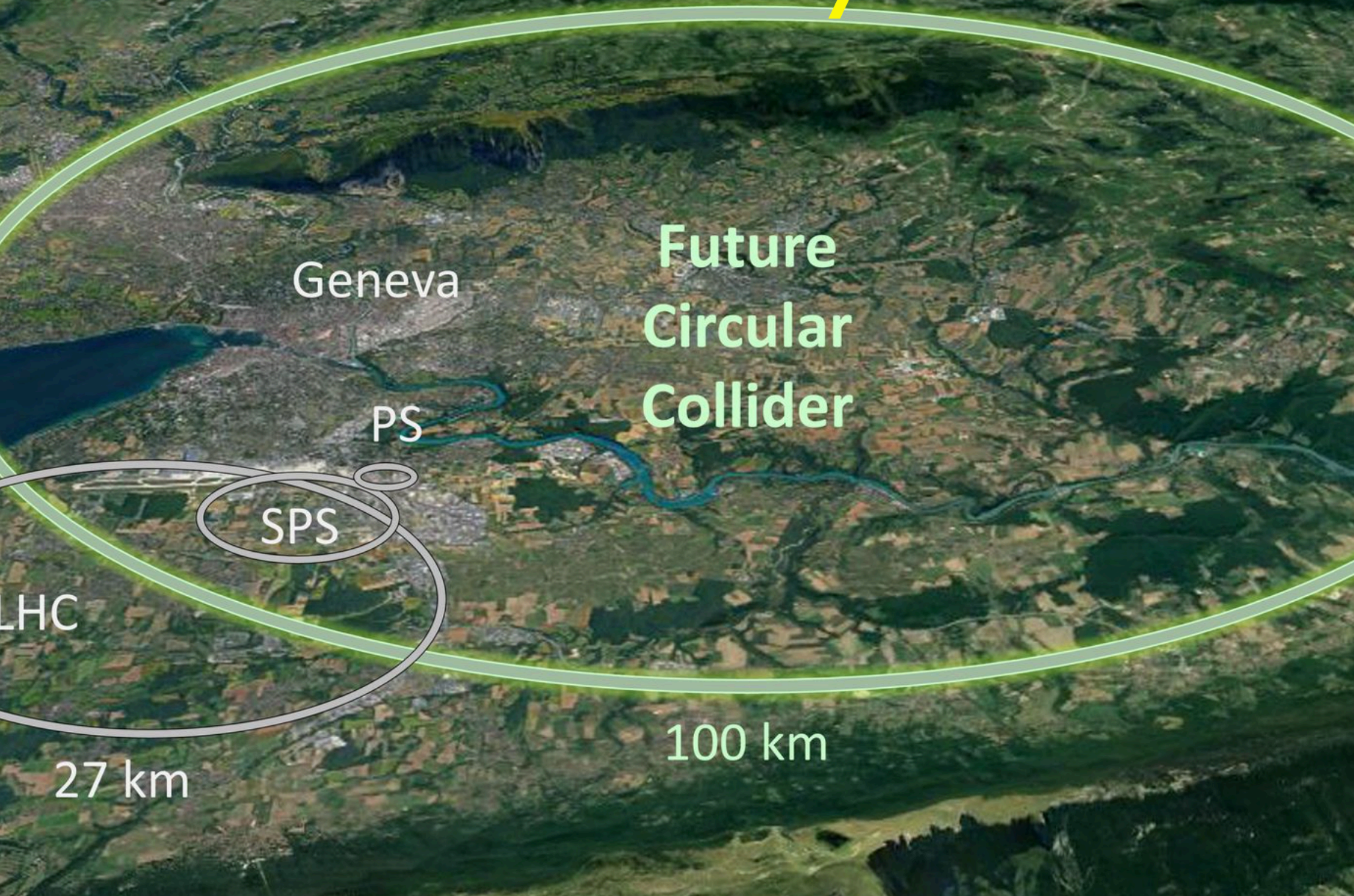
Collider Physics

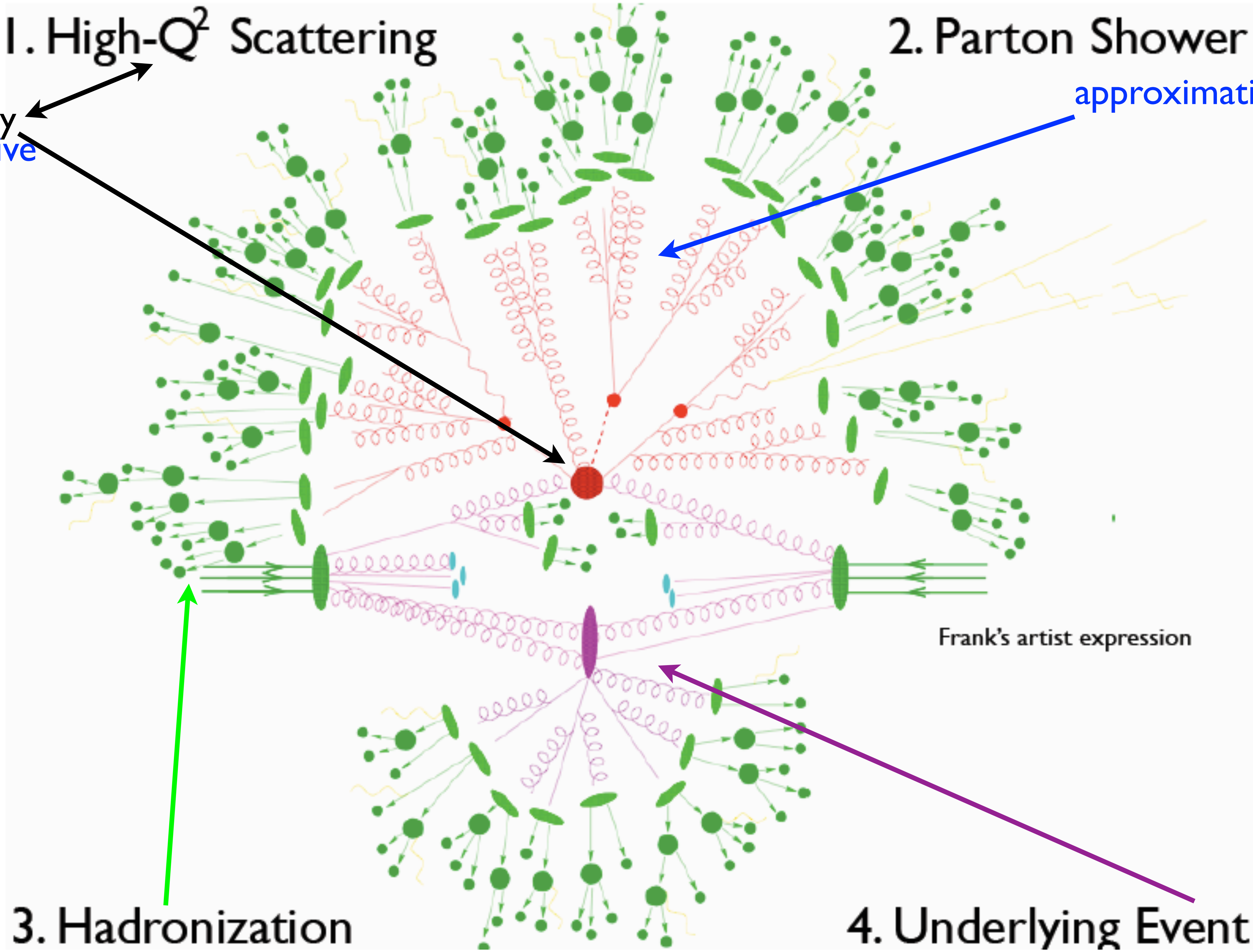


Motivation:

“Elements” of a collision

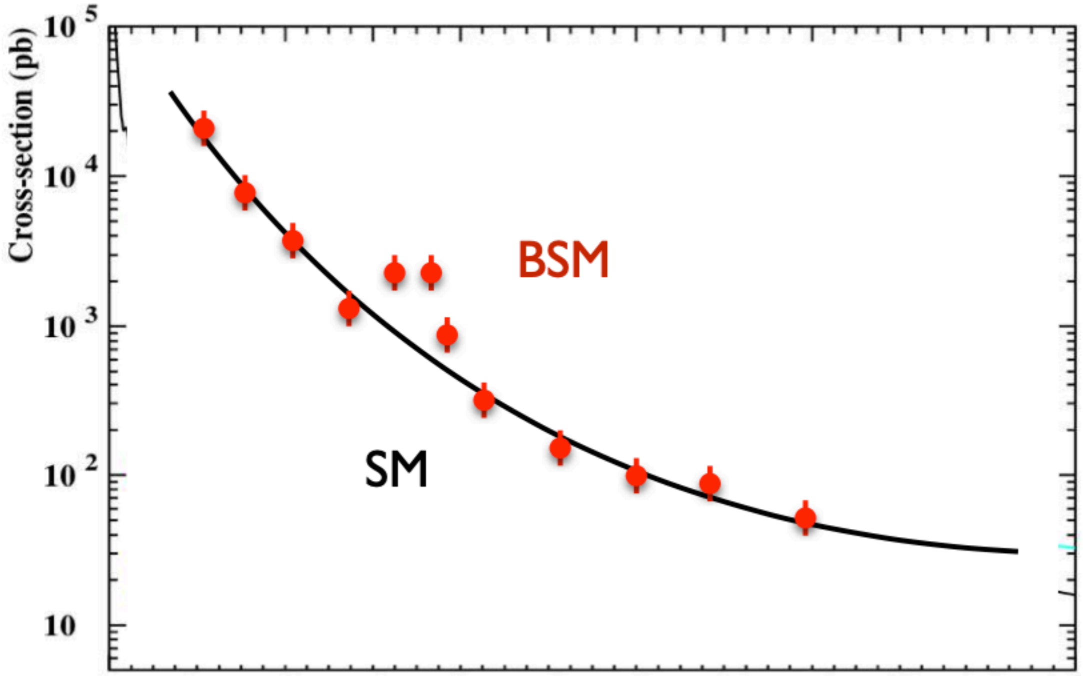
field theory
very inclusive

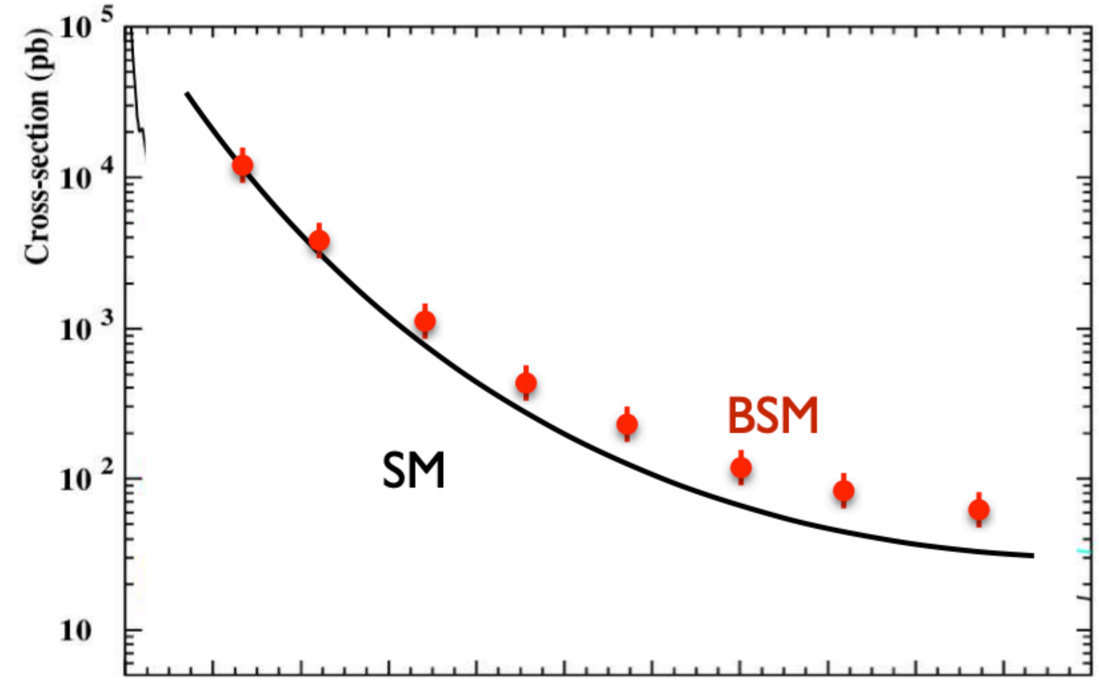
approximation



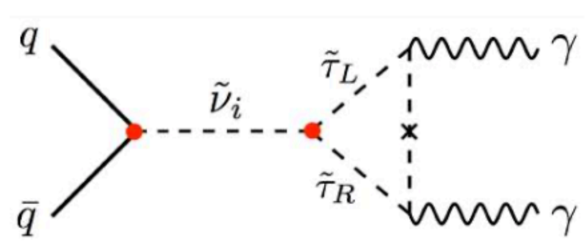
model

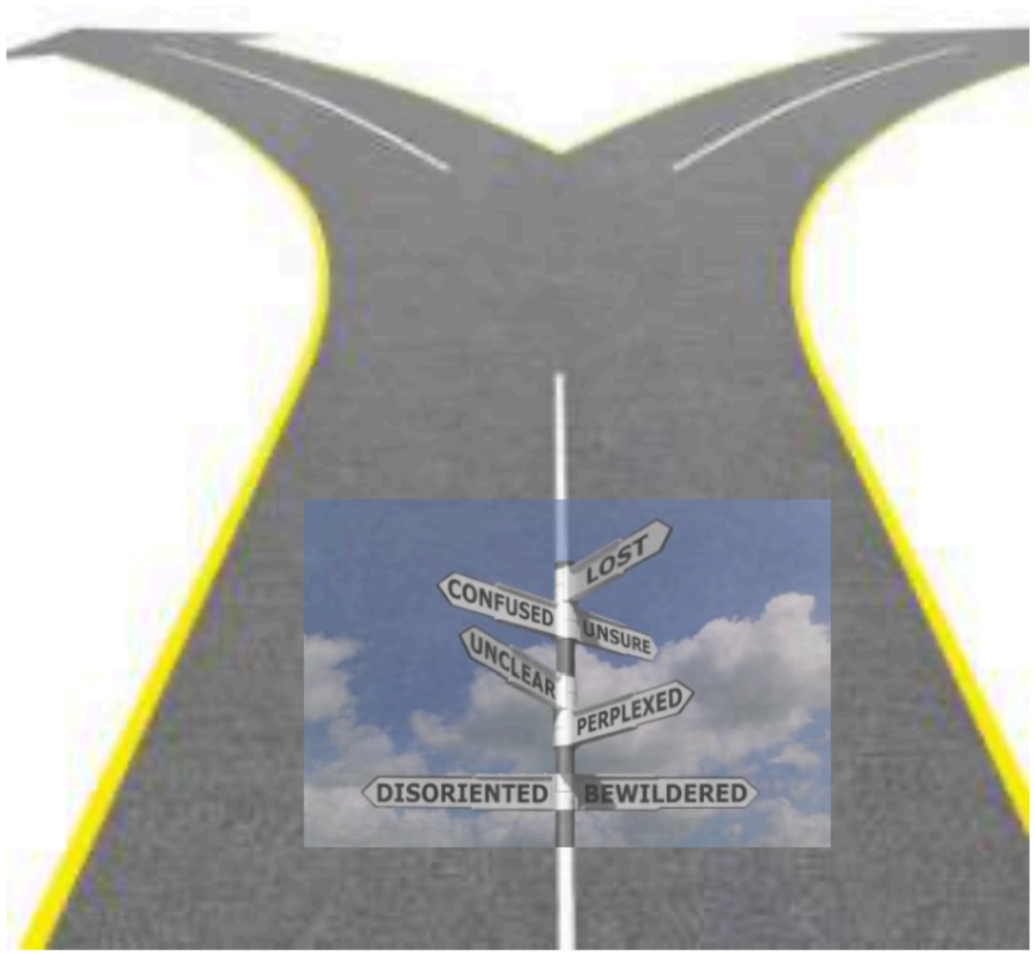
model



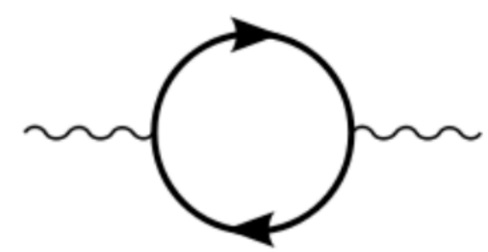


Search for
new *states*
Resonances
"Descriptive TH"



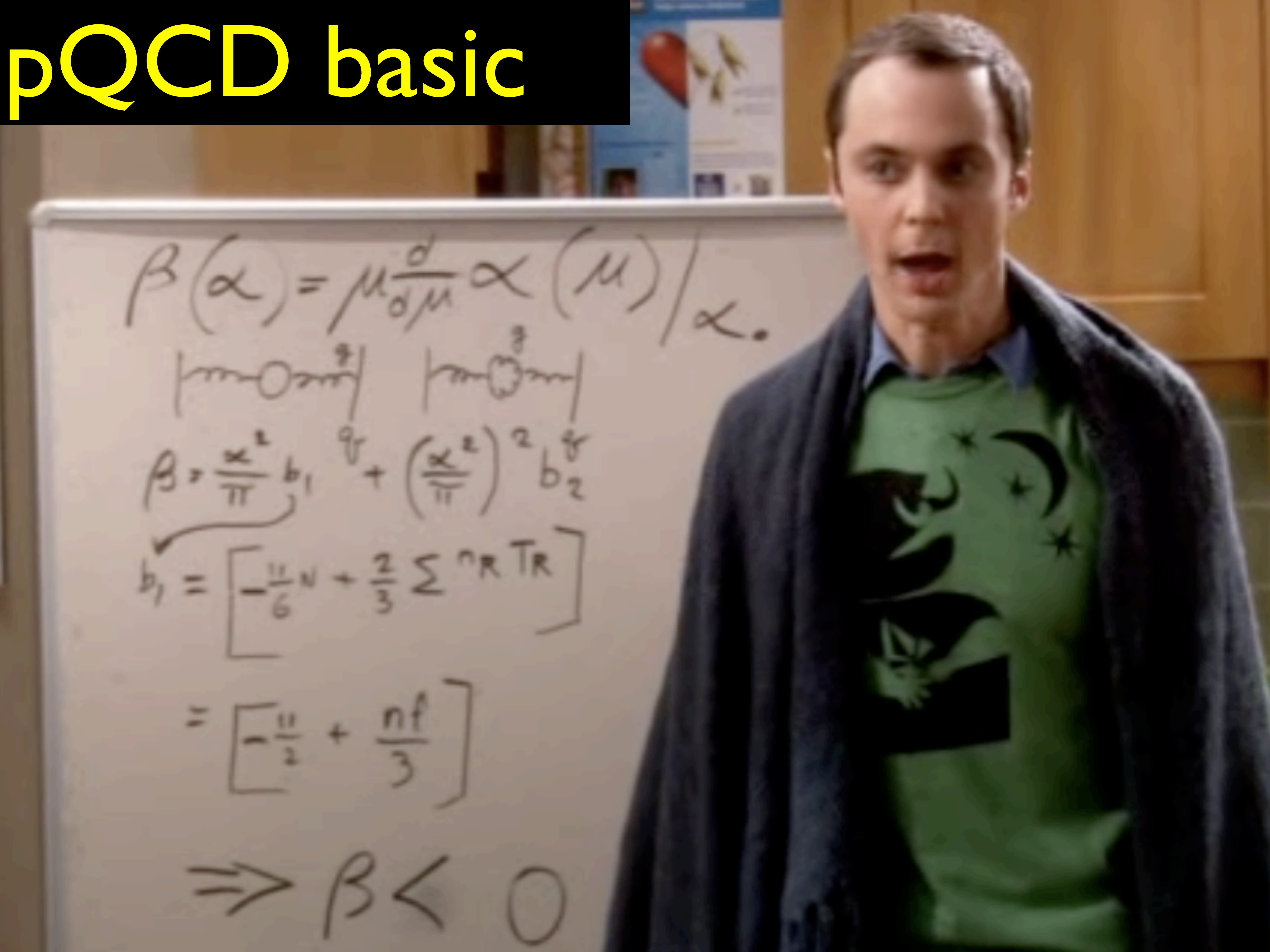


Search for new
interactions
Deviations from TH
"Precision TH"

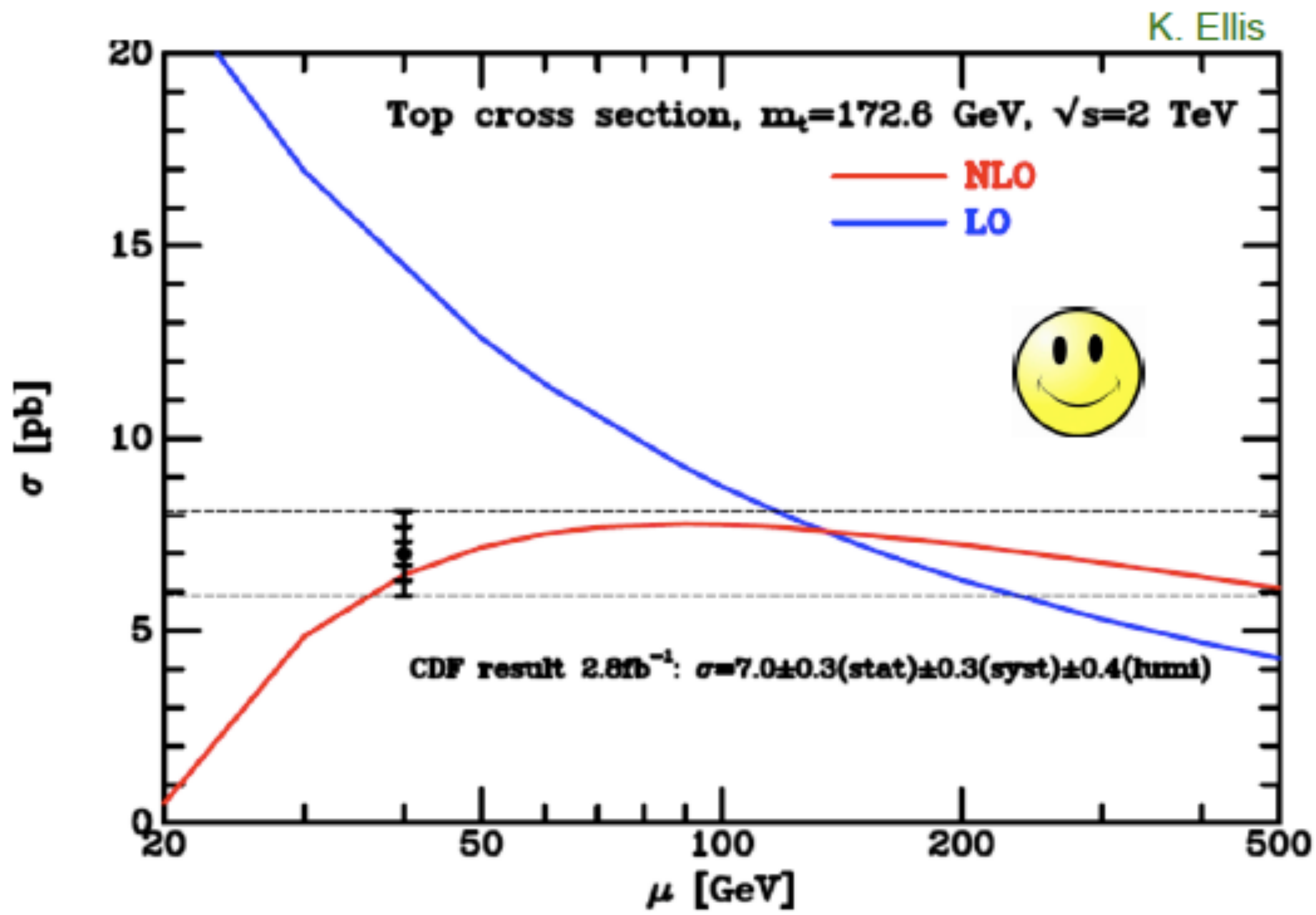


[From Daniel de Florian @ ICTP-SAIFR]

pQCD basic



- In order to have precise predictions working at LO might not be enough



[Stirling-Pascos 2011]

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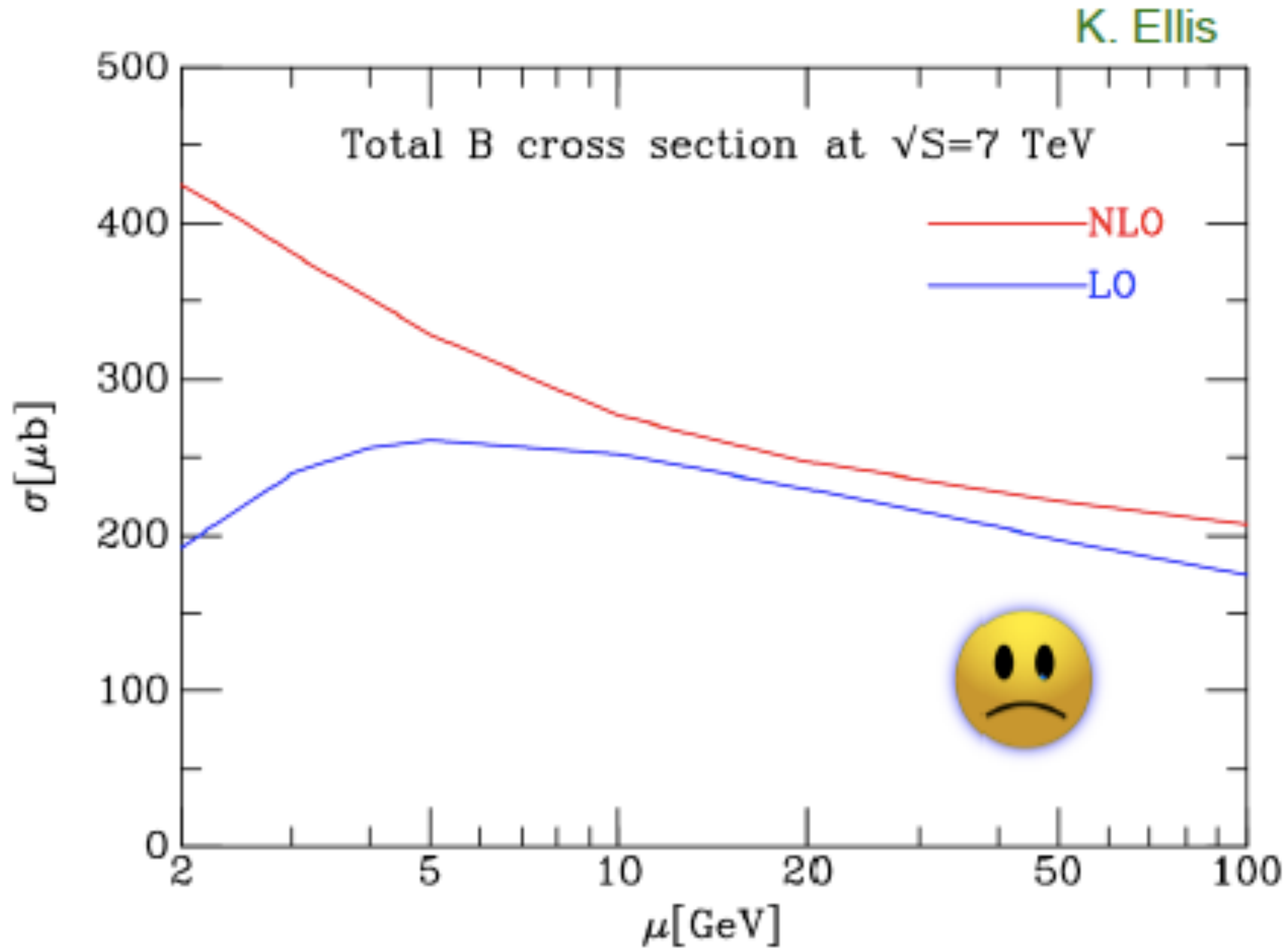
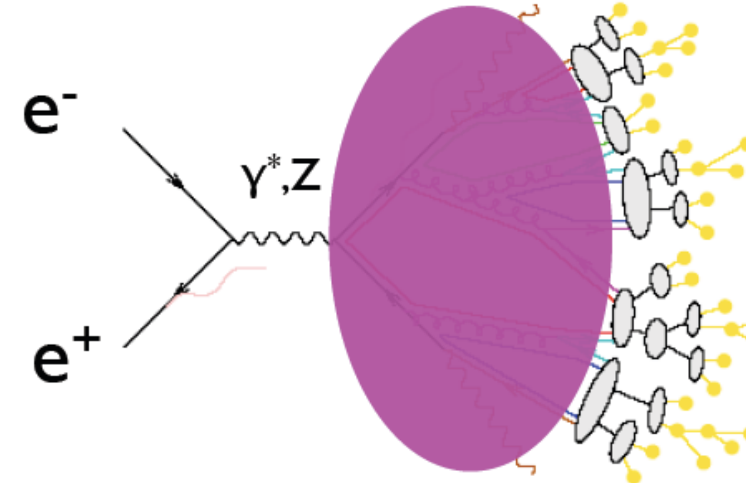


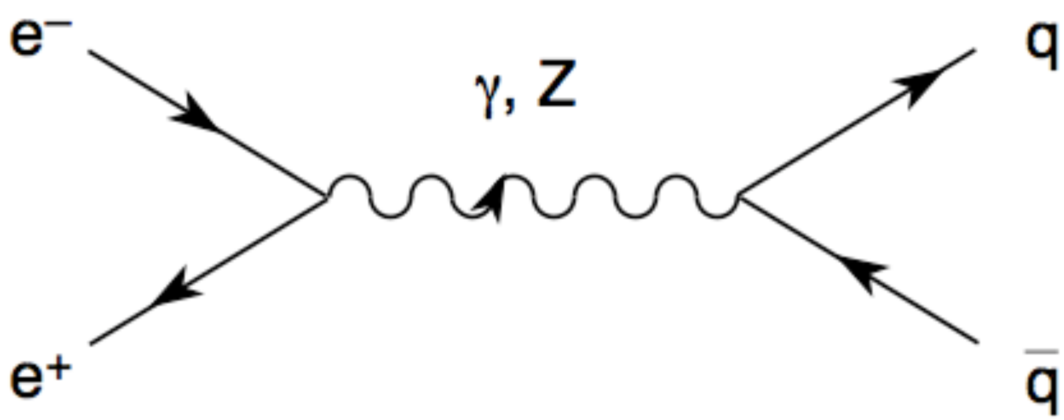
[Stirling-Pascos 2011]

NLO QCD in e^+e^- colliders



Total Cross Section

➡ Can we use pQCD despite confinement? "YES"



- The γ/Z virtuality is $Q = \sqrt{s}$
- Production occurs at a distance $\simeq \frac{1}{Q}$
- Q is large $\Longrightarrow$ pQCD applicable

➡ Hadronization changes quarks and gluons to hadrons.

➡ Hadronization takes place at a scale $\frac{1}{\Lambda}$.

➡ The change in the outgoing state occurs too late to modify the probability of the event to happen!

➡ Details of the final state certainly are changed.

Lowest Order Result (α_s^0)

- For simplicity, we neglect the Z contribution (i.e. $\sqrt{s} << M_Z$)

$$\frac{d\sigma_0}{d\cos\theta} = \frac{\pi\alpha^2 Q_f^2}{2s} N_c \left(1+\cos^2\theta\right) \quad \Longrightarrow \quad \sigma_0 = \frac{4\pi\alpha^2}{3s} N_c\, Q_f^2$$

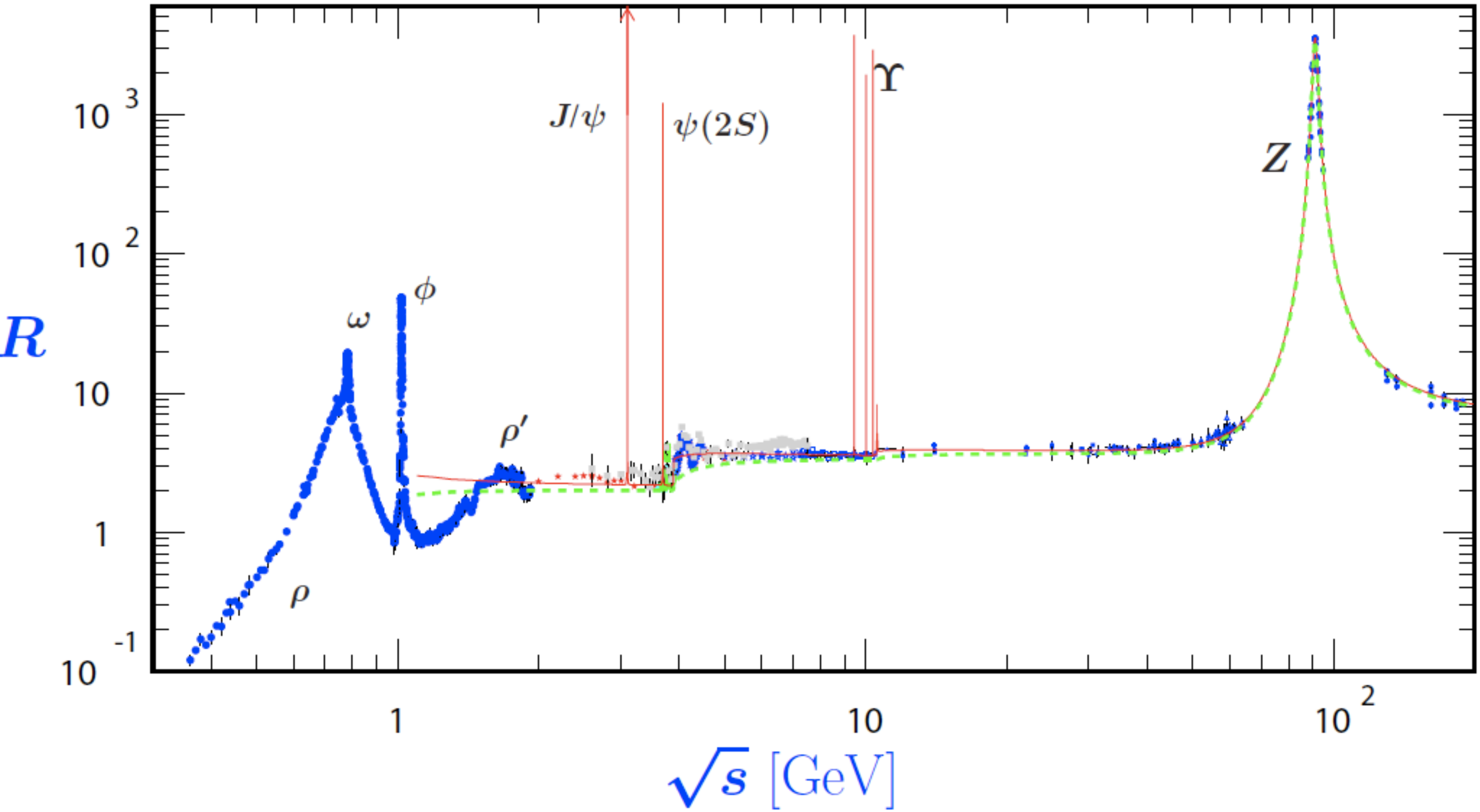
leading to

$$R_0 \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum_q Q_q^2$$

- At the Z pole (i.e. neglecting γ), we have

$$R_0 = N_c \frac{\sum_q \left(A_q^2 + V_q^2\right)}{A_\mu^2 + V_\mu^2}$$

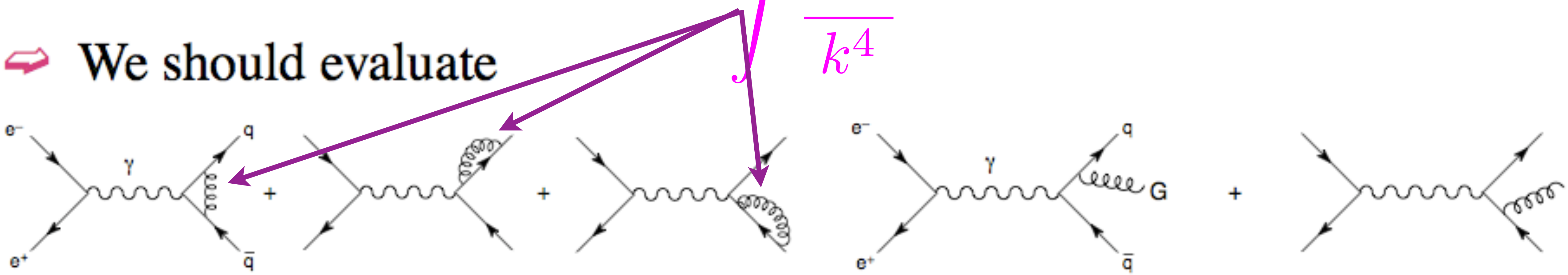
$$R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$$



Next Order Correction (α_s^1)

$$\int \frac{d^4k}{k^4}$$

- We should evaluate



- Writing $\mathcal{M}^P = \mathcal{M}_0^P + \mathcal{M}_1^P$, the α_s contribution has the form

$$\int d\Phi_2 \left[2\,\mathrm{Re}\left(\mathcal{M}_0^{2\to2}\right)^\dagger \mathcal{M}_1^{2\to2}\right] + \int d\Phi_3\, |\mathcal{M}_0^{2\to3}|^2$$

- After adding all contributions the UV divergences cancel out (Ward identity). The same happens for the IR ones!

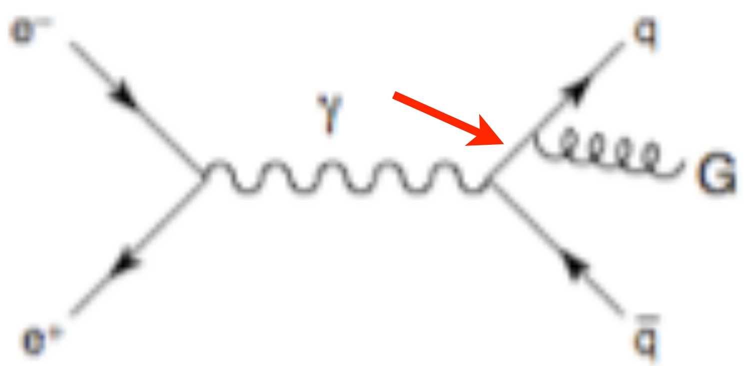
$$R = R_0\left(1 + \frac{\alpha_s(\mu)}{\pi}\right) \longrightarrow R_0\left(1 + \frac{\alpha_s(\sqrt{s})}{\pi}\right)$$

- At the Z pole, the NLO corrections $\simeq 4\%$

There is NO renormalization for IR divergences. They indicate sensitivity to long range physics.

The IR singularities are not physical: they indicate the breakdown of the perturbative approach.

Origin of the IR singularities in $e^+e^- \to q\bar{q}g$

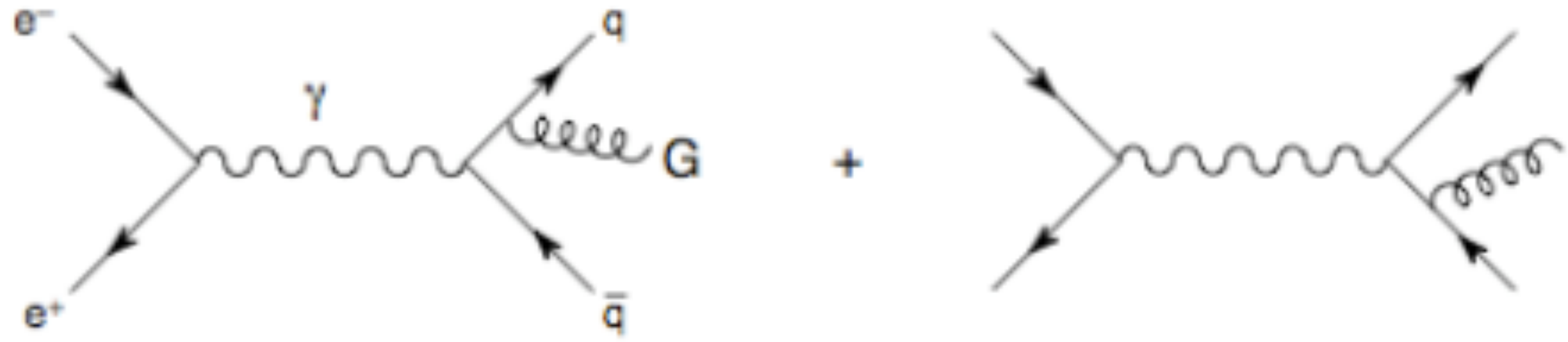


$$\frac{1}{(p_q+p_g)^2} = \frac{1}{2p_q\cdot p_g} = \frac{1}{2E_qE_g(1-\cos\theta_{qg})}$$

The phase space integration helps but doesn't cure the IR divergence

$e^+e^- \to q\bar{q}g$ contribution

➩ We have to evaluate



➩ It is useful to define

$$x_1 = \frac{2E_q}{\sqrt{s}} = 1 - \frac{x_2 E_g}{\sqrt{s}}(1 - \cos\theta_{qg})$$

$$x_2 = \frac{2E_{\bar{q}}}{\sqrt{s}} = 1 - \frac{x_1 E_g}{\sqrt{s}}(1 - \cos\theta_{\bar{q}g})$$

➩ The three body phase space is (I'm sloppy with π's)

$$d\Phi_3 \cong d\alpha \, d\beta \, d\gamma \, dx_1 \, dx_2$$

➯ Doing the algebra and performing the angular integrals, we obtain

$$\sigma^{q\bar{q}g} = \sigma_0 \, N_c \sum_q Q_q^2 \int dx_1 dx_2 \, \frac{2\alpha_s}{3\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

with $0 \leq x_1 \; (x_2) \leq 1$ and $x_1 + x_2 \geq 1$.

➯ This integral is divergent for $x_{1(2)} \to 1$ This corresponds to

- soft gluon limit $E_g \to 0$;
- the gluon is collinear with the quark $\theta qg \to 0$;
- the gluon is collinear with the antiquark $\theta \bar{q} g \to 0$.

➯ The origin of the singularities is the massless quark (antiquark) propagator

$$\frac{1}{(p_q + p_g)^2} = \frac{1}{2p_q \cdot p_g} = \frac{1}{2E_q E_g (1 - \cos\theta_{qg})}$$

⇨ Unlike UV divergences, there is no renormalization for the IR ones. They indicate sensitivity to long range physics like masses, hadronization process, etc.

⇨ The singularities are not physical; they indicate the breakdown of the perturbative approach. Quarks and gluons are never on mass–shell–particles and we can not ignore the effects of confinement at a scale $\simeq 1$ GeV.

⇨ General form of the IR divergences. Writing the born term as

$$\mathcal{M} = \overline{u}(p_q)\epsilon^\mu\gamma_\mu v(p_{\bar{q}}) \quad \text{with} \quad \mathcal{N} = \epsilon^\mu\gamma_\mu v(p_{\bar{q}}) \quad \Longrightarrow \quad \mathcal{M} = \overline{u}(p_q)\mathcal{N}$$

⇨ The gluon emission from the quark line leads to

$$\mathcal{M}_1 = \overline{u}(p_q)(-i)\gamma_\alpha i\frac{\not{p}_q + \not{p}_g}{(p_q + p_g)^2}\mathcal{N}$$

➡ In the limit $p_g \to 0$

$$\mathcal{M}_1 = \overline{u}(p_q)\frac{\gamma_\alpha \not{p}_q}{(p_q+p_g)^2}\mathcal{N} = \overline{u}(p_q)\frac{2p_{q\alpha}}{2p_q\cdot p_g}\mathcal{N} = \frac{p_{q\alpha}}{p_q\cdot p_g}\mathcal{M}$$

➡ The total amplitude for gluon emission is this limit is

$$\mathcal{M}_{q\bar{q}g} = \left(\frac{p_{q\alpha}}{p_q\cdot p_g} - \frac{p_{\bar{q}\alpha}}{p_{\bar{q}}\cdot p_g}\right)\mathcal{M}$$

$$|\mathcal{M}|^2_{q\bar{q}g} = 2\frac{p_q\cdot p_{\bar{q}}}{(p_q\cdot p_g)(p_{\bar{q}}\cdot p_g)}|\mathcal{M}|^2.$$

➡ After including the $d\Phi_3$ we obtain (explain!)

$$\sigma^{q\bar{q}g} = \frac{2\alpha_s}{3\pi}\sigma_{q\bar{q}}\int d\cos\theta_{qg}\frac{dE_g}{E_g}\frac{4}{(1-\cos\theta_{qg})(1+\cos\theta_{qg})}.$$

the quark and antiquark are basically back to back in this limit.

➯ The perturbative expansion for the production of quarks and gluons is ill defined. However, it well defined for the "production of hadrons".

➯ We should introduce a regulator. For us, the number of dimensions $D = 4 - 2\epsilon$. Doing so

$$\sigma^{q\bar{q}g} = \sigma_0 \, N_c \sum_q Q_q^2 H(\epsilon) \int dx_1 dx_2 \, \frac{2\alpha_s}{3\pi} \frac{x_1^2 + x_2^2 - \epsilon(2 - x_1 - x_2)}{(1-x_1)^{1+\epsilon}(1-x_2)^{1+\epsilon}}$$

with

$$H(\epsilon) = \frac{3(1-\epsilon)^2}{(3-2\epsilon)\Gamma(2-2\epsilon)} = 1 + \mathcal{O}(\epsilon)$$

resulting in

$$\sigma^{q\bar{q}g} = \sigma_0 \, N_c \sum_q Q_q^2 H(\epsilon) \, \frac{2\alpha_s}{3\pi} \left[+ \frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} + \mathcal{O}(\epsilon) \right]$$

Virtual corrections to $e^+e^- \to q\bar{q}$

➡ Using dimensional regularization to evaluate the loop corrections we obtain

$$\sigma^{q\bar{q}g^\star} = \sigma_0 \, N_c \sum_q Q_q^2 H(\epsilon) \, \frac{2\alpha_s}{3\pi} \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \mathcal{O}(\epsilon) \right]$$

➡ The cross section for the production of gluons and quarks at order α_s is finite leading to

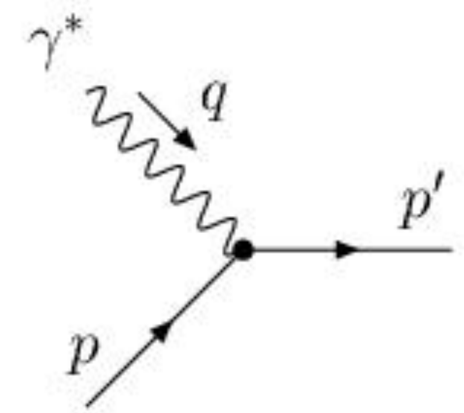
$$R = N_c \sum_q Q_q^2 \left\{ 1 + \frac{\alpha_s}{\pi} \left(1 + 0.448 \, \alpha_s - 1.30 \, \alpha_s^2 \right) + \mathcal{O}(\alpha_s^4) \right\}$$

NLO in DIS

QCD Corrections

➯ Does the parton model make sense when we include QCD corrections?

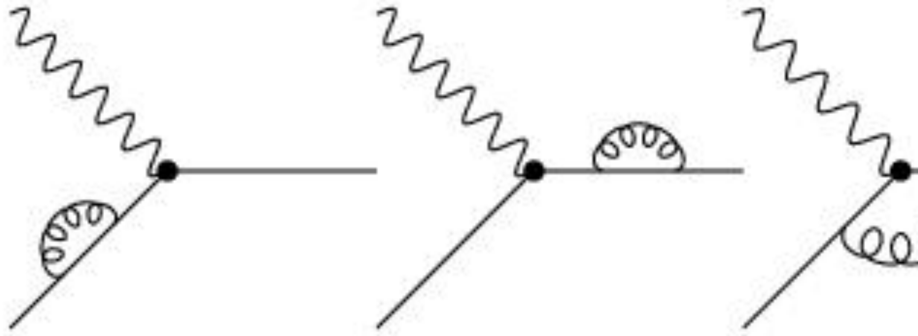
Virtual Gluon Contributions



☆ $M^{\mu}_{virt} = -ie_q\bar{u}(p')\gamma^{\mu}(1+\alpha_s V)u(p)$

☆ Using the Landau gauge $\Longrightarrow$ only the vertex contributes.

☆ There are only IR divergences.



☆ $V = \frac{C_F}{4\pi}\left(\frac{4\pi\mu^2}{Q^2}\right)^{\epsilon}\frac{\Gamma(1+\epsilon)\,\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(-\frac{2}{\epsilon^2}-\frac{3}{\epsilon}-8\right)$

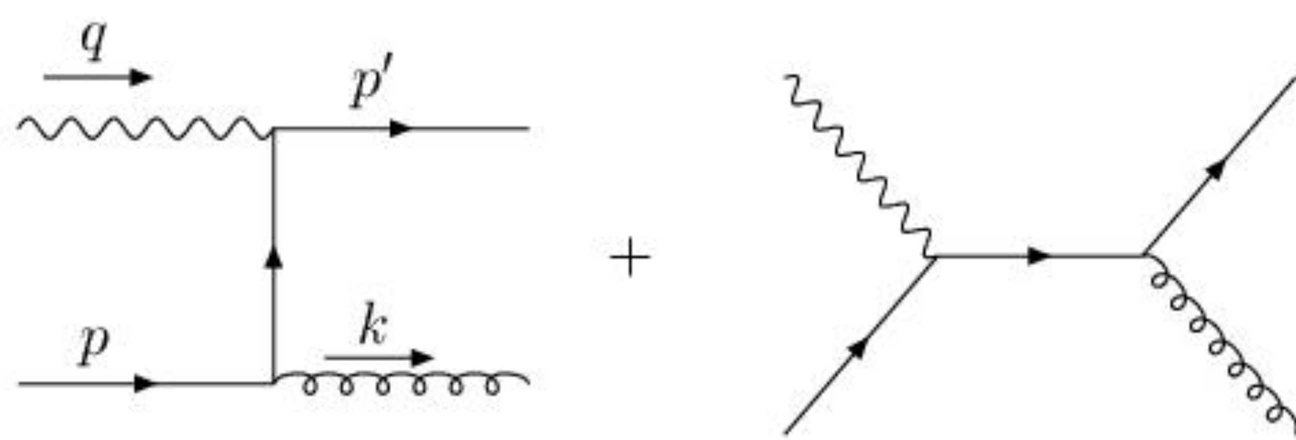
☆ The contribution to $p_\mu p_\nu W^{\mu\nu}_{virt} = 0$ (current conservation).

☆ Finally

$$\left.\frac{F_2}{x}\right|_{virt} = \sum_q Q_q^2 \int_x^1 \frac{dy}{y}\, q_0(y)\delta(1-y/x) \times$$
$$\left\{1 + \frac{\alpha_s}{2\pi} C_F \left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon} \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} \left(-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} - 8 - \frac{\pi^2}{3}\right)\right\} .$$

Real Gluon Emission

- $g_{\mu\nu}W^{\mu\nu}$ and $p_\mu p_\nu W^{\mu\nu}$ contribute.
- There are IR divergences for the 1st diagram $\simeq \frac{1}{2p\cdot k}$



$$\sigma^{(\gamma^* q \to qg)} = \frac{\alpha_s C_F}{2\pi} \int \sigma^{(\gamma^* q \to q)}(zp)\, \frac{1+z^2}{1-z}\, \frac{d\ell_T^2}{\ell_T^2}\, dz$$

After doing the algebra and some tricks, we get $(z = y/x)$

$$\left.\frac{F_2}{x}\right|_q = \frac{\alpha_s}{2\pi} C_F \left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon} \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} \otimes$$

$$\sum_q Q_q^2 \int_x^1 \frac{dy}{y} q_0(y) \left\{ \frac{2}{\varepsilon^2}\delta(1-y/x) + \frac{3}{2\varepsilon}\delta(1-y/x) - \frac{1}{\varepsilon}\frac{1+z^2}{(1-z)_+} \right.$$

$$\left. + \text{finite terms} \right\}$$

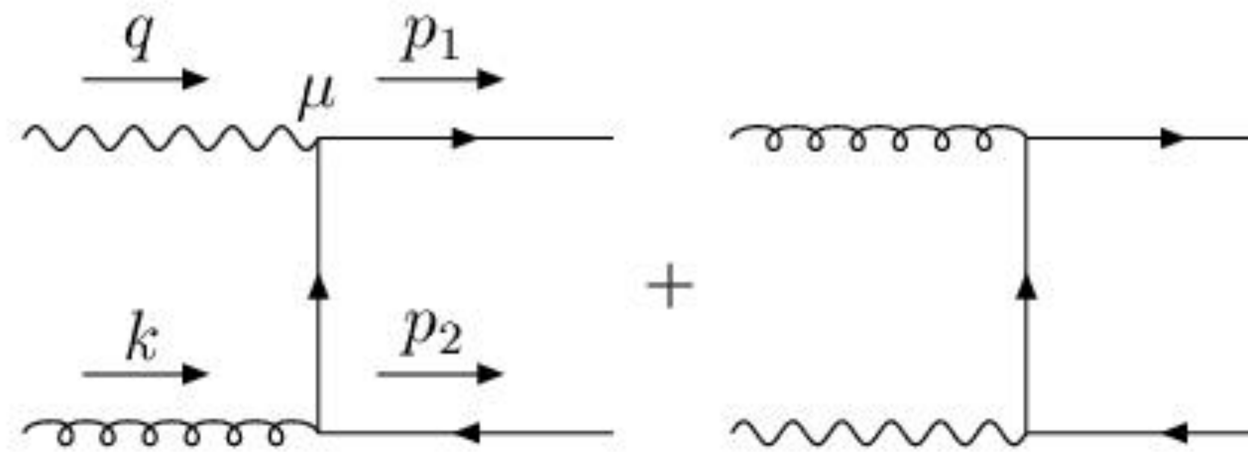
where

$$\int_0^1 dx\, g(x)[f(x)]_+ \equiv \int_0^1 dx\, (g(x) - g(1))\, f(x)$$

Notice: that the IR divergences do NOT cancel when we add these two contributions! Not the whole story yet!!!!

Initial State Gluons

- Inclusive reaction $\Longrightarrow$ we should consider $\gamma^* g \to q\bar{q}$.
- There are collinear divergences in W_T.
- W_L is finite.



- Defining $z = x/y$

$$\left.\frac{F_2}{x}\right|_g = \frac{\alpha_s}{2\pi}\frac{1}{1-\varepsilon}\sum_q Q_q^2 \int_x^1 dy\, g(y) z \times$$

$$\left\{[z^2+(1-z)^2]\left(-\frac{1}{\varepsilon}\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} + \ln\frac{Q^2}{4\pi\mu^2}\right) + 6z(1-z)\right\}$$

- Collinear divergences are not physical (m_q, m_g, etc)

$\Longrightarrow$ sensitivity to low scale physics.

➯ We can save the model $\Longrightarrow$ redefine pdf's! They will depend on Q^2.

➯ We define the quarks pdf's through (DIS scheme) (there are others)

$$F_2(x,Q^2) \equiv \sum_q e_q^2\, x\, [q(x,Q^2) + \bar{q}(x,Q^2)]$$

since

$$\frac{F_2}{x} = \left.\frac{F_2}{x}\right|_{virt} + \left.\frac{F_2}{x}\right|_q + \left.\frac{F_2}{x}\right|_g$$

we have (including $\frac{1}{2}$ of F_g to quarks) explain pieces! $z = \frac{x}{y}$

$$\begin{aligned} q(x,Q_F^2) &= q_0(x) \\ &+ \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\, q_0(y)\left[\left(-\frac{1}{\bar{\varepsilon}} + \ln\frac{Q_F^2}{\mu^2}\right)P_{qq}(z) + F_{qq}(z)\right] \\ &+ \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\, g(y)\left[\left(-\frac{1}{\bar{\varepsilon}} + \ln\frac{Q_F^2}{\mu^2}\right)P_{qg}(z) + F_{qg}(z)\right] \end{aligned}$$

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Dokshitzer–Gribov–Lipatov–Altarelli–Parisi Equation

- Deriving $q(x, Q^2)$ with respect to $\ln Q^2$ leads to

$$\frac{d\, q(x, Q^2)}{d \ln Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[q_0(y) P_{qq}\left(\frac{x}{y}\right) + g(y) P_{qg}\left(\frac{x}{y}\right) \right]$$

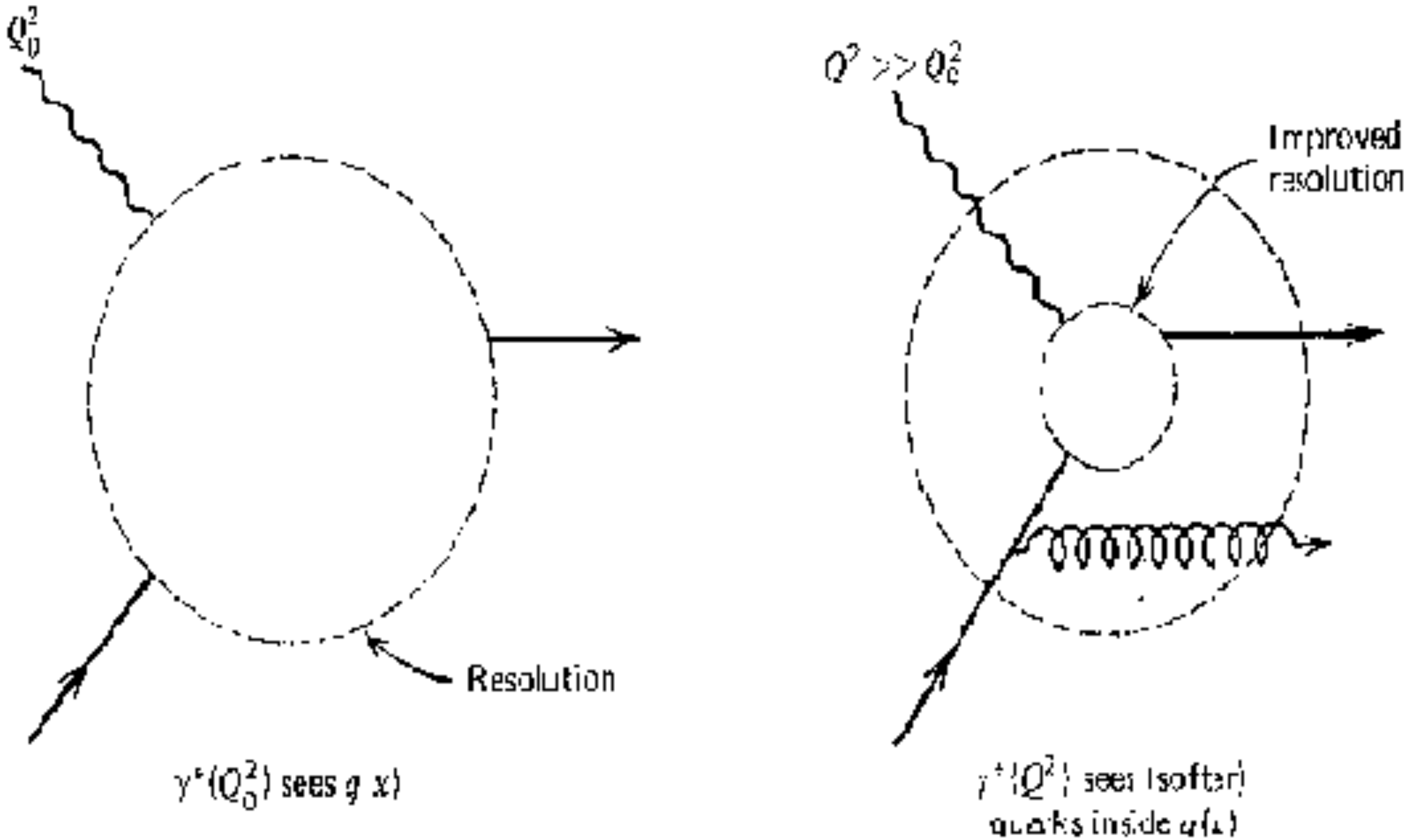
$$= \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} \left[q(y, Q^2) P_{qq}\left(\frac{x}{y}\right) + g(y, Q^2) P_{qg}\left(\frac{x}{y}\right) \right] + \mathcal{O}(\alpha_s^2)$$

- Knowing $q(x, Q_0^2)$ and $g(x, Q_0^2)$ $\Longrightarrow$ we can obtained them for other Q^2 $\Longrightarrow$ predictivity is not lost.

- We can even use it to extract α_s.

- Physical interpretation: γ^* with Q_0^2 probes scales $1/Q_0$. Fluctuations have a different lifetime.

➪ $xp = yp \times \frac{x}{y}$



➪ $\frac{\alpha_s}{2\pi} P_{qq}(z)$ is the "probability" of the quark to keep a fraction z of its momentum.

NLO in hadron colliders

long distance, non-perturbative

- The parton model expression for cross sections is

$$\sigma = \sum_{ij} \frac{1}{1+\delta_{ij}} \int dx_1\, dx_2 \quad \left\{ f_i(x_1, Q_F^2) f_j(x_2, Q_F^2) \ + \ i \leftrightarrow j \right\} \otimes$$

$$\hat{\sigma}_{ij}(\alpha_s(Q_R^2), Q_R^2, Q_F^2; x_1 x_2 s)$$

short distance, perturbative

- Expanding the pdf's and $\hat{\sigma}$ $(X = X^{(0)} + X^{(1)} + \cdots)$ the lowest order term is

$$\sigma = \sum_{ij} \frac{1}{1+\delta_{ij}} \int dx_1\, dx_2 \left\{ f_i^{(0)}(x_1) f_j^{(0)}(x_2) \ + \ i \leftrightarrow j \right\} \otimes \hat{\sigma}_{ij}^{(0)}(x_1 x_2 s)$$

⇨ The NLO contribution is obtained through

$$[f_i^{(1)} f_j^{(0)} + f_i^{(0)} f_j^{(1)} + i \leftrightarrow j] \times \hat{\sigma}^{(0)} \oplus [f_i^{(0)} f_j^{(0)} + i \leftrightarrow j] \times \hat{\sigma}^{(1)}$$

⇨ The red term contains collinear divergences that are canceled by the divergences in the blue term. Let's consider one example.

W^+ production

⇨ For simplicity, let us consider only the initial gluon contribution to $\bar{q}$! What is missing?

$$q_0(x) = q(x, Q_F^2) - \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y}\, g(y) \left[\left(-\frac{1}{\bar{\varepsilon}} + \ln \frac{Q_F^2}{\mu^2} \right) P_{qg}(z) + F_{qg}(z) \right]$$

and

$$\hat{\sigma}^{(0)} = \frac{\pi^2}{3} \frac{\alpha}{\sin^2 \theta_W} \delta(\hat{s} - M_W^2)$$

$$z = \frac{x}{y}$$

➩ Defining $\tau_0 = M_W^2/s$ and $z = \tau_0/(x_1 x_2)$ we have

$$f_i^{(1)} f_j^{(0)} \times \hat{\sigma}^{(0)} = -\frac{\pi \alpha \alpha_s}{12 s \sin^2 \theta_W} \sum_i \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2$$

$$q_{0i}(x_1) g(x_2) \otimes \left\{ [z^2 + (1-z)^2] \left(-\frac{1}{\bar{\varepsilon}} \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} \right) + G_{finite} \right\}$$

➩ Evaluating explicitly $gq \to W^+ q$ gives rise to

$$f_i^{(0)} f_j^{(0)} \times \hat{\sigma}^{(1)} = \frac{\pi \alpha \alpha_s}{12 s \sin^2 \theta_W} \sum_i \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2$$

$$q_{0i}(x_1) g(x_2) \otimes \left\{ [z^2 + (1-z)^2] \left(-\frac{1}{\bar{\varepsilon}} \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} \right) + H_{finite} \right\}$$

➩ The collinear divergences cancel out. We are left with the finite contribution To this order, now, we do $q_0(x) \to q(x, Q_F^2)$

- Scales:
  - The evaluation of $\hat{\sigma}$ contains a UV divergence => renormalization => remnant of the process is the renormalization scale μ_R
  - Full calculation should not depend on μ_R => we can estimate the higher order corrections by the μ_R dependence
  - At each order, the subprocess cross section and the PDF's have a residual factorization scale dependence on μ_F
  - The residual scale dependence should improve with higher order calculations

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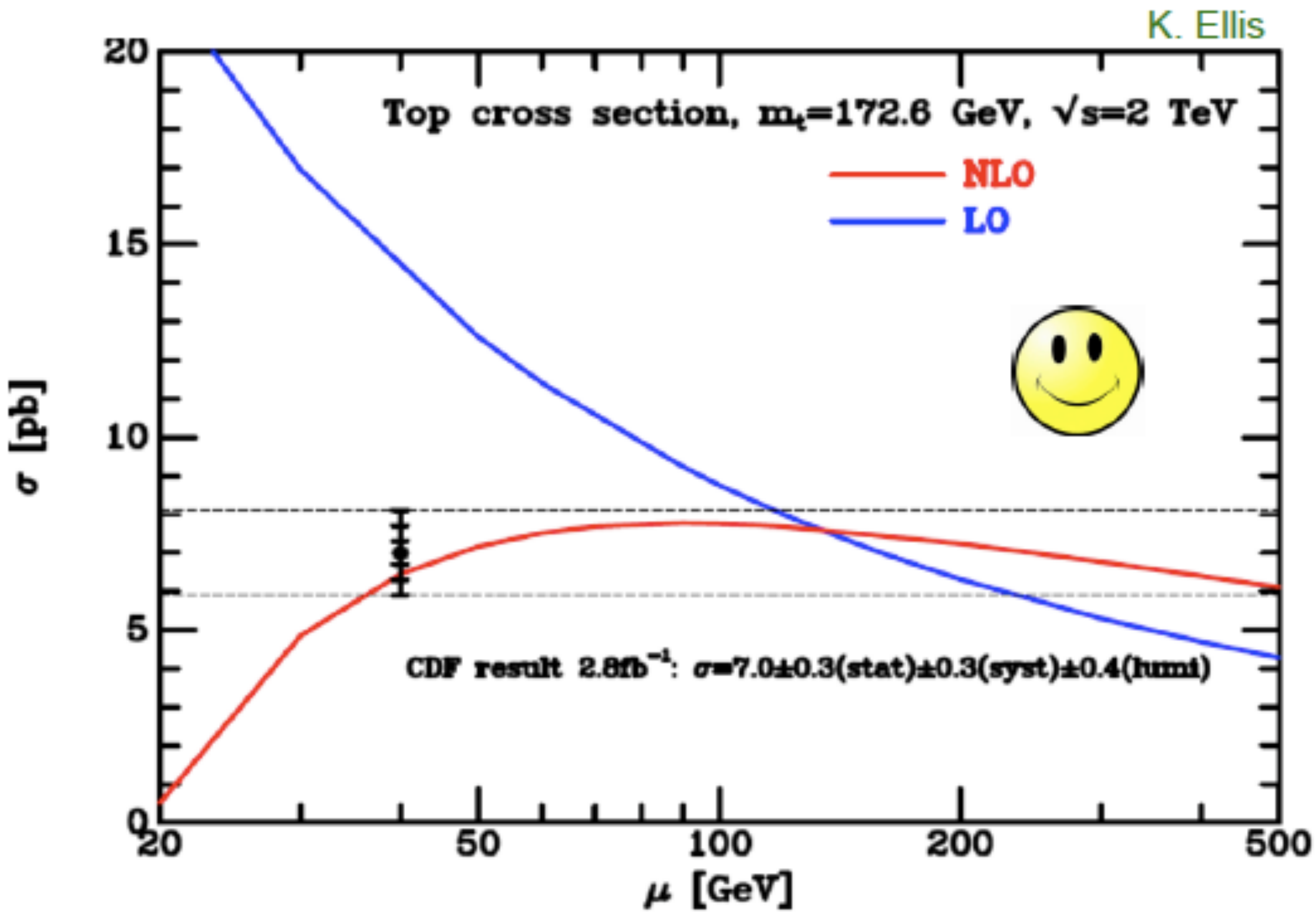
Why should we care about NLO?

- NLO correction can be large, e.g. 30-100%
- NLO reduces the sensitivity to unphysical scales
- More accurate predictions have impact in searches/measurements



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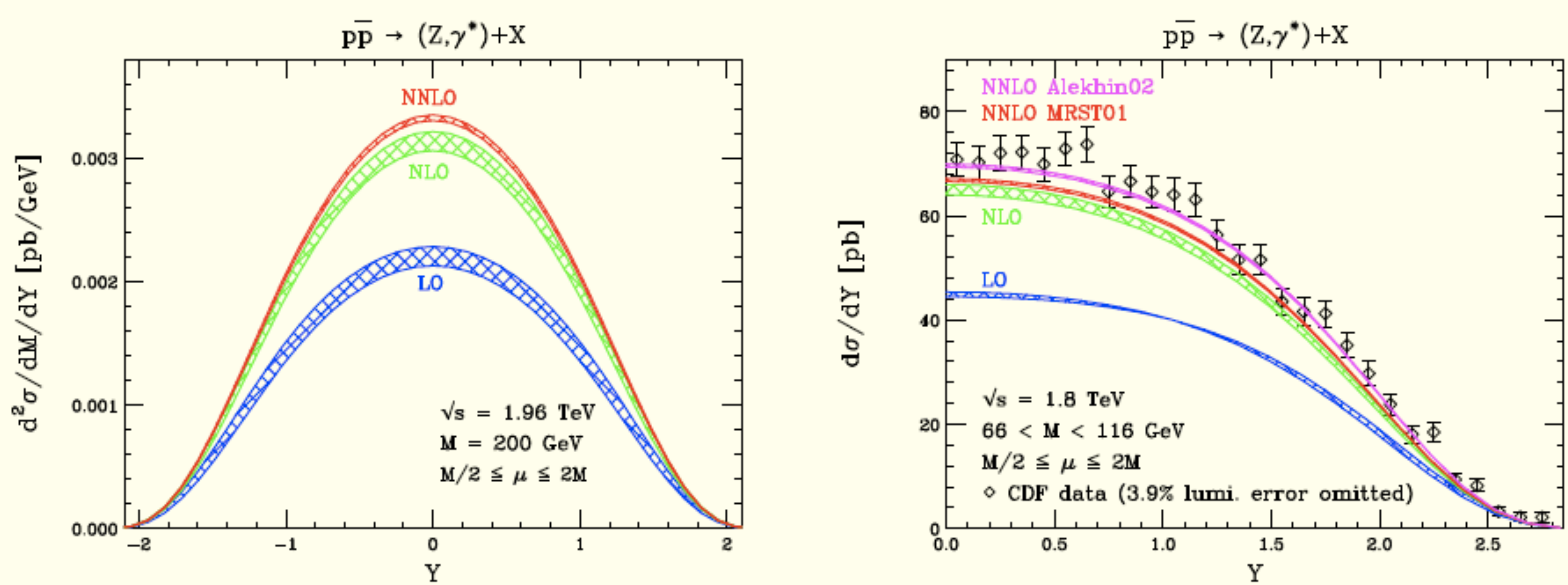

(C. Anastasiou, L. Dixon, K. Melnikov, F. Petriello, PRL 91 (2003) 182002)

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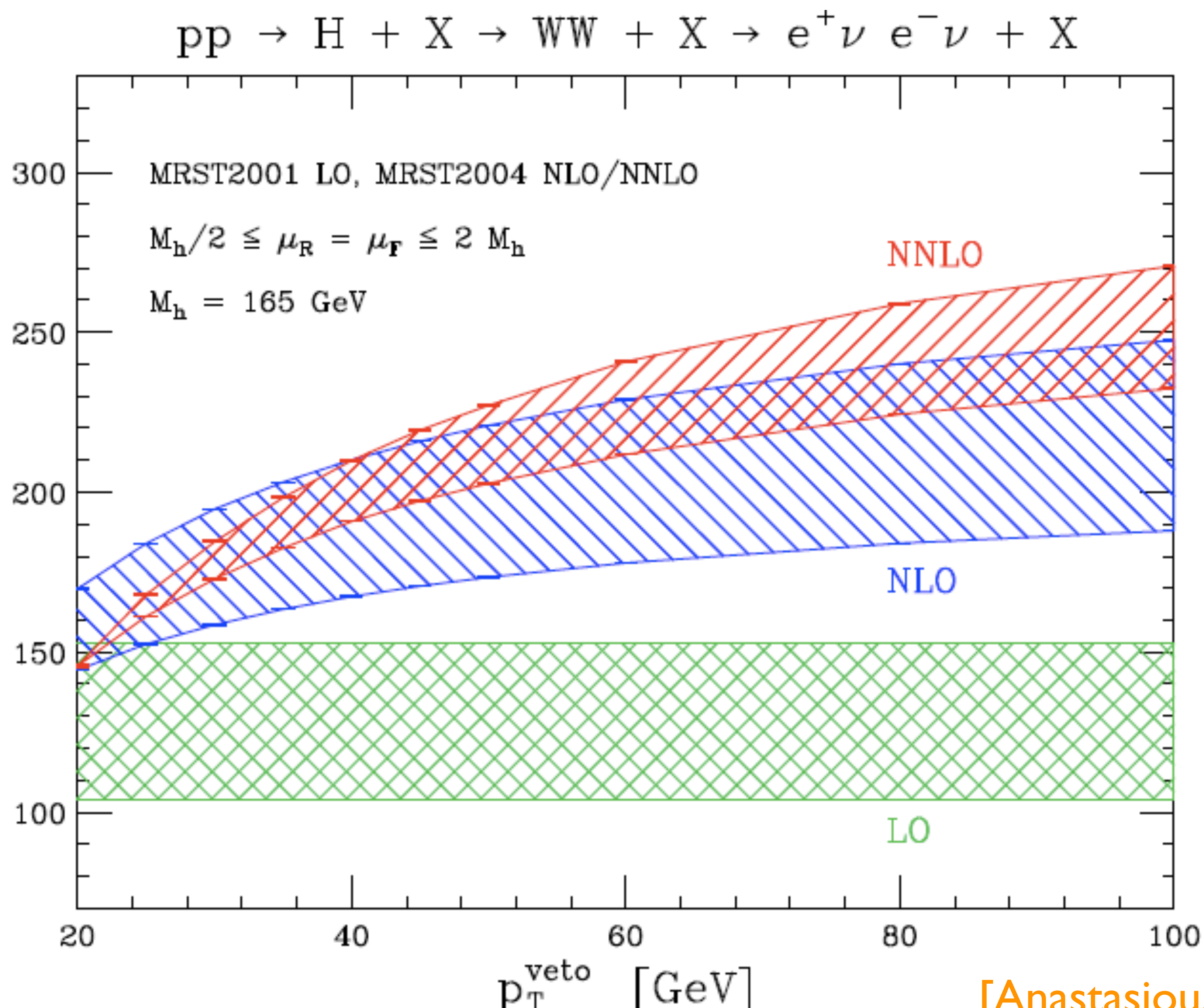
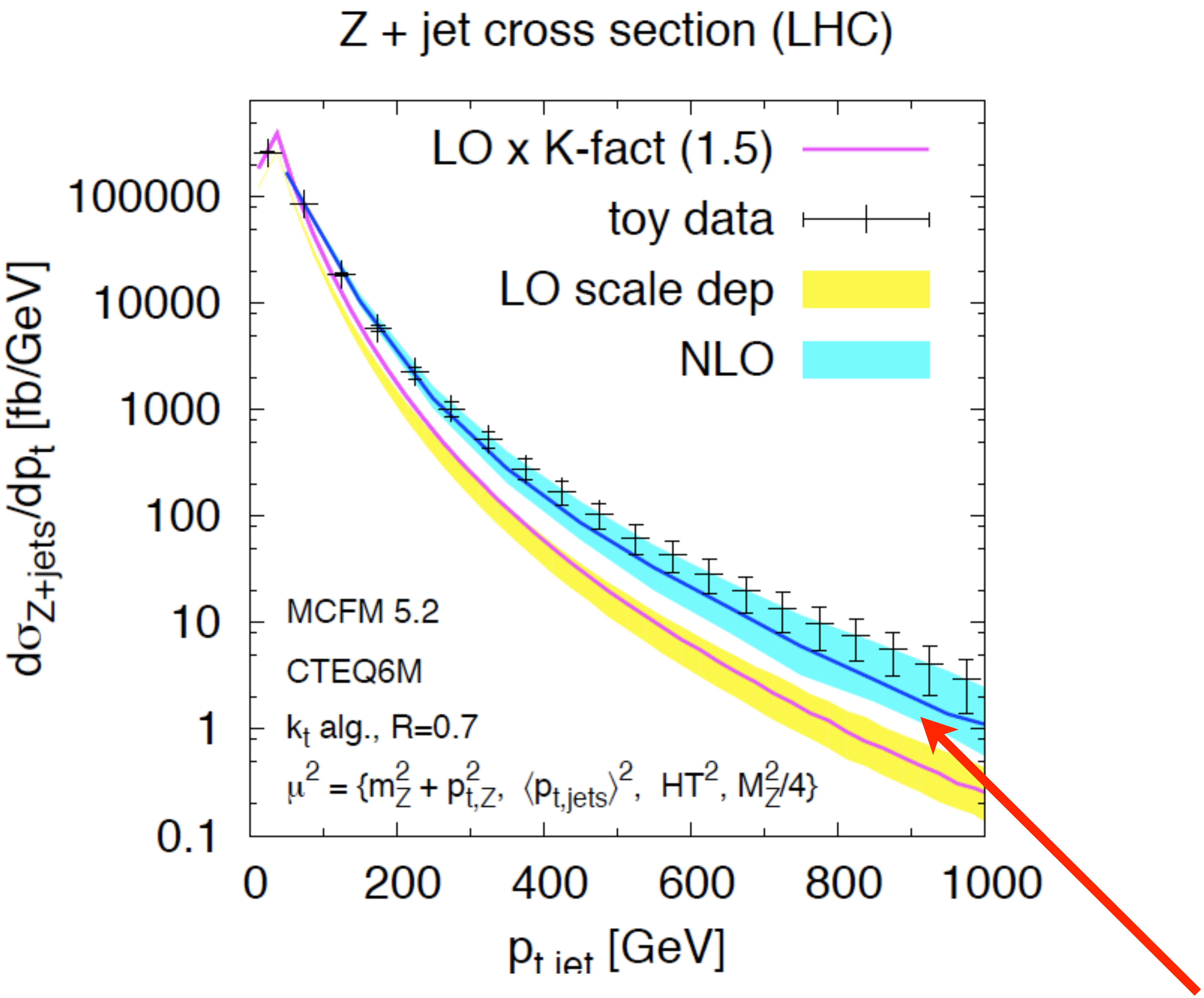


factor of 2!

[Anastasiou, Melnikov, Petriello-04]



Using LO there might be fake excesses

[Zanderighi]

In brief,

$$d\sigma = \sum_{ab} \int dx_a \int dx_b\, f_a(x_a,\mu_F^2) f_b(x_b,\mu_F^2) \times d\hat{\sigma}_{ab}(x_a,x_b,Q^2,\alpha_s(\mu_R^2))$$

LO

Born level partonic cross section

pdfs obtained from a LO analysis and 1 loop AP

$\alpha_s\,(m_q)$ obtained from a LO analysis and evolved with 1 loop β

NLO

1-loop level partonic cross section

pdfs obtained from a NLO analysis and 2 loop AP

$\alpha_s\,(m_q)$ obtained from a NLO analysis and evolved with 2 loop β

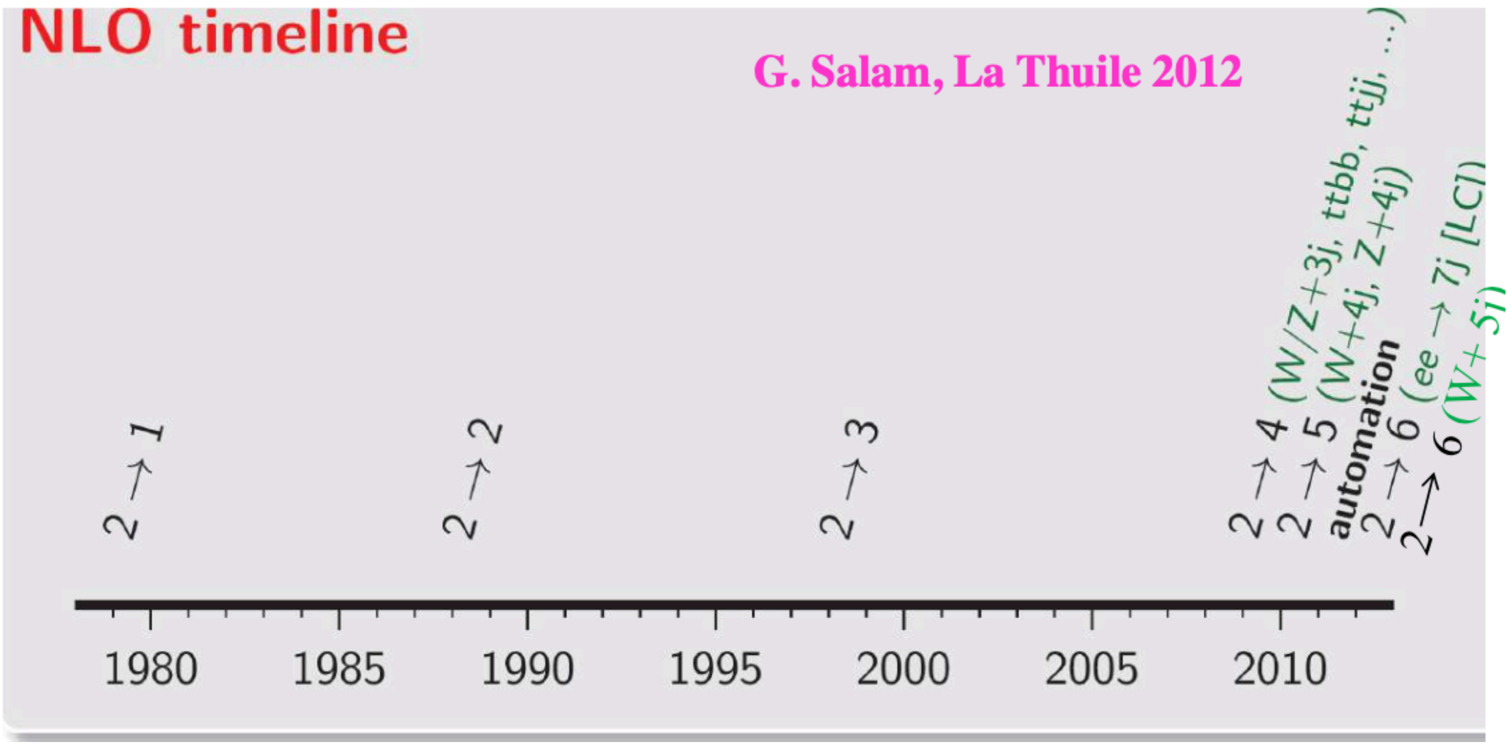
NNLO

2-loop level partonic cross section

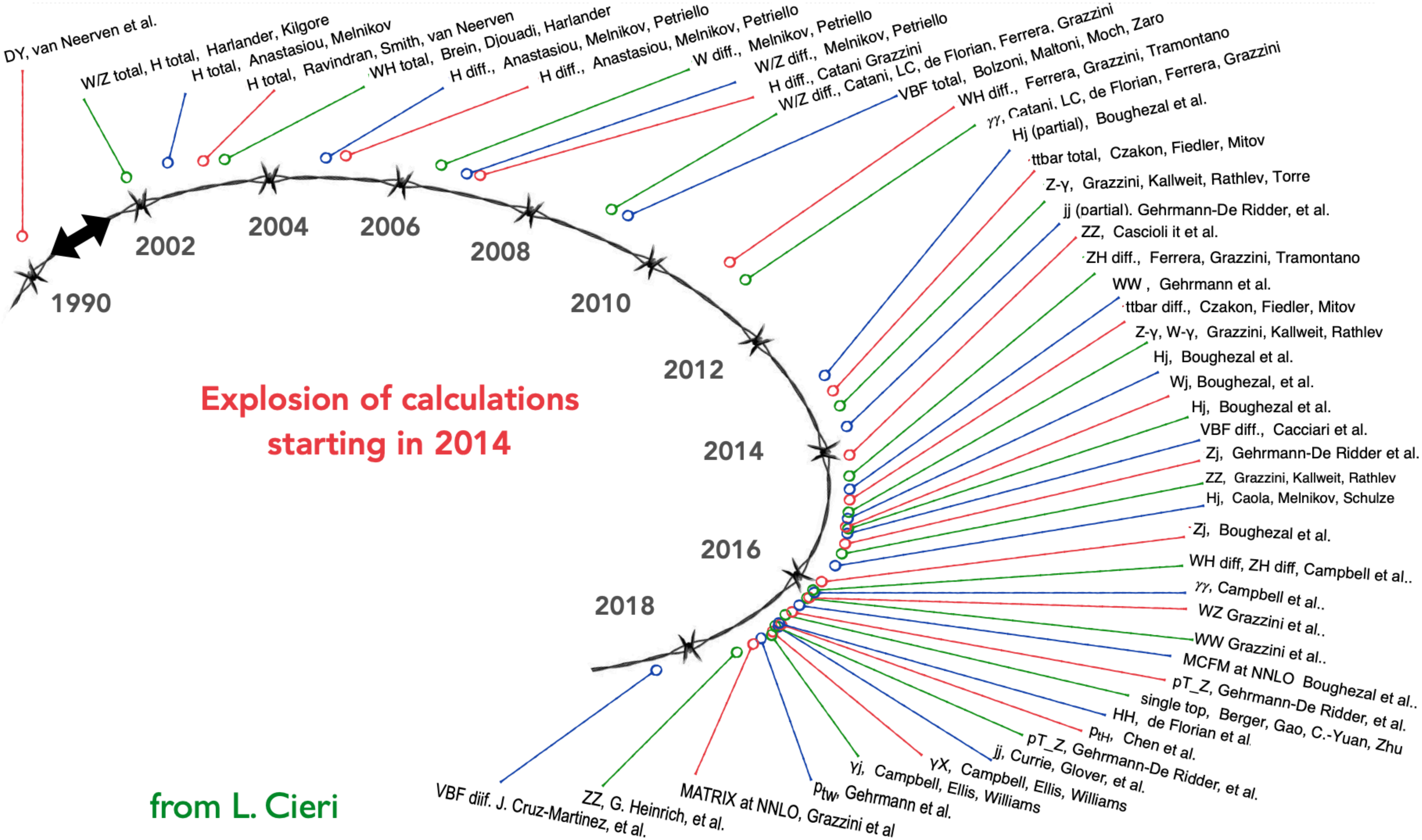
pdfs obtained from a NNLO analysis and 3 loop AP

$\alpha_s\,(m_q)$ obtained from a NNLO analysis and evolved with 3 loop β

- “state of the art”
- automatic NLO



The NNLO revolution

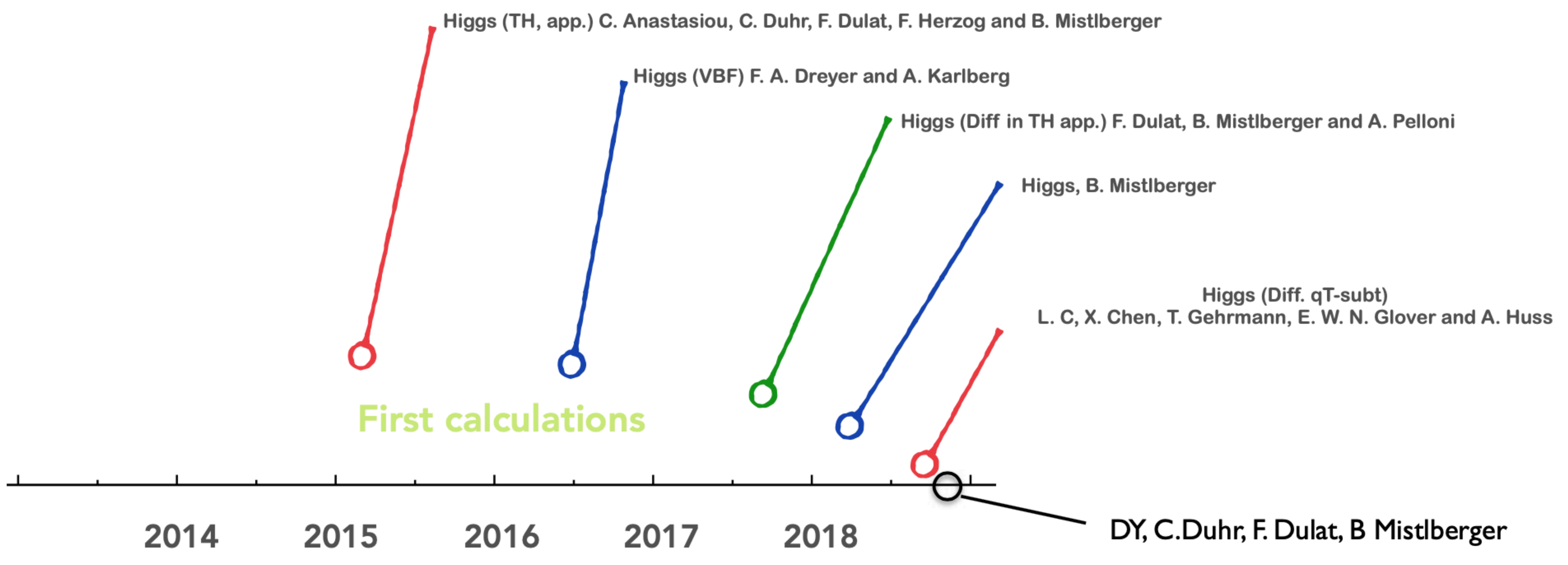


Explosion of calculations starting in 2014

from L. Cieri

N^{3}LO

The new Frontier?

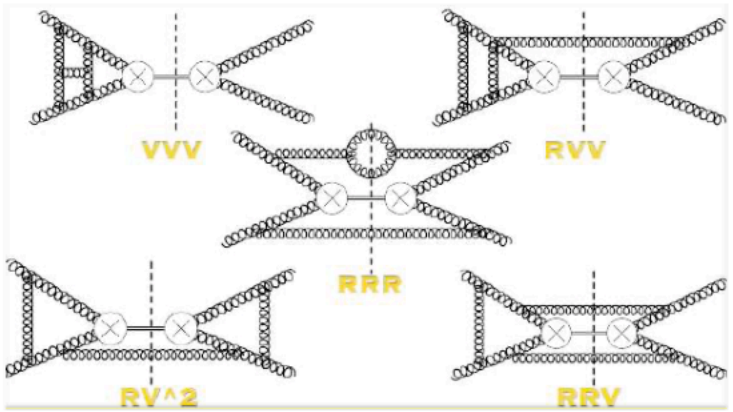


from L. Cieri

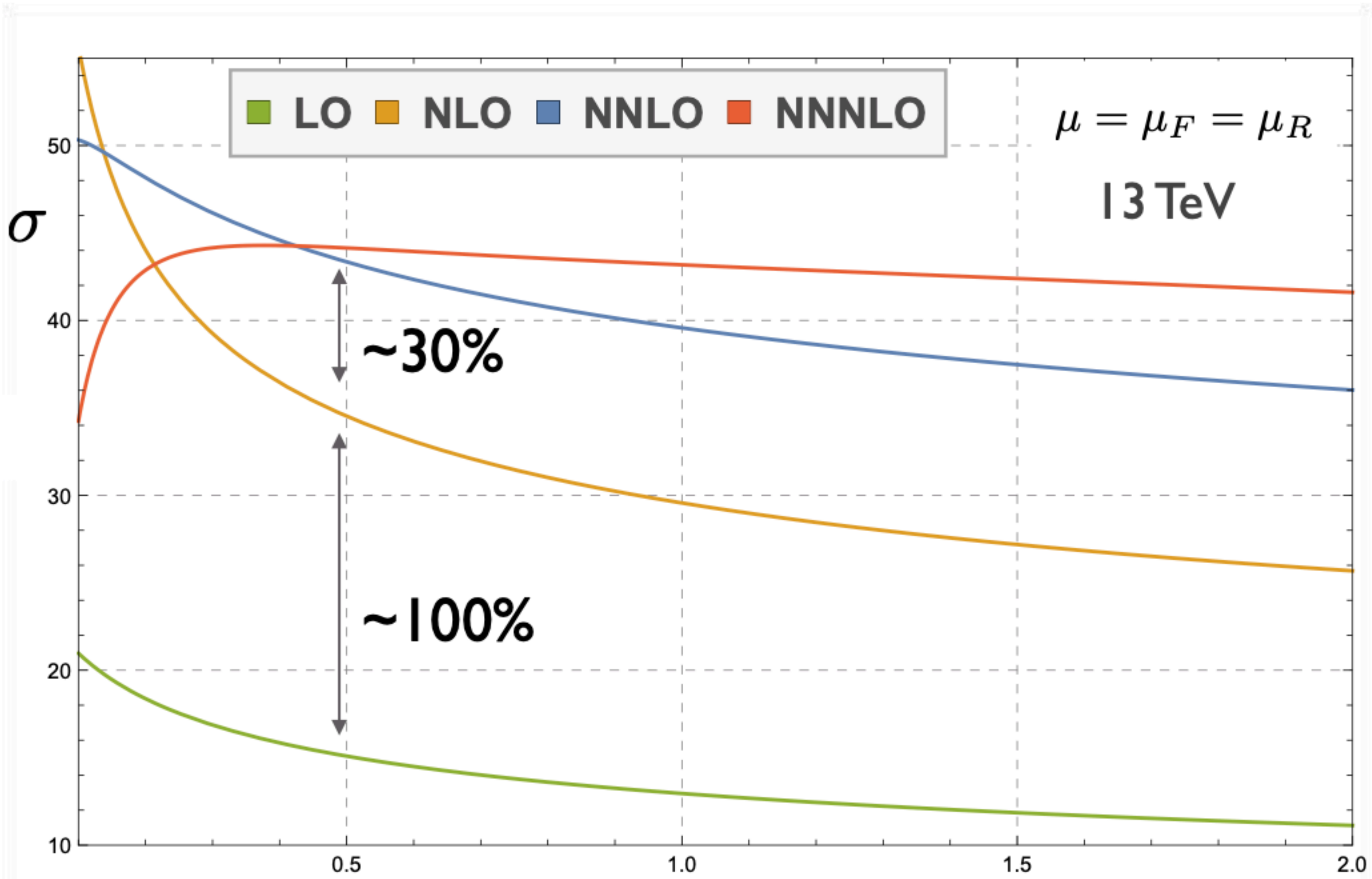
Higgs at N^3LO

C. Anastasiou, C. Duhr, F. Dulat, F. Herzog, B. Mistlberger (2015)
B. Mistlberger (2018)

- Very relevant observable called for higher orders (slow convergence)
- Impressive calculation : new techniques
  - Within (excellent) heavy top approximation



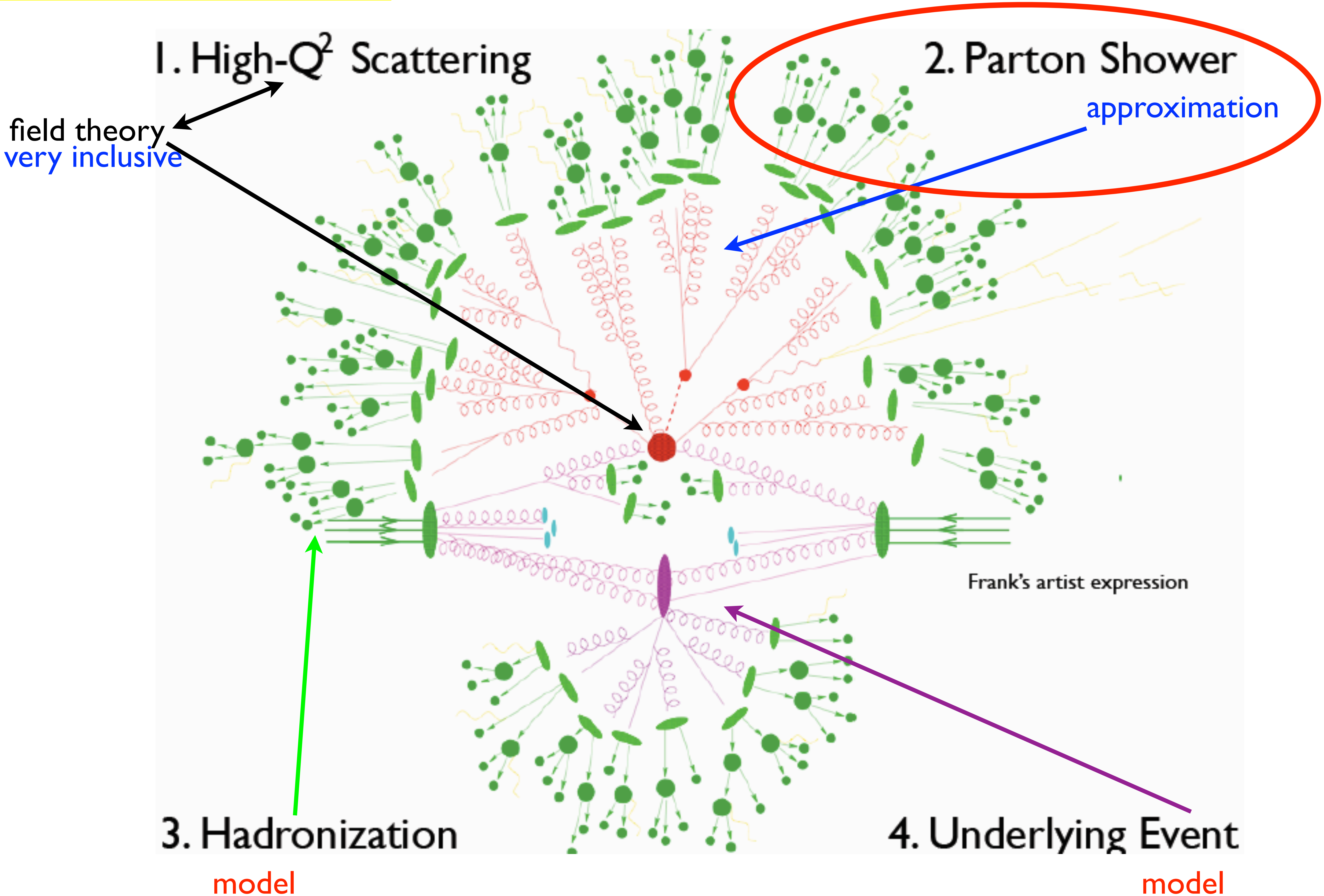
68273802 loop and phase space integrals



- Observe stabilization of expansion
- Small correction (2% at $M_H/2$)
- Scale variation at N^3LO ~2%

Parton shower

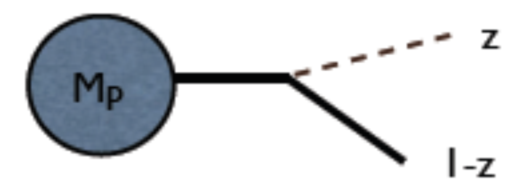
we should describe the whole event!



- Parton shower is an approximation for multi-particle states.

- Matrix elements for $q \to qg$ $(g \to gg \ \cdots)$ due to soft and collinear divergences can be approximated

$$\frac{1}{(p_q + p_g)^2} \simeq \frac{1}{2E_g E_q(1 - \cos\theta)}$$



- We write the approximation:

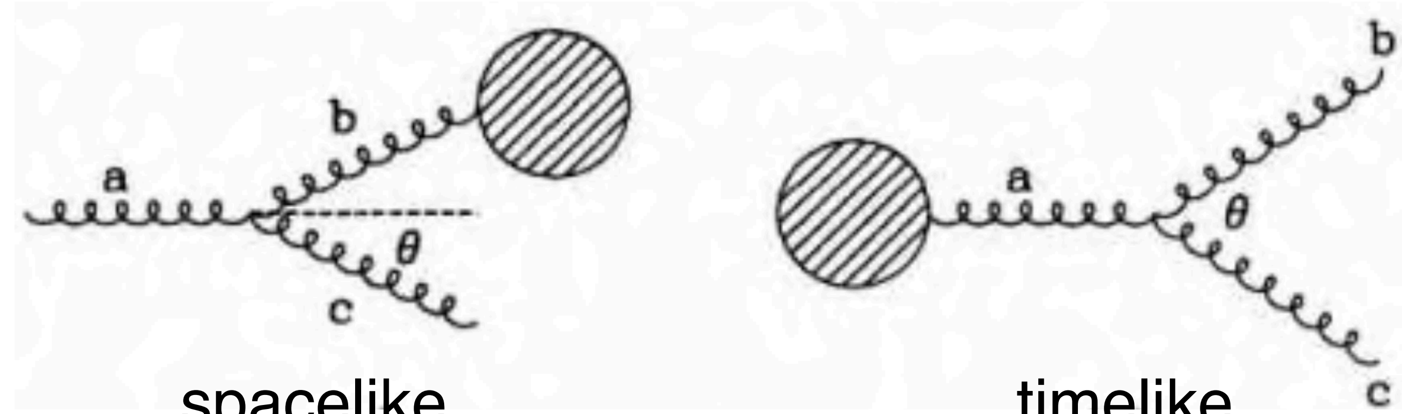
$$|M_{p+1}|^2 d\Phi_{p+1} \simeq |M_p|^2 d\Phi_p \frac{dt}{t} \frac{\alpha_S}{2\pi} P(z) dz d\phi$$

- The parton shower evolution is a Markov process based on this approximation.

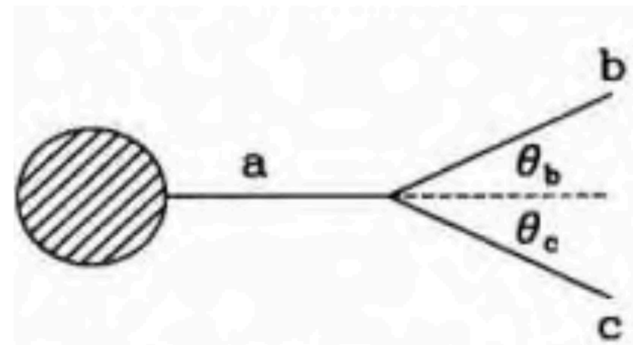
- General purpose MC (eg PYTHIA) use this approximation to simulate higher order processes.

Parton branching

- Two types of branching



- For a timelike branching



$$\theta = \theta_b + \theta_c$$

$$p_b^2 \ , \ p_c^2 << p_a^2 = t$$

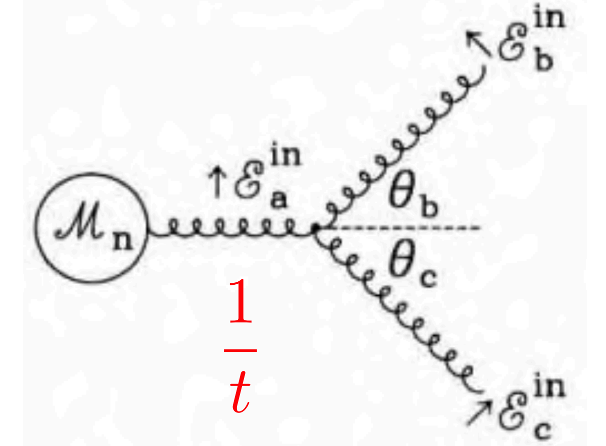
$$E_b = zE_a \quad , \quad E_c = (1-z)E_a$$

- Kinematics (for small angles)

$$t = 2E_bE_c(1-\cos\theta) \simeq z(1-z)E_a^2\theta^2$$

$$\theta = \frac{1}{E_a}\sqrt{\frac{t}{z(1-z)}}$$

p_T conservation $\quad z\theta_b = (1-z)\theta_c$



- Matrix element for g > g g

$$M_{n+1} \simeq M_n \frac{g_s^2}{t} V_{ggg} \Longrightarrow |M_{n+1}|^2 \simeq \frac{4g_s^2}{t} C_A F(z; \epsilon_a, \epsilon_b, \epsilon_c) \; |M_n|^2$$

$$C_A = 3$$

- Non-vanishing contributions

ε_a	ε_b	ε_c	$F(z; \varepsilon_a, \varepsilon_b, \varepsilon_c)$
in	in	in	$(1-z)/z + z/(1-z) + z(1-z)$
in	out	out	$z(1-z)$
out	in	out	$(1-z)/z$
out	out	in	$z/(1-z)$

- Average + sum of polarizations

c soft

$$C\langle F \rangle = C_A \left[\frac{1-z}{z} + \frac{z}{1-z} + z(1-z) \right] \equiv \hat{P}_{gg}(z)$$

b soft

- Matrix element for $g \to q\bar{q}$

$$C\langle F\rangle = T_R\left[z^2(1-z)^2\right] \equiv \hat{P}_{qg}(z)$$ with $T_R = \frac{1}{2}$

- Matrix element for $q \to qg$

$$C\langle F\rangle = C_F\frac{1+z^2}{1-z} \equiv \hat{P}_{gq}(z)$$ with $C_F = \frac{4}{3}$

- Phase space:

we showed that $d\Phi_{n+1} = d\Phi_n \otimes dM^2 \otimes d\Phi_2$

here $dM^2 = dt$

Now $$d\Phi_2 = \int \frac{d^3p_b}{2E_b}\delta((p_a - p_b)^2) = \int \frac{d^3p_b}{2E_b}\delta(t - 2p_a \cdot p_b)$$

$$= \frac{E_b^2 dE_b \ d\varphi \ d\cos\theta_b}{2E_b}\delta(t - 2E_aE_b(1\cos\theta_b))$$

$$= \frac{1}{2} \ \frac{1}{2E_aE_b} \ E_b dE_b \ d\varphi = \frac{1}{4}\frac{dE_b}{E_a} \ d\varphi$$

$$= \frac{1}{4}dzd\varphi$$

Finally $$d\Phi_{n+1} = d\Phi_n \otimes dt \otimes \frac{1}{4}dzd\varphi$$

- Cross section

$$d\sigma_{n+1} = \frac{1}{\mathcal{F}_{n+1}} \, |M_{n+1}|^2 d\Phi_{n+1}$$

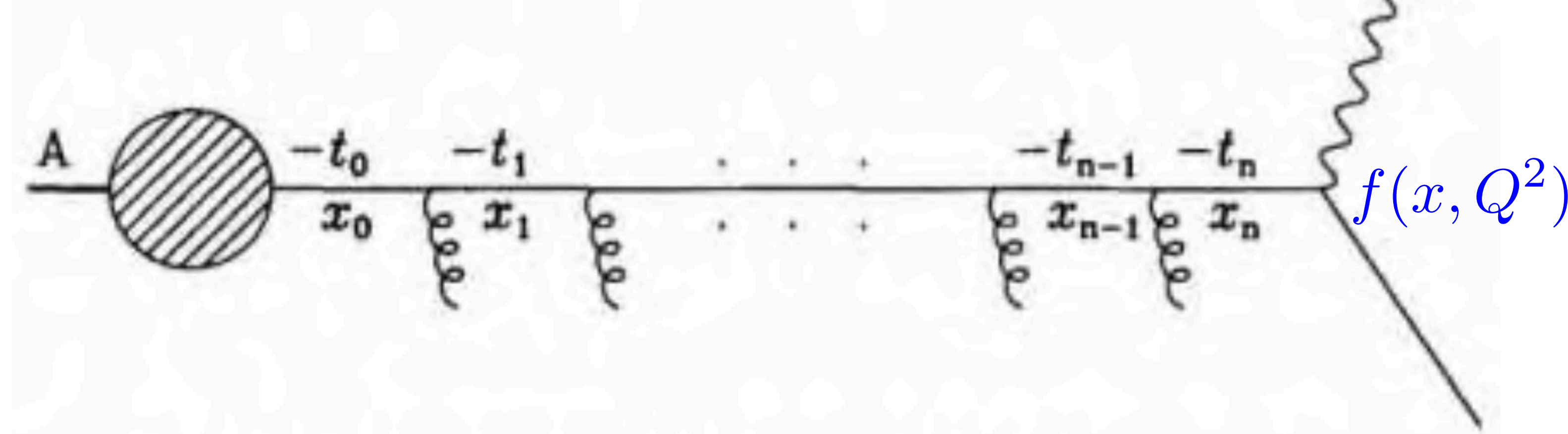
putting all together and remembering that $\mathcal{F}_{n+1} = (2\pi)^3 \mathcal{F}_n$

$$d\sigma_{n+1} = d\sigma_n \, \frac{dt}{t} \, \frac{g_s^2}{(2\pi)^3} \, dz \, d\varphi$$

$$= d\sigma_n \, \frac{dt}{t} \, dz \, \frac{\alpha_s}{2\pi} \hat{P}_{ba}(z)$$

- For spacelike branchings the expression for t changes

Evolution equations

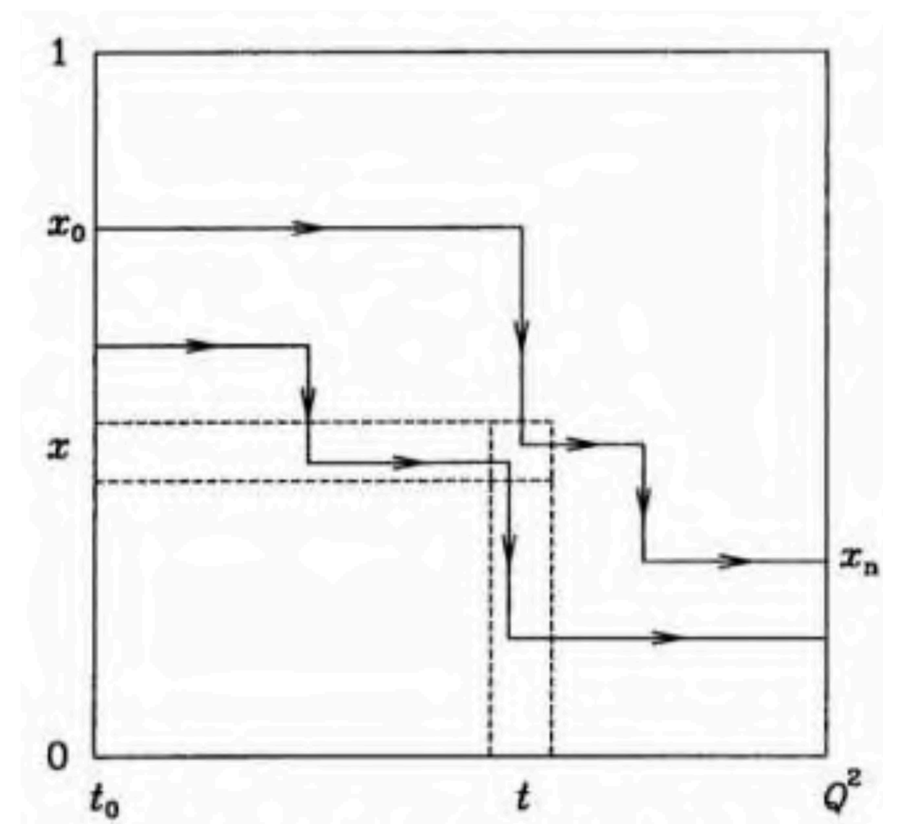


- Let's consider a space like evolution (e.g. DIS)

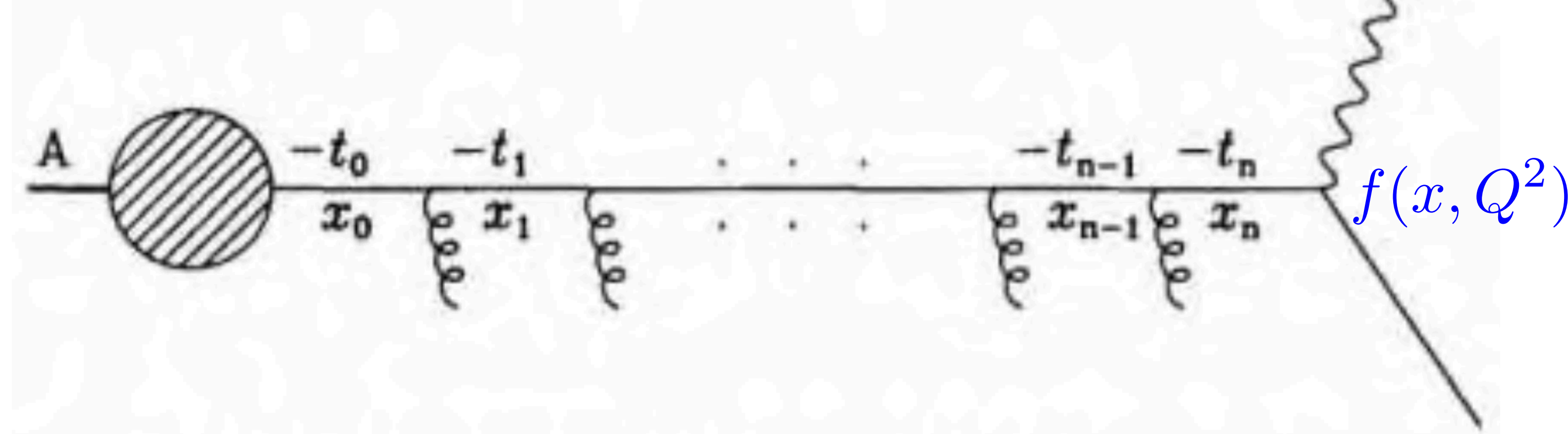
- Pictorial representation

$$\delta f_{in}(x,t) = \frac{\delta t}{t} \int_x^1 dx' \; f(x',t) \; \frac{\alpha_s}{2\pi} \; \hat{P}(z) \; \delta(x - zx') \; dz$$

$$= \frac{\delta t}{t} \int_0^1 \frac{dz}{z} \; \frac{\alpha_s}{2\pi} \; \hat{P}(z) \; f(x/z,t)$$



Evolution equations

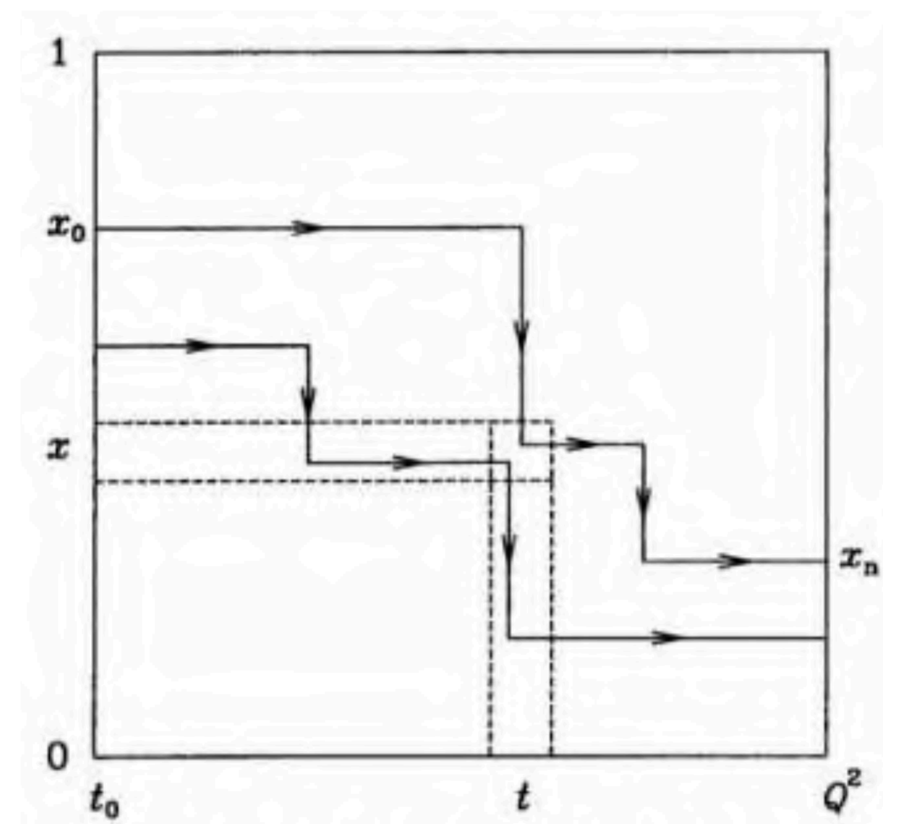


- Let's consider a space like evolution (e.g. DIS)

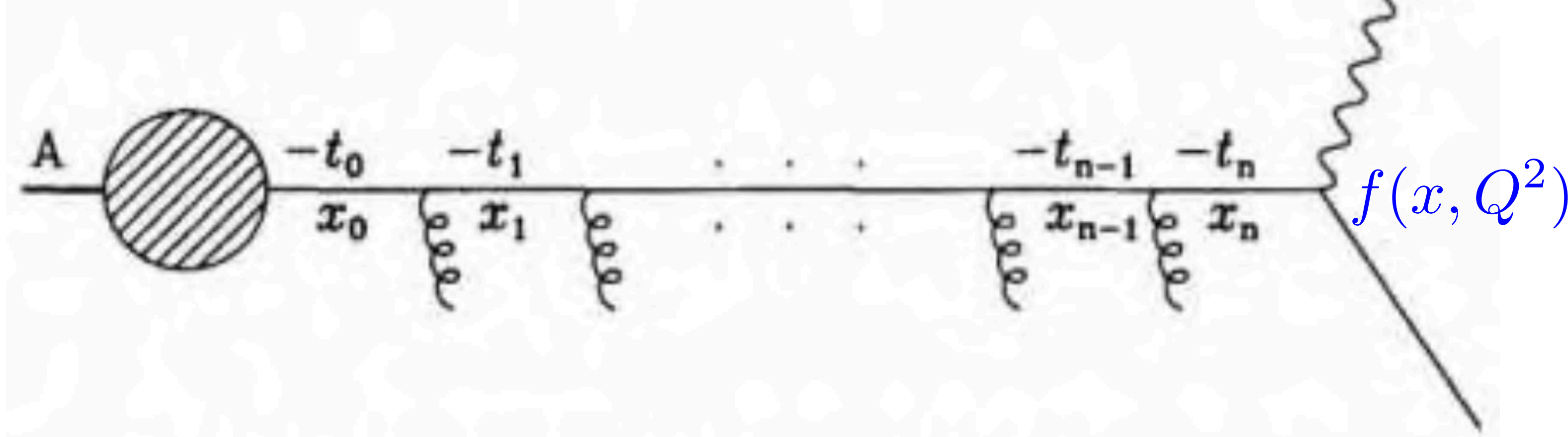
- Pictorial representation

$$\delta f_{out}(x,t) = \frac{\delta t}{t}\; f(x,t) \int_0^x dx' \,\frac{\alpha_s}{2\pi}\, \hat{P}(z)\; \delta(x' - zx)\; dz$$

$$= \frac{\delta t}{t}\; f(x,t) \int_0^1 dz\, \frac{\alpha_s}{2\pi}\, \hat{P}(z)$$



Evolution equations

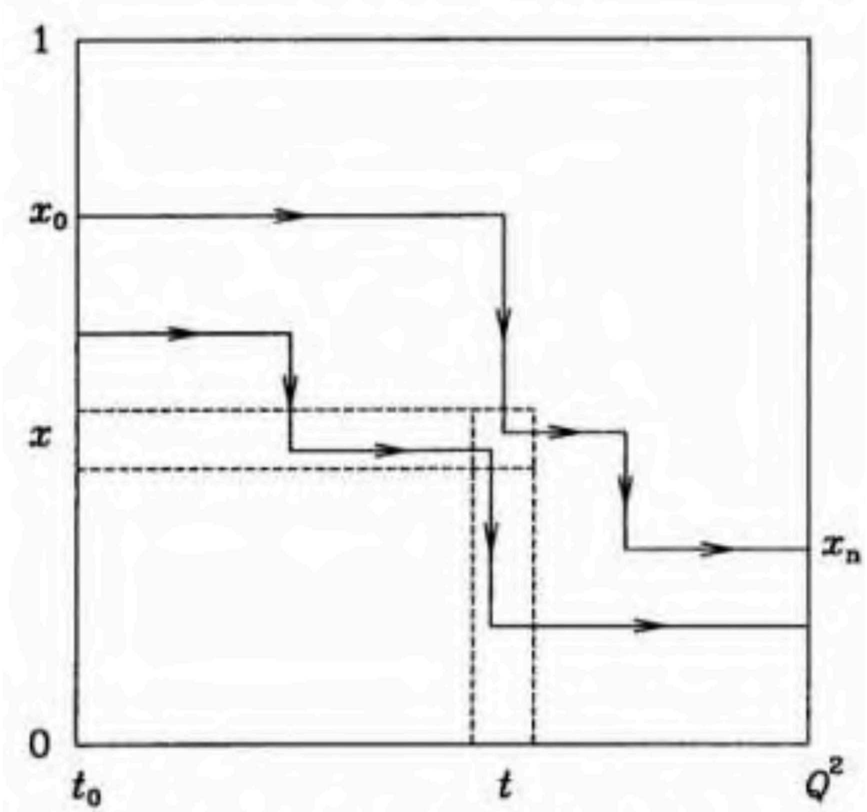


- Let's consider a space like evolution (e.g. DIS)

- Pictorial representation

$$\delta f_{in}(x,t) = \frac{\delta t}{t} \int_0^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} \hat{P}(z)\ f(x/z,t)$$

$$\delta f_{out}(x,t) = \frac{\delta t}{t}\ f(x,t) \int_0^1 dz\ \frac{\alpha_s}{2\pi} \hat{P}(z)$$



So

$$\delta f(x,t) = \delta f_{in} - \delta f_{out} = \frac{\delta t}{t} \int_0^1 dz \; \frac{\alpha_s}{2\pi} \; \hat{P}(z) \; \left[\frac{1}{z} f\left(\frac{x}{z}, t \right) - f(x,t) \right]$$

$$t \frac{\partial}{\partial t} f(x,t) = \int_0^1 \frac{dz}{z} \; \frac{\alpha_s}{2\pi} \; P(z) \; f\left(\frac{x}{z}, t \right)$$

with $P(z) = \hat{P}(z)_+$

- Sudakov factor

$$\Delta(t) \equiv \exp\left[- \int_{t_0}^{t} \frac{dt'}{t'} \; \int dz \frac{\alpha_s}{2\pi} \hat{P}(z) \right]$$

$$t \frac{\partial}{\partial t} f(x,t) = \int_0^1 \frac{dz}{z} \; \frac{\alpha_s}{2\pi} \; \hat{P}(z) \; f\left(\frac{x}{z}, t \right) + \frac{f(x,t)}{\Delta(t)} \; t \frac{\partial}{\partial t} \Delta(t)$$

- Sudakov factor

$$\Delta(t) \equiv \exp\left[-\int_{t_0}^{t} \frac{dt'}{t'} \int dz \frac{\alpha_s}{2\pi} \hat{P}(z)\right]$$

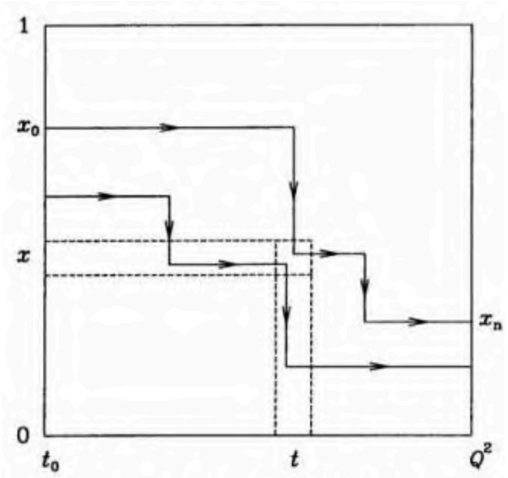
$$t\frac{\partial}{\partial t} f(x,t) = \int_0^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} \hat{P}(z)\, f\left(\frac{x}{z}, t\right) + \frac{f(x,t)}{\Delta(t)}\, t\frac{\partial}{\partial t}\Delta(t)$$

$$\Longrightarrow\ t\frac{\partial}{\partial t}\left(\frac{f(x,t)}{\Delta(t)}\right) = \frac{1}{\Delta(t)} \int_0^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} \hat{P}(z)\, f\left(\frac{x}{z}, t\right)$$

$$\Longrightarrow\ f(x,t) = \Delta(t)\, f(x,t_0) + \int \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \frac{\alpha_s}{2\pi} \hat{P}(z)\, f\left(\frac{x}{z}, t'\right)$$

$\Delta(t) f(x,t_0)$ paths that did not branch in $t_0 \to t$

probability of not branching!



- Generalization

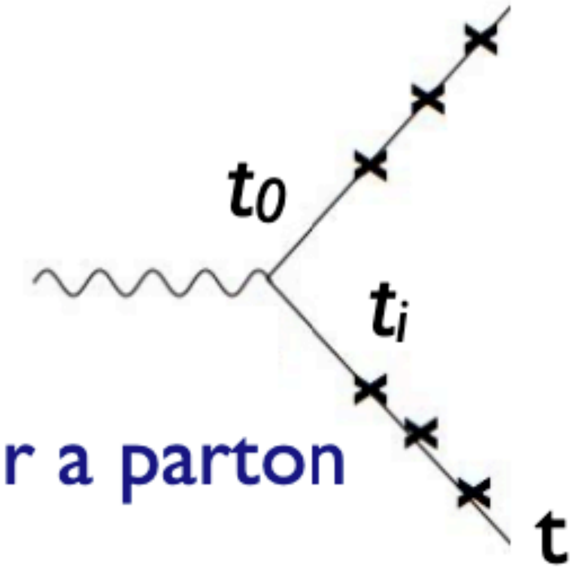
$$\Delta_i(t) = \exp\left[-\sum_j \int_{t_0}^{t} \frac{dt'}{t'} \int dz \, \frac{\alpha_s}{2\pi} \, \hat{P}_{ij}(t)\right]$$

$$t\frac{\partial}{\partial t}\left(\frac{f_i}{\Delta_i}\right) = \frac{1}{\Delta_i}\sum_j \int \frac{dz}{z} \, \frac{\alpha_s}{2\pi} \, \hat{P}_{ij} \, f_j\left(\frac{x}{z}, t\right)$$

- Comments

1. there are infrared divergences
2. we need a regularization prescription
3. ad hoc cut off to define resolvable emissions $\int_{\epsilon}^{1-\epsilon} dz$
4. virtual corrections cure the problem

Parton Shower basics



- Now, consider the non-branching probability for a parton at a given virtuality t_i:

$$\mathcal{P}_{\text{non-branching}}(t_i) = 1 - \mathcal{P}_{\text{branching}}(t_i) = 1 - \frac{\delta t}{t_i}\frac{\alpha_s}{2\pi}\int_z^1 \frac{dz}{z}\hat{P}(z)$$

- The total non-branching probability between virtualities t and t_0:

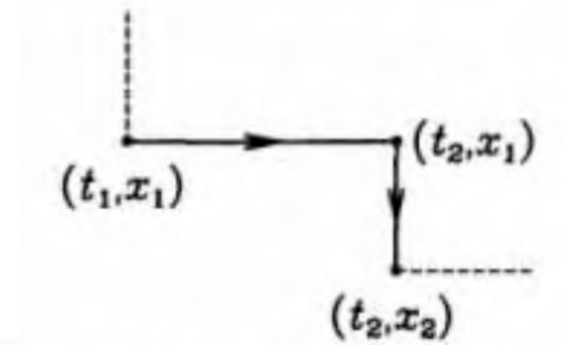
$$\mathcal{P}_{\text{non-branching}}(t, t_0) \simeq \prod_{i=0}^{N}\left(1 - \frac{\delta t}{t_i}\frac{\alpha_s}{2\pi}\int_z^1 \frac{dz}{z}\hat{P}(z)\right)$$

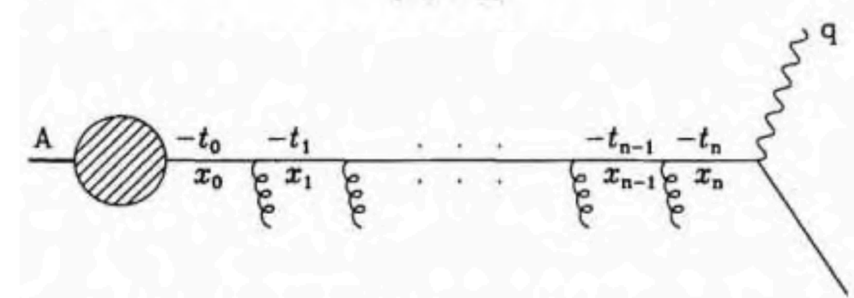
$$= e^{\sum_{i=0}^{N}\left(-\frac{\delta t}{t_i}\frac{\alpha_s}{2\pi}\int_z^1 \frac{dz}{z}\hat{P}(z)\right)}$$

$$\simeq e^{-\int_t^{t_0}\frac{dt'}{t'}\frac{\alpha_s}{2\pi}\int_z^1 \frac{dz}{z}\hat{P}(z)} = \Delta(t, t_0)$$

- This is the famous “Sudakov form factor”

- Monte Carlo
  - Goal: given (x_1,t_1) generate (x_2,t_2)
  - To obtain t_2 solve





space like evolution
$$\frac{\Delta(t_2)}{\Delta(t_1)} = r \in [0,1]$$

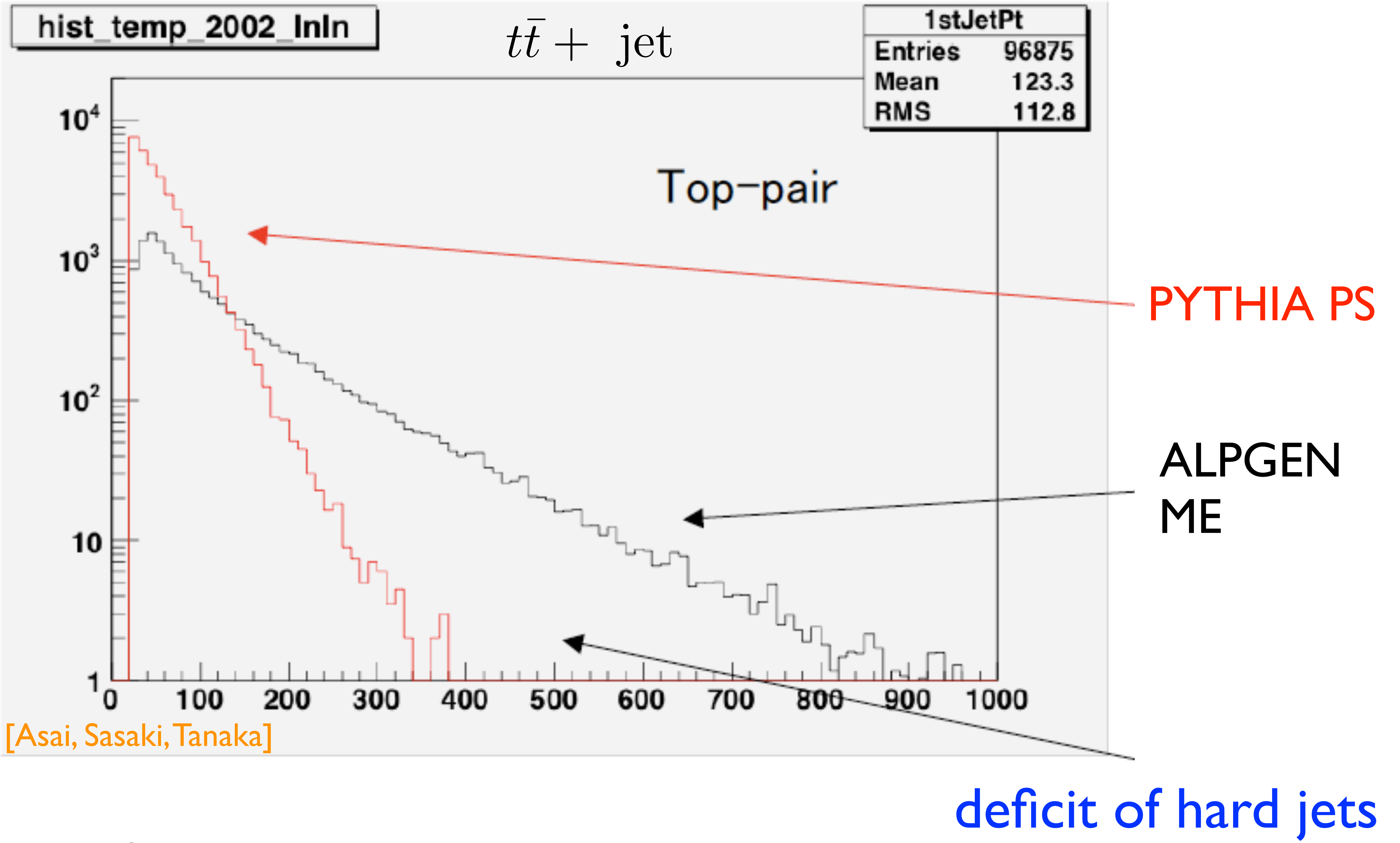
  - To obtain $z=x_2/x_1$ solve

$$\int_{\epsilon}^{x_2/x_1} dz\ \frac{\alpha_s}{2\pi}\ P(z) = r' \int_{\epsilon}^{1-\epsilon} dz\ \frac{\alpha_s}{2\pi}\ P(z) \quad \text{with} \quad r' \in [0,1]$$

- Stop the process if $t_2 > Q^2$

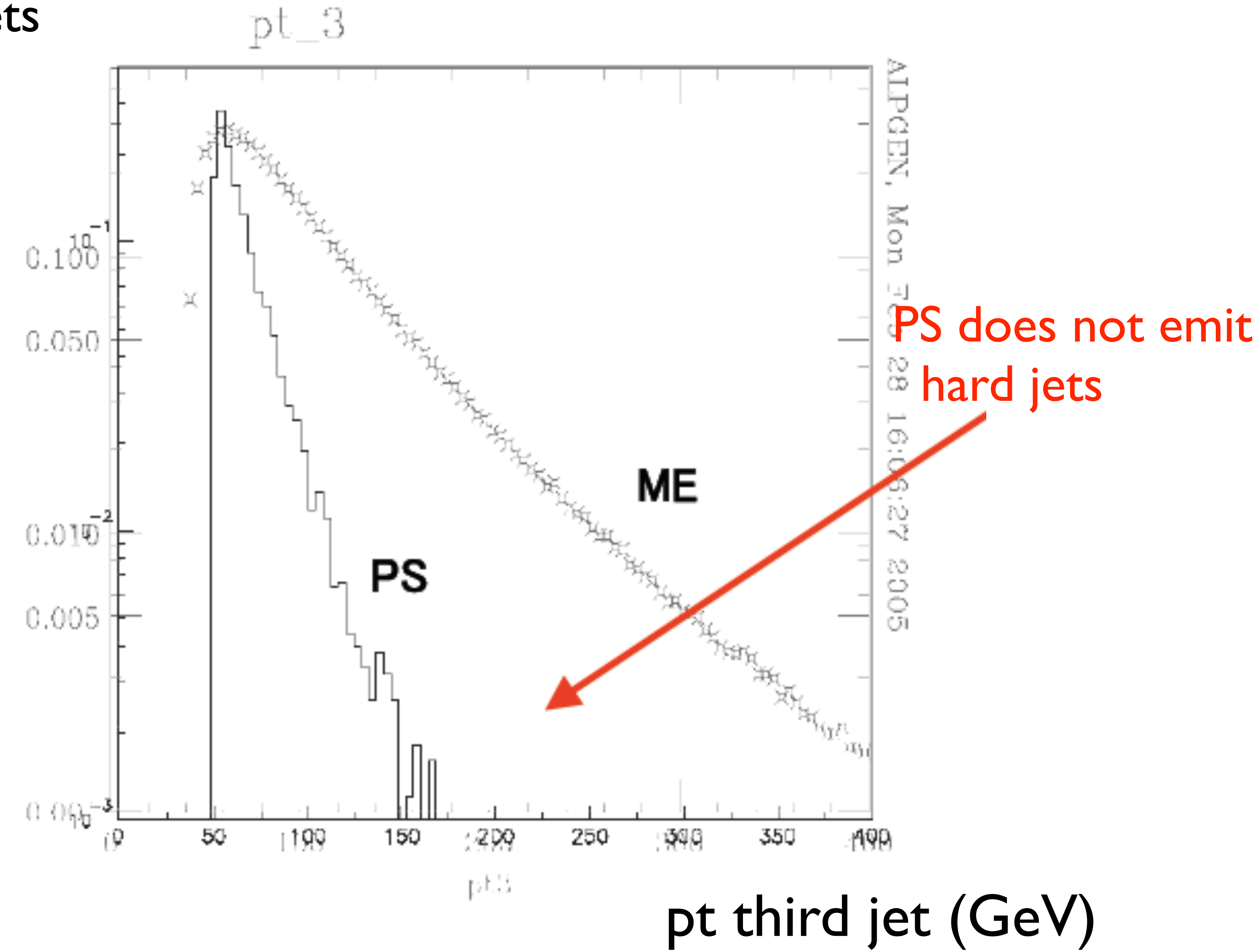
- For time like evolution switch t_2 with t_1 and stop at t_0

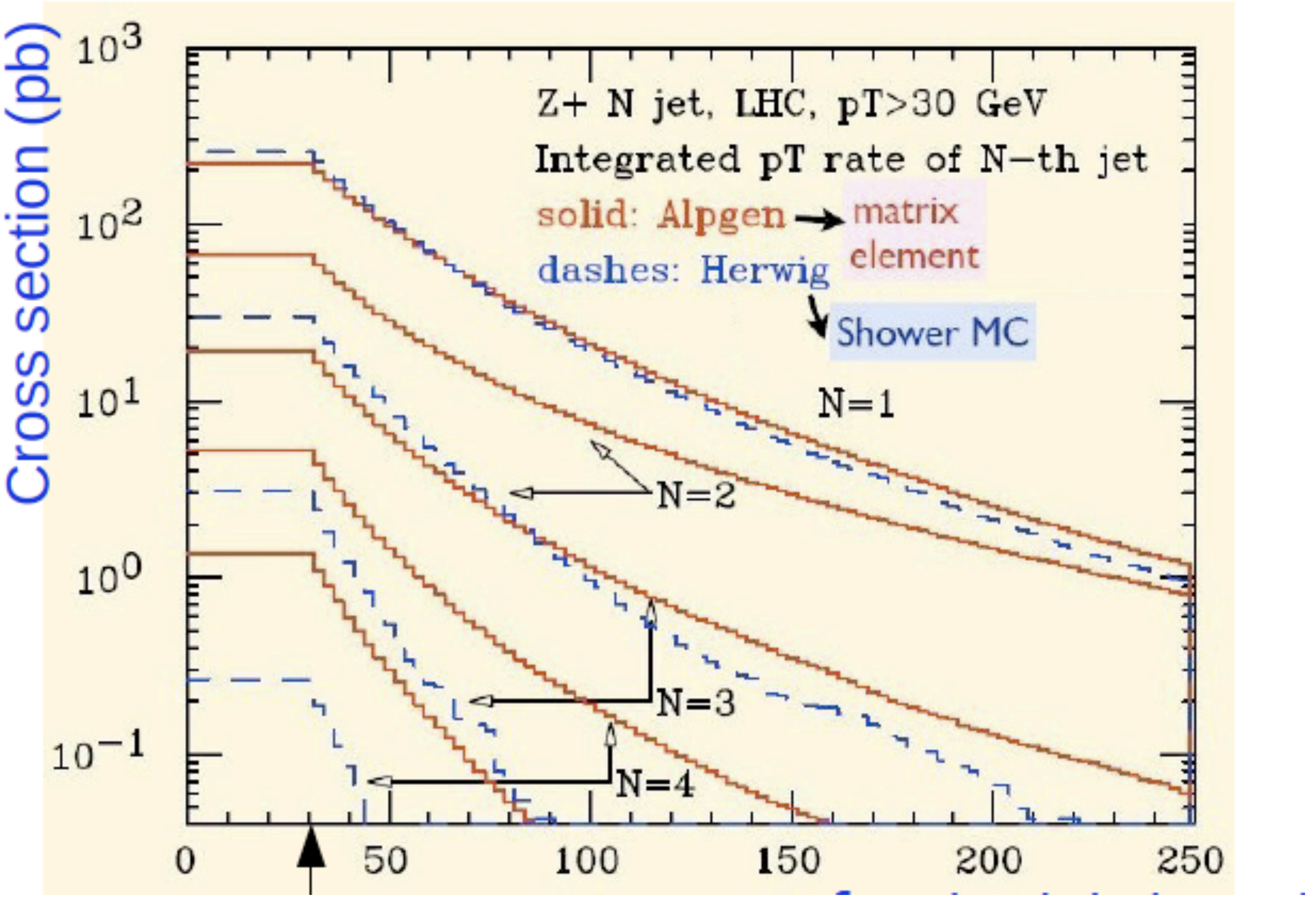
- PS is not accurate to describe extra hard jets, by construction!



p_T of additional jet

Z + N jets


pt_3
AlPGEN, Mon Nov 28 16:06:27 2005
0.100
0.050
0.010
0.005
0
50
100
150
200
250
300
350
400
ME
PS
PS does not emit
hard jets
pt third jet (GeV)

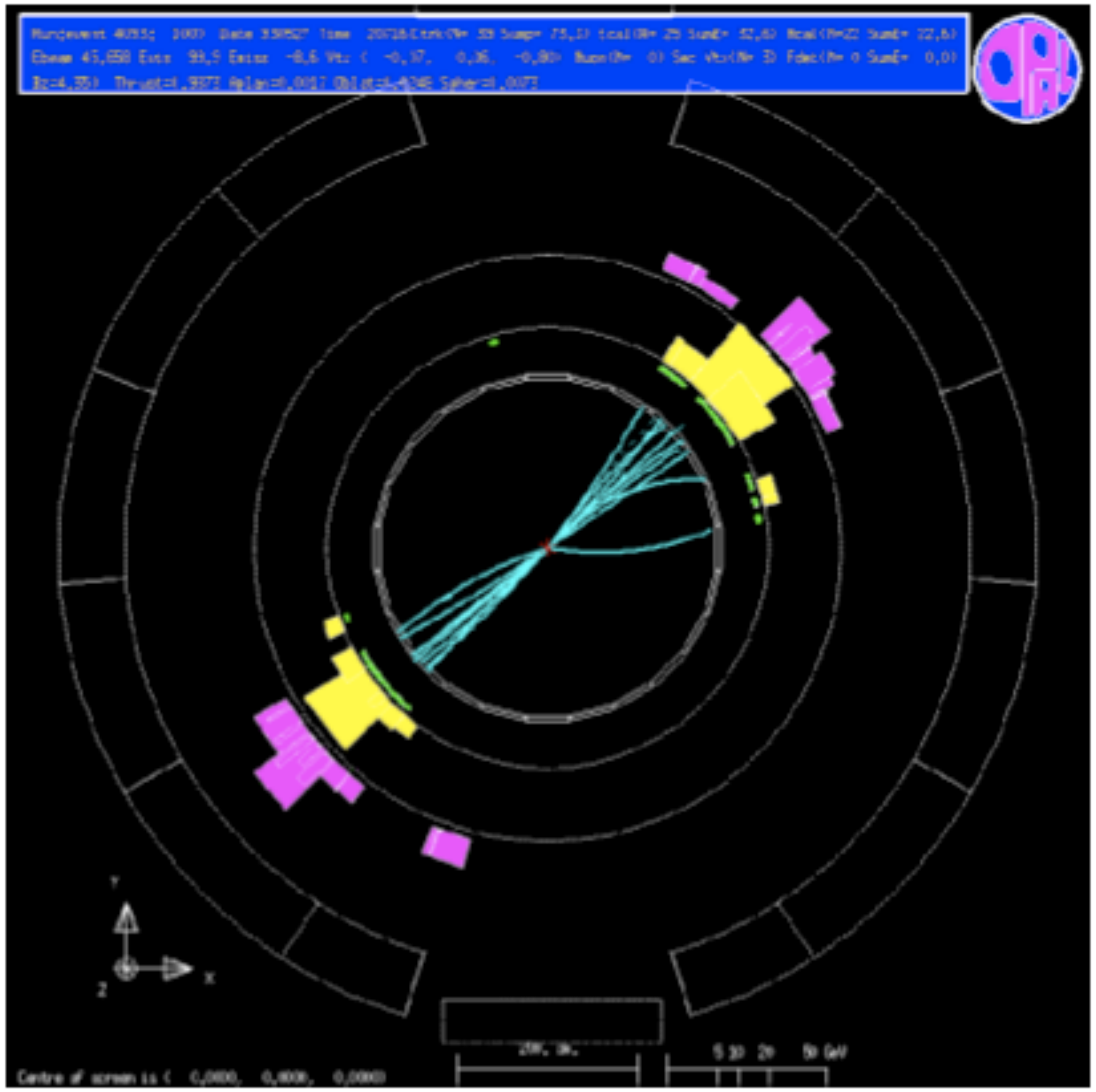


important for SUSY searches

[Mangano]

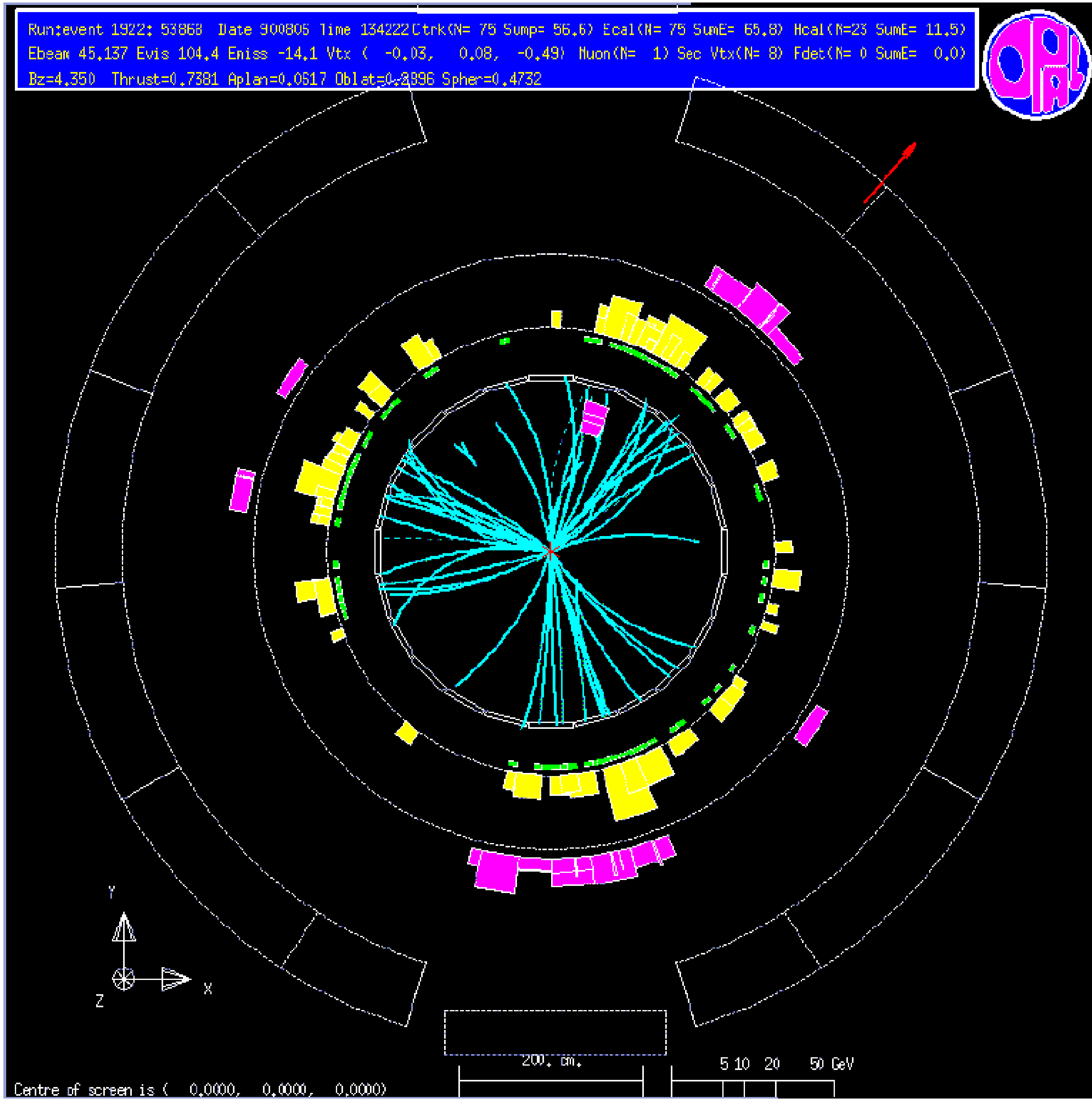
IV. Jets

⇨ Can we obtain more information on the hadron production besides the total cross section?



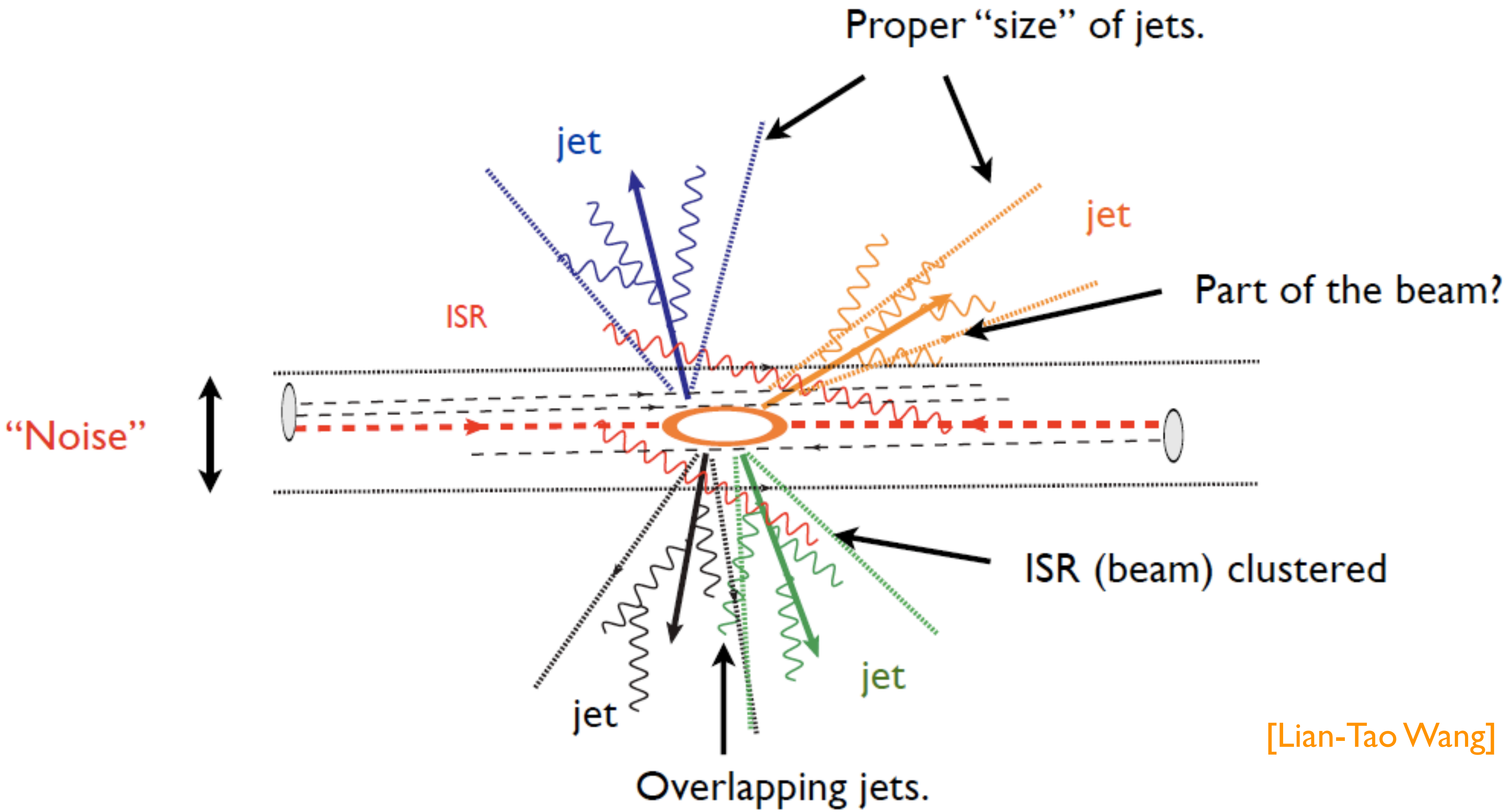
⇨ We expect that soft process don't change completely the high energy features $\Longrightarrow$ a spray of hadrons follows the direction of the original quarks and gluons.

Three jet event:



- why not 4?
- Which particles belong to a jet?
- how to get $p_{parton} \simeq p_{jet}$?

Not an easy task:



[Lian-Tao Wang]

Criteria for a good jet recipe: [Snowmass]

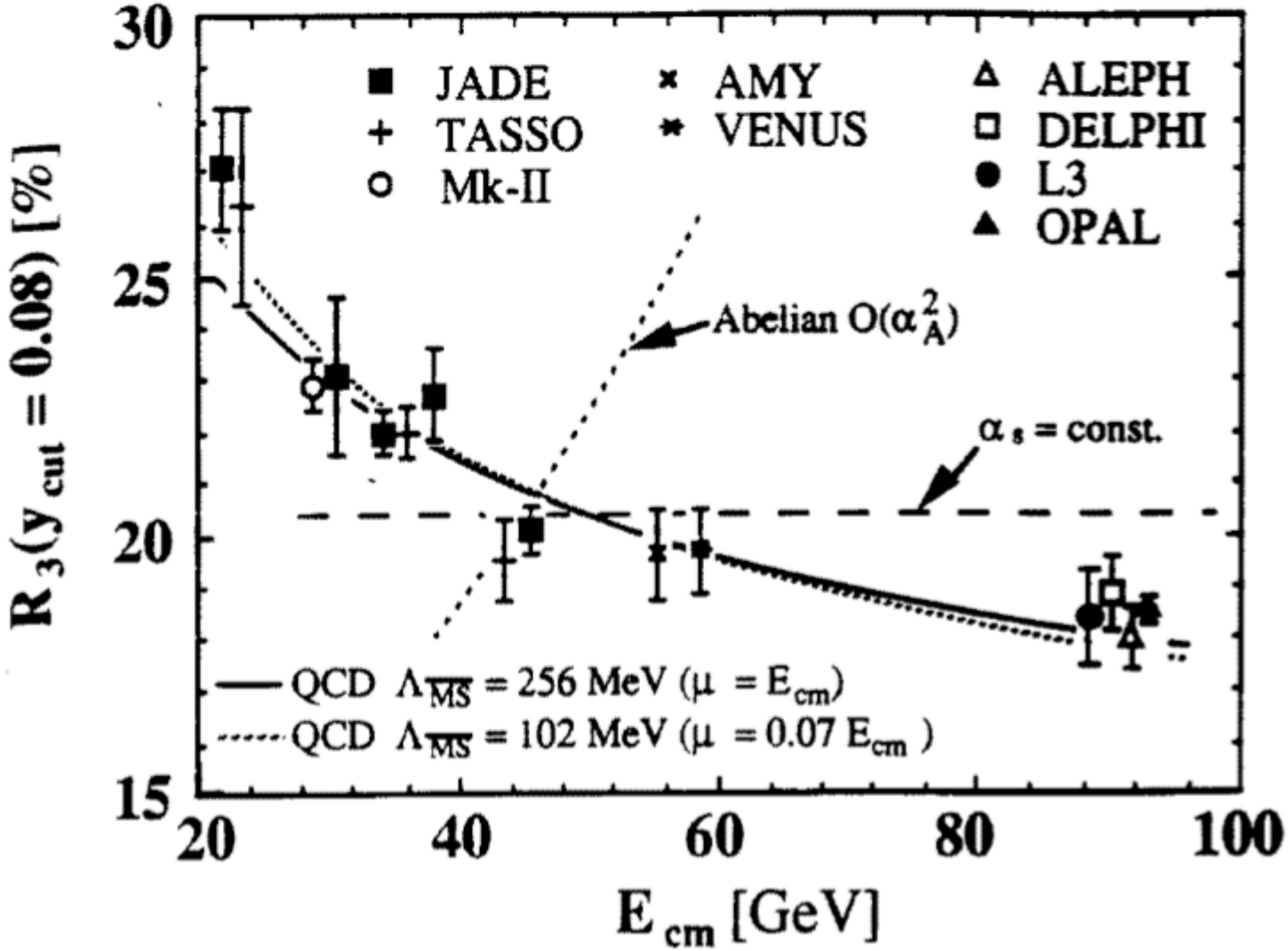
1. Simple to implement in an experimental analysis
2. Simple to implement in a theoretical calculation
3. Defined at any order of perturbation theory
4. Yields finite cross sections at any order of PT
5. Yields a cross section rather insensitive to hadronization

controls the number of jets

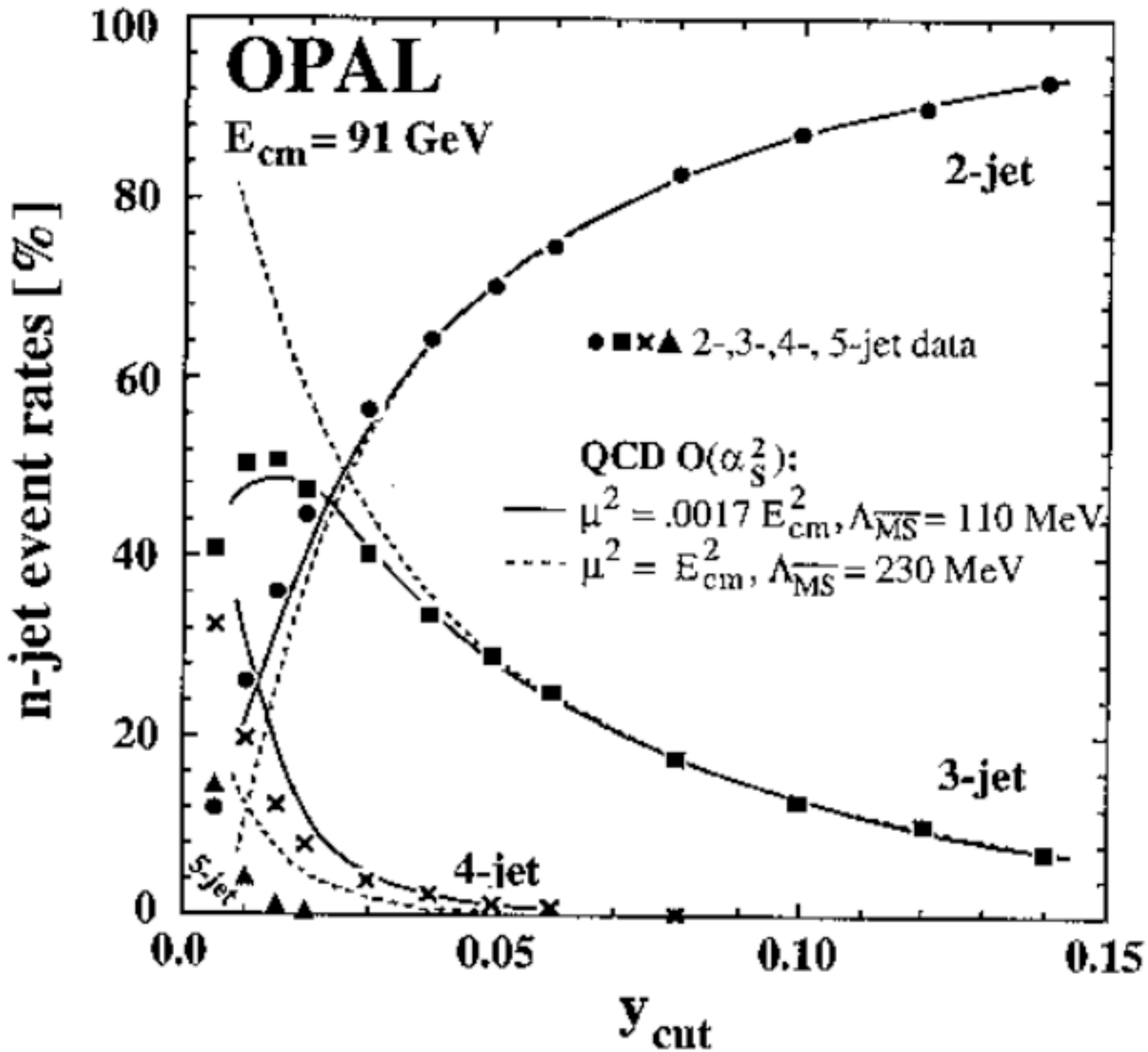
⇨ The JADE jet algorithm is the following: Consider n particles/partons and a cut y

- Identify the pair with minimal invariant mass $\bar{m}$. If $\bar{m}^2 < ys$ join the two particle into a single cluster.
- Apply the previous procedure to the $n-1$ particles and clusters until we can not form new clusters.
- The number of particles/clusters at the end of the process is the number of jets.

➡ This expression describes well the energy dependence of R_3



➩ This expression also describes well the y dependence



A few jet algorithms

- Three popular jet algorithms are kT, anti-kT, and Cambridge/Aachen
- The distance and rule to join objets is

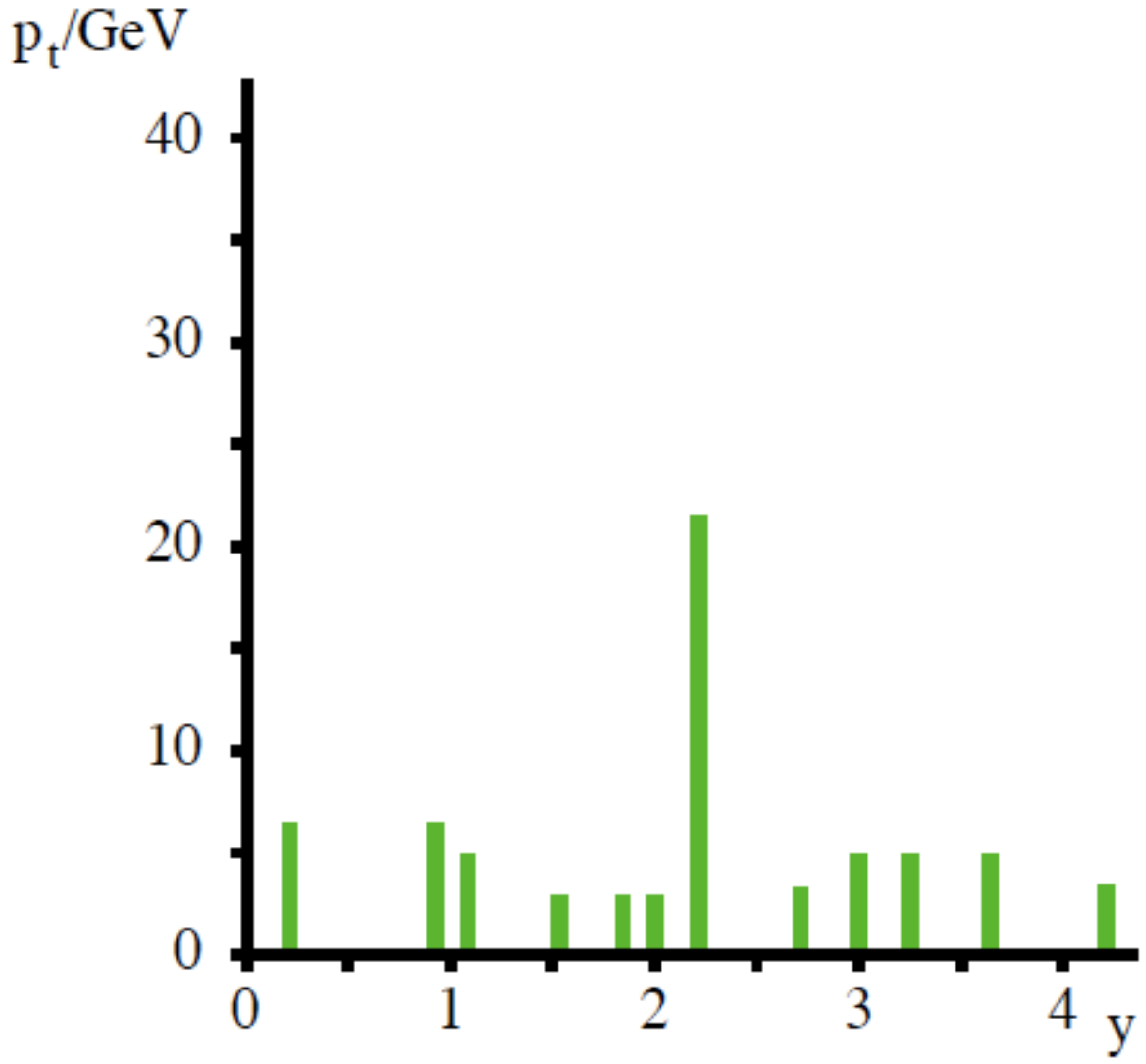
$$\mathbf{d_{ij}} = \min[\mathbf{p_{Ti}^{2\alpha}}, \mathbf{p_{Ti}^{2\alpha}}] \left(\frac{\mathbf{\Delta R_{ij}}}{\mathbf{R}}\right)^{\mathbf{2}} \quad \text{and} \quad \mathbf{d_{iB}} = \mathbf{p_{Ti}^{2\alpha}}$$

with $\mathbf{\Delta R_{ij}} = \sqrt{\mathbf{\Delta}\eta^{\mathbf{2}}_{\mathbf{ij}} + \mathbf{\Delta}\varphi^{\mathbf{2}}_{\mathbf{ij}}}$

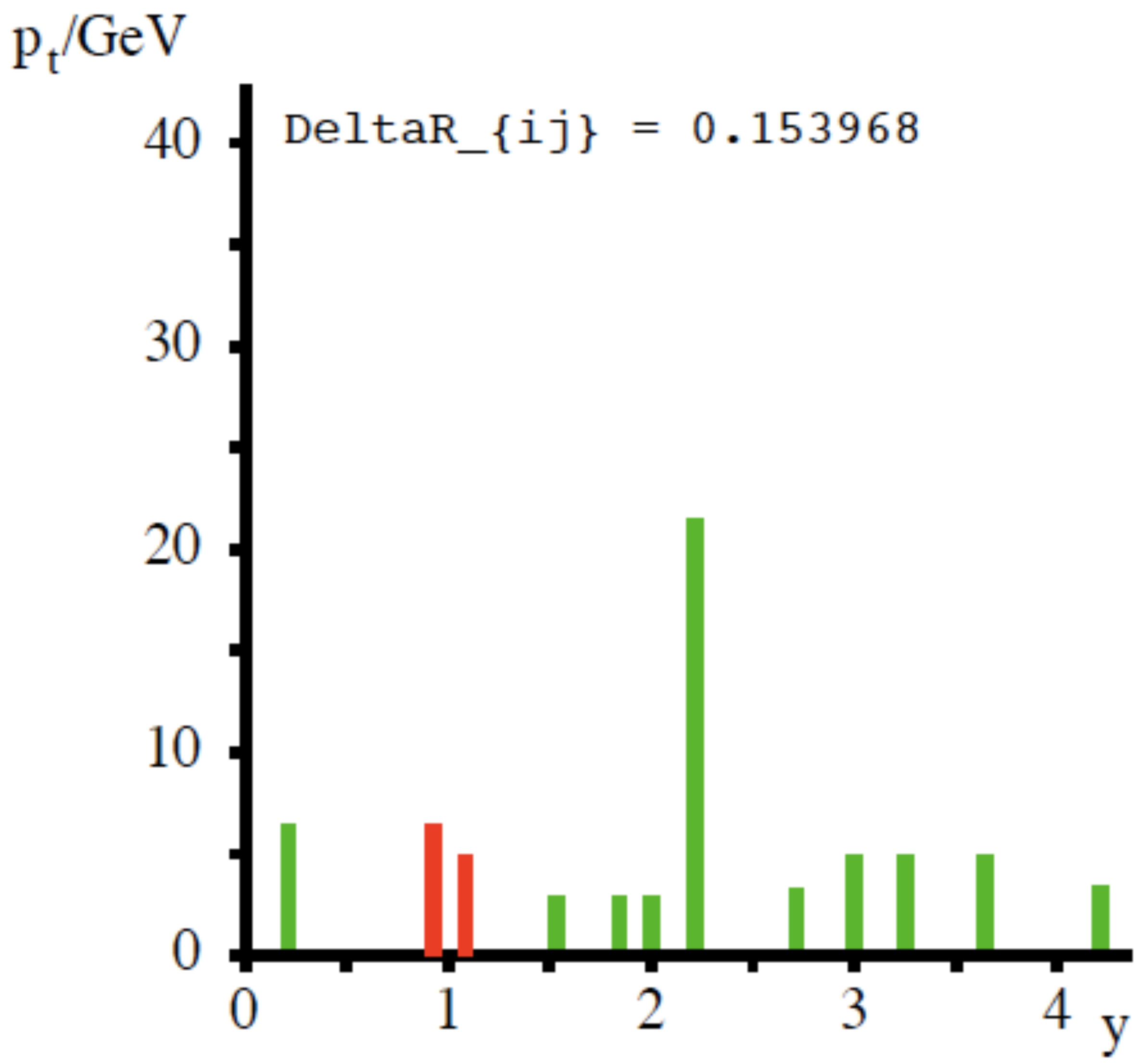
repeatedly combine objets until $\mathbf{d_{iB}}$ is the smaller distance. Then call it a jet, remove from the list and start again

- The choices are: kT ($\alpha = \mathbf{1}$); anti-kT ($\alpha = -\mathbf{1}$); C/A ($\alpha = \mathbf{0}$)

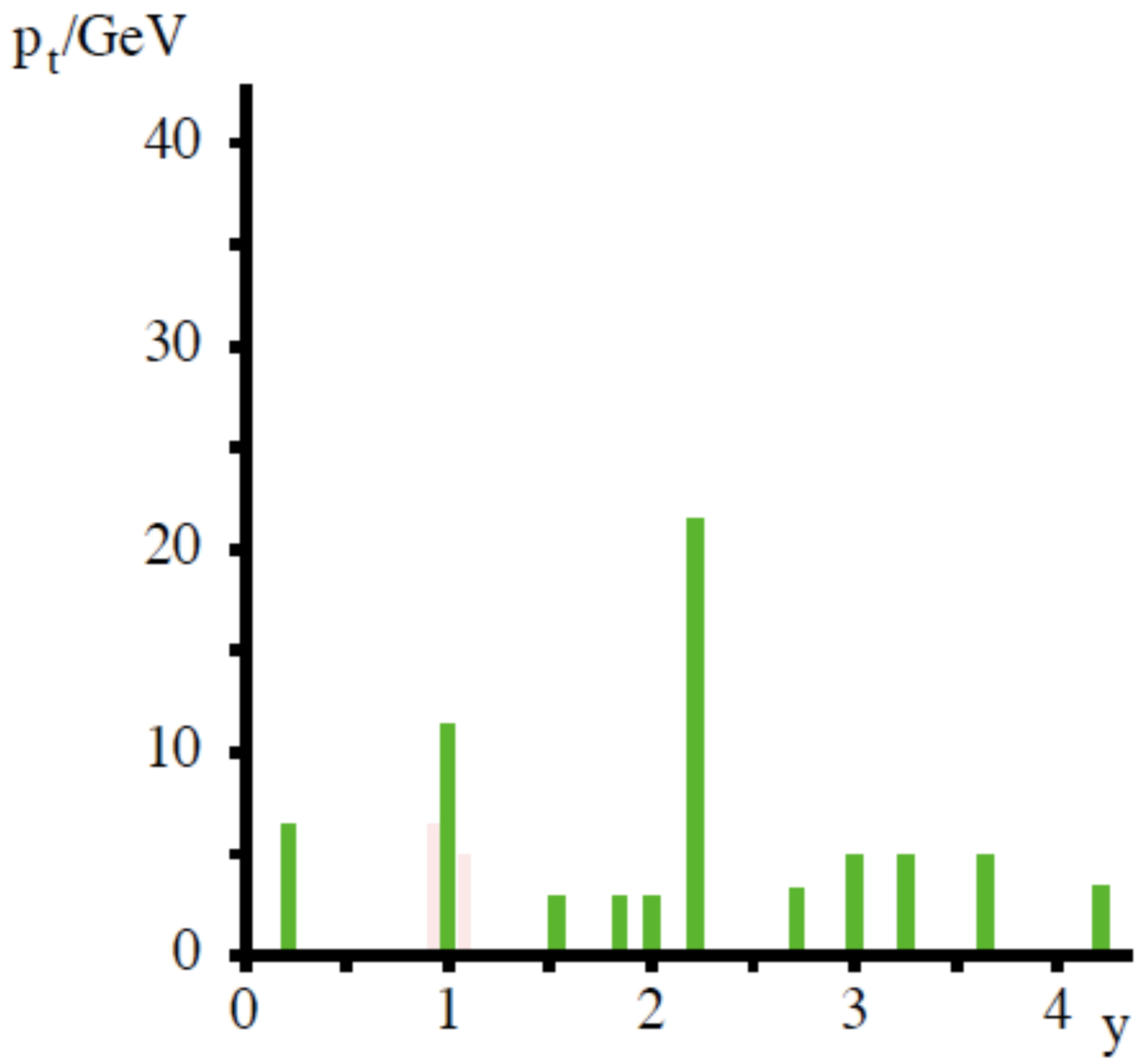
- Example with C/A algorithm [borrow from G. Salam]



• Example with C/A algorithm [borrow from G. Salam]



- Example with C/A algorithm [borrow from G. Salam]



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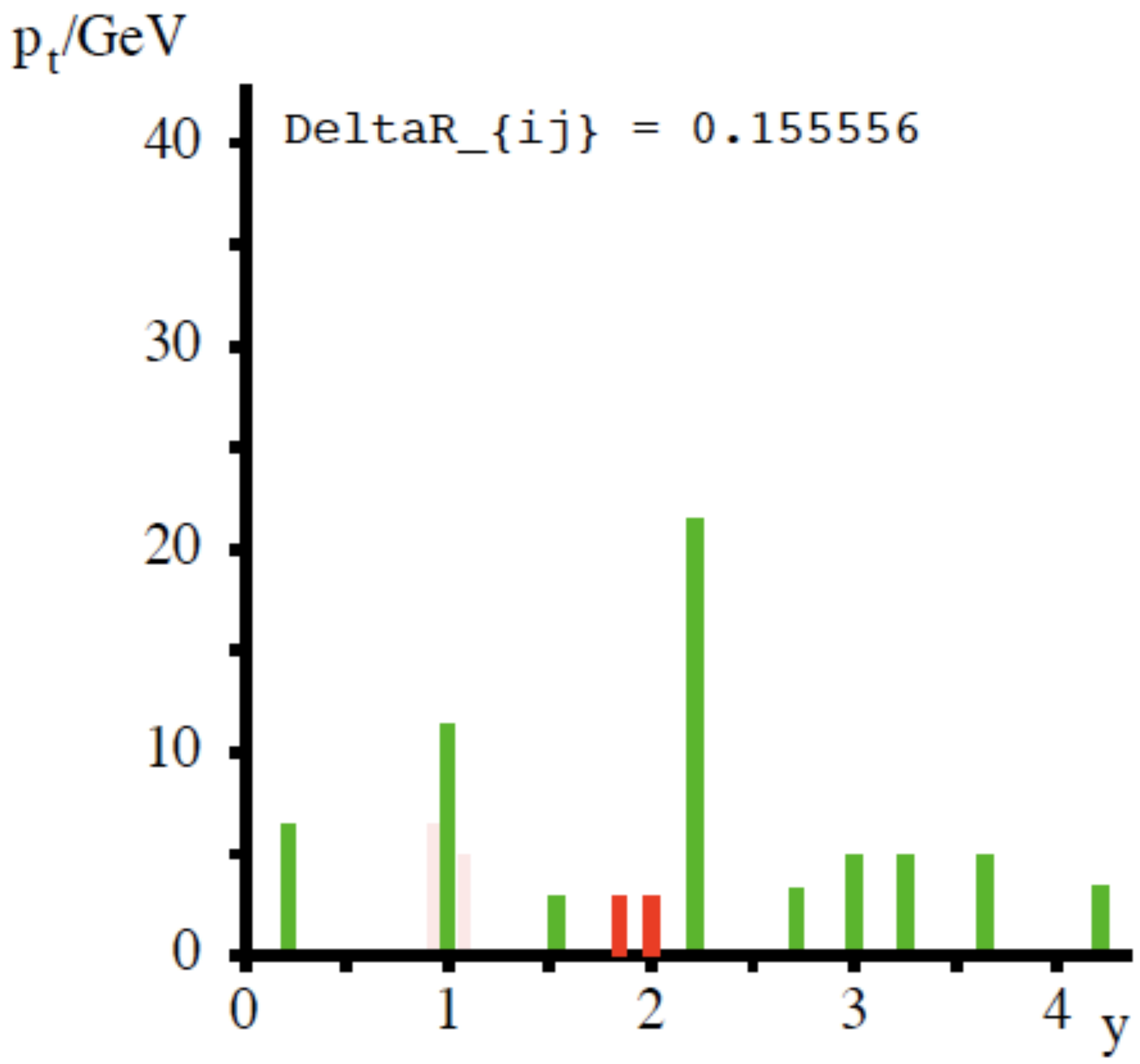

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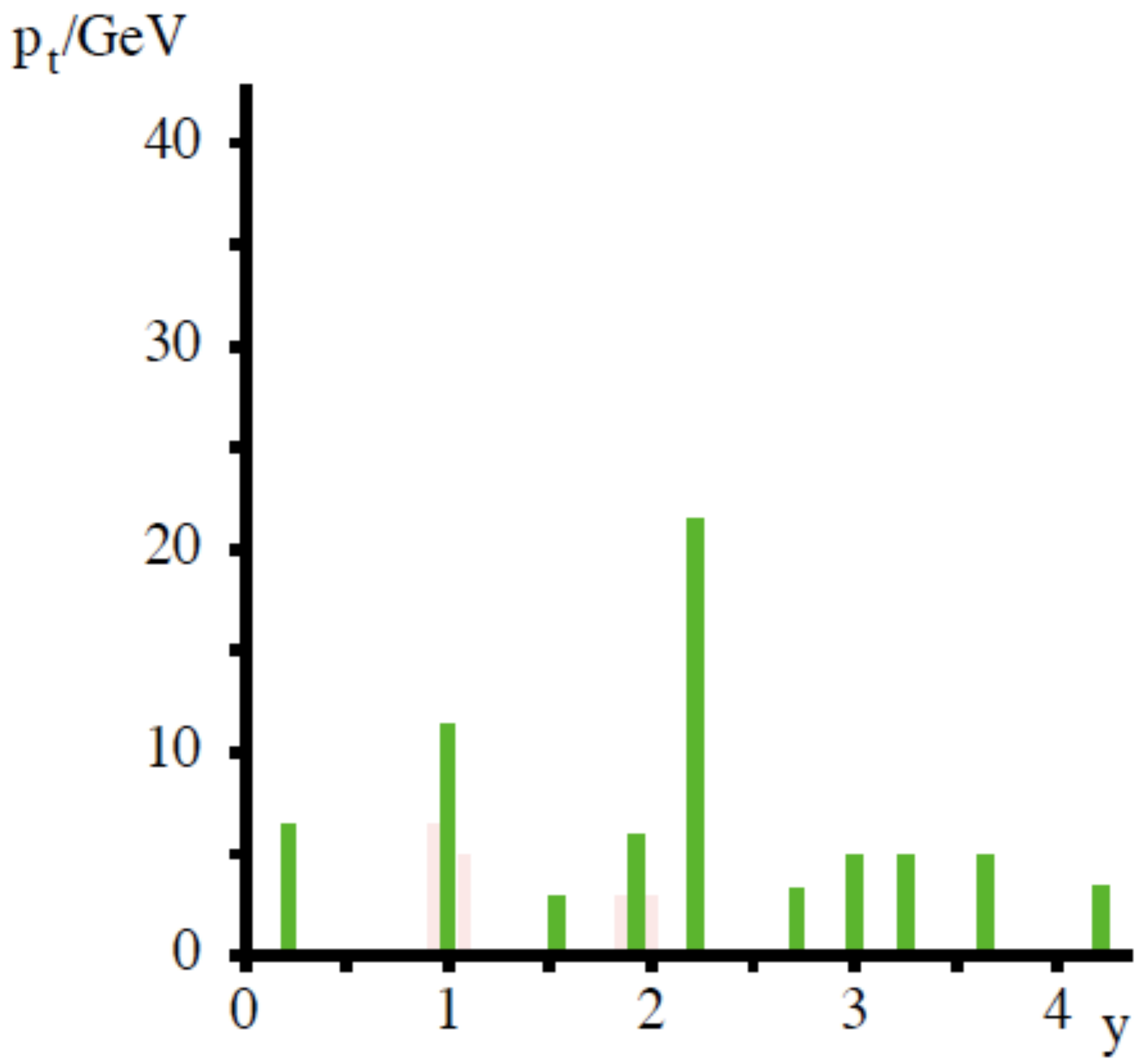

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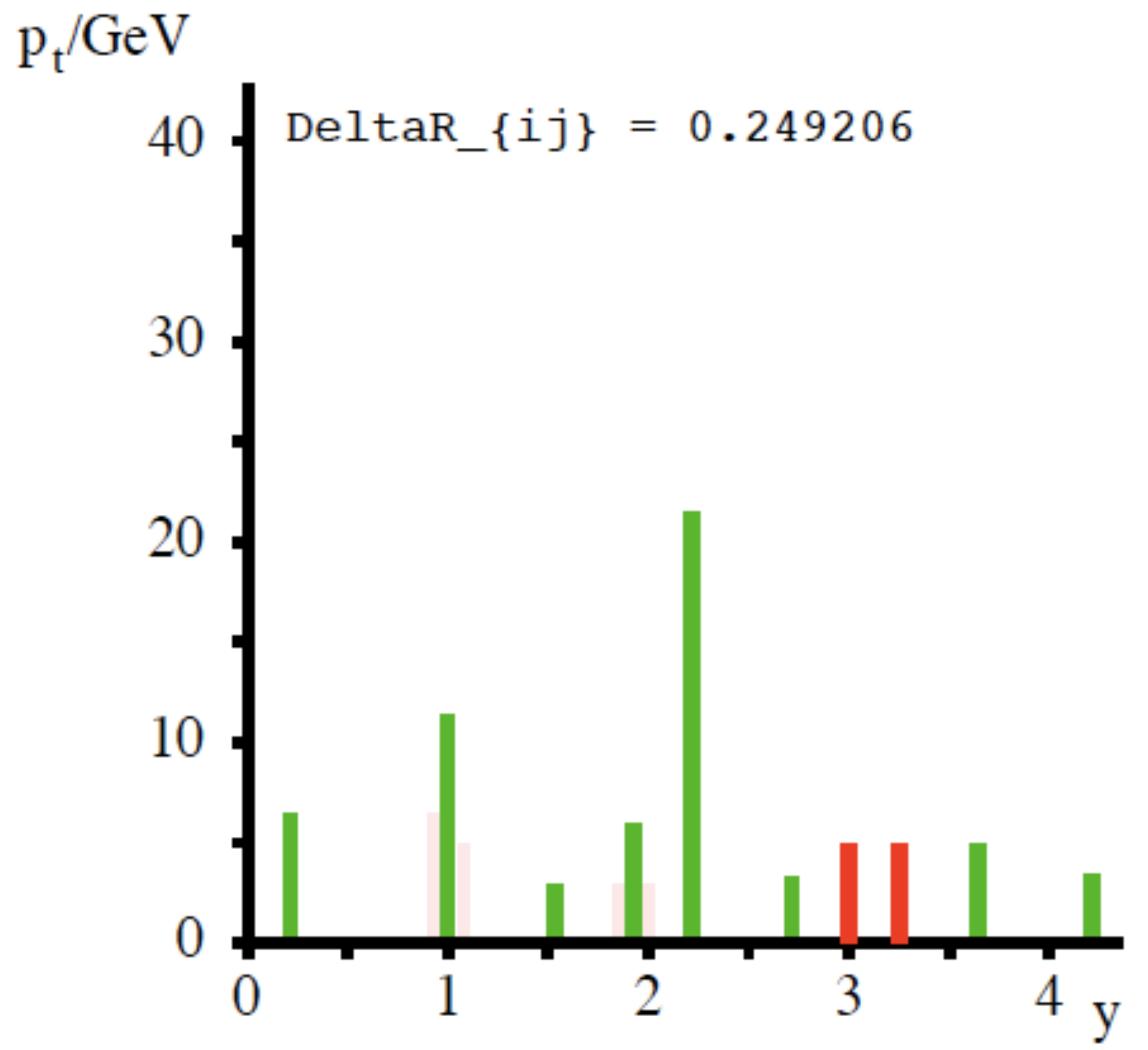

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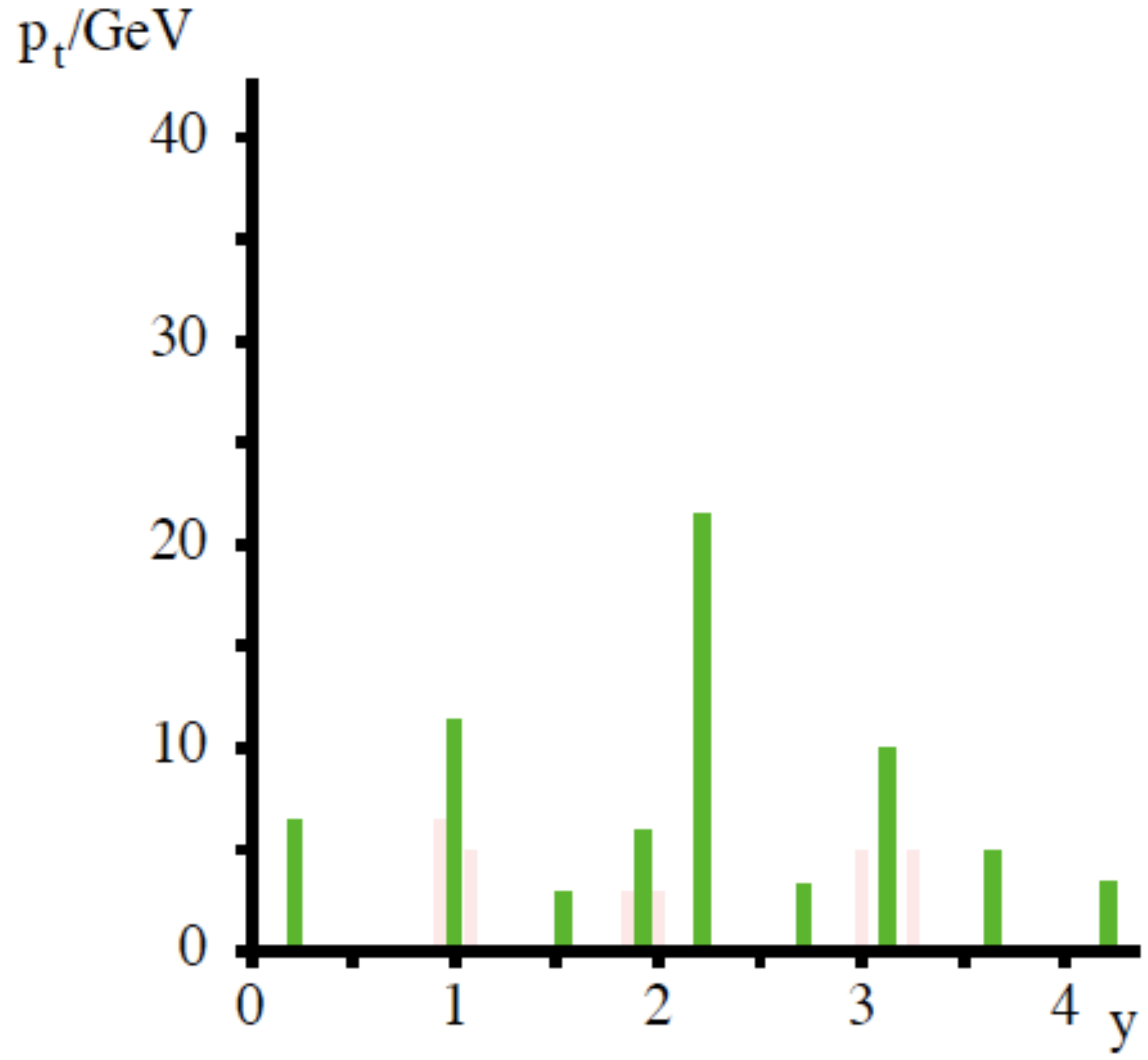

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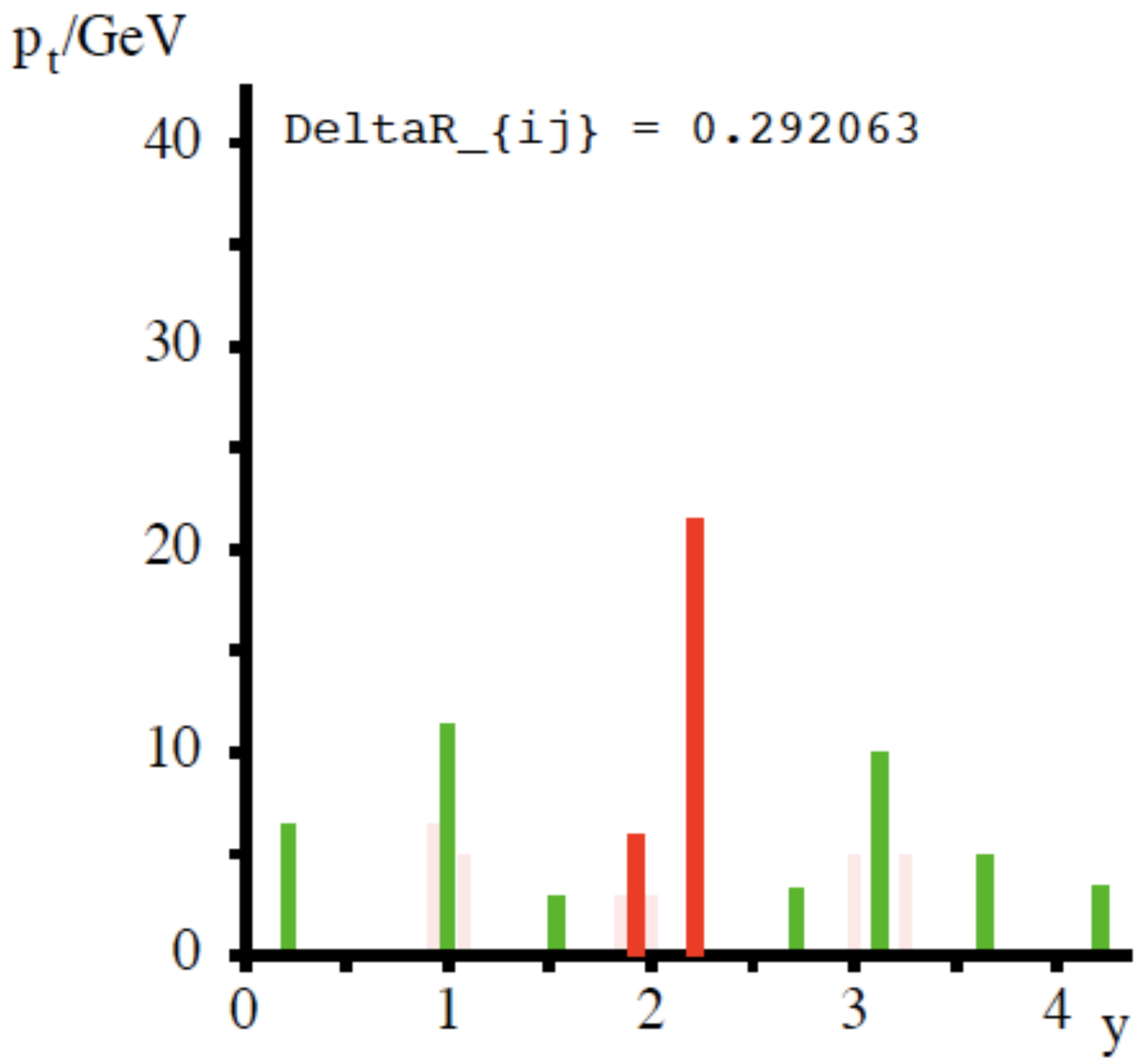

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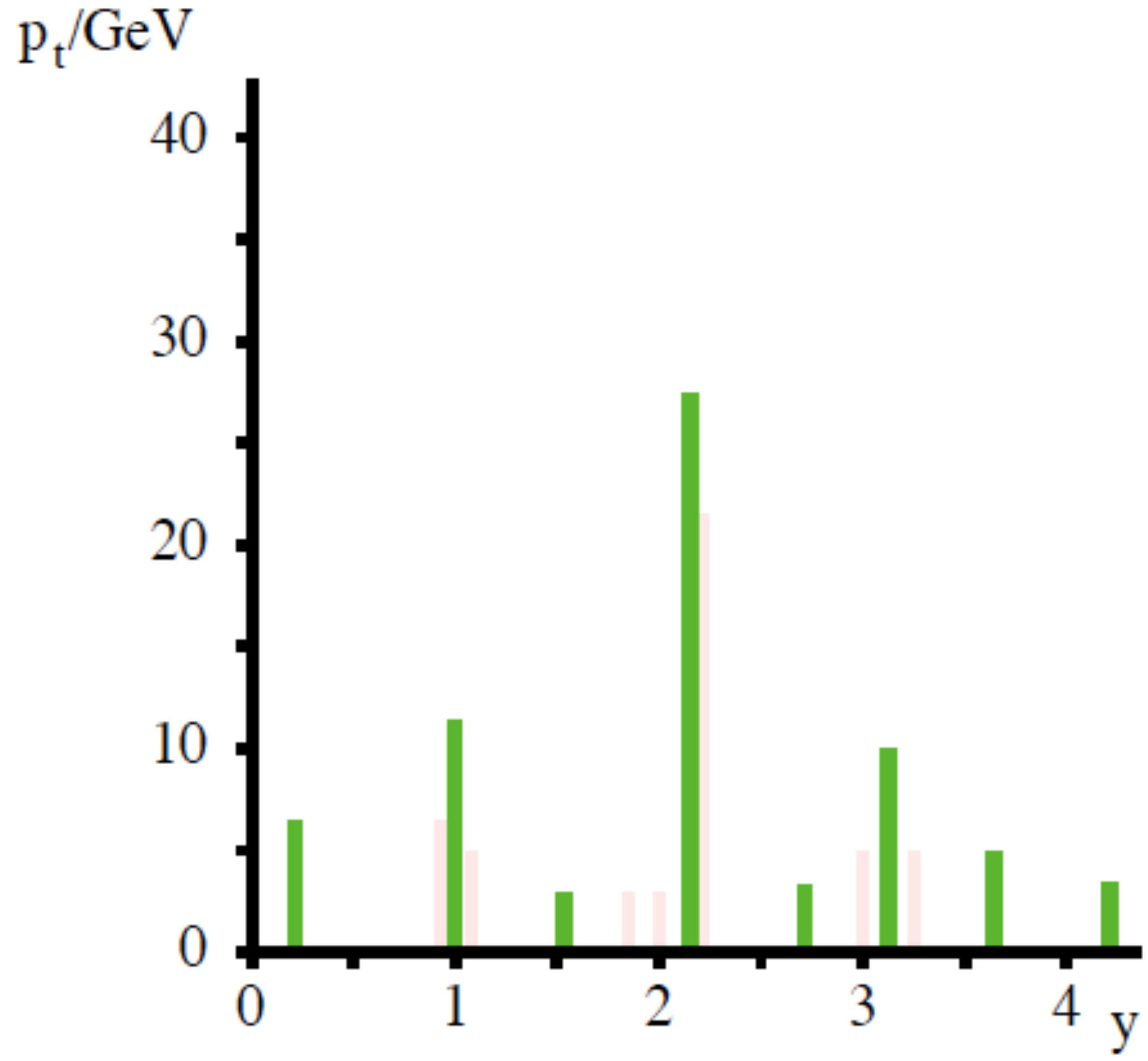

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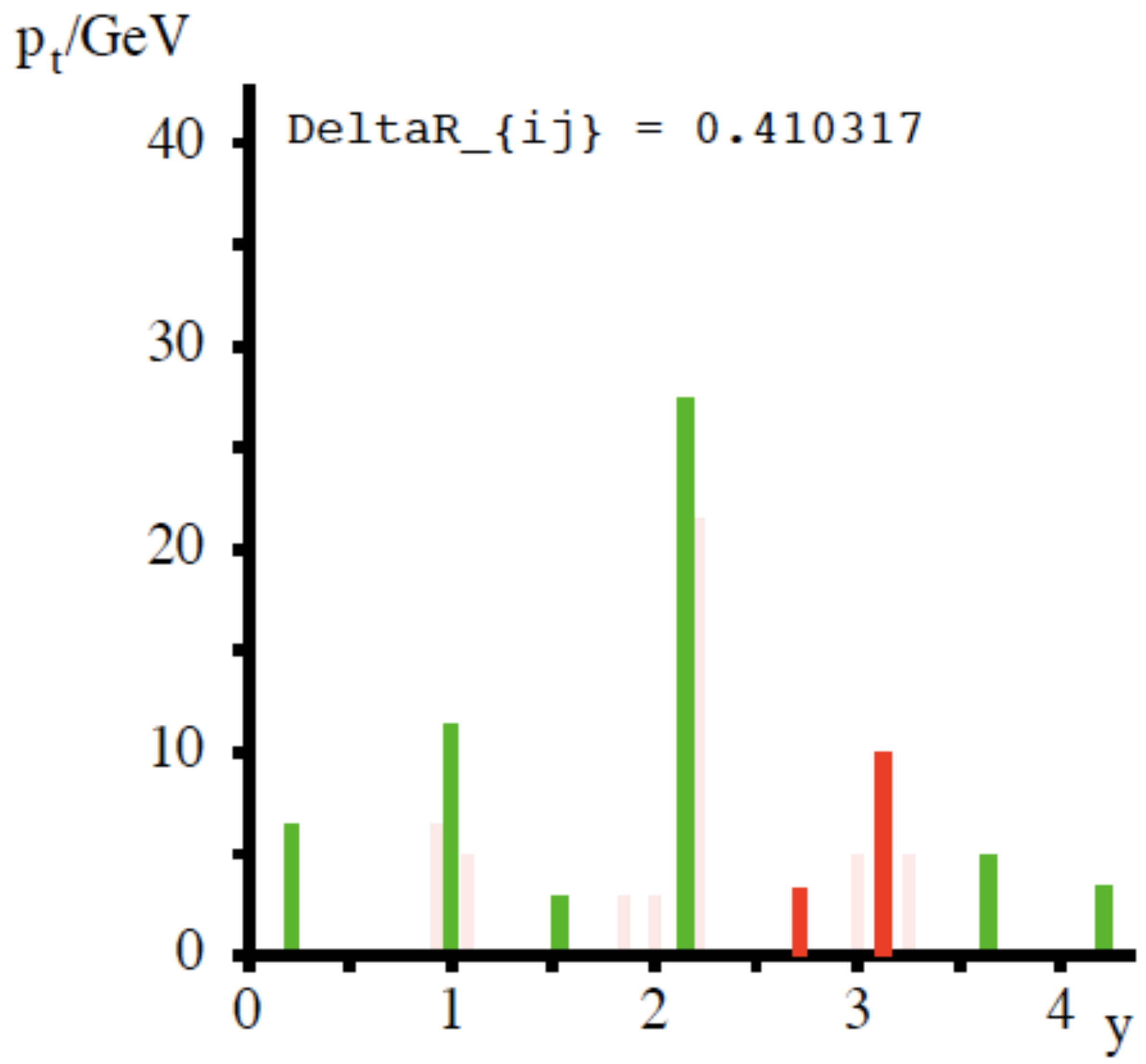

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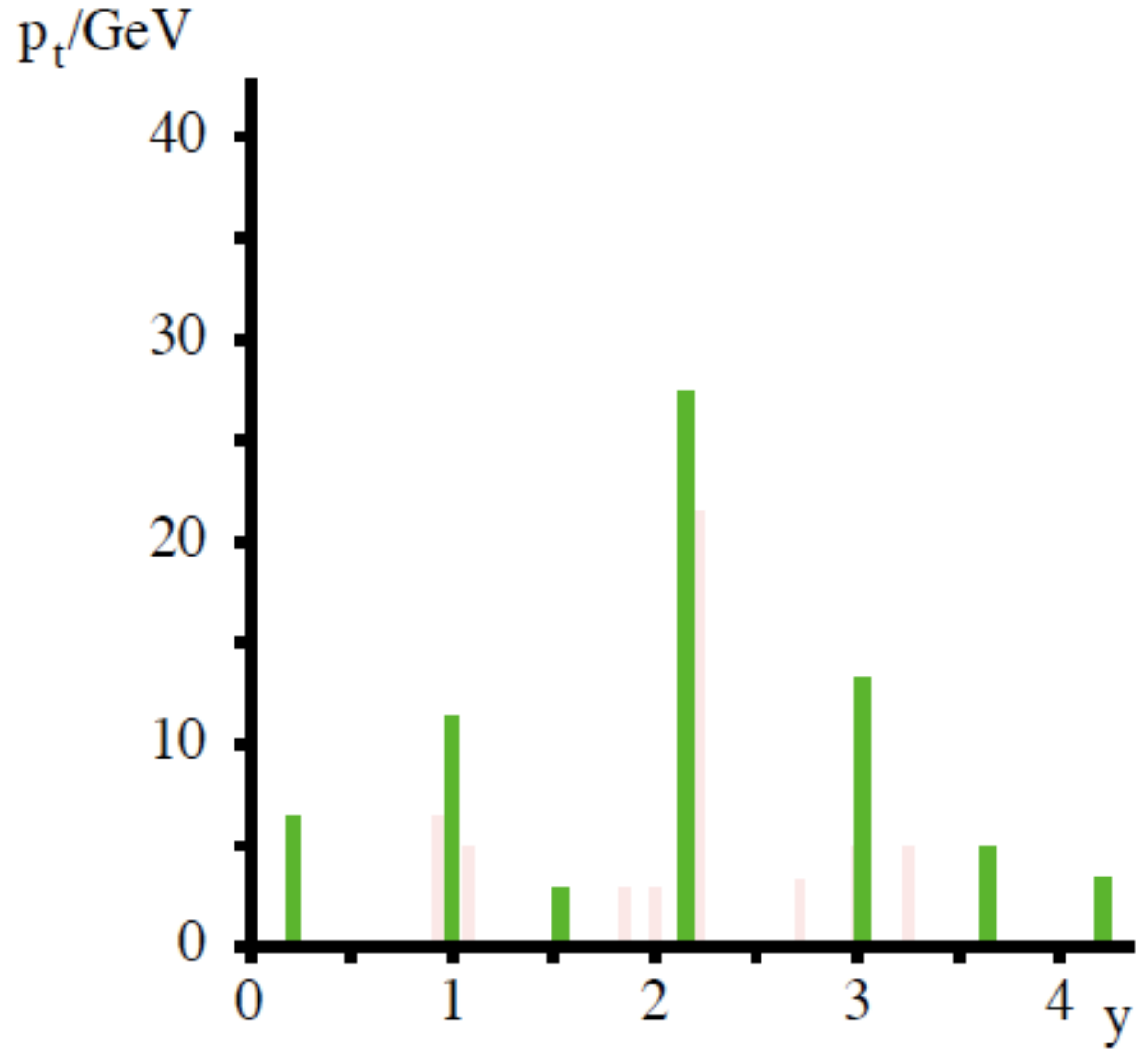

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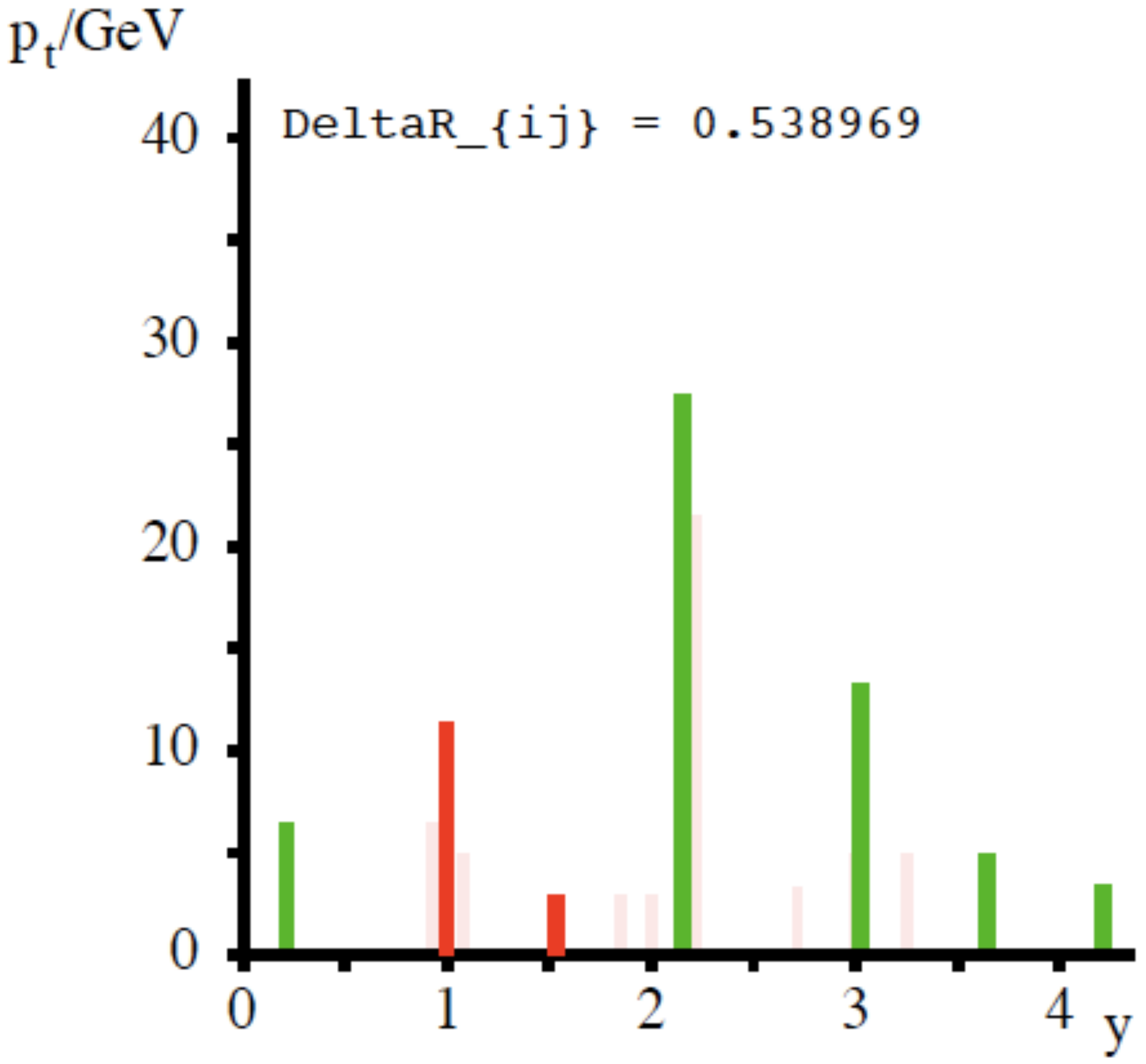

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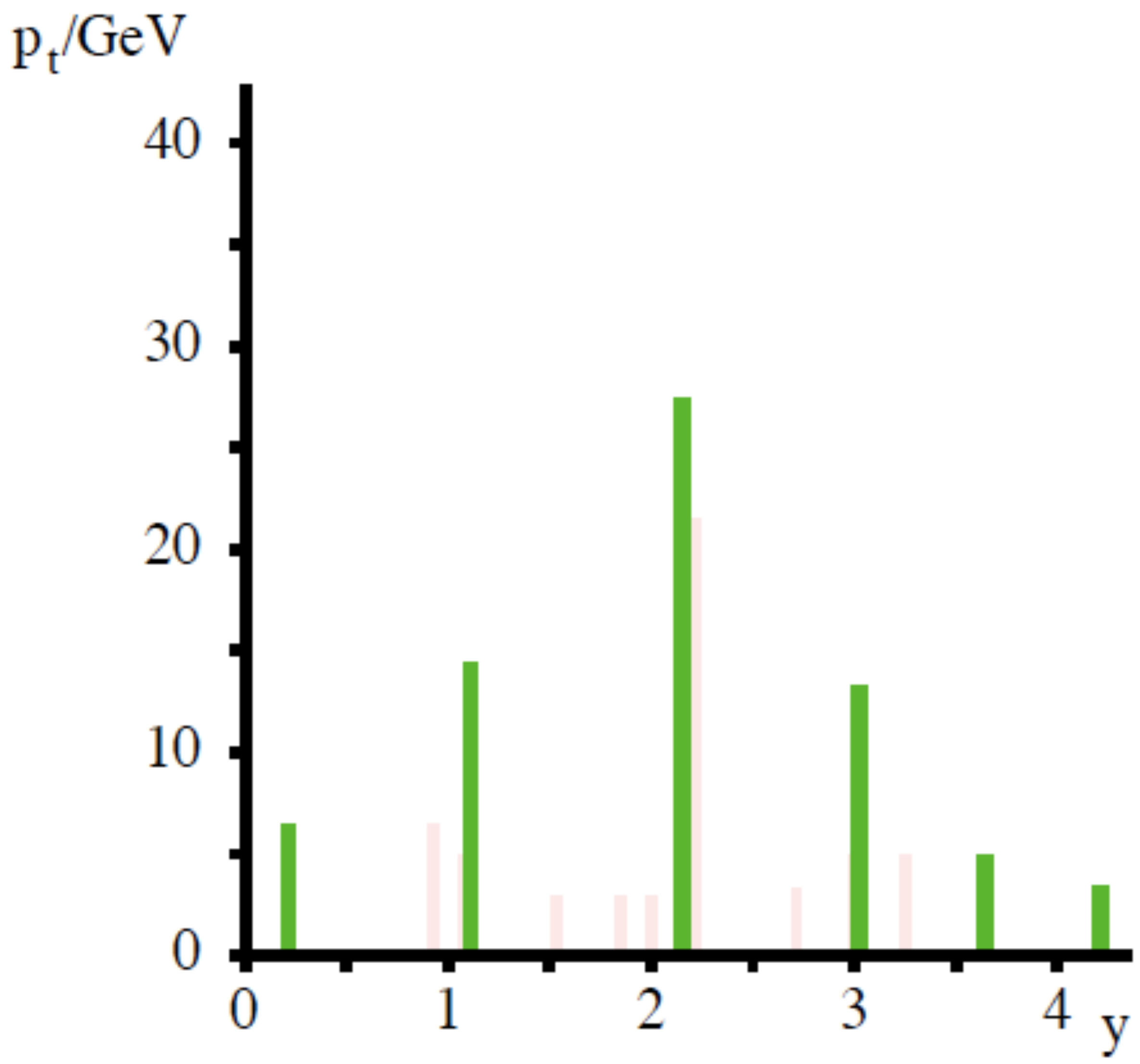

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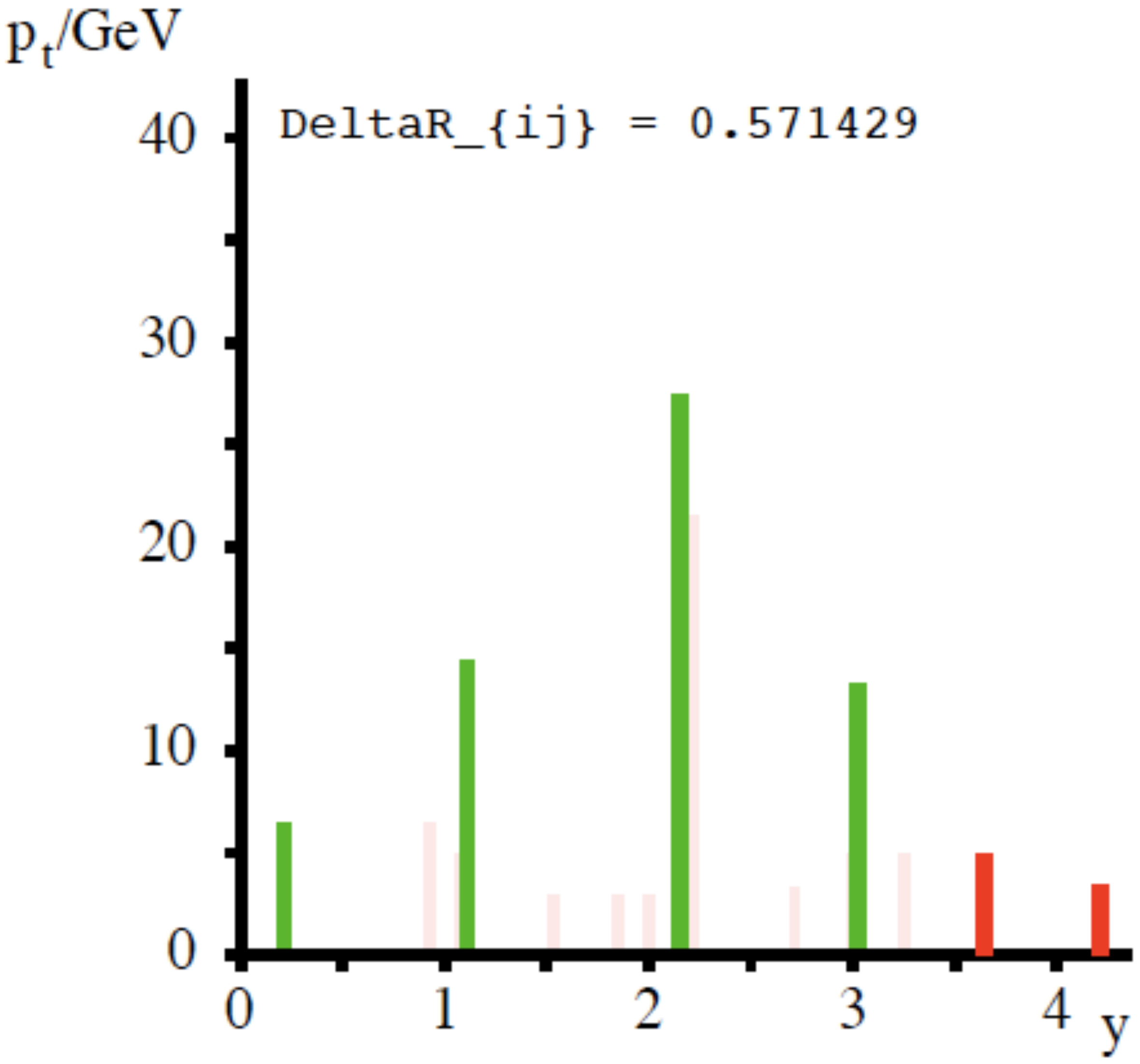

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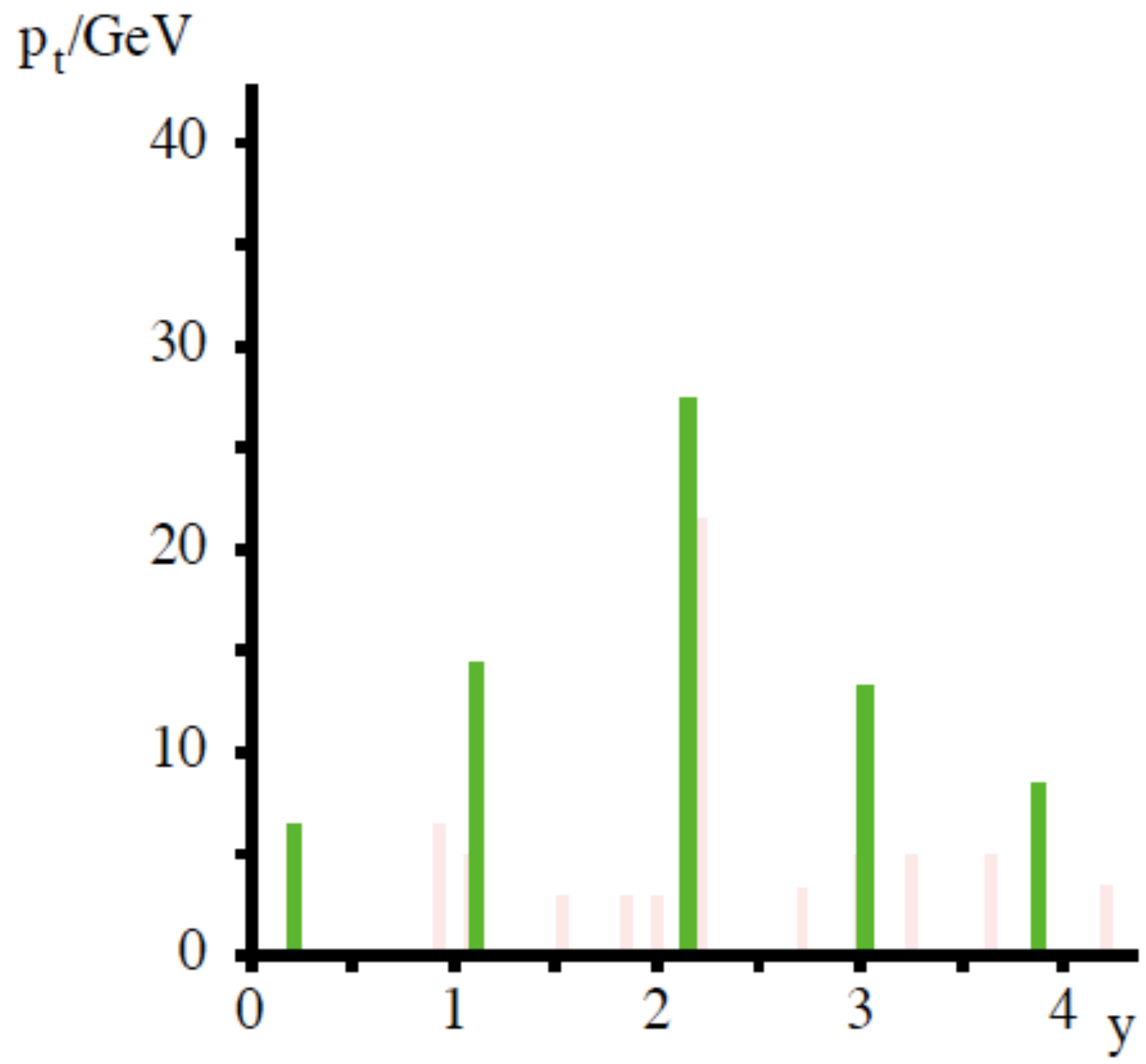

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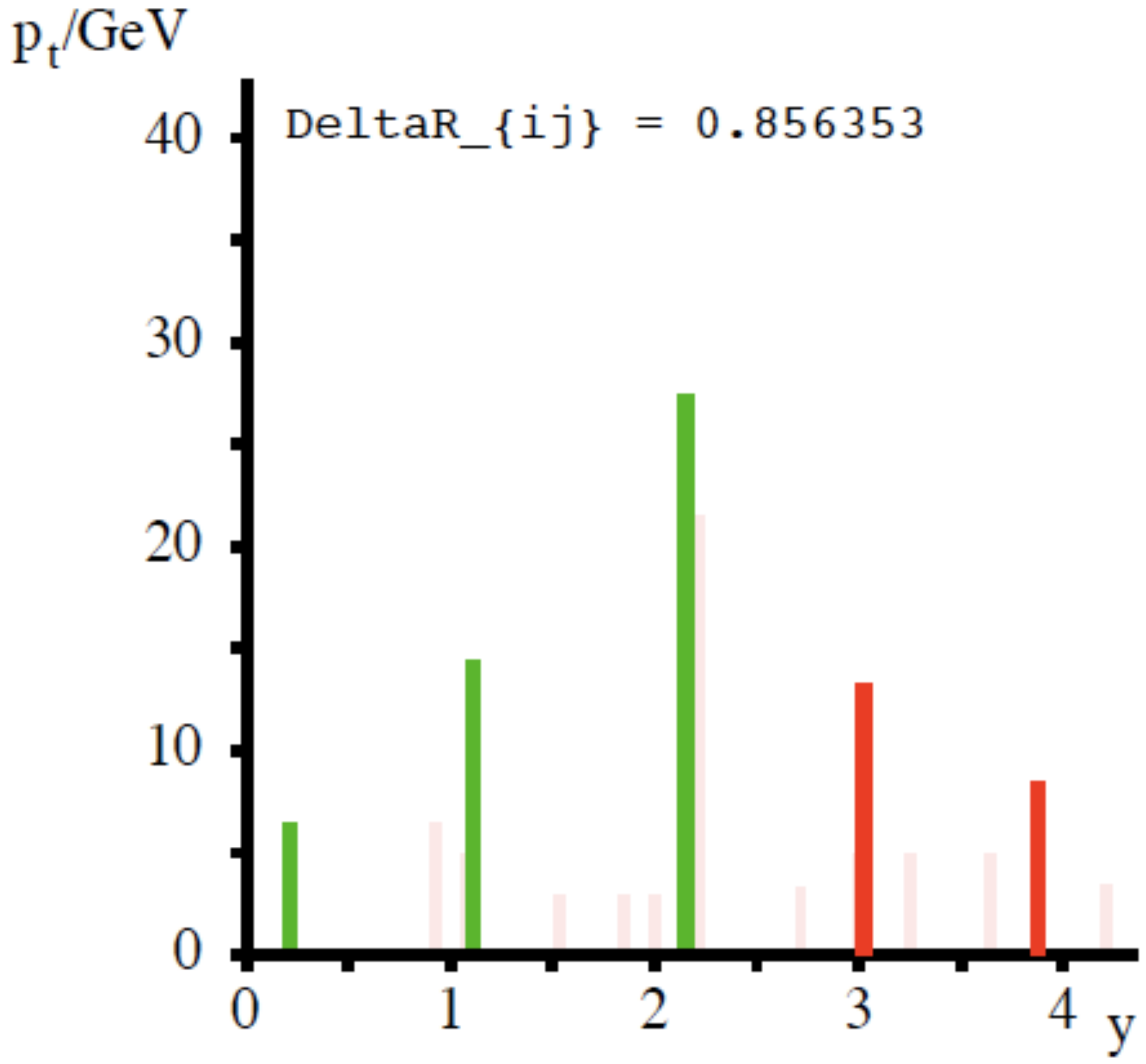

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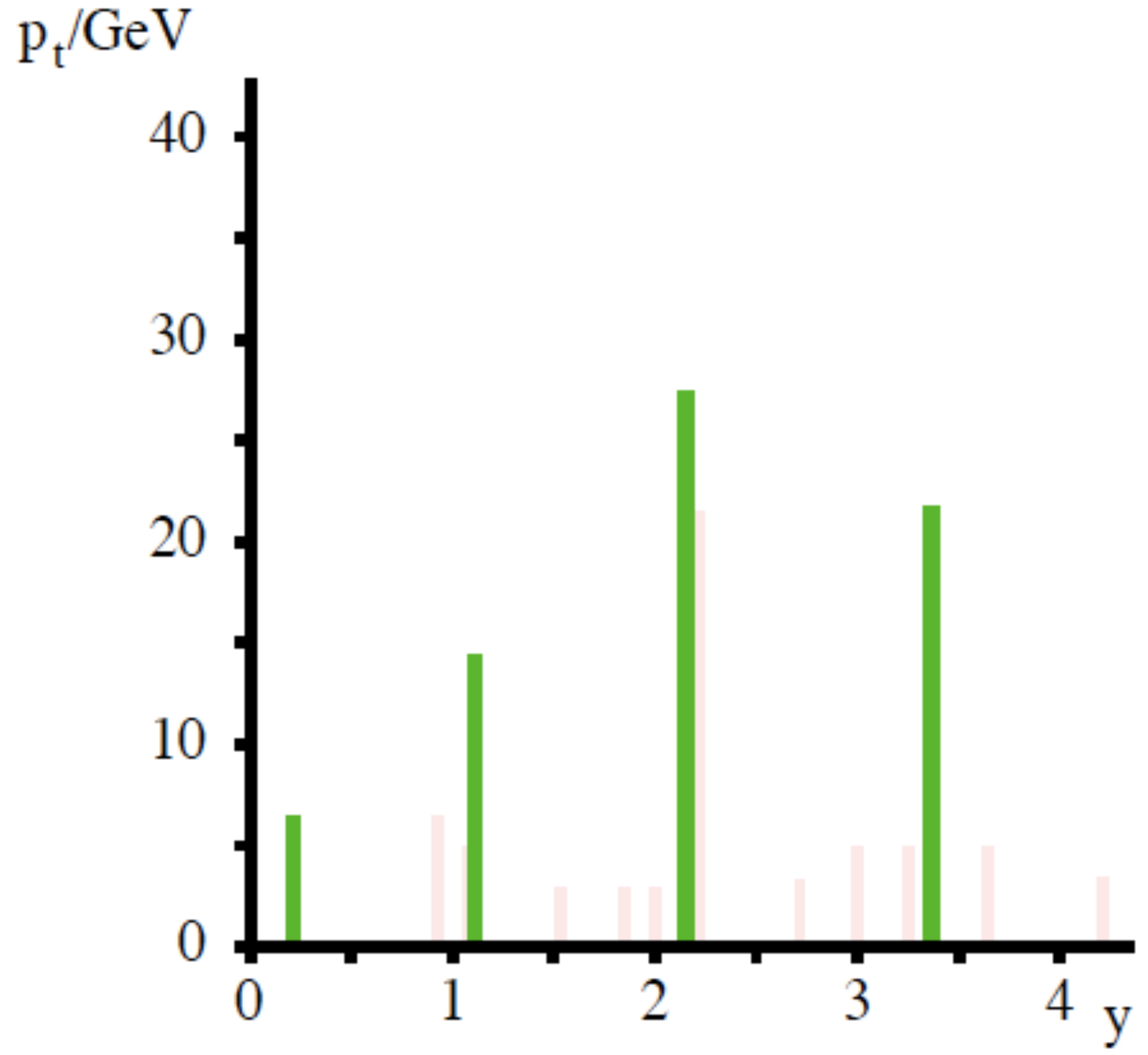

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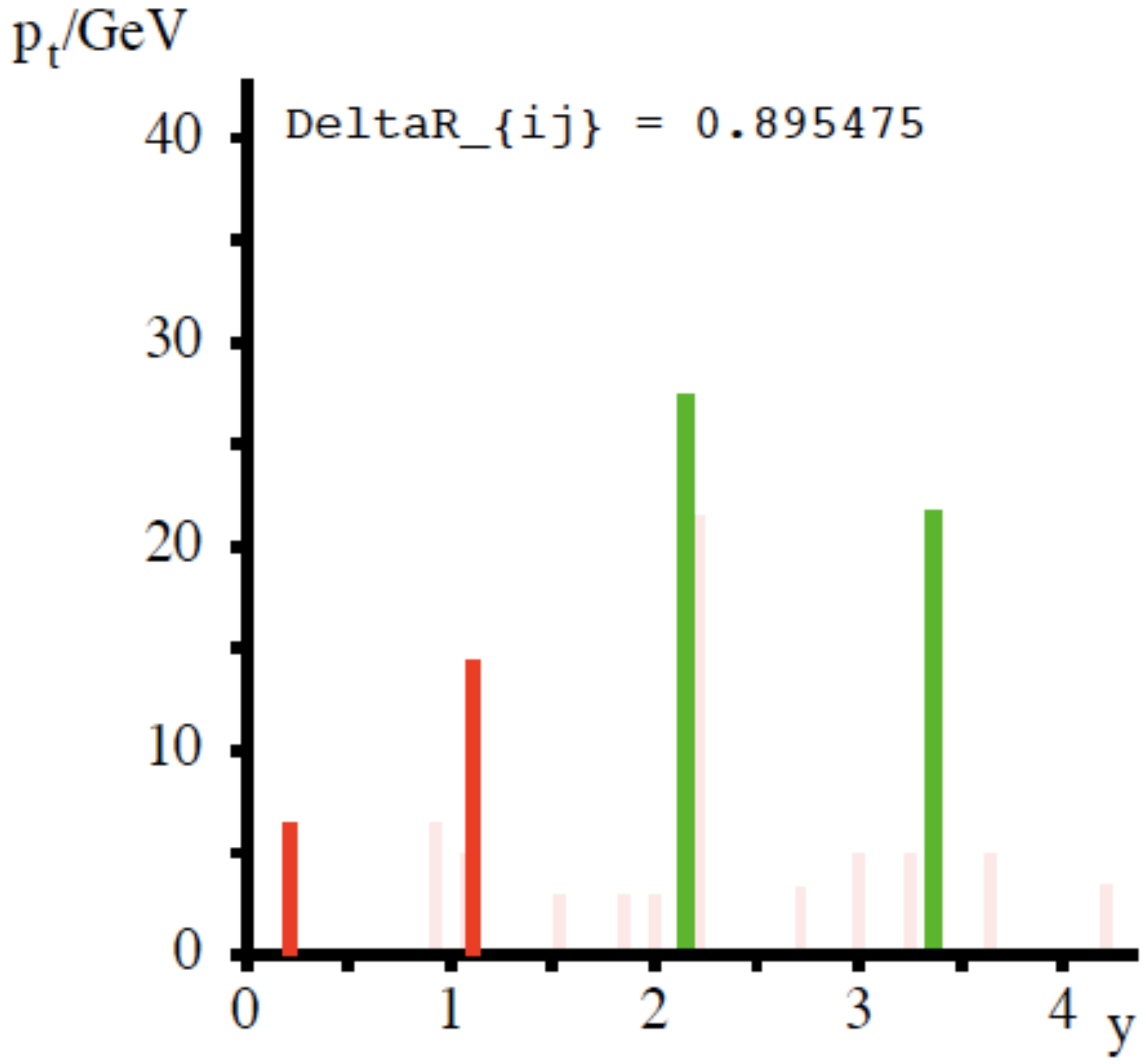

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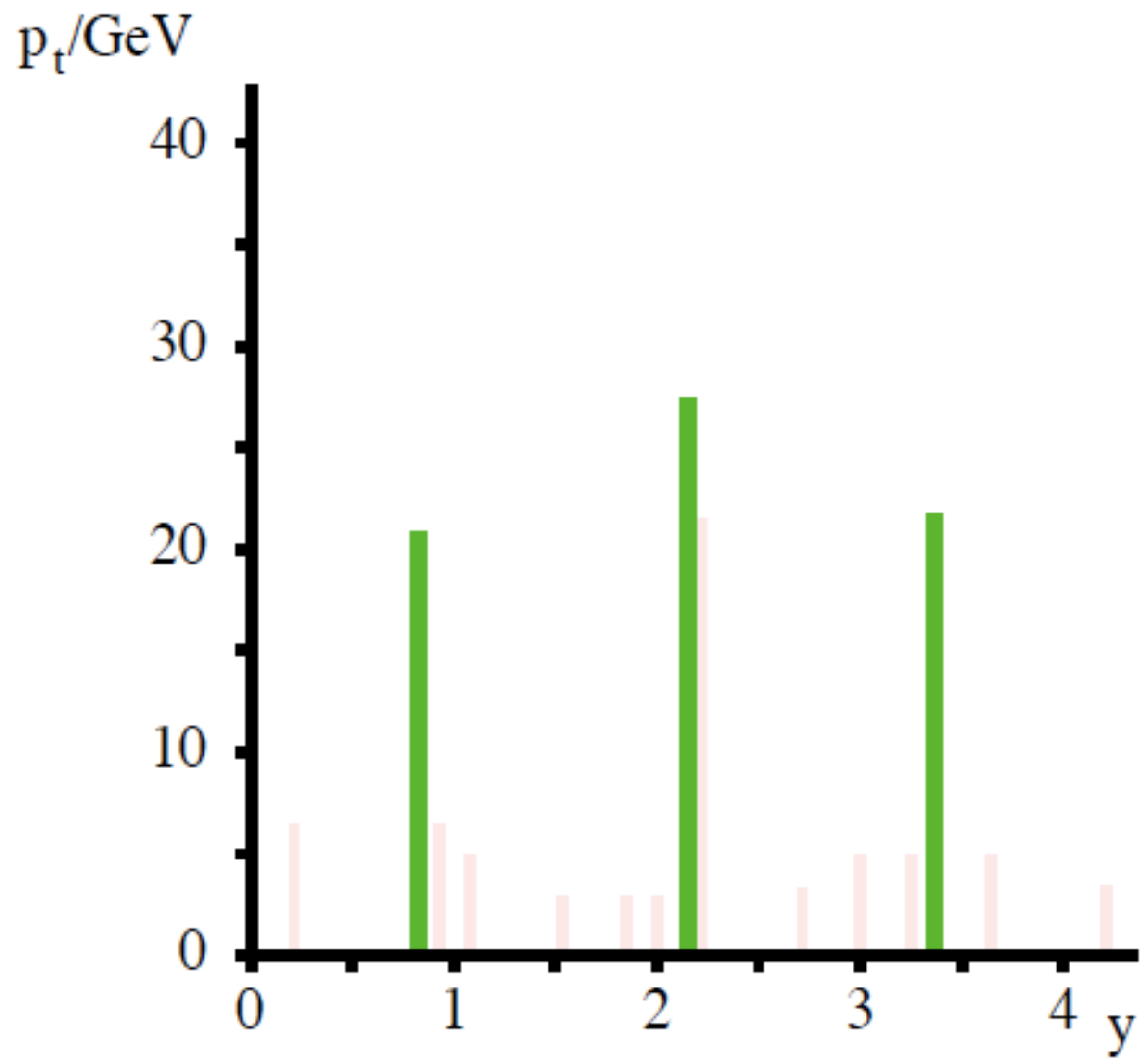

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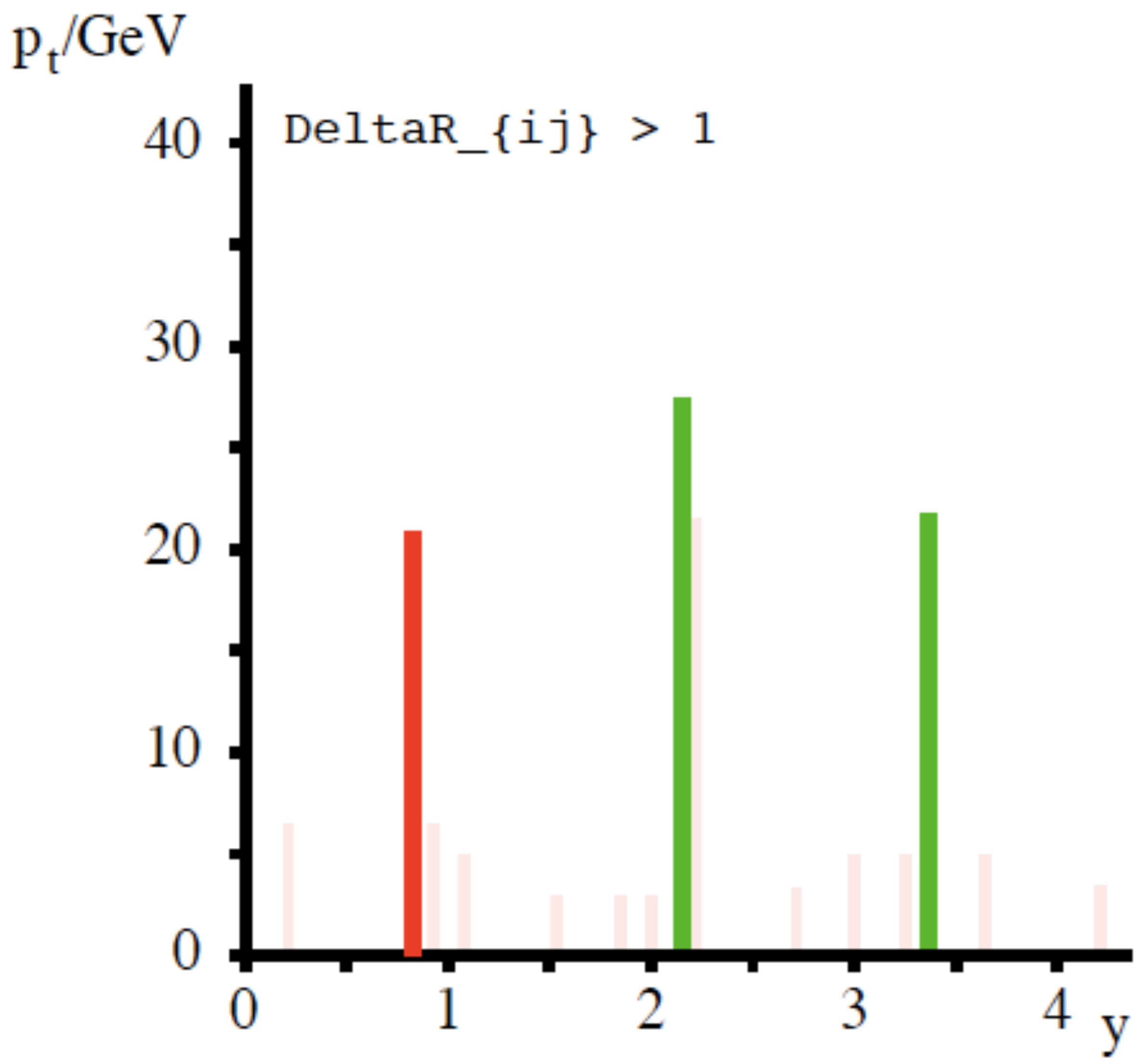

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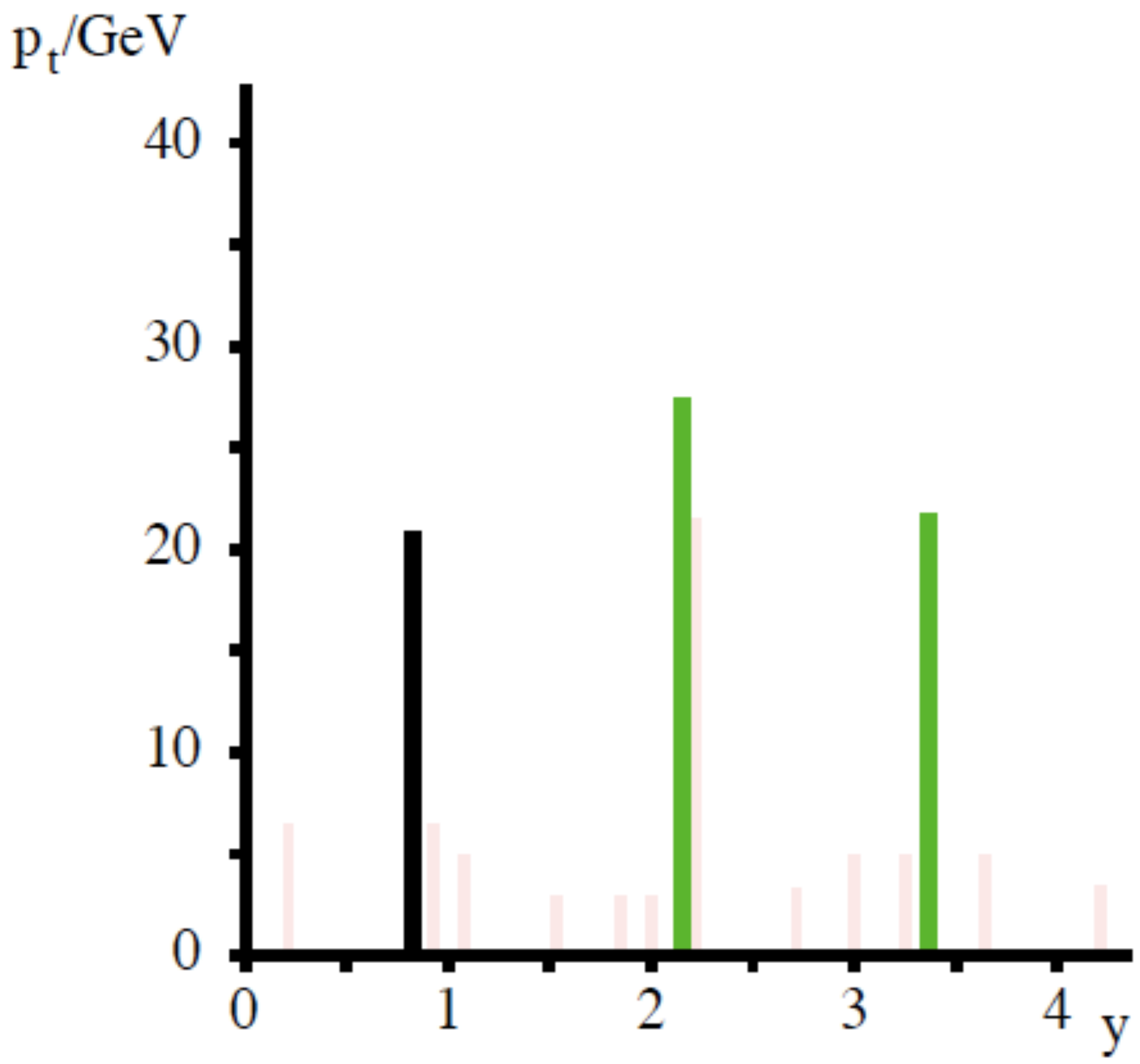

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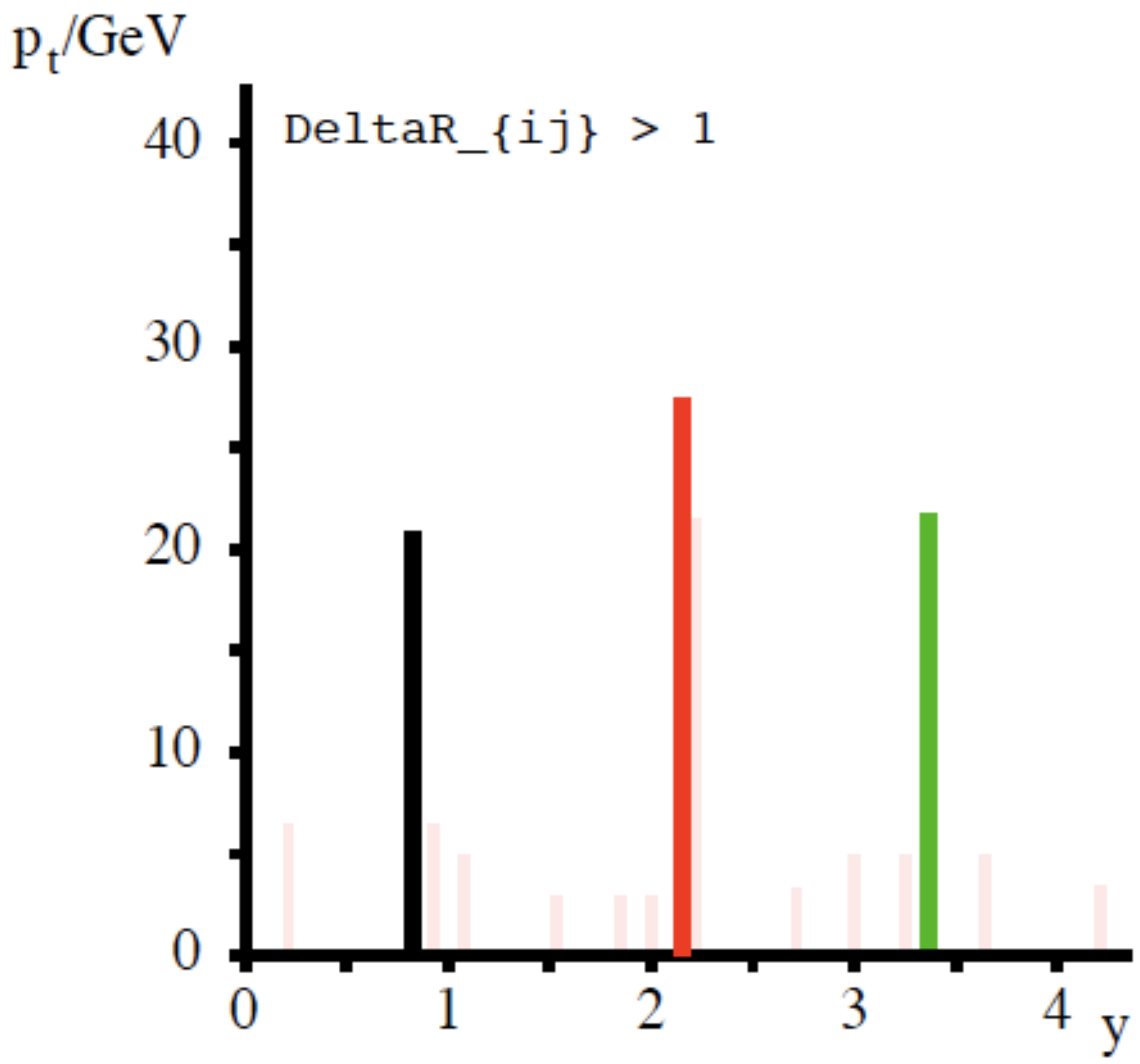

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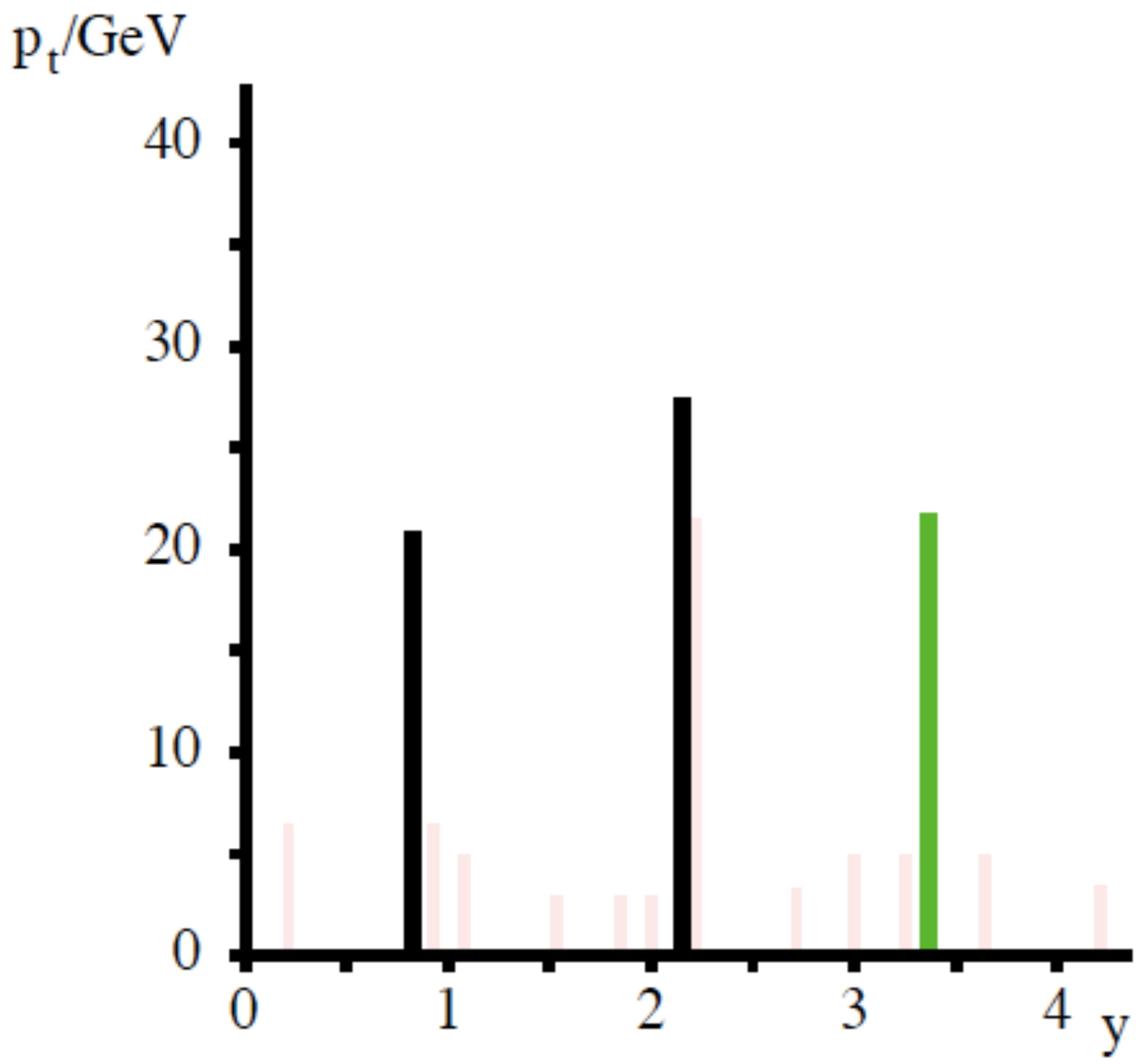

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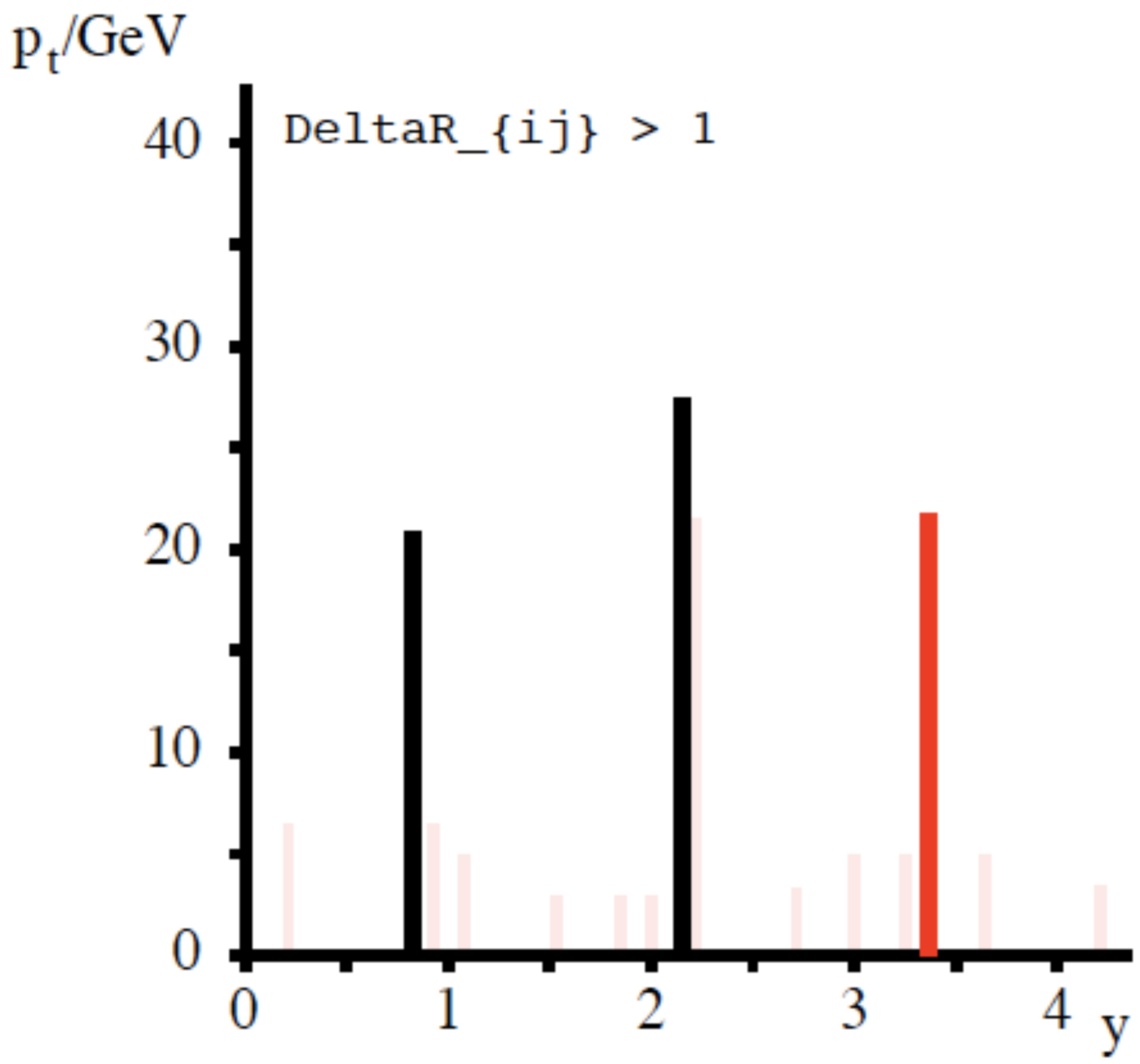

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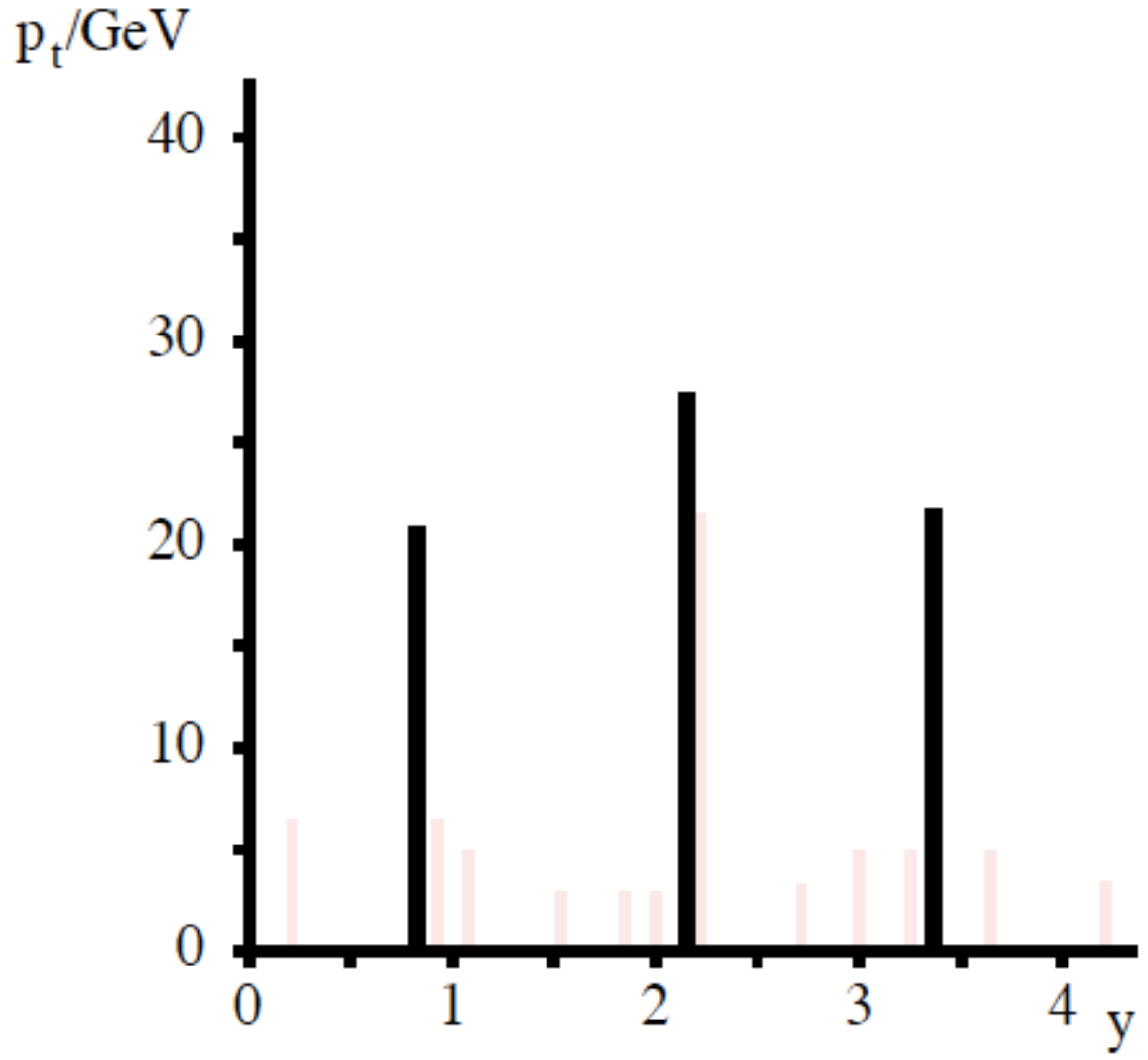
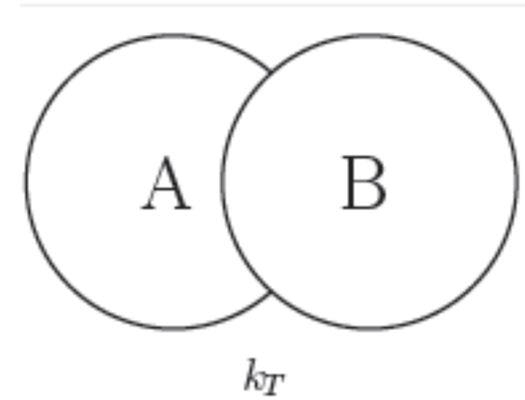
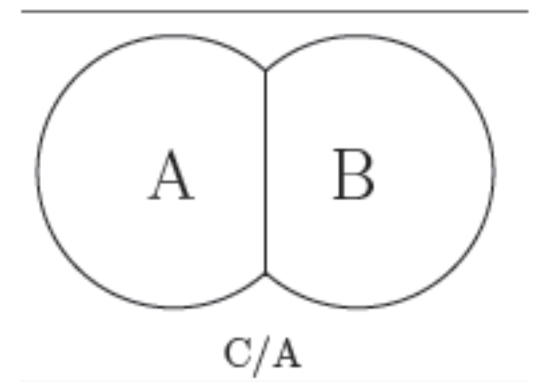


• The different algorithms lead to distinct jets shapes when they overlap

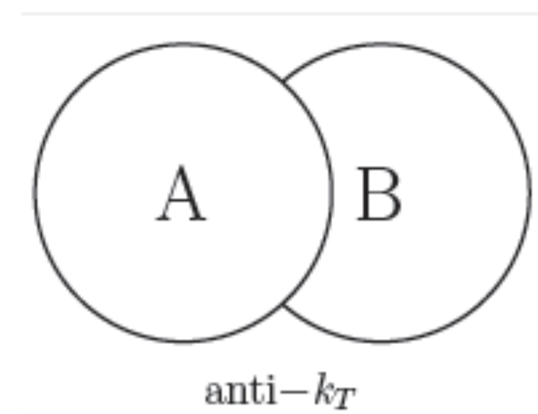
kT (1) starts around softer objects



C/A (0) cares only about distances

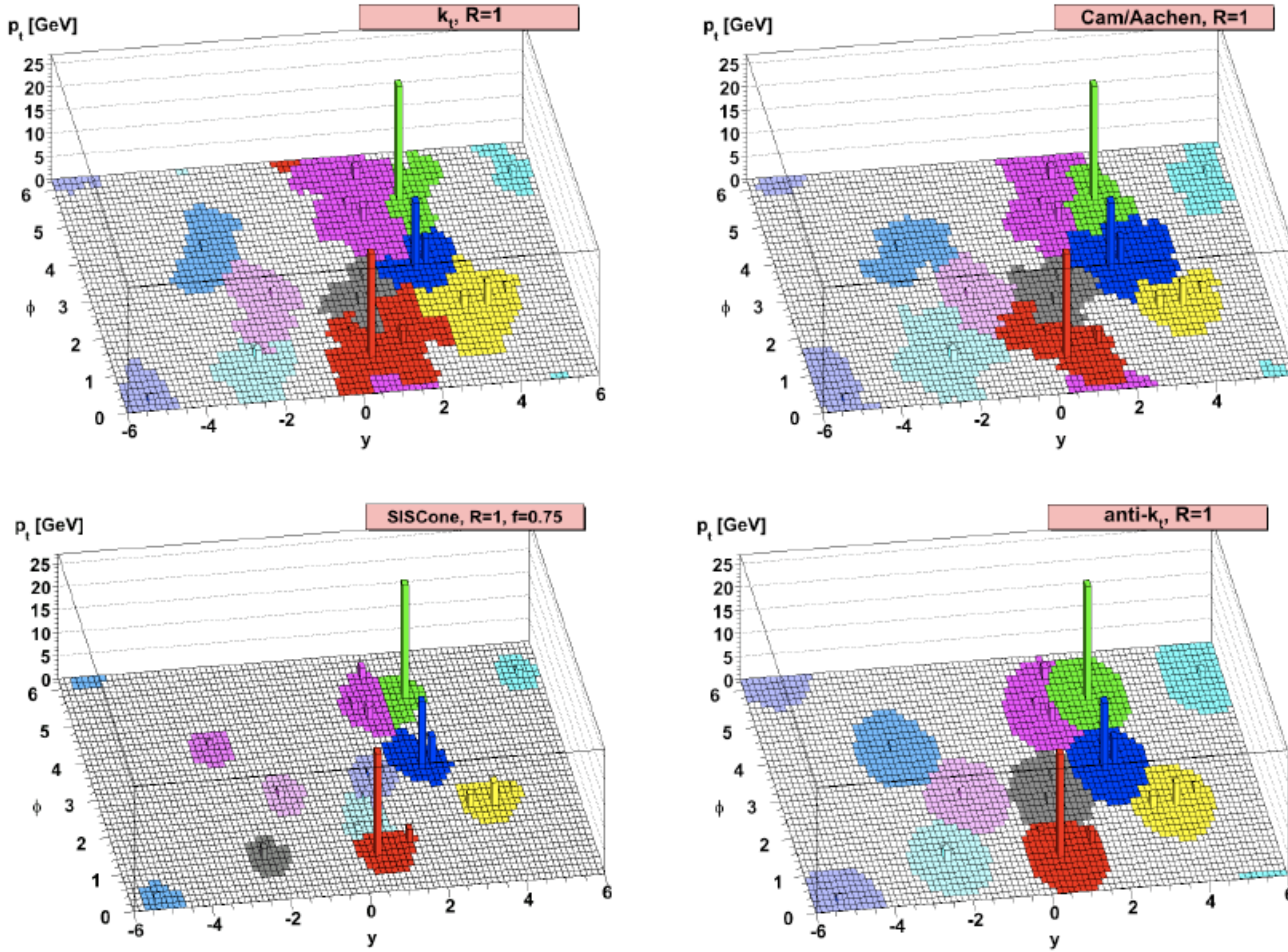


anti-kt (-1) clusters around hard objects



$$p_T^A > p_T^B$$

$$\mathbf{d_{ij}} = \min[\mathbf{p_{Ti}^{2\alpha}}, \mathbf{p_{Ti}^{2\alpha}}] \left(\frac{\mathbf{\Delta R_{ij}}}{\mathbf{R}}\right)^{\mathbf{2}} \quad \text{and} \quad \mathbf{d_{iB}} = \mathbf{p_{Ti}^{2\alpha}}$$



[JHEP04 (2008) 063]

Jet production

- The basic expression for 2 to 2 processes is

$$\frac{d\sigma}{dp_T^2} = \sum_{ij} \int dx_1 dx_2 \frac{f_i(x_1, Q_F^2) f_j(x_2, Q_F^2)}{(1+\delta_{ij})} \times \frac{d\hat{\sigma}}{dp_T^2}$$

- In the jet–jet CMS $\Longrightarrow$ $dy_1 dy_2 dp_T^2 = \frac{1}{2} s dx_1 dx_2 d\cos\theta^*$

$$\frac{d^3\sigma}{dy_1 dy_2 dp_T^2} = \frac{1}{16\pi s^2} \sum_{ij} \frac{f_i(x_1, Q_F^2) f_j(x_2, Q_F^2)}{(1+\delta_{ij}) x_1 x_2} \times \overline{\sum} |M(ij \to kl)|^2$$

with

$$x_1 = \frac{x_T}{2}\,(e^{y_1} + e^{y_2}) \quad ; \quad x_2 = \frac{x_T}{2}\,(e^{-y_1} + e^{-y_2}) \qquad \mathbf{x_T} = \frac{\mathbf{2p_T}}{\sqrt{\mathbf{s}}}$$

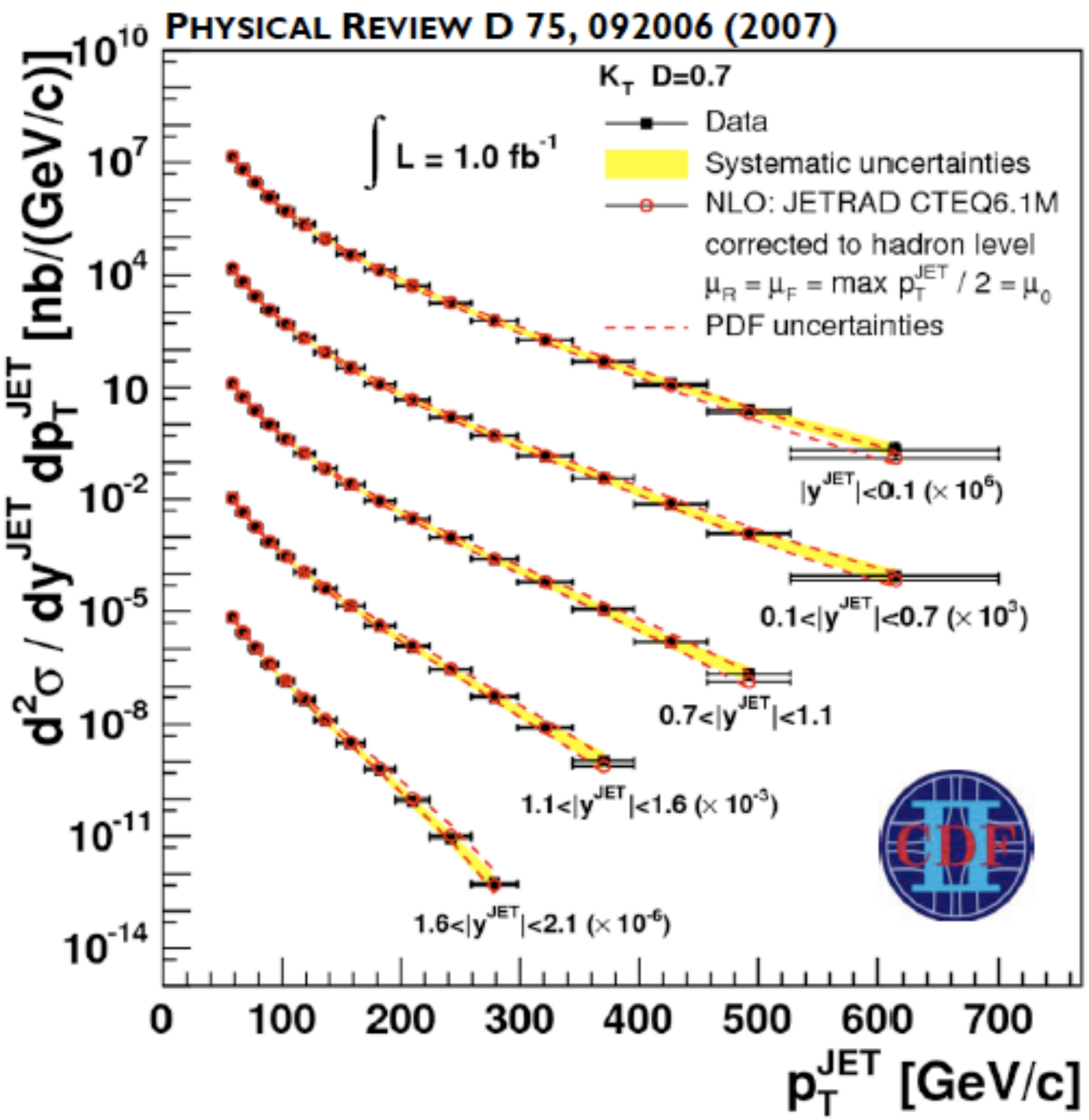
✛ The LO processes leading to jets are (gluon in the t–channel)

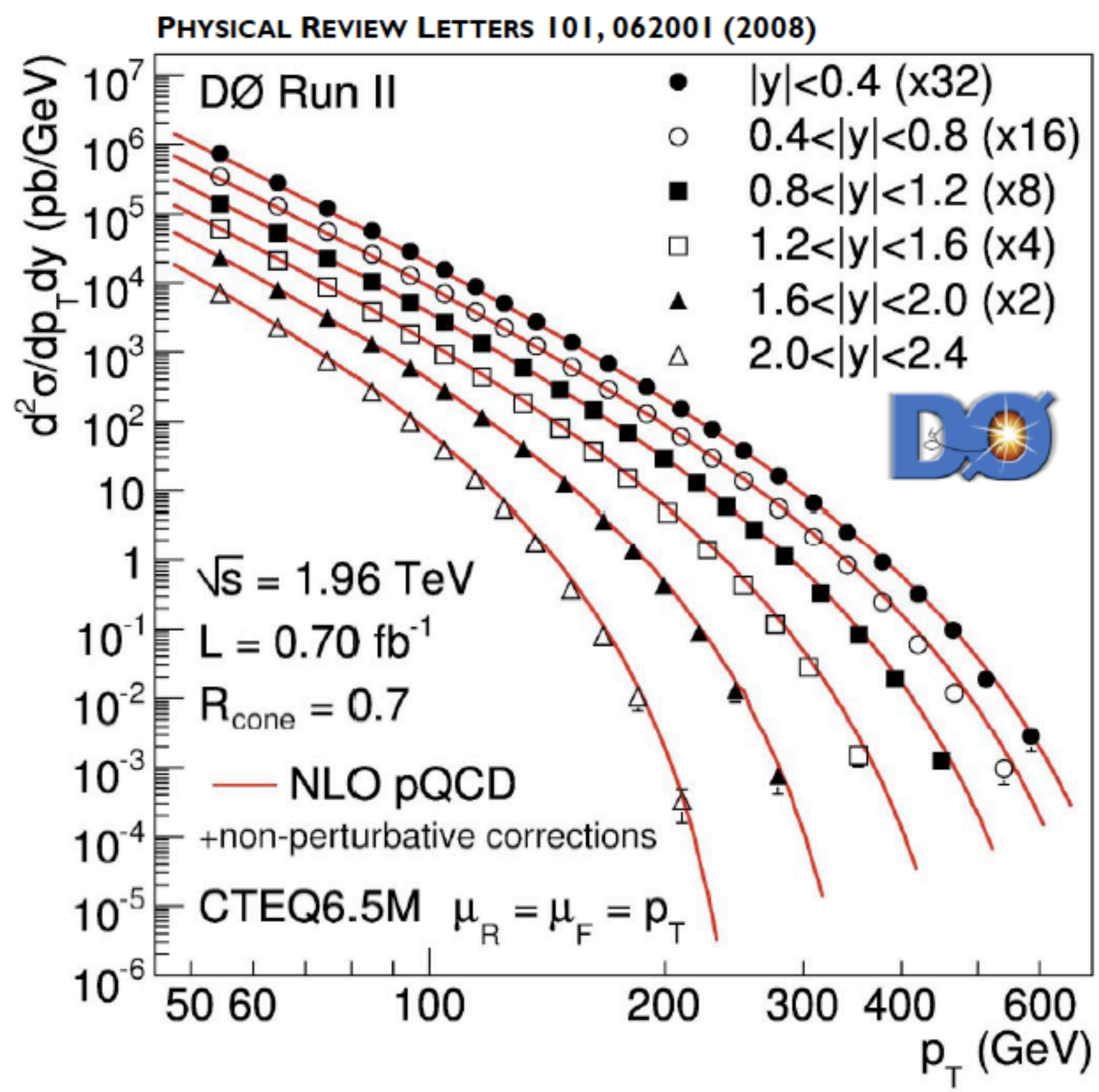
Process	$\frac{32\pi^2}{\alpha_s^2}\,\frac{d\hat{\sigma}}{d\Omega}$	at 90 degrees
$qq' \to qq'$	$\frac{1}{2\hat{s}}\frac{4}{9}\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$	2.2
$qq \to qq$	$\frac{1}{2}\frac{1}{2\hat{s}}\left[\frac{4}{9}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right)-\frac{8}{27}\frac{\hat{s}^2}{\hat{u}\hat{t}}\right]$	3.3
$q\bar{q} \to q'\bar{q}'$	$\frac{1}{2\hat{s}}\frac{4}{9}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}$	0.2
$q\bar{q} \to q\bar{q}$	$\frac{1}{2\hat{s}}\left[\frac{4}{9}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right)-\frac{8}{27}\frac{\hat{u}^2}{\hat{s}\hat{t}}\right]$	2.6
$q\bar{q} \to gg$	$\frac{1}{2}\frac{1}{2\hat{s}}\left[\frac{32}{27}\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}}-\frac{8}{3}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right]$	1.0
$gg \to q\bar{q}$	$\frac{1}{2\hat{s}}\left[\frac{1}{6}\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}}-\frac{3}{8}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right]$	0.1
$gq \to gq$	$\frac{1}{2\hat{s}}\left[-\frac{4}{9}\frac{\hat{s}^2+\hat{u}^2}{\hat{s}\hat{u}}+\frac{\hat{u}^2+\hat{s}^2}{\hat{t}^2}\right]$	6.1
$gg \to gg$	$\frac{1}{2}\frac{1}{2\hat{s}}\frac{9}{2}\left(3-\frac{\hat{t}\hat{u}}{\hat{s}^2}-\frac{\hat{s}\hat{u}}{\hat{t}^2}-\frac{\hat{s}\hat{t}}{\hat{u}^2}\right)$	30.4

with $\hat{t} = -\hat{s}\,(1-\cos\theta)/2$ and $\hat{u} = -\hat{s}\,(1+\cos\theta)/2$

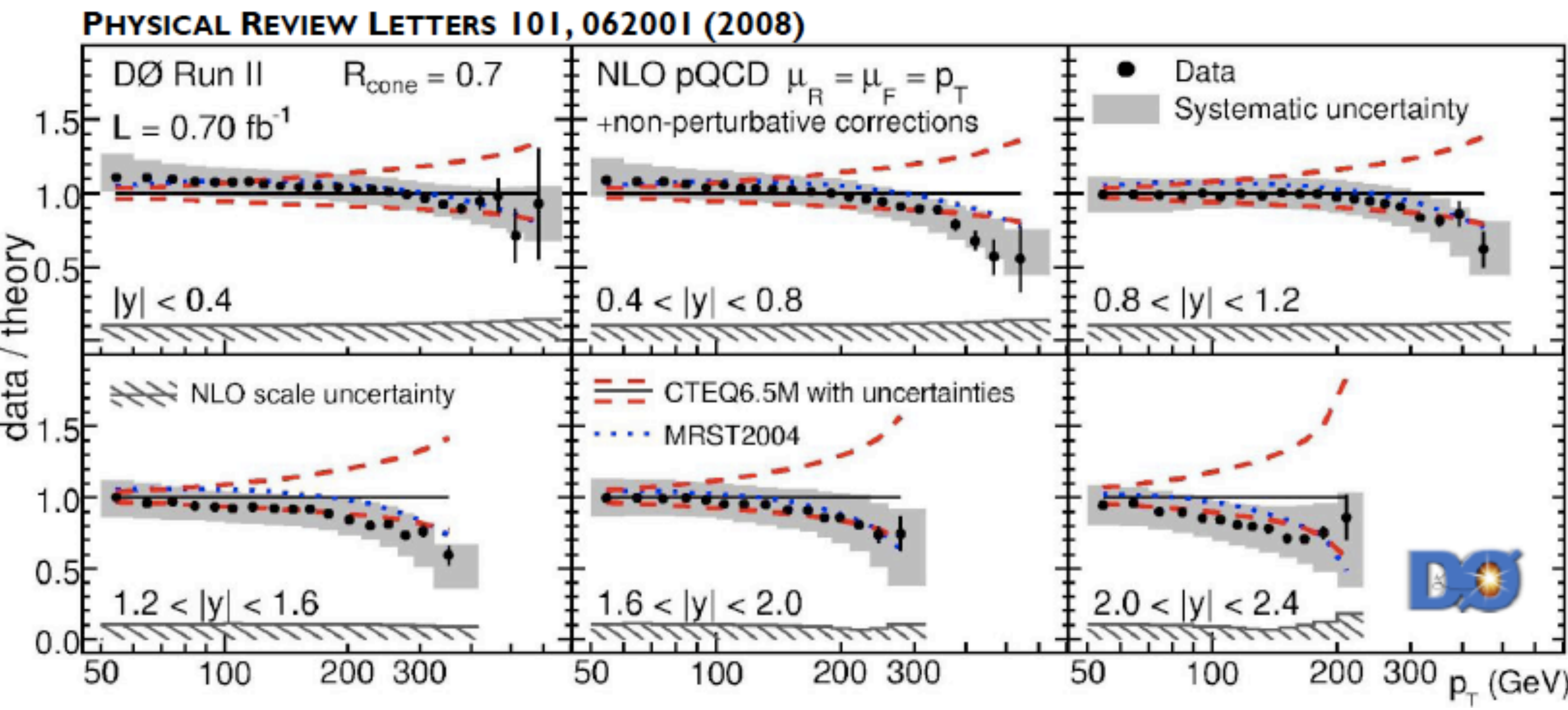
Tevatron results

the inclusive jet cross section does agree with NLO QCD over 8 orders of magnitude!





•Let's look the results without the dirt trick of log plots



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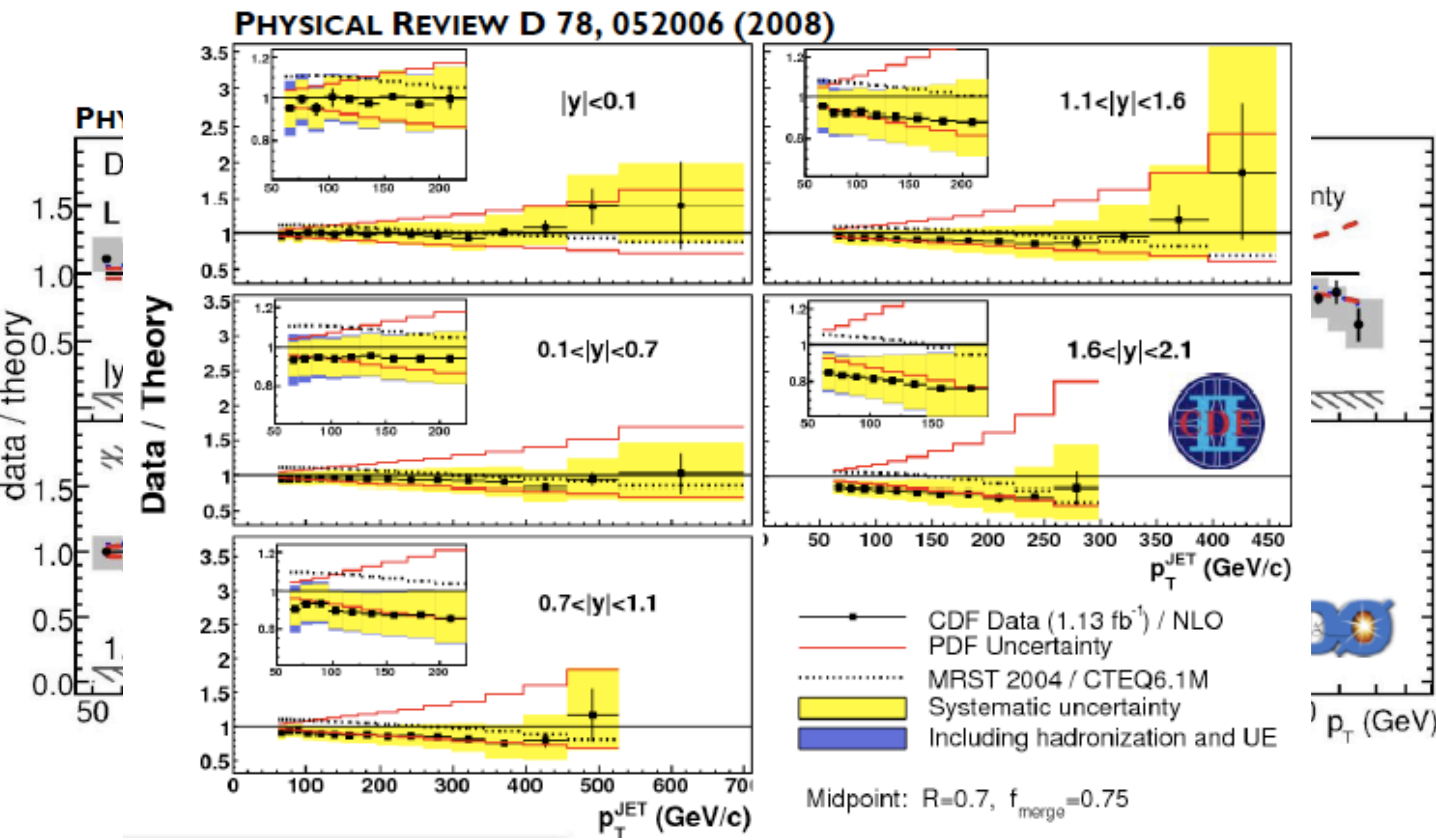
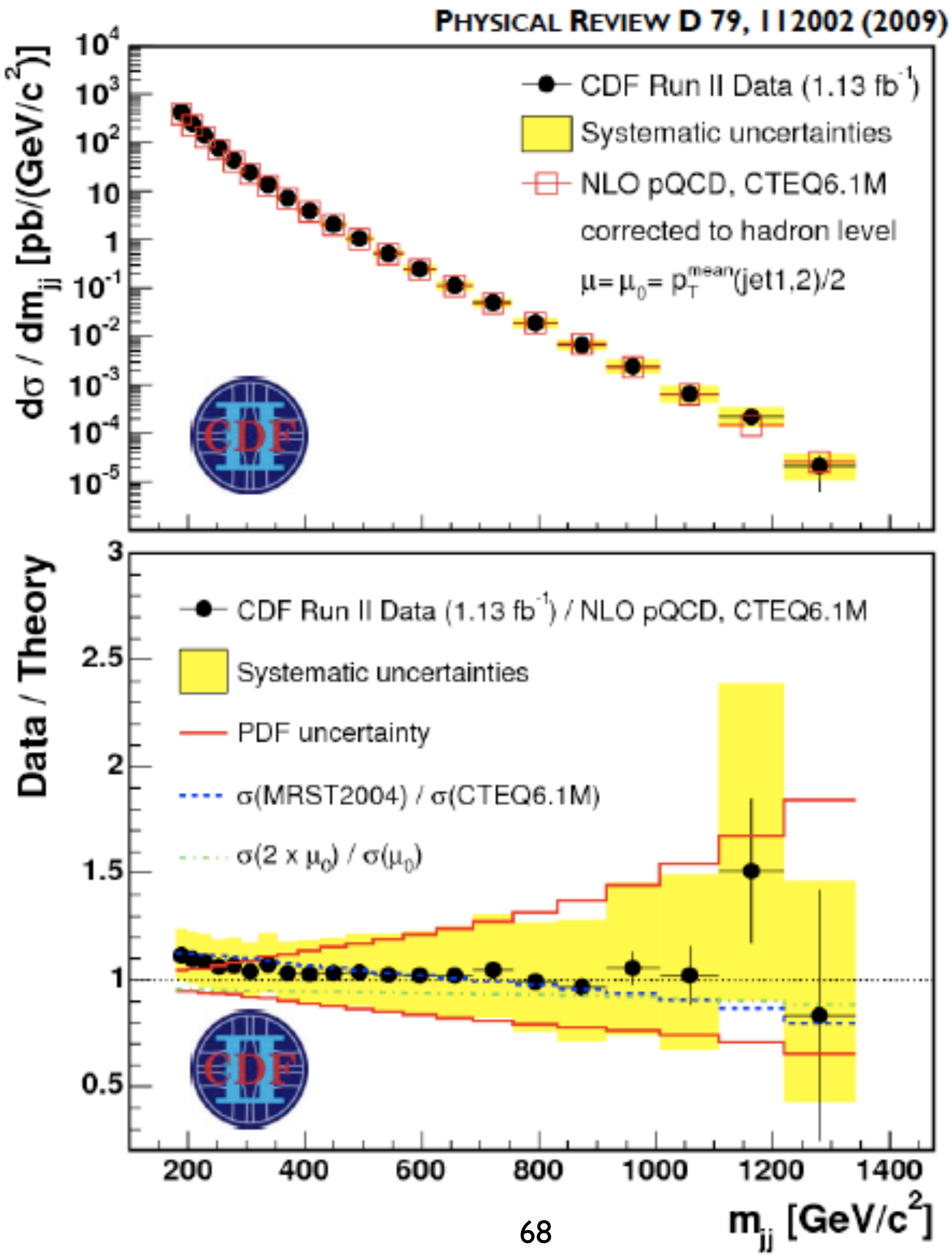
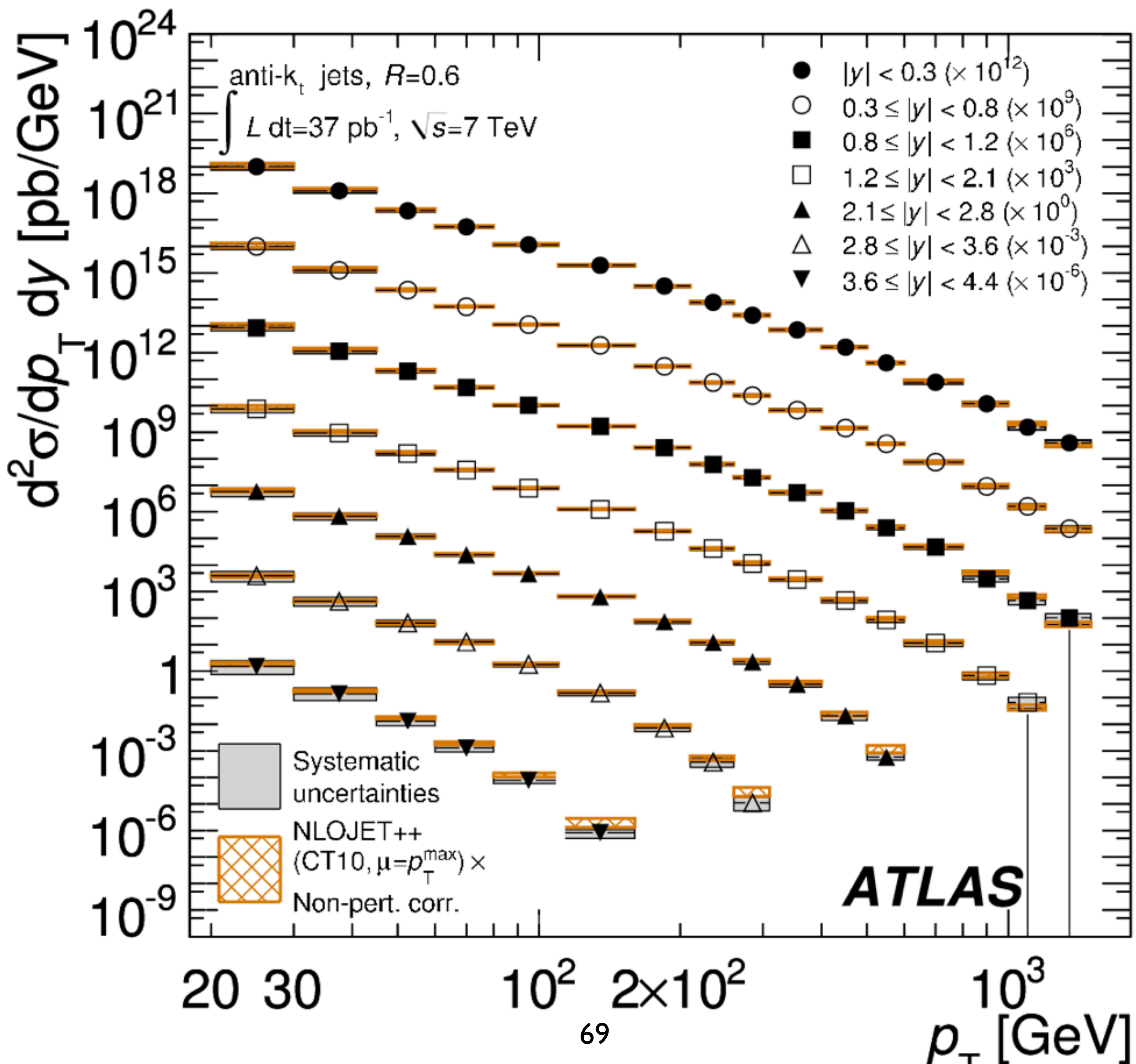


• The agreement is also nice for the dijet invariant mass, eg, at CDF

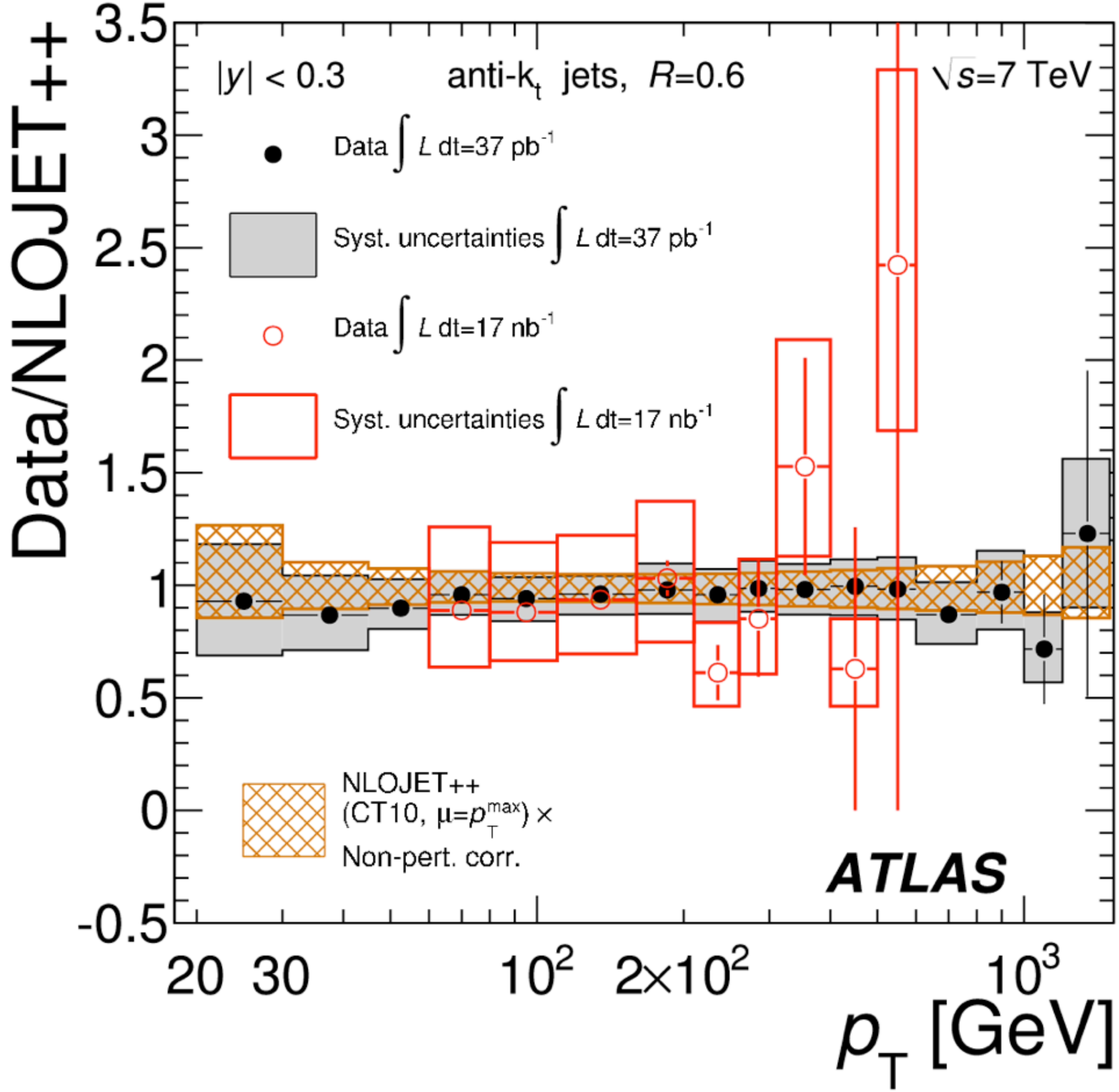


Jets at the LHC

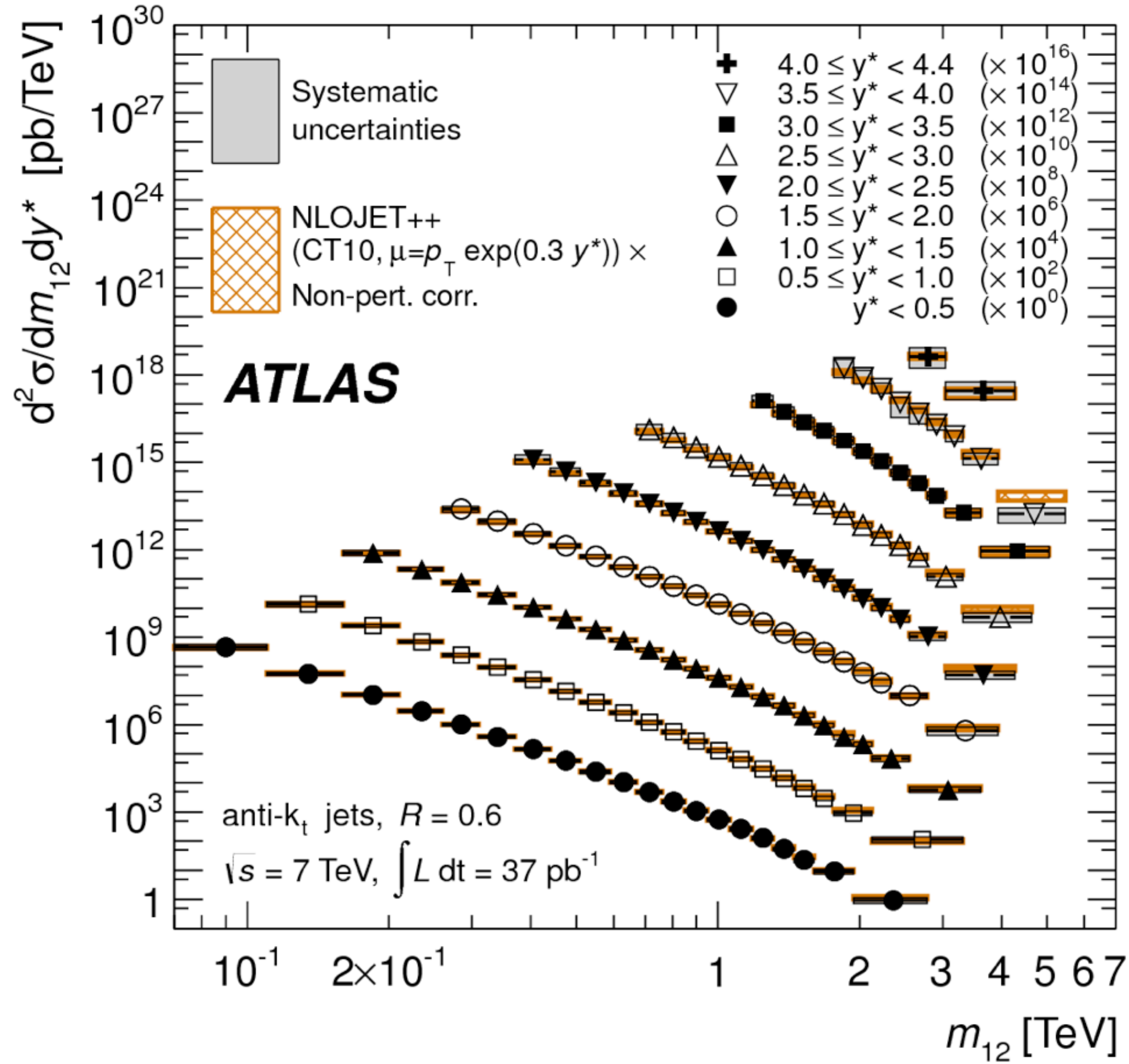
the inclusive jet cross section is nicely described by NLO QCD

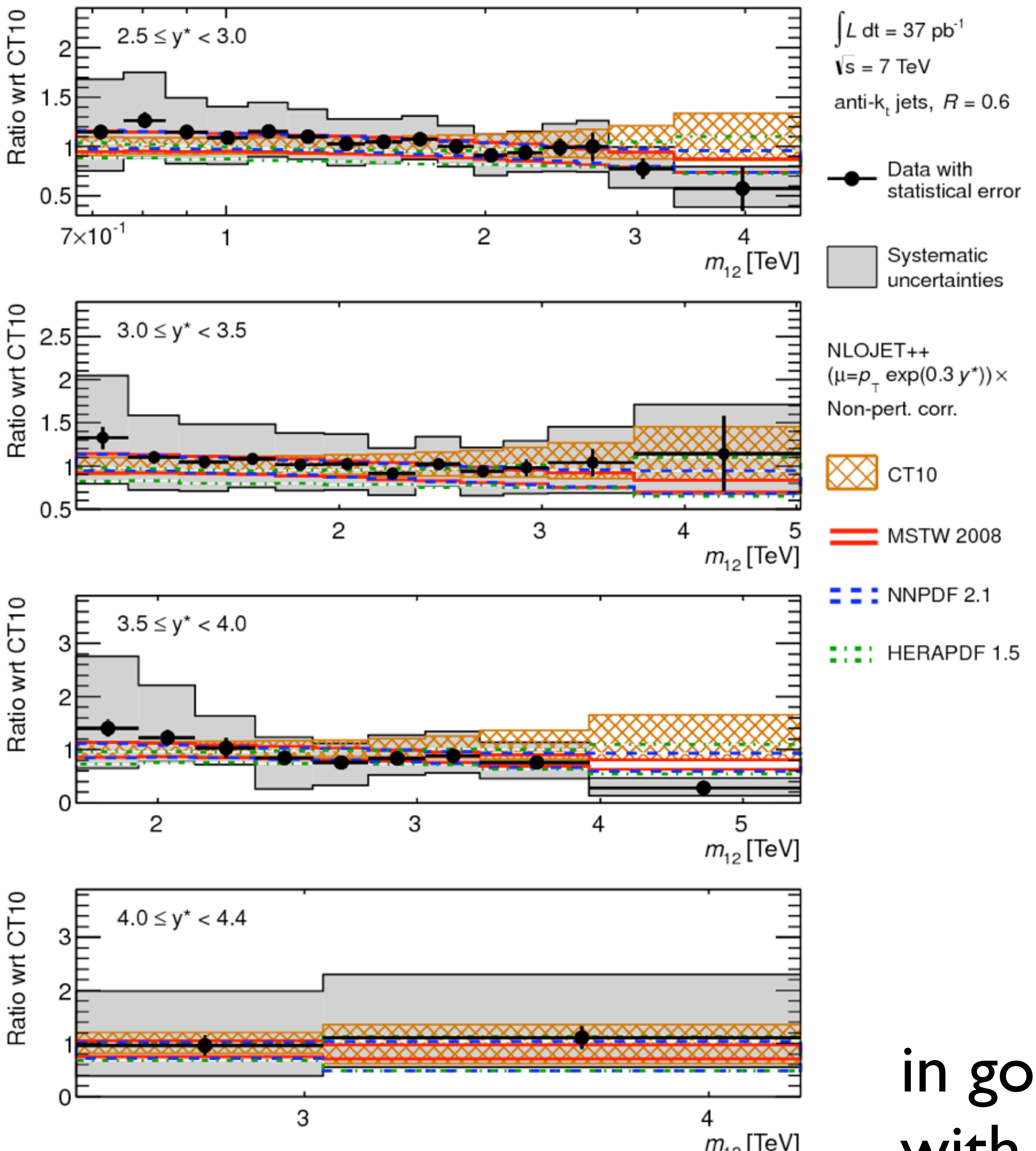


a more serious comparison



again we can study dijet invariant masses

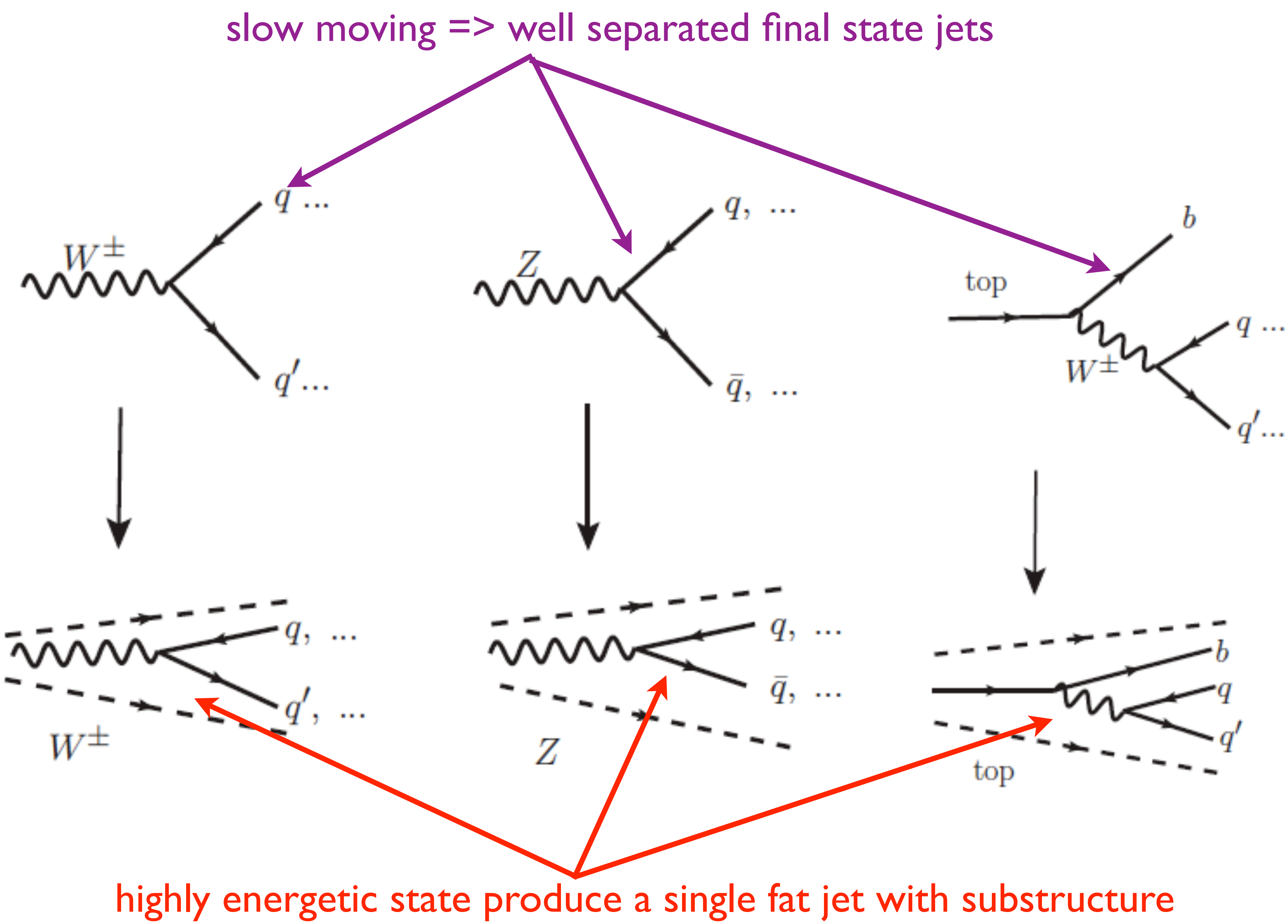




in good agreement with QCD

Highly boosted objects: fat jets

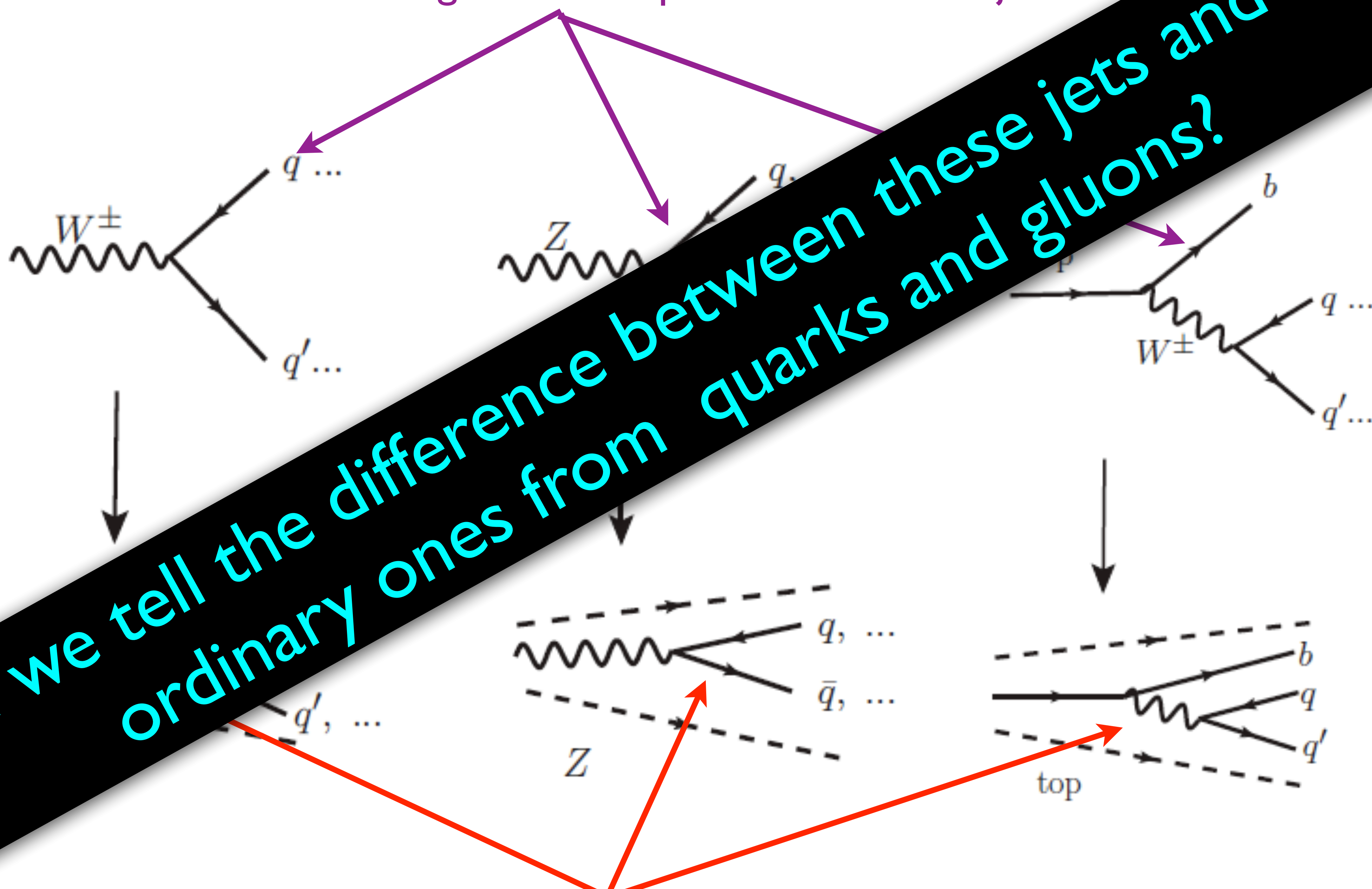
- At the LHC W's, Z's, H's, and tops can be very energetic such that their decay products are collinated, merging the final state jets!



Highly boosted objects: fat jets

- At the LHC W's, Z's, H's, and tops can be very energetic such that their decay products are collinated, merging the final state jets!

slow moving => well separated final state jets

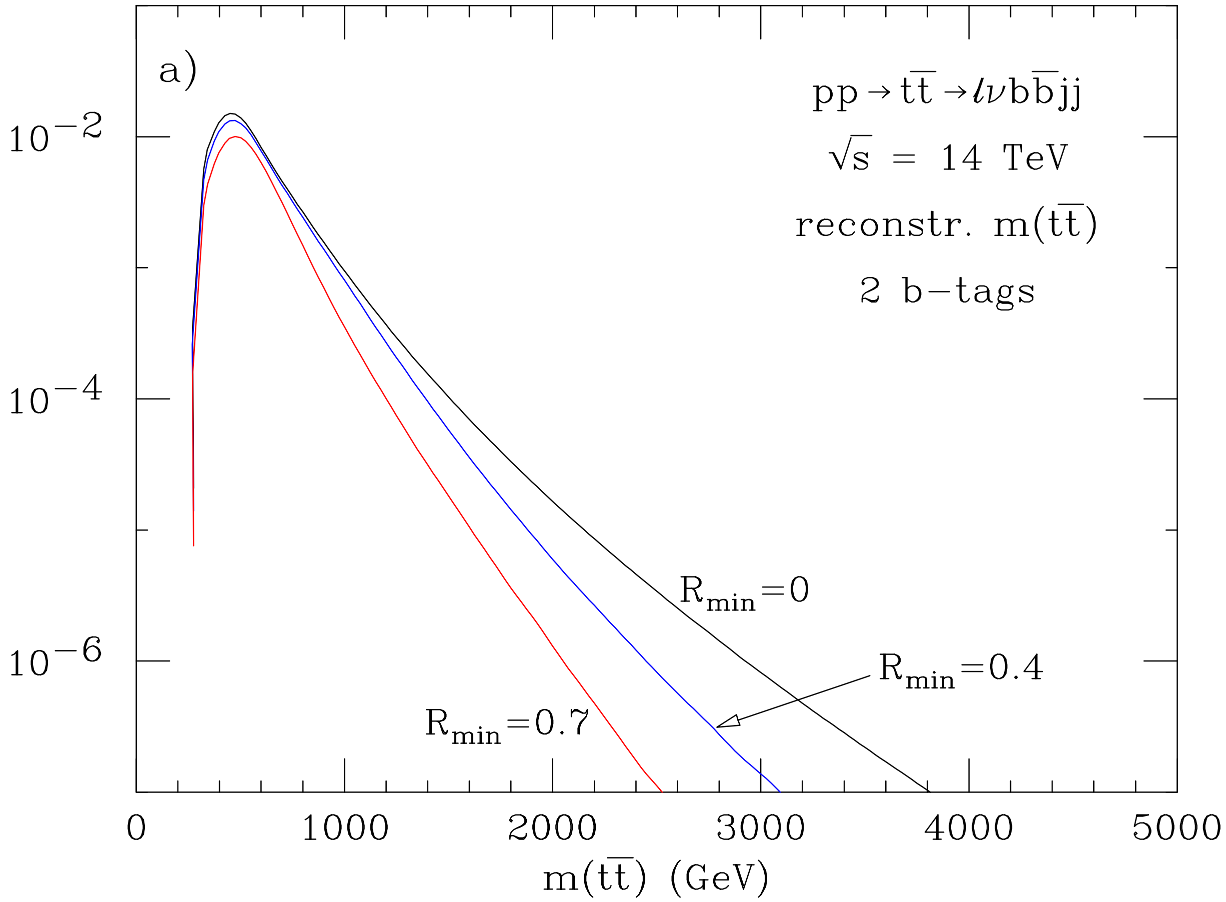


highly energetic state produce a single fat jet with substructure

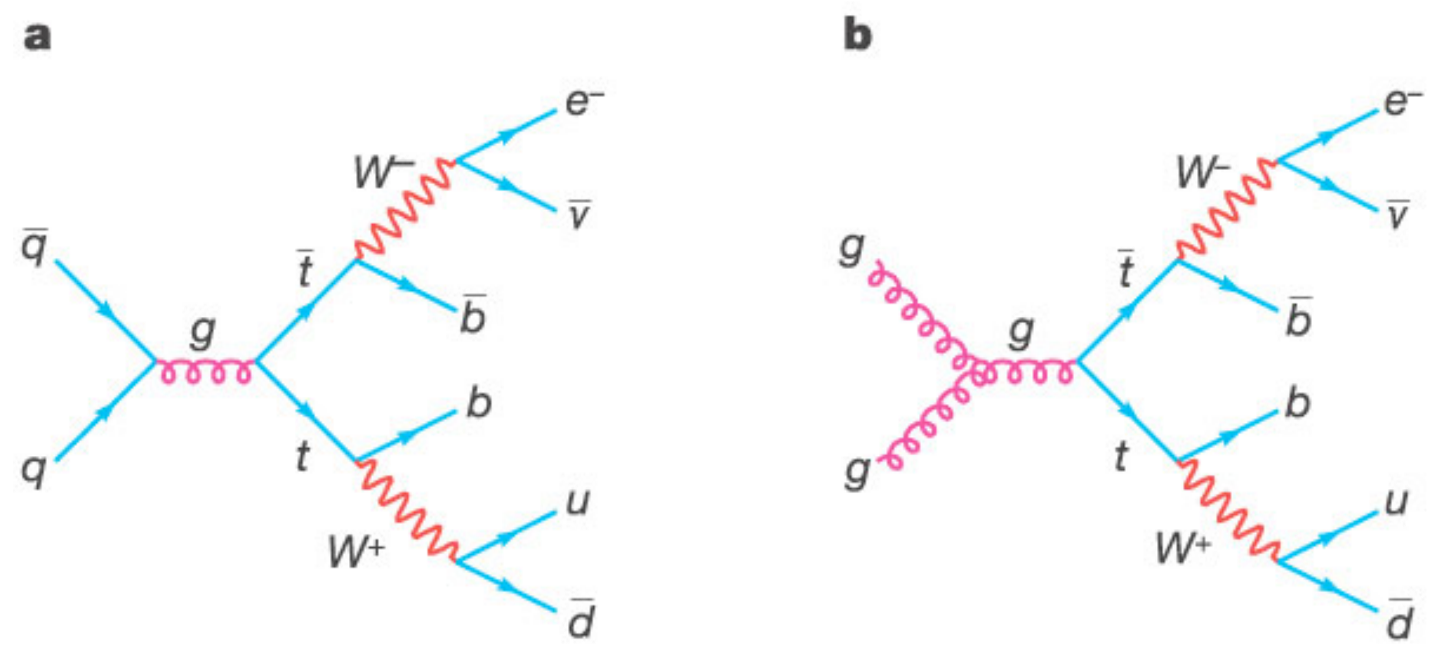
can we tell the difference between these jets and the ordinary ones from quarks and gluons?

Why should we care?

[Baur, Orr 0707.2066]

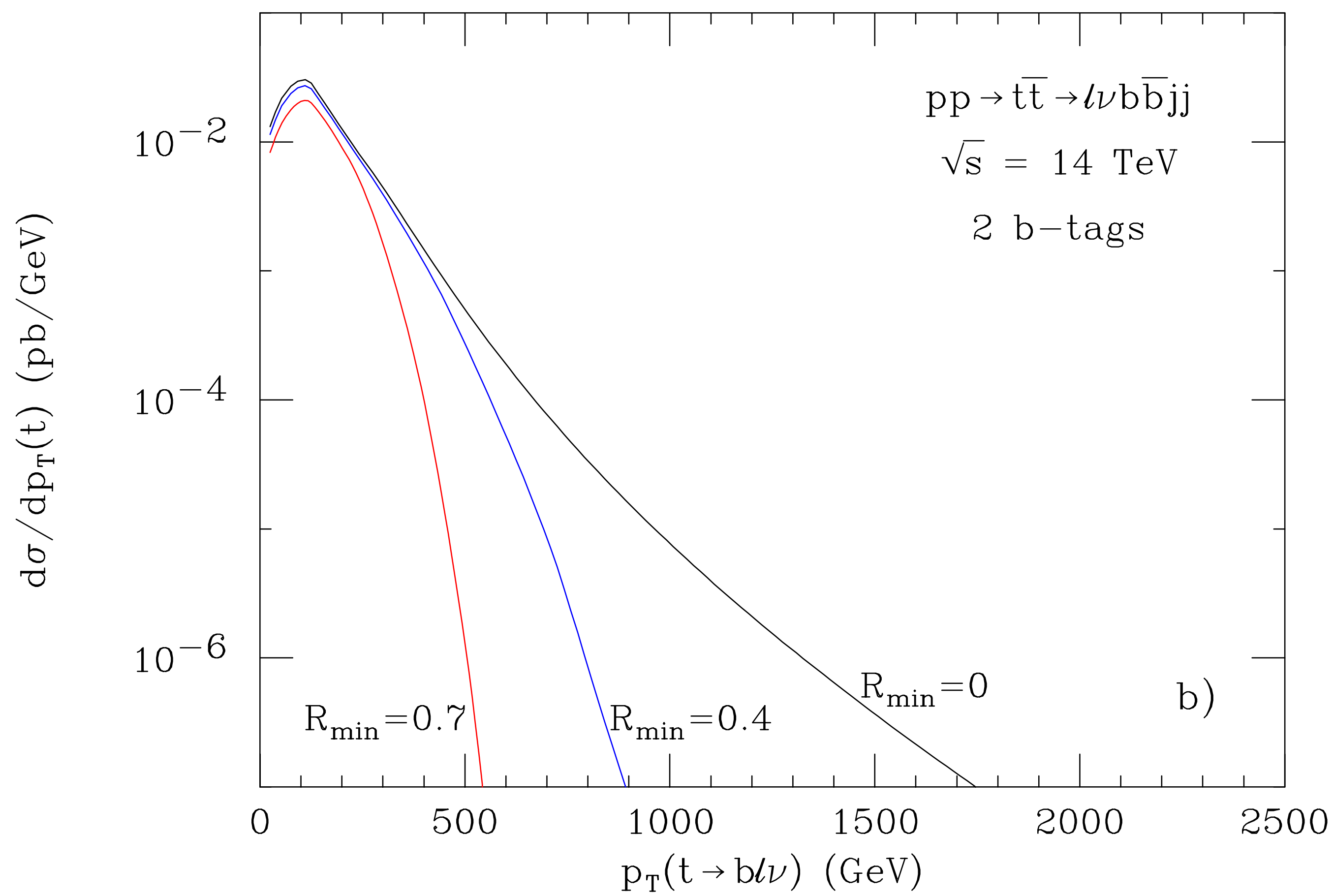


top pair production at the LHC



$$\Delta R_{jj} \geq R_{min}$$

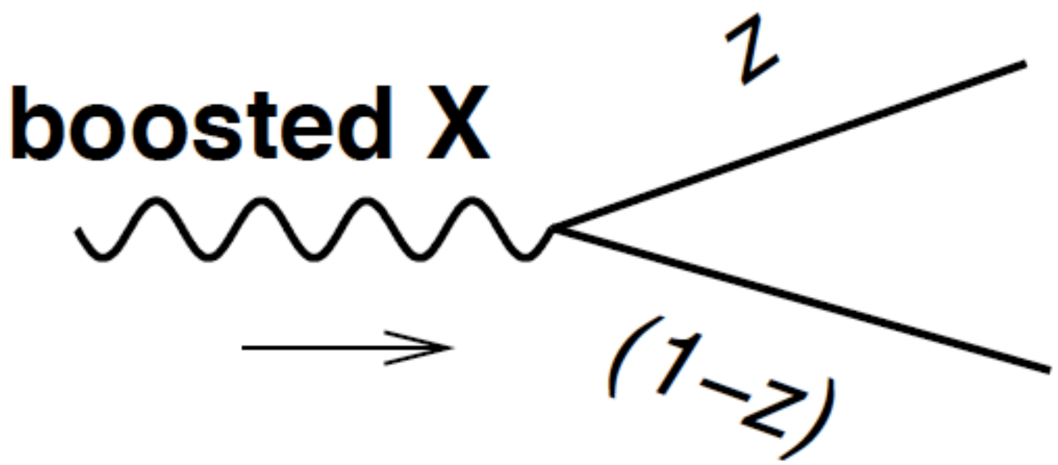
$$\Delta R_{jj} = \sqrt{\Delta\varphi_{jj}^2 + \Delta\eta_{jj}^2}$$



- requiring separated jets suppress the signal at high invariant masses

Basic idea

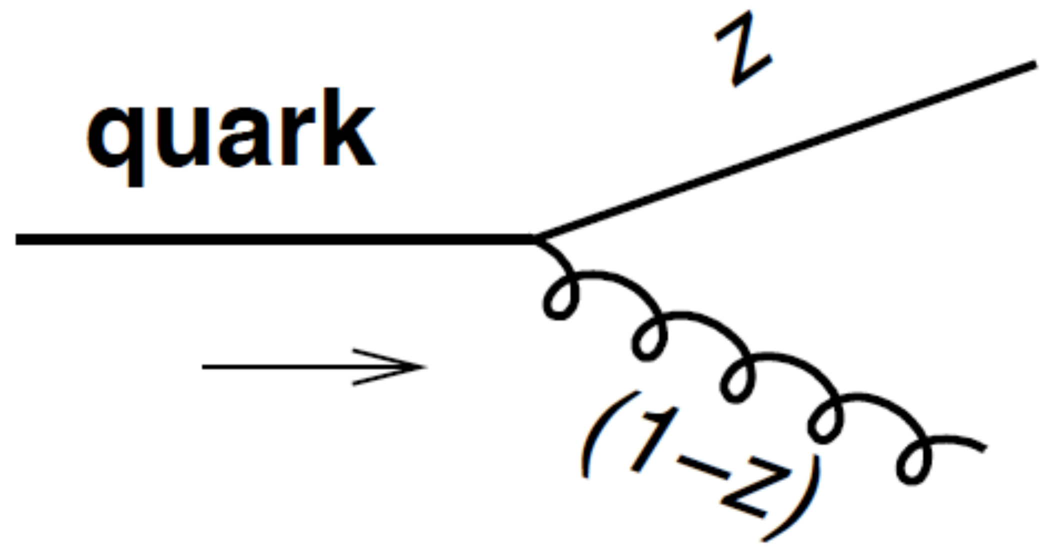
signal



$$P(z) \propto 1$$

daughters have similar momenta

background (QCD)



$$P(z) \propto \frac{1+z^2}{1-z}$$

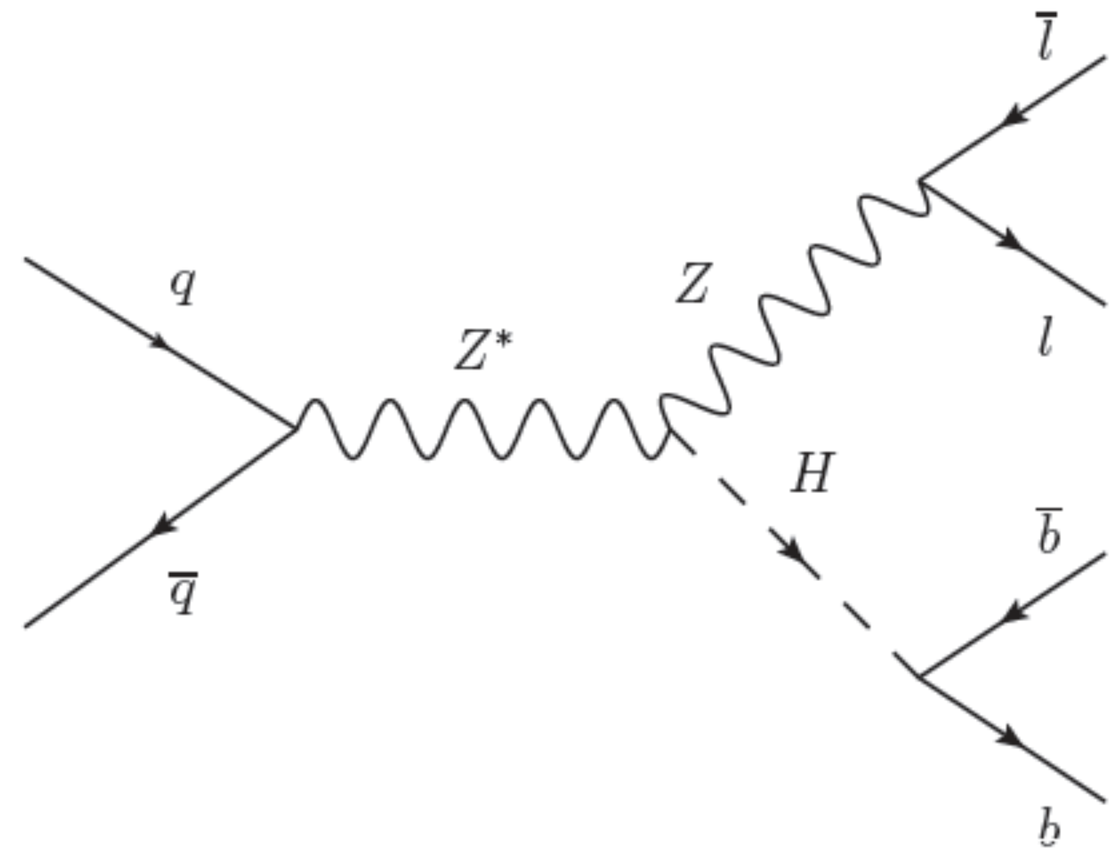
QCD prefers softer radiation

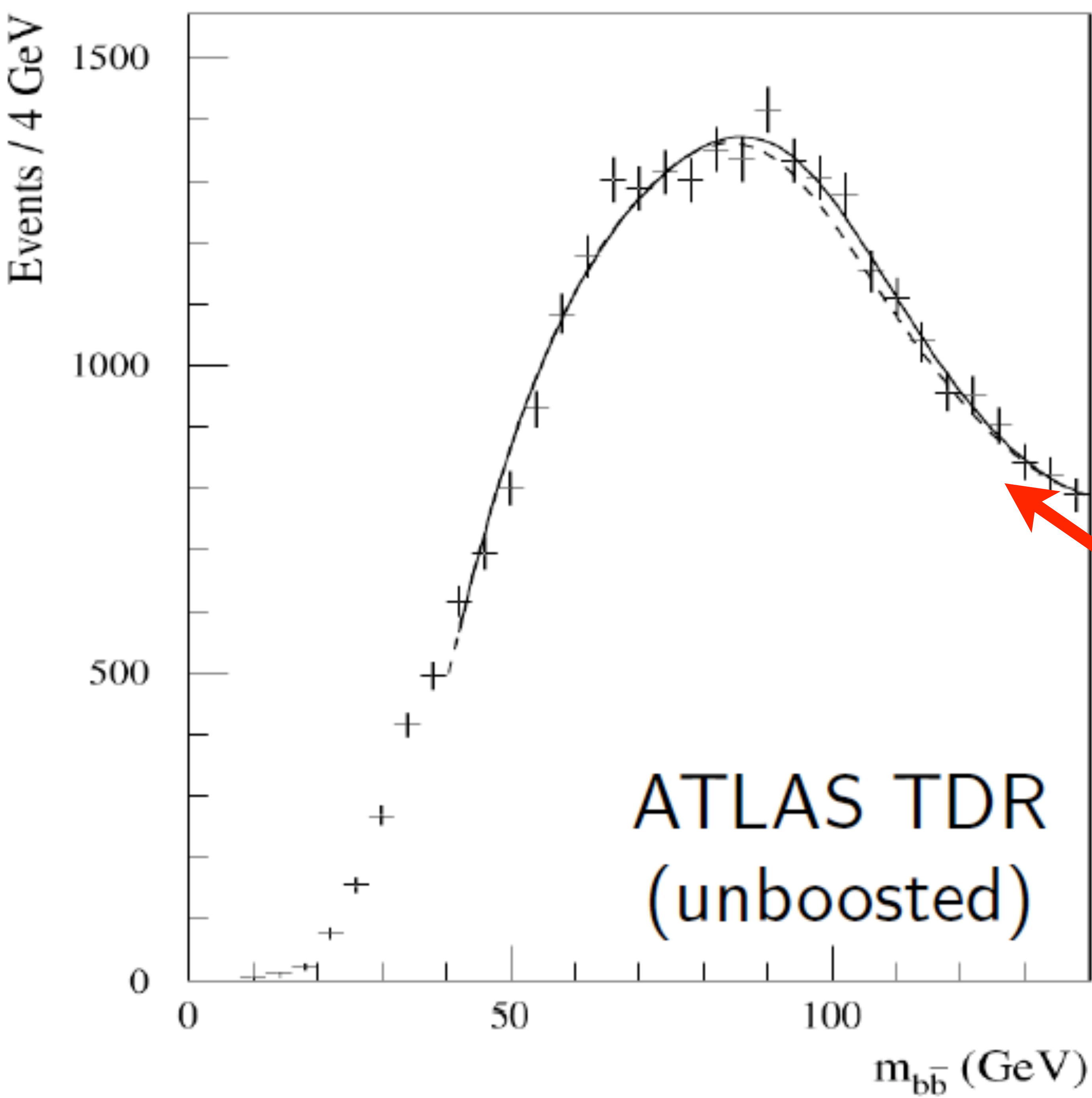
idea: reverse the jet algorithm analyzing the daughters

- first proposed by Seymour (1993) for W's

Example: VH at the LHC [Butterworth et al arXiv 0802.2470]

Unboosted analysis

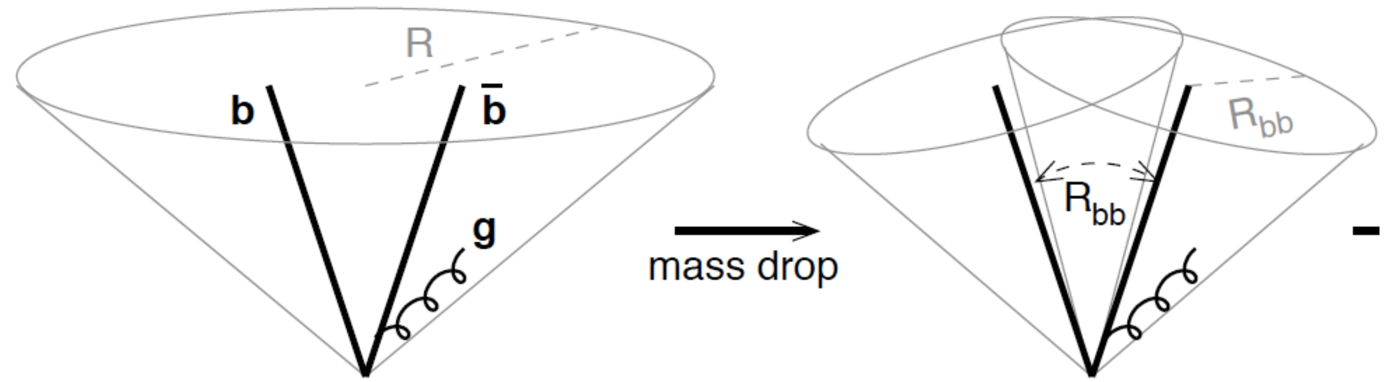




H is hard to see.

Boosted analysis

Search for boosted Higgs:
for jet j with size R



- Undo the clustering and label j_1 , j_2 with $m_{j_1} > m_{j_2}$

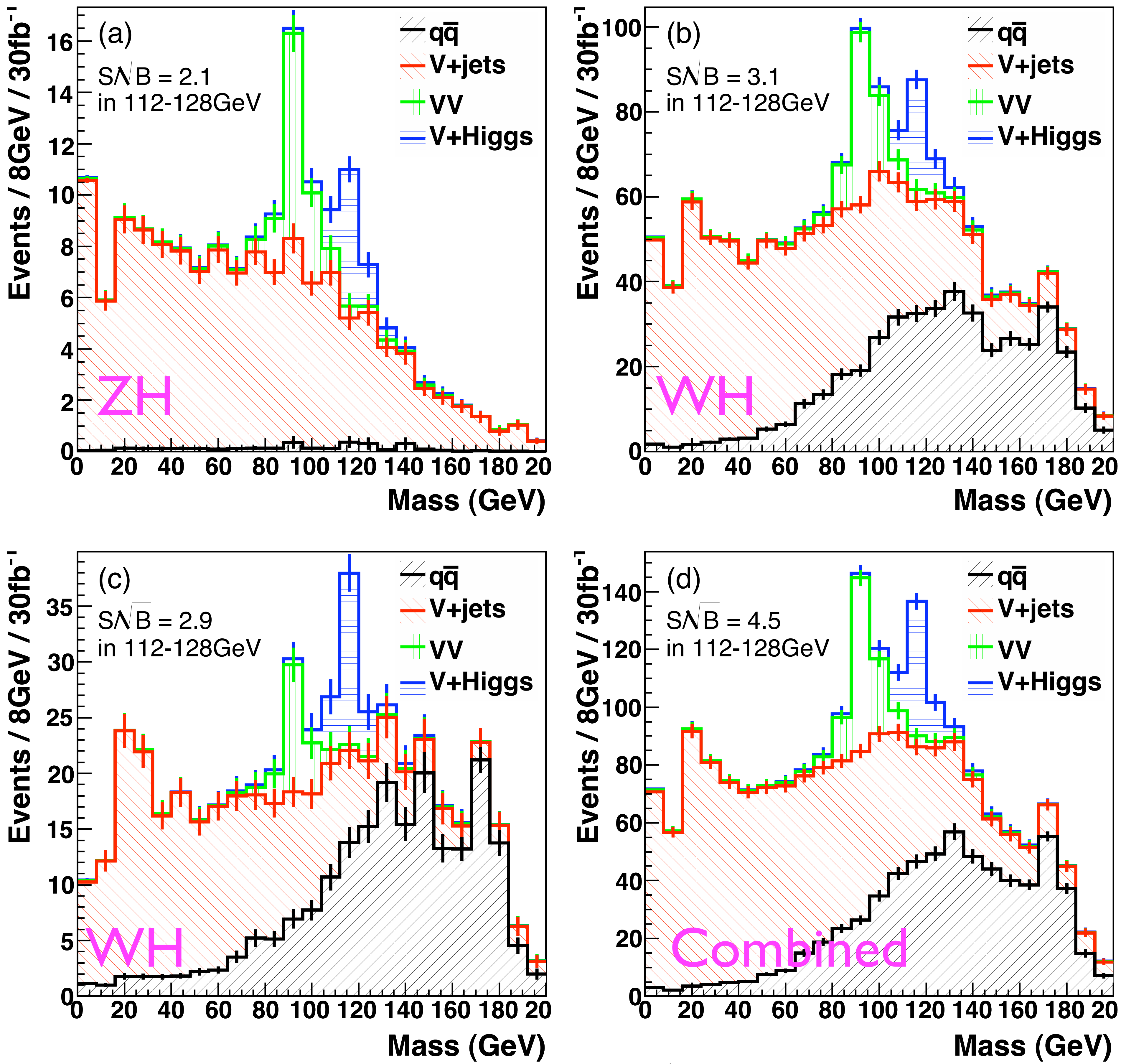
- j is a heavy particle if there is a mass drop such that

$$m_{j_1} < \mu \, m_j$$ [mass drop]

$$y \simeq \frac{\min(p_{tj_1}, p_{t,j_2})}{\max(p_{tj_1}, p_{t,j_2})} > y_{cut}$$ [symmetric splitting]

- if there is no mass drop rename and redo the analysis

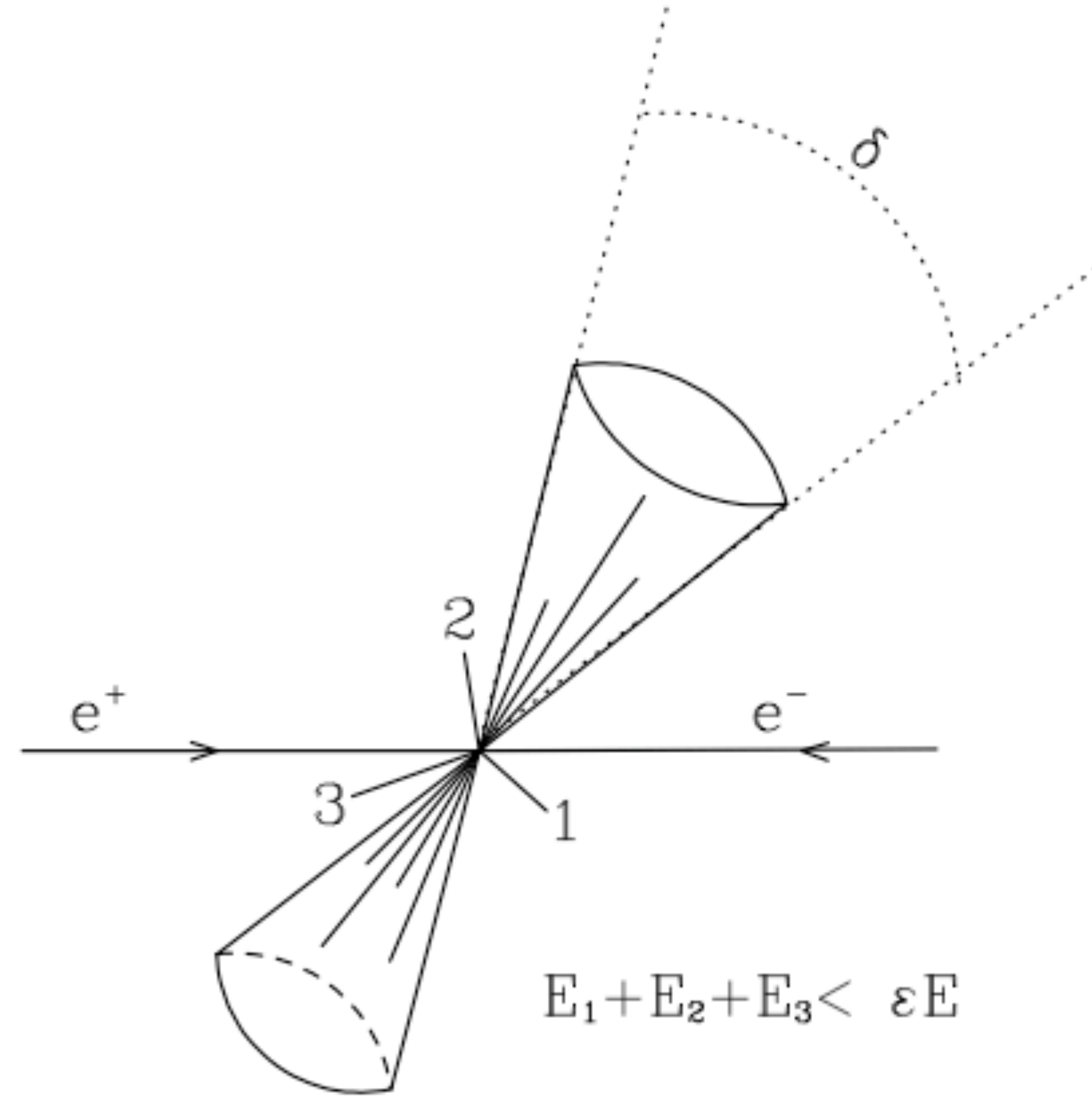
For instance $y_{cut} = 0.15$ and $\mu = 0.67$



$R = 1.2$ and $p_T^j > 200$ GeV

First answer

- Sterman and Weinberg: 2 jet event if a fraction $1-\epsilon$ of the total energy is contained in two cones of size δ.



- This can be applied to hadrons (experimental data) and quarks/gluons (theory).

- This is IR finite: sums collinear and soft gluons and virtual corrections.

General form of the IR divergences

$$\sigma^{q\bar{q}g} = \frac{2\alpha_s}{3\pi}\sigma_{q\bar{q}} \int d\cos\theta_{qg} \frac{dE_g}{E_g} \frac{4}{(1-\cos\theta_{qg})(1+\cos\theta_{qg})}$$

- The integral diverges for

$$E_g \to 0$$ (soft gluon limit)

$$\theta_{gq} \to 0$$

$$\theta_{g\bar{q}} \to 0$$ (colinear limit)

II. Connecting theory and experiment