

HOMEWORK Lec 02 (MATRIX ALGEBRA)

(10)

① Solve the Lin vec sys $Ax=b$ with

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} -1 \\ -2 \\ 8 \\ -1 \end{bmatrix}$$

a) Via elimination techniques directly;

b) Partition the system and solve blockwise

② Solve the following matrix Lin sys

$$AX + XB = C$$

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 6 & -2 \\ 5 & 1 \end{bmatrix}$$

③ Find the F_i 's blocks in terms of A, B, C, D

$$\begin{bmatrix} F_1 & F_2 \\ F_3 & F_4 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1}$$

④ Create a nontrivial Lin sys of 4×4 elements with Kronecker structure then solve it, counting the flops:

a) Directly (elimination)

b) the fast method (low complexity)

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⑤ show that

$$\text{Tr}(AB) = \text{vec}^T(A^T) \text{vec}(B)$$