Lista 14 - MAT-121 - Sobre derivadas parciais

(I) Calcule $\dfrac{\partial z}{\partial x}$ e $\dfrac{\partial z}{\partial y}$, se existir:

1. $z = 4x^2y^3 - \dfrac{1}{xy} + 4$
2. $z = sen(x^3 + y^2)$
3. $z = e^{xseny}$
4. $z = xy^2 ln(x^4 + y^4 + 3)$
5. $z = arctg\dfrac{y}{x}$
6. $z = \dfrac{x^3 + 2y^2}{2x^2 + 4y}$
7. $z = \begin{cases} \dfrac{xy}{x^2+y^2} & \text{se } (x,y) \neq (0,0) \\ 0 & \text{se } (x,y) = (0,0) \end{cases}$
8. $z = sec(x^4y)$
9. $f(x,y) = \begin{cases} \dfrac{(x-1)^3}{(x-1)^2+(y-2)^2} + 5x & \text{se } (x,y) \neq (1,2) \\ 5 & \text{se } (x,y) = (1,2) \end{cases}$
10. $z = x^y$
11. $f(x,y) = \begin{cases} \dfrac{3(x+3)^2y}{(x+3)^2+y^2} + 2x + y & \text{se } (x,y) \neq (-3,0) \\ -6 & \text{se } (x,y) = (-3,0) \end{cases}$
12. $z = ln(xy^7)$

(II) Seja $z = \dfrac{x^2y}{x^2+y^2}$. Verifique que $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = z$.

(III) Seja $g : \mathbb{R} \longrightarrow \mathbb{R}$ uma função derivável tal que $g(0) = 2$ e $g'(0) = -1$.
Considere a função $f(x,y) = 3x.g(x^2 - y^2)$. Calcule $\dfrac{\partial f}{\partial x}(1,1)$ e $\dfrac{\partial f}{\partial y}(1,1)$.

(IV) Seja $\phi : \mathbb{R} \longrightarrow \mathbb{R}$ uma função derivável tal que $\phi(1) = 3$ e $\phi'(1) = 2$.

Considere a função $g(x, y) = (2x^3 + 3y)\phi(\frac{y}{x})$. Calcule $\frac{\partial g}{\partial x}(3, 3)$ e $\frac{\partial g}{\partial y}(3, 3)$.

(V) Seja $f(x, y) = \int_0^{x^2-y^4} e^{-t^2} dt$. Calcule $\frac{\partial f}{\partial x}(4, 2)$ e $\frac{\partial f}{\partial y}(4, 2)$.

(VI) Calcule $\frac{\partial f}{\partial x}$ e $\frac{\partial f}{\partial y}$, sendo

$$f(x, y) = \begin{cases} \frac{x^2 - y^3}{x^2 + y^2} & \text{se } (x, y) \neq (0, 0) \\ 0 & \text{se } (x, y) = (0, 0) \end{cases}$$