Escola Politécnica da Universidade de São Paulo
Departamento de Engenharia Mecatrônica e de Sistemas Mecânicos - PMR

Aula 3 – Análise de Sistemas Não Lineares

Funções Descritivas

Prof. Eduardo A. Tannuri

PMR 5014

Controle Não Linear Aplicado a Sistemas Mecânicos e Mecatrônicos

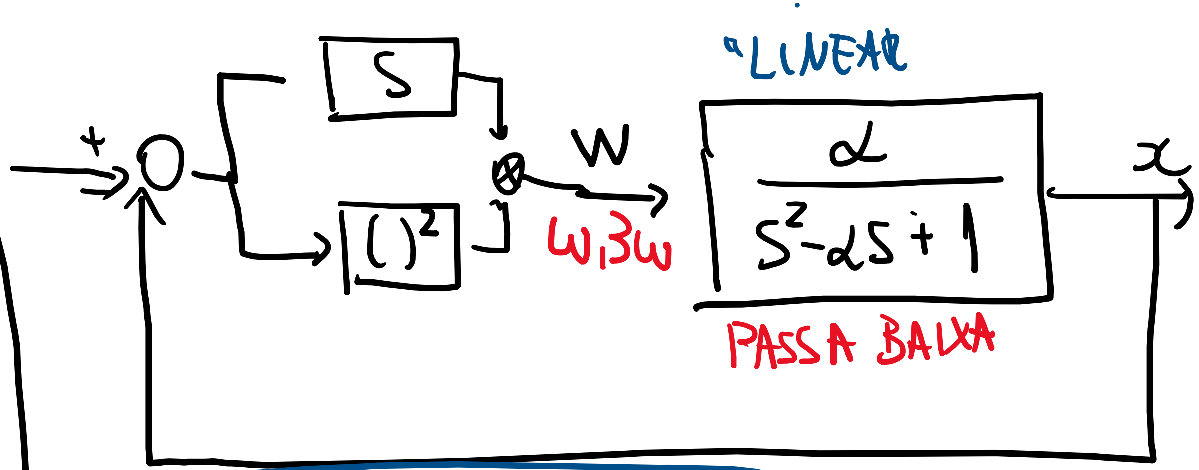
FUNÇÕES DESCRITIVAS

- ANÁLOGO À F. TRANSF. P/ SISTEMA NÃO LINEAR
- 1º HARMÔNICA OSCILAÇÕES DE SIST. NÃO LINEAR

$$\ddot{x} + \alpha(x^2 - 1)\dot{x} + x = 0$$

$$\ddot{x} + \underbrace{\alpha x^2 \dot{x}}_{\text{NÃO LINEAR}} - \alpha\dot{x} + x = 0 \qquad -w$$

$$\underbrace{\ddot{x} - \alpha\dot{x} + x}_{\text{SIST. LINEAR}} = \underbrace{\alpha \cdot w}_{\text{EXCITAÇÃO NÃO LINEAR}}$$



$$\boxed{\begin{aligned} x(t) &= A \operatorname{sen}(\omega t) \\ \dot{x}(t) &= A\omega \cos(\omega t) \end{aligned}}$$

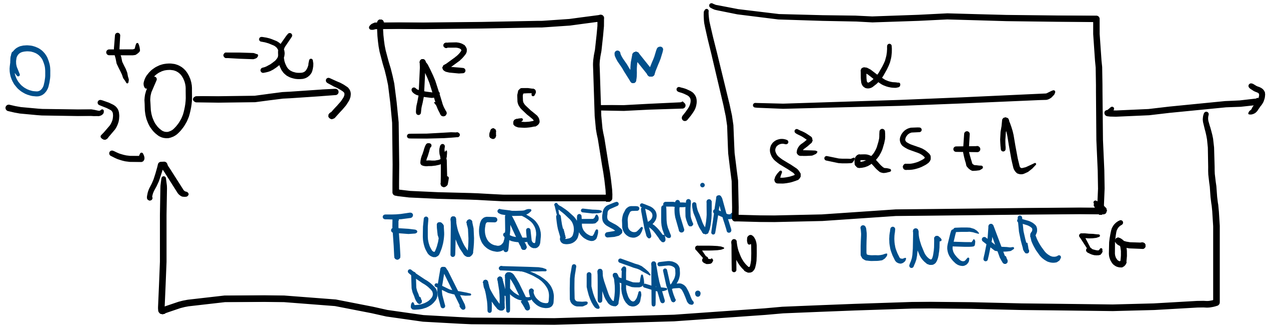
$$\operatorname{sen}^2 \omega t = \frac{1 - \cos 2\omega t}{2}$$

$$W = -x^2 \dot{x} = -A^2 \operatorname{sen}^2 \omega t \, A\omega \cos \omega t$$

$$W = -\frac{A^3\omega}{4}\left(\cos \omega t - \cos 3\omega t\right)$$

CONSIDERANDO QUE A HARMÔNICA EM 3ω SERÁ FILTRADA PELA PARTE LINEAR

$$W \cong -\frac{A^3\omega}{4}\cos \omega t = \frac{A^2}{4} \cdot d[-A \operatorname{sen} \omega t]$$



$W = N(A,\omega)\cdot(-x)$

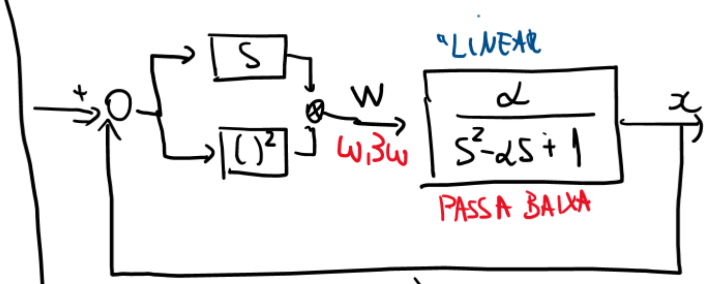
CONSIDERANDO QUE ∃ UM CICLO LIMITE

$FTMF = \frac{N\cdot G}{1+N\cdot G}$ ⇝ P/ TER CICLO LIMITE, POLOS DEVEM SER $\pm j\omega$

⇒ LOGO $1 + \frac{A^2}{4}\cdot j\omega\left(\frac{\alpha}{-\omega^2 - \alpha j\omega + 1}\right) = 0$

$-4\omega^2 - 4\alpha\omega j + 4 + A^2\alpha\cdot j\omega = 0$

$\begin{cases} -4\omega^2 + 4 = 0 \\ -4\cancel{\alpha}\omega + A^2\cancel{\alpha}\omega = 0 \end{cases} \Rightarrow \begin{cases} \omega = 1 \\ A = 2 \end{cases} \rightarrow$ $\boxed{\text{Solução} \cong 2\,\text{sen}(1\cdot t)}$



$x(t) = A\,\text{sen}(\omega t)$

$\dot{x}(t) = A\omega\cos(\omega t)$

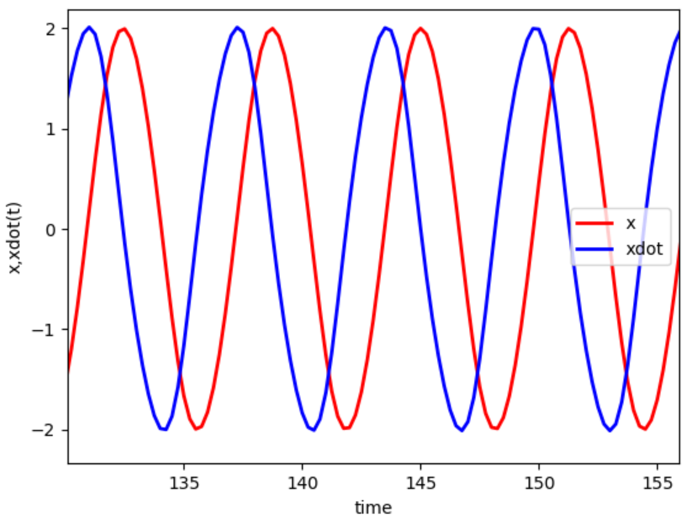
$\text{sen}^2\omega t = \frac{1-\cos 2\omega t}{2}$

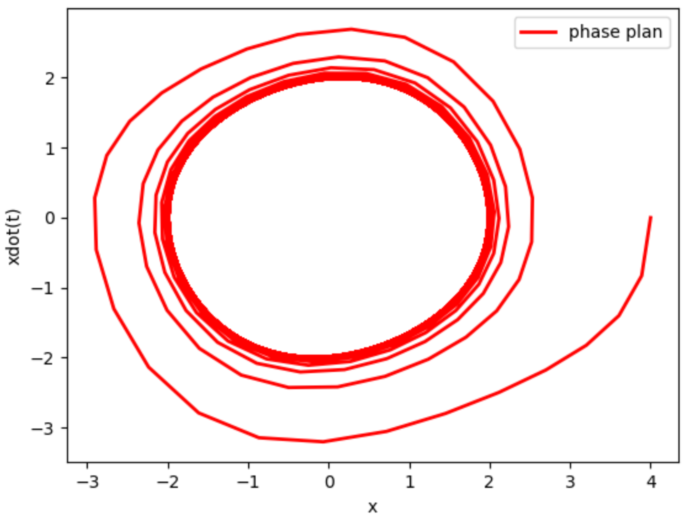
$W = -x^2\cdot\dot{x} = -A^2\text{sen}^2\omega t\, A\omega\cos\omega t$

$W = -\frac{A^3\omega}{4}(\cos\omega t - \cos 3\omega t)$

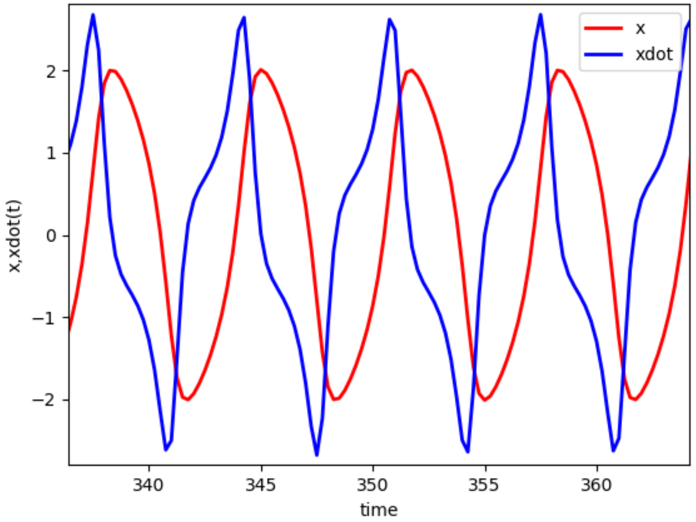
CONSIDERANDO QUE A HARMÔNICA EM 3ω SERÁ FILTRADA PELA PARTE LINEAR

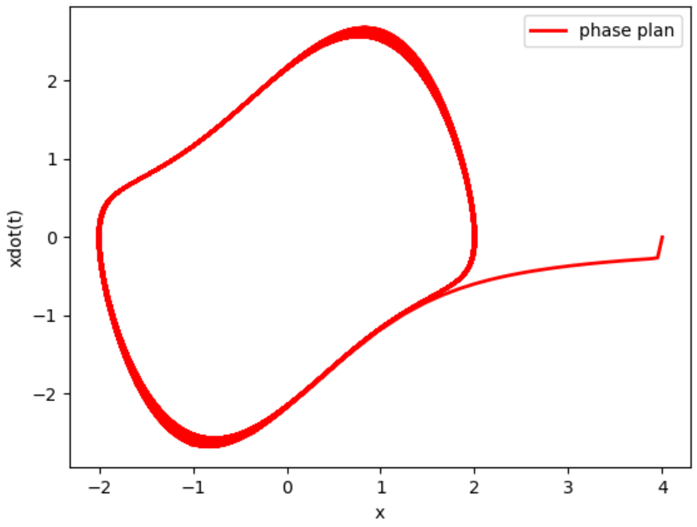
$W \cong -\frac{A^3\omega}{4}\cos\omega t = \frac{A^2}{4}\cdot d[-A\,\text{sen}\,\omega t]$





$\alpha = 0{,}1$



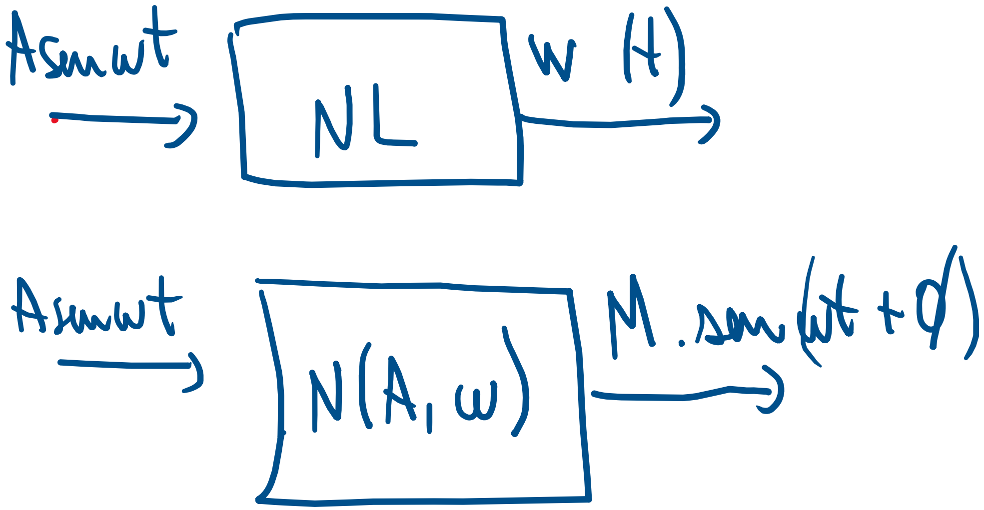


$\alpha = 1$

GENERALIZANDO

$$w(t) = M(A,\omega)\cdot \operatorname{sen}(\omega t + \phi(A,\omega))$$

$$\boxed{N(A,\omega) = M(A,\omega)\,\angle\,\phi(A,\omega)}$$



$$M(A,\omega) = \sqrt{a_1^2 + b_1^2}$$

$$a_1 = \frac{1}{\pi}\int_{-\pi}^{\pi} w(t)\cos\omega t \, d\omega t$$

$$b_1 = \frac{1}{\pi}\int_{-\pi}^{\pi} w(t)\operatorname{sen}\omega t \, d\omega t$$

P/ CASO ANTERIOR $x^2\dot{x}$

$$w(t) = \frac{-A^3\omega}{4}(\cos\omega t - \cos 3\omega t)$$

$$\left.\begin{aligned}\int_{-\pi}^{\pi}\cos^2 x\,dx &= \pi\\ \int_{-\pi}^{\pi}\operatorname{sen}^4 x\,dx &= \frac{3}{4}\pi\end{aligned}\right\} \Rightarrow \left\{\begin{aligned} a_1 &= \frac{-A^3\omega}{4}\cdot\pi\cdot\frac{1}{\pi} = \frac{-A^3\omega}{4}\\ b_1 &= 0\end{aligned}\right.$$

$$\Rightarrow \boxed{w(t) \approx \frac{-A^3\omega}{4}\cdot\cos\omega t}$$

MOLA NÃO LINEAR

$$K = 1 + \frac{3x^2}{2}$$

$$W = x + \frac{x^3}{2}$$

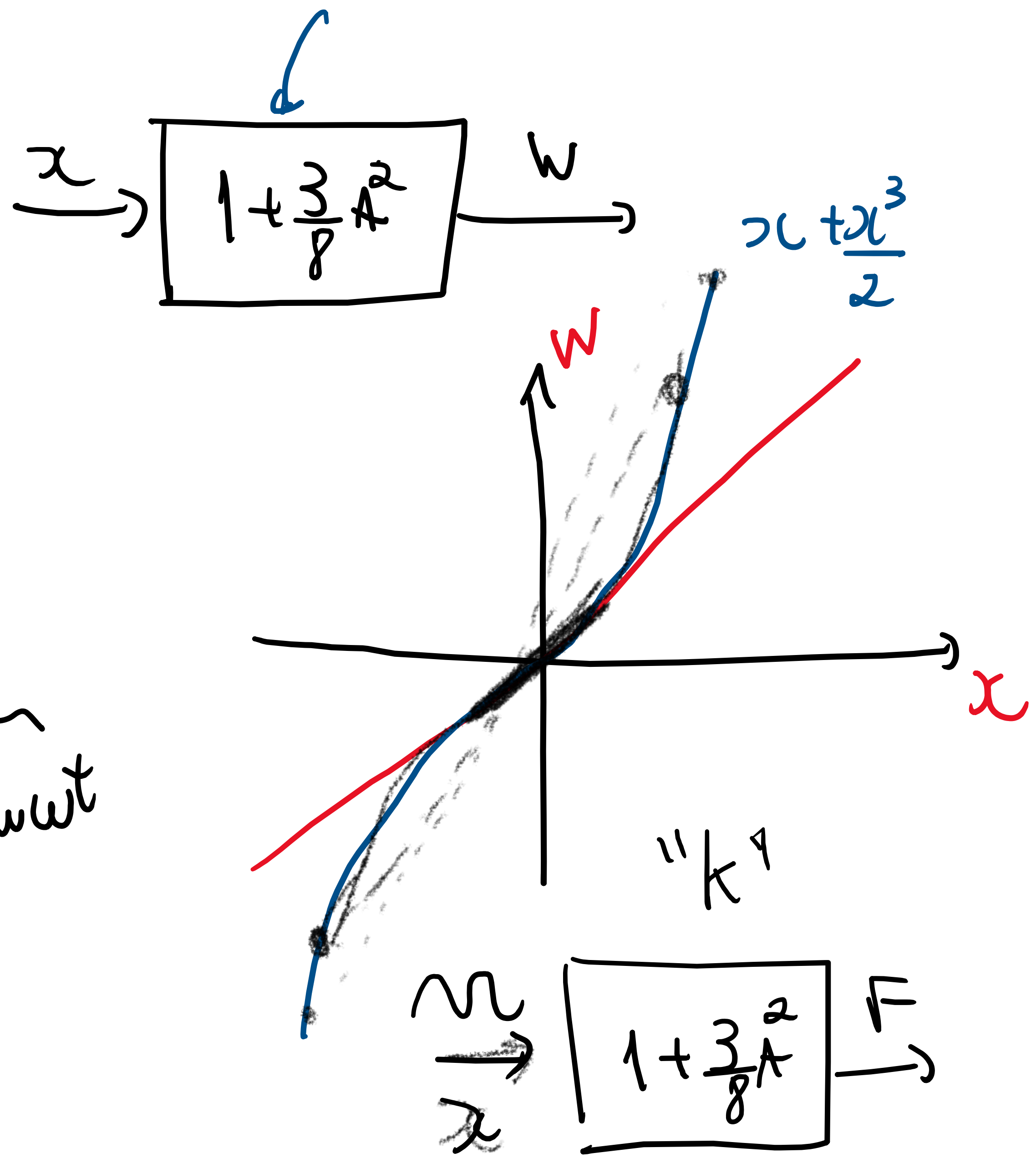
$$W = A\,\text{sen}\,\omega t + \frac{A^3}{2}\,\text{sen}^3\,\omega t$$

↳ $a_1 = 0$

$$b_1 = A + \frac{3}{8}A^3$$

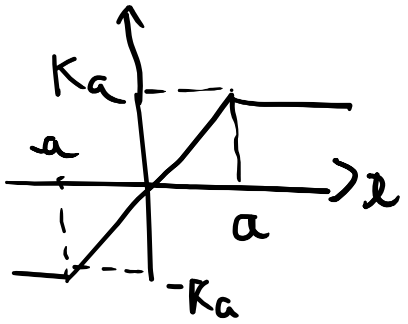
$$W = \left(A + \frac{3}{8}A^3\right)\text{sen}\,\omega t = \left(1 + \frac{3}{8}A^2\right)\cdot \overbrace{A\,\text{sen}\,\omega t}^{x}$$

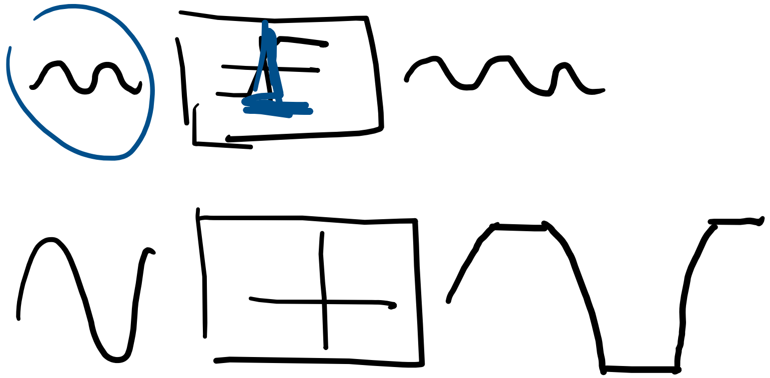
$$\boxed{N(A,\omega) = 1 + \frac{3}{8}A^2}$$



NÃO LINEARIDADES COMUNS

SATURAÇÃO

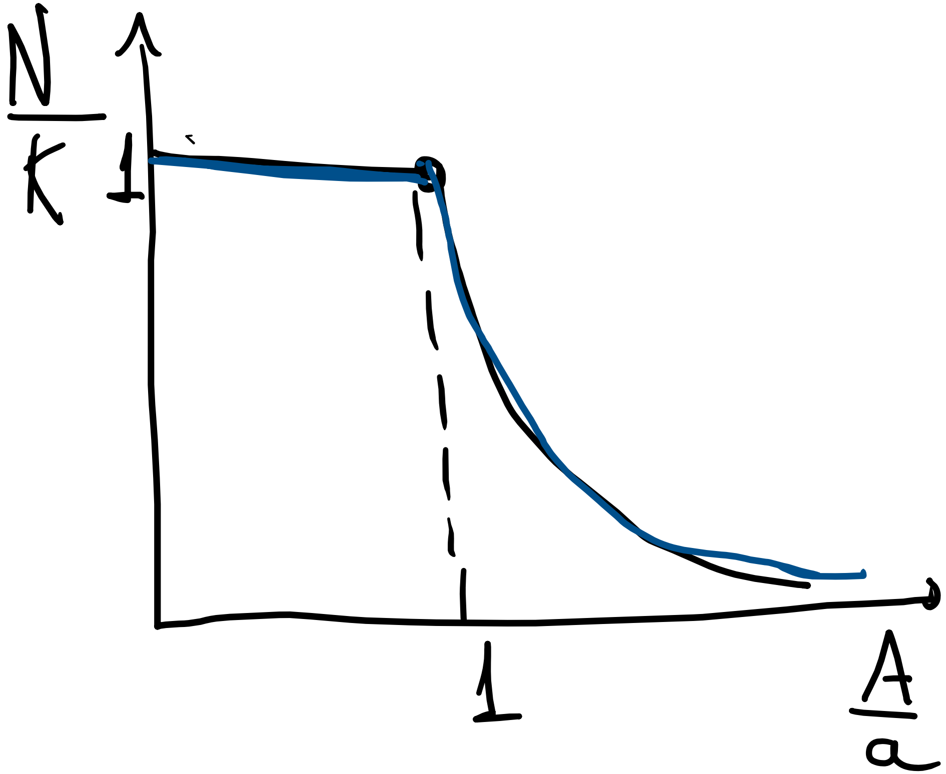




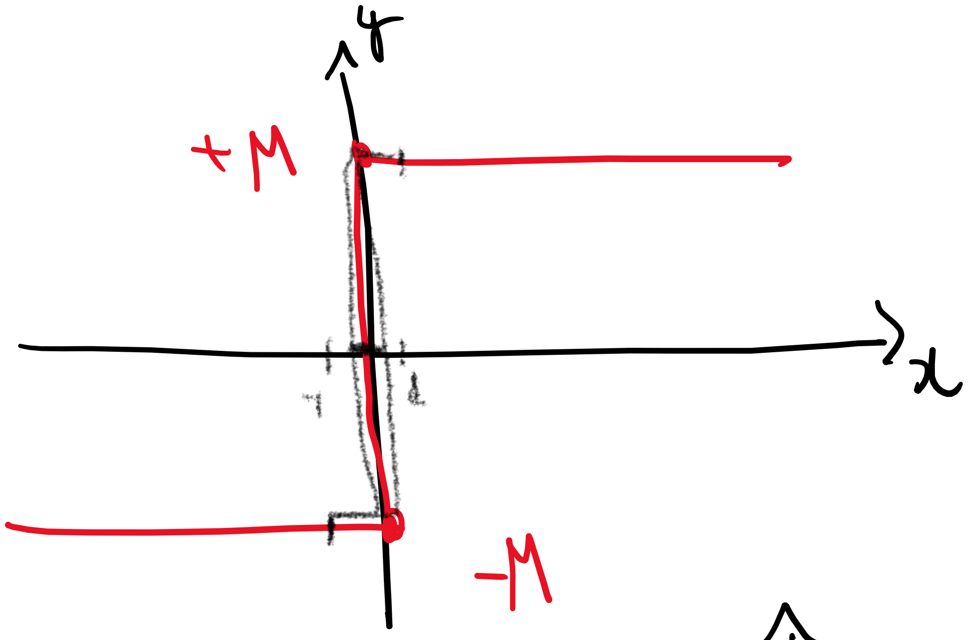
$$x(t) = A \operatorname{sen} \omega t$$

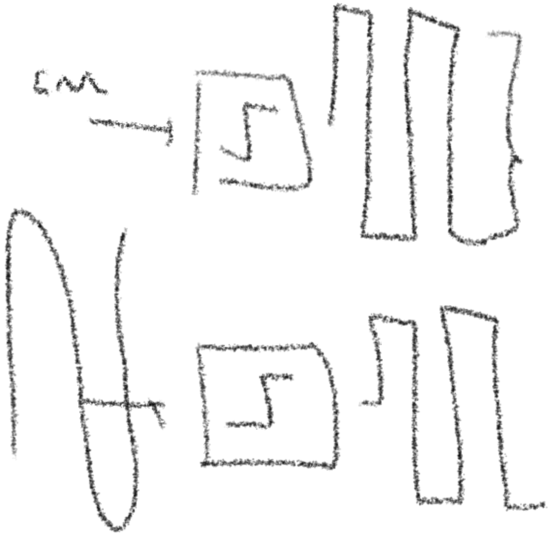
$$w = \begin{cases} \begin{cases} KA \operatorname{sen} \omega t & \text{p/ } 0 \leq \omega t \leq \gamma \\ K.a & \text{p/ } \gamma \leq \omega t \leq \pi/2 \end{cases} & \text{se } A > a \\ Ka \operatorname{sen} \omega t & \text{se } A < a \end{cases}$$

$$N(A,\omega) = \frac{2K}{\pi}\left(a \operatorname{sen} \frac{a}{A} + \frac{a}{A}\sqrt{1-\frac{a^2}{A^2}}\right)$$

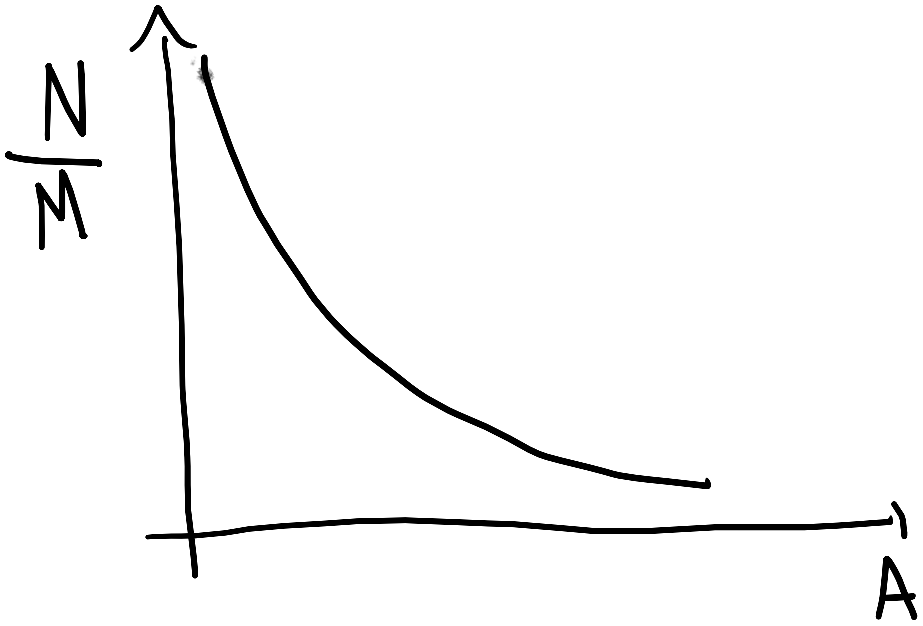


ON-OFF

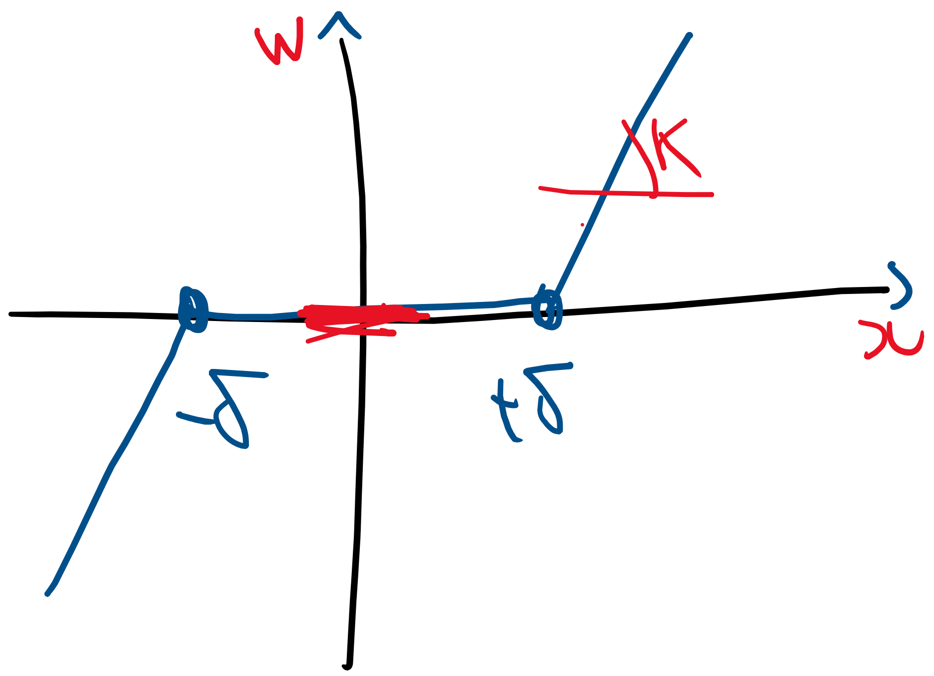




$$N(A) = \frac{4}{\pi} \cdot \frac{M}{A}$$



DEAD ZONE



$$N(A) = \begin{cases} 0 & \text{p/ } \dfrac{A}{\delta} < 1 \\ \dfrac{2K}{\pi}\left(\dfrac{\pi}{2} - \operatorname{asin}\dfrac{\delta}{A} - \dfrac{\delta}{A}\sqrt{1-\dfrac{\delta^2}{A^2}}\right) & \end{cases}$$