Phenomenological Evidence for the Phonon Hall Effect

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In the electrical Hall effect, a magnetic field, applied perpendicular to an electrical current, induces through the Lorentz force a voltage perpendicular to the field and the current. It is generally assumed that an analogous effect cannot exist in the phonon thermal conductivity, as there is no charge transport associated with phonon propagation. In this Letter, we argue that such a magnetotransverse thermal effect should exist and experimentally demonstrate this "phonon Hall effect" in $Tb_3Ga_5O_{12}$.

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It is well established that a magnetic field can affect the thermal conductivity: Less than a decade after the first observation of the electrical Hall effect, Leduc reported the thermal conductivity of metals to behave similarly [1–4]. This is due to the electronic contribution to the thermal conductivity and is thus another direct consequence of the Lorentz force acting on free electrons. With the Beenakker effect, another magnetotransverse thermal conductivity effect was reported [5]. This effect is due to an anisotropic scattering cross section of the diffusing gas molecules. Later, the scattering of phonons from spins was observed in the longitudinal thermal conductivity of dielectric crystals [6]. The Righi-Leduc effect, the spin phonon scattering, and the Beenakker effect can be used as tools for the investigation of the quasiparticle number, magnetic excitations [7], and molecular gyromagnetic ratios [8]. But, so far, no magnetotransverse effect was proposed nor observed for the phonon contribution to the thermal conductivity. Here we argue that the coupling of certain phonon modes to the magnetic field will lead to anisotropic phonon scattering and to a magnetotransverse phonon thermal conductivity, i.e., a phonon Hall effect.

Diffusive phonon transport is described by Fourier's law in which the heat current j_i is linearly related to the driving temperature gradient $\partial T/\partial x_j$ through the thermal conductivity tensor k_{ij}: $j_i = -k_{ij}\partial T/\partial x_j$. The temperature distribution $T(\mathbf{x})$ of the system is given by the heat equation: $kij\partial/\partial x_i\partial/\partial x_j T = 0$. This equation is analogous to the Poisson equation, describing the electrical potential distribution, with the thermal conductivity replacing the electrical conductivity. In dielectrics, the diagonal part of k_{ij} can be successfully modeled in terms of a phonon gas [9], assuming isotropic scattering [10]. A magnetic field can be included in this model [11]. There exists, however, no microscopic theory for the off-diagonal part of k_{ij} for the phonon thermal conductivity. Onsager pointed out that one can conclude on the existence and symmetry of an off-diagonal contribution in any kind of diffusive transport, making use of the invariance under time reversal of the microscopic equations of motion [12]. He showed that k_{ij} is always symmetric in zero field, but may contain an antisymmetric off-diagonal contribution if an external field B is applied: $k_{ij}(B) = k_{ji}(-B)$. The diagonal part is therefore an even function of B, whereas the off-diagonal part must be an odd function of B. An off-diagonal contribution to the thermal conductivity means a heat current perpendicular to the original temperature gradient. In a finite sample, however, a transverse current cannot exist but must be balanced by a transverse temperature difference $\Delta T_\perp$. Figure 1(b) shows numerical solutions of the heat equation, taking into account the existence of an off-diagonal contribution to the thermal conductivity. The result is an obvious analogy to the electrical Hall effect. Accordingly, we can define the thermal Hall angle $\alpha(B) = k_{xy}(B)/k_{xx}(B)$ which is a measure of the inclination of the isotherms with respect to the zero field gradient along the sample.

Even though no microscopic theory on magnetotransverse phonon transport exists, one can gain insight into a possible realization of this effect by exploring the close analogy between transverse acoustic phonons and photons.

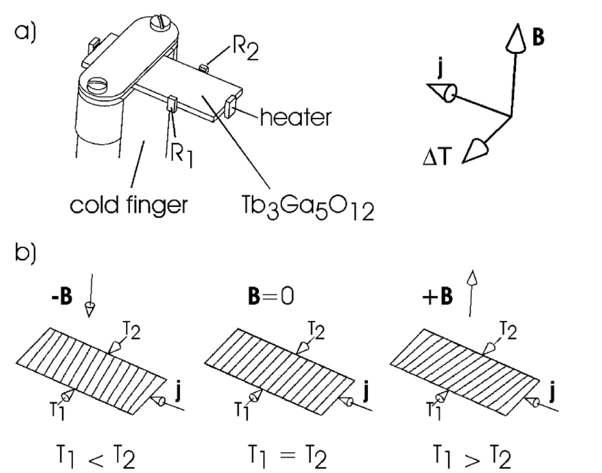

FIG. 1. (a) Setup and geometry of the magnetotransverse phonon transport. (b) Phenomenology: Isotherms without and with a magnetic field.

(As longitudinal acoustic phonons are effectively Rayleigh scattered into transverse modes due to the ratio of their respective sound velocities [13], transverse phonons dominate the phonon thermal conductivity.) For the photon case, a magnetotransverse diffusion effect was demonstrated [14] and was explained as follows: the magnetic field splits the photon dispersion relation into circularly polarized branches and causes a rotation of the Rayleigh pattern in the differential scattering cross section [15]. In multiple scattering this leads to a transverse photon flux. For this optical case a simple scaling law has been predicted theoretically [16] and verified experimentally [14]: $D_\perp/D_{||} = V_{\text{eff}} B l^*$ where $D_\perp$ and $D_{||}$ are the transverse and the longitudinal diffusivities, V_{eff} is the effective Verdet constant, and l^* is the scattering mean free path. The similarities for the transversal phonon case are clear: It is known that the magnetic field also splits the diamagnetic [17] and paramagnetic [18] phonon dispersion relations leading to the acoustic Faraday effect. The acoustic Faraday effect was demonstrated experimentally for metals [19] and recently in superfluid ^{3}He-B [20]. In dielectrics, it is expected to be of comparable order of magnitude as its optical counterpart. Furthermore, phonons scattered at impurities show the characteristic Rayleigh pattern [13]. We can therefore expect that a similar magnetic field effect on the phonon scattering cross section will occur and by consequence that a magnetotransverse effect will exist in the phonon thermal conductivity of dielectrics, obeying a similar scaling law as for the optical case: $k_{xy}(B)/k_{xx}(B) = V_{\text{eff}} B l^*$, where V_{eff} is the effective acoustic Verdet constant. (This mechanism operates, of course, also in metals, but there its contribution is overshadowed by the Righi-Leduc effect.) This Letter describes the experimental observation of this new effect, which we shall call the phonon Hall effect.

We have used crystals of paramagnetic terbium gallium garnet, $Tb_3Ga_5O_{12}$ (TGG), to search for the magnetotransverse phonon thermal conductivity. These crystals are dielectric, are cubic, and contain ions that carry both a high charge and a large magnetic moment, both factors that have led us to expect a strong coupling to the magnetic field. We have verified that the zero field magnetic susceptibility shows no anomaly between 3.5 and 120 K and that therefore the material is paramagnetic in this temperature range. The rectangular bar-shaped samples (size $15.7 \times 5.7 \times 0.67$ mm^3) are mounted on a cold finger protruding in the vacuum can, immersed in a ^{4}He-bath cryostat [Fig. 1(a)]. The cold finger (a LiF crystal) and the vacuum chamber (Araldite plastic) are nonmetallic in order to avoid artifacts due to the occurrence of the Righi-Leduc effect in these parts. A temperature gradient can be established by means of an electrical heater. A set of thermometers R_1 and R_2 is placed on opposite sides of the sample in a direction perpendicular to the field and the original gradient. We have used paired ruthenium oxide thick film chip resistors ($1.5 \times 0.9 \times 0.5$ mm^3), which show a thermally activated hopping behavior. The thermometers themselves are also subject to the magnetic field and behave according to Onsager's relations. We can separate the field-even longitudinal magnetoresistance from the field-odd magnetotransverse transport effect we are looking for by using the following differential technique (Fig. 2): The resistance difference $\Delta R(B, \Delta T_\perp) = R_1(B, T_1) - R_2(B, T_2)$ between the thermometers is evaluated (by means of a resistance bridge) at positive field $[\Delta R(+B, \Delta T_+)]$ and negative field $[\Delta R(-B, \Delta T_-)]$, subtracted from each other and then converted to a temperature difference using the sensitivity s of the resistances: $\Delta T_\perp = 1/2[\Delta R(+B, \Delta T_+) - \Delta R(-B, \Delta T_-)]/s$. This eliminates the field-even contribution of the thermometers' electrical magnetoresistance and contributions of the thermal magnetoresistance of the sample that appear due to misalignment of the thermometers. This procedure is repeated through several cycles for each measurement and the average is taken. We have also measured the longitudinal thermal conductivity of TGG as a function of temperature. At the temperature of our experiments (5.45 K) the thermal conductivity was found to be 4.5×10^{-3} W K cm^{-1}, which is about 2 orders of magnitude below the expected value for nonscattering garnet samples. It is inferred that the observed low thermal conductivity is due to Rayleigh scattering by Tb ions at Ga sites [21] and due to zero field split energy levels. In our measurements of the longitudinal thermal conductivity as a function of magnetic field, we have observed changes of 30% in a field of 4 T. From these measurements we conclude that (i) the phonon transport in this temperature range is reduced far below the Debye value by scattering and is therefore diffusive, and (ii) the thermal magnetoresistance witnesses the influence of the magnetic field on the phonon scattering. Both conditions that seem necessary for the observation of transverse phonon transport are therefore fulfilled in this material. We have measured the magnetotransverse temperature difference using the protocol described above, measuring at least 3 full cycles for each data point (Fig. 2). By this method, we achieve a sensitivity of 10 μK for the magnetotransverse temperature difference, in the presence of a longitudinal temperature difference of up to 1 K.

In some cases after changing parameters, the first data point was different by more than the experimental error from the succeeding data points at the same field strength and direction in further cycles. These points remain unexplained and have been deliberately discarded. All further data points varied only within the errors due to electrical noise and base temperature fluctuations.

Figure 3 shows the magnetotransverse temperature difference for heat currents parallel and perpendicular to the magnetic field. The vanishing of the effect for the heat current parallel to the magnetic field is a very characteristic property of magnetotransverse transport effects and allows

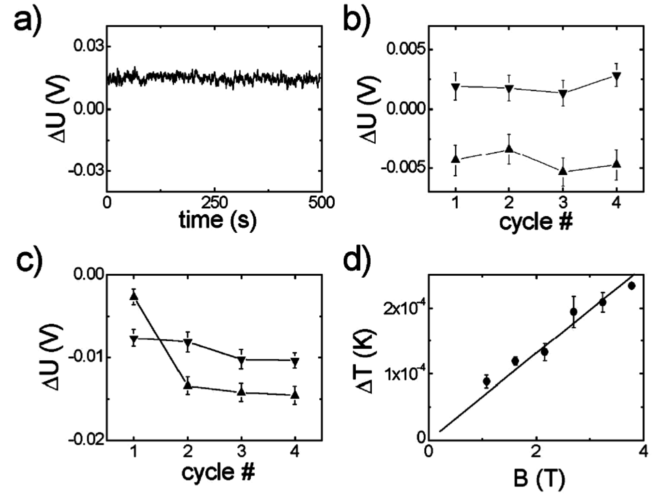

FIG. 2. Measurement protocol. (a) Output of the differential bridge at positive field (typical). (b) Mean values of bridge output for positive and reversed field directions. Up triangles: positive field direction. Down triangles: reversed field direction. The error bars result from averaging measurements as in (a) at positive and negative field values. (c) As in (b), but for a case in which the first value is distinct from the long term average. (d) Magnetotransverse temperature difference, obtained by subtracting the upper and lower curves obtained in a measurement cycle like (b) and converting the voltage difference to a temperature difference, as a function of magnetic field.

one to rule out artifacts of the electrical heater, the electronics, and the thermometers.

In Fig. 4 we show the linear dependence of the transverse temperature difference on magnetic field for different heater powers, i.e., for different longitudinal gradients. The inset shows the dependence of the slopes of these curves on the heater power. A clear linear relation is obtained, which is a characteristic signature of any diffusive effect. The slope in the inset corresponds to a thermal Hall angle α of about 1×10^{-4} rad T^{-1} at 5.45 K, comparable to the values observed for the case of the photon Hall effect [14]. According to the analogy with the optical case, α should equal $V_{\mathrm{eff}} l^*$. Assuming a phonon mean free path $l^* = 1\ \mu$m, this results in an effective acoustic Verdet constant of $V_{\mathrm{eff}} = 100\ \mathrm{rad\,m^{-1}\,T^{-1}}$, which is of the same order of magnitude as observed for the optical case.

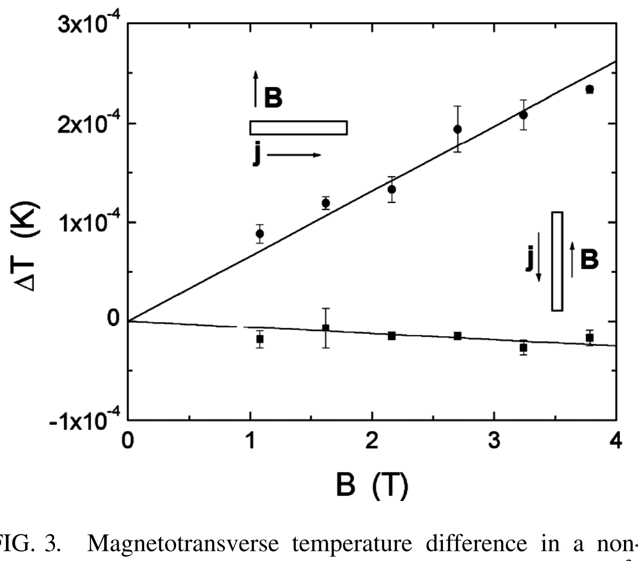

FIG. 3. Magnetotransverse temperature difference in a nonoriented sample of $Tb_3Ga_5O_{12}$ (size $15.7 \times 5.7 \times 0.67\ \mathrm{mm}^3$) for heat currents perpendicular (circles) and parallel to the field (squares). The mean sample temperature was adjusted to 5.45 K at $B = 0$ for all points. The heater power amounts to 0.14 mW, generating a longitudinal gradient of about 1 mK/cm. The straight lines are linear fits through the origin.

In conclusion, we have argued that, contrary to general belief, a magnetotransverse effect should occur in the phonon thermal conductivity of dielectrics. We have set up an experiment that allows us to detect the corresponding transverse temperature differences, odd in the magnetic field. In measurements of the thermal conductivity of TGG we have observed a transverse temperature difference that is odd and linear in the field between 0 and 4 T and scales linearly with the longitudinal temperature gradient. Furthermore, this temperature difference vanishes if the heat current is parallel to the applied magnetic field. These observations constitute the complete phenomenological evidence for the existence of this new effect, which we propose to call the phonon Hall effect, and underline the universality of magnetotransverse diffusion phenomena.

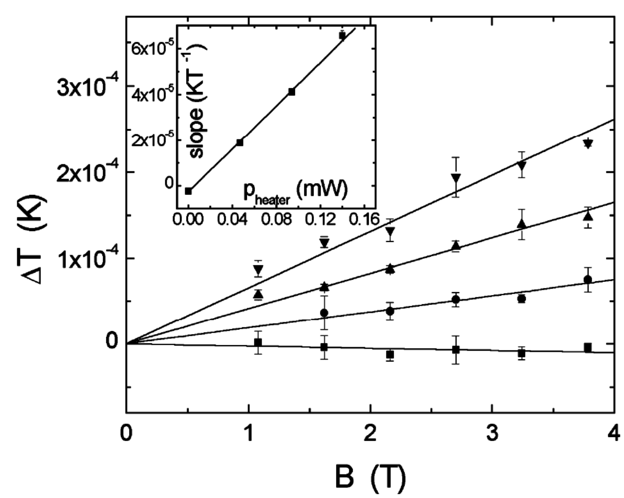

FIG. 4. The magnetotransverse temperature difference of $Tb_3Ga_5O_{12}$ as a function of magnetic field for different heater powers (squares: 0 mW; circles: 0.05 mW; up triangles, 0.09 mW; down triangles: 0.14 mW). The mean sample temperature was adjusted to 5.45 K at $B = 0$ for all points. The straight lines are linear fits through zero. The upper curve is identical with the upper curve in Fig. 2. Inset: The magnetotransverse temperature difference normalized to the magnetic field (slopes of the lines in the main graph) as a function of heater power. The straight line is a linear fit. Errors are slightly larger than the symbols

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