - g_A\gamma_5)$.

The Higgs sector

We introduce a Higgs field that is a doublet of $SU(2)_L$, and so complex, and has hypercharge $Y=1$.

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

Since

$$Q = T_3 + \frac{Y}{2}$$

we get

$$Q = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Therefore ϕ^+ has $Q=1$ and ϕ^0 $Q=0$

So, if we do not want to break the $U(1)_{em}$, generated by Q, the vacuum has to have $\phi^+ = 0$.

We have a self-interacting Higgs potential given by

$$V = \mu^2 \phi^+\phi + \lambda (\phi^+\phi)^2$$

and the covariant derivative as:

$$D_\mu \phi = \partial_\mu \phi + i\frac{g}{2}\sigma_i W^i_\mu \phi + i\frac{g'}{2} \underset{Y=1}{\uparrow} B_\mu \phi$$

The Higgs part of the Lagrangian is:

$$\mathcal{L}_\phi = \frac{1}{2}(D_\mu\phi)^\dagger D^\mu\phi - V$$

Note that we can write the potential as

$$V = \frac{\lambda}{4}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2 - \frac{v^4\lambda}{4}$$

with $v^2 = -\frac{\mu^2}{\lambda}$ and so to have a minimum we need $\mu^2 < 0$.

For $\mu^2 < 0$,
There is a tree-approximation vacuum expectation value at the stationary point of the Lagrangian

$$\langle\phi^\dagger\rangle\langle\phi\rangle = \frac{v^2}{2} = \frac{1}{2}\frac{|\mu^2|}{\lambda}$$

We can always perform an $SU(2)\otimes U(1)$ gauge transformation to a unitary gauge where

$\phi^+ = 0$ and ϕ^0 is hermitian with a positive v.e.v.

That is why we normalized the complex doublet ϕ so that an unconventional factor $\frac{1}{2}$ appears in the kinetic term.

$\mathrm{Re}\,\phi^0$ is the only physical scalar field. In the unitary gauge the v.e.v. are

$$\langle\phi^+\rangle = 0 \qquad \langle\phi^0\rangle = \frac{v}{\sqrt{2}} > 0$$

We can parametrize the Higgs as:

$$\Phi = e^{i\sigma^i \chi_i/2v} \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix}$$

$$\simeq \langle\Phi\rangle_0 + \frac{1}{2\sqrt{2}}\begin{pmatrix} \chi_2 + i\chi_1 \\ 2H - i\chi_3 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} i\sqrt{2}\, w^+ \\ v + H - i z^0 \end{pmatrix}$$

where w^+ and z^0 are the Goldstone bosons. w^+ is complex and z^0 real.

We now make the gauge transformation:

$$\phi \to \phi' = e^{-i\sigma^i\chi^i/2v}\,\phi = \frac{v+H}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

and with such a transformation the Higgs Lagrangian becomes:

$$\mathcal{L}_\phi = \left|\left(\partial_\mu + i\frac{g}{2}\sigma^i W^i_\mu + i\frac{g'}{2}B_\mu\right)\frac{(v+H)}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right|^2$$

$$-\mu^2\frac{(v+H)^2}{2} - \lambda\frac{(v+H)^4}{4}$$

Now

$$g\sigma^i W^i_\mu + g' B_\mu = \begin{pmatrix} gW^3_\mu + g'B_\mu & g(W^1_\mu - iW^2_\mu) \\ g(W^1_\mu + iW^2_\mu) & -gW^3_\mu + g'B_\mu \end{pmatrix}$$

and

$$(g\sigma^i W^i_\mu + g' B_\mu)\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \sqrt{2}g W^+_\mu \\ -gW^3_\mu + g'B_\mu \end{pmatrix} = g\begin{pmatrix} \sqrt{2}\, W^+_\mu \\ \frac{Z_\mu}{\cos\theta_W} \end{pmatrix}$$

$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$, $Z_\mu = \frac{gW^3_\mu - g'B_\mu}{\sqrt{g^2+g'^2}}$, $W^+_\mu = \frac{W^1_\mu - iW^2_\mu}{\sqrt{2}}$

Thus, the kinetic part of the Higgs Lagrangian becomes

$$\mathcal{L} = \left| \begin{pmatrix} 0 \\ \frac{\partial_\mu H}{\sqrt{2}} \end{pmatrix} + i\frac{g}{2}(v+H) \begin{pmatrix} W_\mu^+ \\ \frac{Z_\mu}{\sqrt{2}\cos\theta_W} \end{pmatrix} \right|^2 =$$

$$= \frac{1}{2}(\partial_\mu H)^2 + \frac{g^2}{4}(v+H)^2 \left(W_\mu^+ W_\mu^- + \frac{Z_\mu Z^\mu}{2\cos^2\theta_W} \right)$$

since the crossed term vanishes.

So, we have the terms:

$$\frac{g^2 v^2}{4} W_\mu^+ W^{\mu -} + \frac{g^2 v^2}{8\cos^2\theta_W} Z_\mu Z^\mu$$

which may be identified with

$$\frac{1}{2} M_W^2 (W_\mu^{1\,2} + W_\mu^{2\,2}) + \frac{1}{2} M_Z^2 Z_\mu^2$$

since $W_\mu^+ W_\mu^- = \frac{1}{2}(W_\mu^1 - iW_\mu^2)(W_\mu^1 + iW_\mu^2) = \frac{1}{2}(W_\mu^{1\,2} + W_\mu^{2\,2})$

We get

$$M_W = \frac{gv}{2} \qquad M_Z = \frac{gv}{2\cos\theta_W} = \frac{M_W}{\cos\theta_W}$$

Note that there is no A_μ^2 term and so the mass of the photon is indeed zero.

From the relation

$$\frac{g}{2\sqrt{2}} = \sqrt{\frac{M_W^2 G_F}{\sqrt{2}}} \rightarrow M_W^2 = \frac{\sqrt{2}}{8}\frac{g^2}{G_F}$$

we get

$$v^2 = \frac{1}{\sqrt{2}G_F} \rightarrow v = (\sqrt{2}G_F)^{-1/2}$$

Therefore

$$v \approx 247 \text{ GeV}$$

Therefore we have: $(e = g\sin\theta_W = g'\cos\theta_W)$

$$M_W^2 = \frac{g^2v^2}{4} = \frac{e^2}{\sin^2\theta_W}\frac{v^2}{4} = \underbrace{\frac{e^2}{4\pi}}_{\frac{1}{128} \text{ and not } \frac{1}{137}} \pi \frac{v^2}{\sin^2\theta_W}$$

1/128 and not 1/137 (for energies ~ 100 GeV)

Then

$$M_W = \frac{38.4}{\sin\theta_W} \text{ GeV}$$

and:

$$M_Z = \frac{M_W}{\cos\theta_W} = \frac{38.4}{\sin\theta_W\cos\theta_W} = \frac{76.9}{\sin 2\theta_W} \text{ GeV}$$

By 1984 processes like

$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$
$$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$$

gives

$$\sin^2\theta_W = 0.222 \pm 0.011$$

giving $M_W = 81.5$ GeV and $M_Z = 92.5$ GeV

The values in 2018 are:

$$M_W = 80.379 \pm 0.012 \text{ GeV}$$

$$M_Z = 91.1876 \pm 0.0021 \text{ GeV}$$

$$\sin^2\theta_W = 0.22343 \pm 0.00007$$

The value of $\sin^2\theta_W$ is not predicted by the standard model. It comes, for instance from neutrino/lepton scattering

The Higgs mass

We have performed the gauge transformation (see page 35)

$$\phi \to \phi' = e^{-i\sigma^i x^i/2v}\phi = \frac{v+H}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}$$

Then the Higgs potential becomes:

$$V = \mu^2\frac{(v+H)^2}{2} + \lambda\frac{(v+H)^4}{4} = \boxed{\cancel{\frac{\mu^2 H^2}{2}}}$$

$$\boxed{\cancel{= \frac{\mu^2 v^2}{2} + \lambda}}$$

$$= \frac{\mu^2}{2}(v^2 + 2Hv + H^2) + \frac{\lambda}{4}(v^2 + 2vH + H^2)^2$$

$$= \frac{\mu^2}{2}v^2 + \frac{\lambda}{4}v^4 + (\mu^2 v + v^3\lambda)H$$

$$+ \left(\frac{\mu^2}{2} + \frac{\lambda}{2}v^2 + \lambda v^2\right)H^2 + O(H^3)$$

But

$$v^2 = \frac{|\mu^2|}{\lambda} = -\frac{\mu^2}{\lambda}$$

So:

$$V = \frac{v^2}{2}\left(\mu^2 + \frac{\lambda}{2}v^2\right) + v\,\cancel{(\mu^2 + v^2\lambda)}\,H$$

$$+ \left(\frac{\mu^2}{2} + \frac{3}{2}(-\mu^2)\right)H^2 + O(H^3)$$

So we have a mass term

$$\frac{1}{2}M_H^2 H^2 = -\mu^2 H^2$$

and so:

$$M_H = \sqrt{-2\mu^2}$$

μ is not predicted by the standard Model

The value as in 2018 (PDG) is

$$M_H = 125.18 \pm 0.16 \text{ GeV}$$

The lepton masses: the Yukawa coupling

Note that by adding a mass term for the leptons:

$$M_e \bar{\ell}\ell = M_e(\bar{\ell}_R \ell_L + \bar{\ell}_L \ell_R)$$

mixes L and R components and so breaks gauge symmetry $(SU(2)_L)$

A way to do it is via the so-called Yukawa coupling.

$$\mathcal{L}_{Yuk} = -G_\ell \left(\bar{R} (\phi^\dagger L) + (\bar{L} \phi) R \right)$$

$$= -G_\ell \frac{(v+H)}{\sqrt{2}} \left[\bar{\ell}_R (0\ 1) \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} + (\bar{\nu}_L\ \bar{\ell}_L) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \ell_R \right]$$

$$= -\frac{G_\ell}{\sqrt{2}} (v+H) \left[\bar{\ell}_R \ell_L + \bar{\ell}_L \ell_R \right]$$

$$= -\frac{G_\ell v}{\sqrt{2}} \bar{\ell} \ell - \frac{G_\ell}{\sqrt{2}} \bar{\ell} \ell H$$

Therefore one gets the mass of the lepton as:

$$M_\ell = \frac{G_\ell v}{\sqrt{2}}$$

G_ℓ is not predicted by the standard model.

Note that we also get a coupling of the lepton to the Higgs

$$C_{\bar{\ell}\ell H} \ \bar{\ell} \ell H$$

with

$$C_{\bar{\ell}\ell H} = \frac{M_\ell}{v}$$

That is a prediction of the standard model that has to be verified experimentally.

Note that the more massive the lepton the stronger the coupling to the Higgs.

Introducing the quarks

In the mid 60's it was clear that weak interaction processes in which charge is exchanged between leptons and hadrons are well described by the low energy Lagrangian:

$$\frac{G_F}{\sqrt{2}}\left[\bar{e}\,\gamma_\lambda(1-\gamma_5)\nu_e+\bar{\mu}\,\gamma_\lambda(1-\gamma_5)\nu_\mu\right]J^\lambda + \text{h.c.}$$

where

$$J^\lambda=\bar{u}\,\gamma^\lambda(1-\gamma_5)\,d\,\cos\theta_c+\bar{u}\,\gamma^\lambda(1-\gamma_5)\,s\,\sin\theta_c$$

with $\theta_c \equiv$ Cabibbo angle due to the flavor change of weak interactions

and $\sin\theta_c \simeq 0.220 \pm 0.003$

Process like

$$O^{14}\to N^{14*}+e^+ +\nu_e$$

$$K^+\to \pi^0+e^+ +\nu_e$$

confirm that G_F has about the same value as it has for leptonic processes.

$$\mu^+\to \bar{\nu}+e^+ +\nu_e$$

Then one conclude quarks form a doublet of $SU(2)\times U(1)$

$$\frac{(1-\gamma_5)}{2}\begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix} \qquad (*)$$

and the right-handed quarks are singlets

One must have

$$Y_{Lq} = \frac{1}{3} \qquad Y_{Ru} = \frac{4}{3} \qquad Y_{Rd} = -\frac{2}{3}$$

since $Q = T_3 + \frac{Y}{2}$

$$Q_u = \frac{1}{2} + \frac{1}{2}\frac{1}{3} = \frac{2}{3}$$
$$= 0 + \frac{1}{2}\frac{4}{3} = \frac{2}{3}$$

$$Q_d = -\frac{1}{2} + \frac{1}{2}\frac{1}{3} = \frac{-3+1}{6} = -\frac{1}{3}$$
$$= 0 + \frac{1}{2}\left(-\frac{2}{3}\right) = -\frac{1}{3}$$

etc..

However, there are difficulties:

Z_0 interacts with quark neutral current:

$$\sum_{\substack{q \\ \text{doublet}}} \bar{q}\gamma^\mu (T_3\cos\theta_w + Y\sin\theta_w) q =$$

$$= \sum_q \bar{q}\gamma^\mu \left(T_3\cos\theta_w + 2(Q - T_3)\sin\theta_w\right) q$$

$$= 2\cos\theta_w \sum_q \bar{q}\gamma^\mu \left(\left(\frac{1}{2} - \frac{\sin\theta_w}{\cos\theta_w}\right) T_3 + Q\tan\theta_w\right) q$$

Q is diagonal in quark flavors, but if ~~the~~ the doublet (*) on page 42 ~~was~~ were the only one then the term involving T_3 would contain cross terms like

$$\bar{s}\,\gamma^\mu(1-\gamma_5)\,d \quad \text{and} \quad \bar{d}\,\gamma^\mu(1-\gamma_5)\,s$$

leading to Z exchange interactions like

$$s + \bar{d} \leftrightarrow d + \bar{s} \quad \text{and} \quad s + \bar{d} \leftrightarrow \mu^+ + \mu^-$$

with a strength of ordinary first order weak interactions.

That would lead to rates for processes like

$K^0 - \bar{K}^0$ oscillations

and $K^0 \to \mu^+ + \mu^-$

many orders of magnitude greater than observed.

There would be problems in the charged current sector too at one loop level.

Glashow, Iliopoulos and Maiani (1970) proposed that ~~h~~ there is a further term in J^λ:

$$\bar{c}\,\gamma^\lambda(1-\gamma_5)\,[-d\sin\theta_c + s\cos\theta_c]$$

where c is a fourth quark (the charm quark) with charge $\frac{2}{3}$

The charged current becomes:

$$J^\lambda = (\bar{u}\cos\theta_c - \bar{c}\sin\theta_c)\gamma^\lambda(1-\gamma_5)d + (\bar{u}\sin\theta_c + \bar{c}\cos\theta_c)\gamma^\lambda(1-\gamma_5)s$$

- This current does not conserve strangeness due to the mass difference between c and u

- Loop diagrams for the effective interactions

$$s + \bar{s} \leftrightarrow d + \bar{d}$$

are suppressed by additional factors (since $m_u \ll m_s$) $\frac{m_c^2}{m_W^2}$ bringing the ratio for $K^0-\bar{K}^0$ oscillations into agreement with experiment.

- It was noted (by Weinberg (1971)) that this also solves the problem of strangeness changing Z_0 interactions.

Then there is a further doublet

$$\frac{(1-\gamma_5)}{2}\begin{pmatrix} c \\ -d\sin\theta_c + s\cos\theta_c \end{pmatrix}$$

The addition of this doublet in the neutral current it cancels the strangeness non-conserving terms

$$\bar{s}\gamma^\mu(1-\gamma_5)d \quad \text{and} \quad \bar{d}\gamma^\mu(1-\gamma_5)s$$

Removing the excess of Z exchange in

$K^0-\bar{K}^0$ oscillations and $K^0 \to \mu^+ + \mu^-$

Mesons $c\bar{c}$ were discovered in 1974, and indicated that $m_c = 1.5$ GeV.

So we have 2 generations of quarks and leptons:

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \text{(mixed)}, \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$$

The lepton τ was discovered and indicated a third generation.

A fifth quark was discovered (b) with charge $-\frac{1}{3}$ and $m_b = 4.5$ GeV.

A sixth quark (top) with charge $\frac{2}{3}$ was a theoretical necessity and it was discovered in 1995 with

$$m_t = 181 \pm 12 \text{ GeV}$$

Now the quark current is expressed as:

$$J^\lambda = \overline{\begin{pmatrix} u \\ c \\ t \end{pmatrix}} \gamma^\lambda (1-\gamma_5) V \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$V \equiv$ unknown 3×3 unitary matrix

Kobayashi - Maskawa matrix (1972)

QCD

- Gell-Mann, Ne'eman flavor $SU(3)_F$ did not have the 3 and $\bar{3}$ reps. By assuming the existence of particles transforming by them one can understand

$$\text{mesons} \equiv q\bar{q} \qquad 3 \times \bar{3} = 8 + 1$$

$$\text{baryons} \equiv qqq \qquad 3 \times 3 \times 3 = 10 + 8 + 8 + 1$$

- The magnetic moment of the proton is

$$\mu_p = 2.79 \times \frac{e\hbar}{2m_p}$$

instead of

$$\mu_p = \frac{e\hbar}{2m_p}$$

if it was elementary.

- The $\Delta^{++} = uuu$ led Greenberg to postulate a new quantum number → color.

- hadrons with 2 or 4 quarks were not observed and that could be explained by postulating that hadrons are colorless, i.e. invariant under color rotation:

$$q^i \to \sum_{k=1}^{3} U^{ik} q^k \qquad U U^+ = 1$$

If $\det U = 1$ then one gets $U \in SU(3)$.
So $SU(3)$ color is born.

The singlet rep. only appear in

$$3_c \times 3_c \times 3_c = 10_c + 8_c + 8_c + 1_c$$

and

$$3_c \times \bar{3}_c = 8_c + 1_c$$

and explain why we do not have exotic particles with (2 or 4 quarks)

- Gell-Mann postulated (1962) that the current algebra for quarks was such that the commutation relations appear as if the quark fields entering into them were <u>free</u>.
That showed to be in agreement with sum rules.

- Deep inelastic ~~s~~ scattering (Bjorken scaling) amplitudes do not depend upon energy (for short distances) but on ratio of energies.

  Feynman showed how this could be interpreted if we consider as $Q^2, E \to \infty$, we consider the proton to be made of "partons" that do not interact among themselves.

- Strong interactions: What is the theory?

  It is observed that it is flavor independent

  So should act equally well on u, d, c, s..

- Weinberg and Nanopoulos (1973) showed that (to avoid catastrophic violation of parity) strong interaction should ~~not~~ act on quantum numbers other than flavor.

  → look for coupling with color, which is blind to weak and electromagnetic interactions.

- Take 8 vector gluons A_μ^a, $a = 1 \dots 8$ in the adjoint of $SU(3)_c$ interacting universally with all quark flavors

$$\mathcal{L}_I = \mathcal{L}_0 + \underset{\substack{\uparrow \\ \text{universal}}}{g} \sum_q \bar{q}^i \gamma^\mu \underset{\substack{\uparrow \\ SU(3)_c \text{ generator (in triplets)}}}{T^a_{ik}} q^k A^a_\mu$$

- It was realized that the massless vector fields of non-abelian YM theory possesses ~~hidden~~ (highly) hidden infrared singularities that could prevent the liberation of quarks and gluons (infrared slavery) → confinement ~~hyp~~ hypothesis.

Now take a gauge theory

$$\mathcal{L}_{QCD} = \mathcal{L}_I - \frac{1}{4} \operatorname{Tr} F_{\mu\nu}^2$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig [A_\mu, A_\nu]$$

It takes us from the real world of mesons and baryons to the fictitious realm of quarks and gluons

- 't Hooft (unpublished) Politzer (1973), Gross and Wilczek (1973) proved that in a YM theory the effective coupling constant vanishes at short distances (<u>asymptotic freedom</u>) and increases at long distances.

  It makes confinement probable.

  QCD is an YM theory in agreement with experiment at short distances (large energies)

- It is a local field theory leading to local observables.

  Hilbert space of quarks and gluons, but only a subspace is physical (colorless states)

  Only color singlet operators survive like

$$\sum \bar{q}^i \gamma^\mu (1 \pm \gamma_5) q^i$$

$$\sum \bar{u}_i \gamma_5 d^i \qquad \sum \varepsilon^{ijk} u^i u^j d^k$$

  i, j, k = color indices.

- Must live to see if this picture is correct!

Why is SU(3) color an exact symmetry? Why it can not be broken?

The answer has to do with confinement. As mentioned before the singularities at the infrared regime could lead to the so-called infrared slavery, and perhaps that is linked to confinement. Generating mass to gluons, and so breaking the SU(3) color symmetry (by hand or spontaneously) would kill that infrared behavior of the theory.

In addition, if one adds Higgs fields, to break the SU(3) color spontaneously and generate mass to the gluons, would affect the ~~asymptotic~~ asymptotic freedom, since the Higgs scalar would contribute with positive terms to the beta function $\beta(g)$ and that could flip its sign.

Magnetic monopoles

An idea put forward by 't Hooft and Polyakov is that confinement could be produced by a dual Meissner effect in QCD. That effect could be produced by a condensate of magnetic monopoles with zero mass perhaps.

However, no one was able to show that in QCD

Witten and Vafa have shown (1994) that in $N=2$ supersymmetric gauge theories that condensate can happen, since monopoles are massless in some point of moduli space.