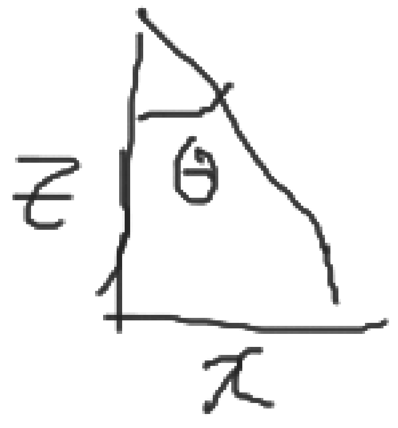
1. **2.4** Encontre o campo elétrico a uma distância z acima do centro de um circuito quadrado de lado a (Fig. 2.8), em que cada fio tem densidade linear de carga λ.



$$\tan\theta = \frac{x}{z}$$

$$d(\tan\theta)\,d\theta = \frac{dx}{z}$$

$$\frac{1}{\cos^2\theta}\,d\theta = \frac{dx}{z}$$

$$dx = z\frac{d\theta}{\cos^2\theta} \qquad \frac{1}{z^3}\int \frac{z\,d\theta}{\cos^2\theta} =$$

$$I = \int \frac{dx}{(x^2+z^2)^{3/2}} = \frac{dx}{z^3\left(1+\frac{z^2}{x^2}\right)^{3/2}}$$

$$I = \frac{1}{z^3}\int \frac{z\,d\theta}{\cos^2\theta\,(1+\tan^2\theta)^{3/2}}$$

$$I = \frac{1}{z^3}\int \frac{z\,d\theta}{\cos^2\theta\left(\frac{1}{\cos^3\theta}\right)^{3/2}} = \frac{1}{z}\int \cos\theta\,d\theta$$

$$I = \frac{1}{z^3}\int \cos\theta\,d\theta = \frac{1}{z^3}\left[\operatorname{sen}\theta\right]$$

$$I = \frac{1}{z^2}\left[\frac{z\,x}{\lambda\sqrt{x^2+z^2}}\right]$$

Exercício 1

$$d\vec{E}_1 = dE \cos\theta \, \vec{e}_z \qquad dE = \frac{dq}{4\pi\varepsilon_0 PR^2}$$

$$\cos\theta = \frac{z}{PR} \qquad PR^2 = z^2 + r^2 \quad e \quad r^2 = x^2 + \frac{a^2}{4}$$

$$PR^2 = z^2 + x^2 + \frac{a^2}{4}$$

$$dE_1 = dE\cos\theta = \frac{dq\, z}{4\pi\varepsilon_0 PR^3} = \frac{\lambda z\, dx}{4\pi\varepsilon_0 \left(z^2 + x^2 + \frac{a^2}{4}\right)^{3/2}}$$

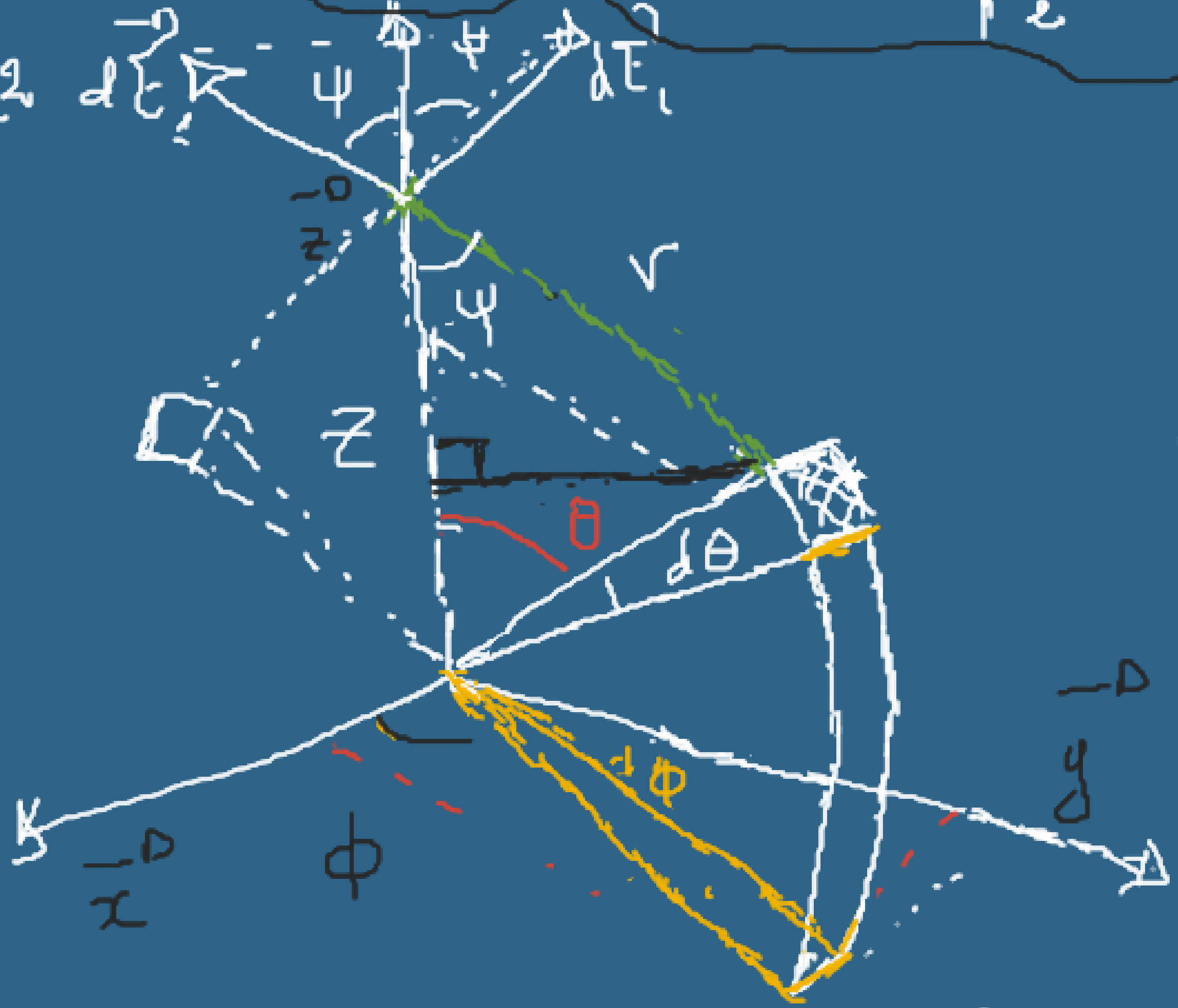
$$dE_1 = \frac{z\lambda\, dx}{4\pi\varepsilon_0 \left(z^2 + x^2 + \frac{a^2}{4}\right)^{3/2}} \Rightarrow dE_1 = \frac{z\lambda\, dx}{4\pi\varepsilon_0 \left(z^2 + x^2 + \frac{a^2}{4}\right)^{3/2}}$$

$$E_1 = \frac{z\lambda}{4\pi\varepsilon_0}\int_0^a \frac{dx}{\left(z^2 + x^2 + \frac{a^2}{4}\right)^{3/2}} = \frac{z\lambda}{4\pi\varepsilon_0 \left(z^2 + \frac{a^2}{4}\right)} \left[\frac{x}{\sqrt{z^2 + x^2 + \frac{a^2}{4}}}\right]_0^a$$

$$E_1 = \frac{z\lambda}{4\pi\varepsilon_0\left(z^2 + \frac{a^2}{4}\right)}\left(\frac{a}{\sqrt{z^2 + a^2 + \frac{a^2}{4}}} - 0\right) = \frac{z\lambda a}{4\pi\varepsilon_0\left(z^2 + \frac{a^2}{4}\right)\sqrt{z^2 + \frac{a^2}{2}}}$$

$$\vec{E} = 4\vec{E}_1 = \frac{4 h a Z}{4\pi\varepsilon_0 \sqrt{\frac{a^2}{2} + z^2}\left(\frac{a^2}{4} + z^2\right)} \vec{e_z}$$

Exercício 2



$$d\vec{E} = dE \cos\psi \, \vec{e_z}$$

$$d\vec{E} = \frac{dq \cos\psi}{4\pi\varepsilon_0 r^2} \vec{e_z} \qquad \vec{r} = |\vec{R} - \vec{Z}|$$

$$r^2 = R^2 + z^2 - 2\vec{R}\cdot\vec{Z} = R^2 + z^2 - 2RZ\cos\theta$$

$$\cos\psi = \frac{Z - R\cos\theta}{r}$$

$$d\vec{E} = \frac{\sigma\, ds(z - R\cos\theta)\, \vec{e_z}}{4\pi\varepsilon_0 \left(R^2 + z^2 - 2Rz\cos\theta\right)^{3/2}}$$

$$d\vec{S} = R^2 \operatorname{sen}\theta \, d\theta \, d\phi \; ; \; d\vec{E} = \frac{\sigma R^2 \operatorname{sen}\theta \, d\theta \, d\phi \, (z - R\cos\theta)\, \vec{e_z}}{4\pi\varepsilon_0 \left(R^2 + z^2 - 2Rz\cos\theta\right)^{3/2}}$$

$$d\vec{E} = \frac{2\pi R^2 \operatorname{sen}\theta \, d\theta \, (z - R\cos\theta)}{4\pi\varepsilon_0 \left(R^2 + z^2 - 2Rz\cos\theta\right)^{3/2}} \; ; \; u = \cos\theta \Rightarrow du = \operatorname{sen}\theta \, d\theta$$

$$\operatorname{sen}\theta \, d\theta \, (z - R\cos\theta) = (z - Ru)\, du$$

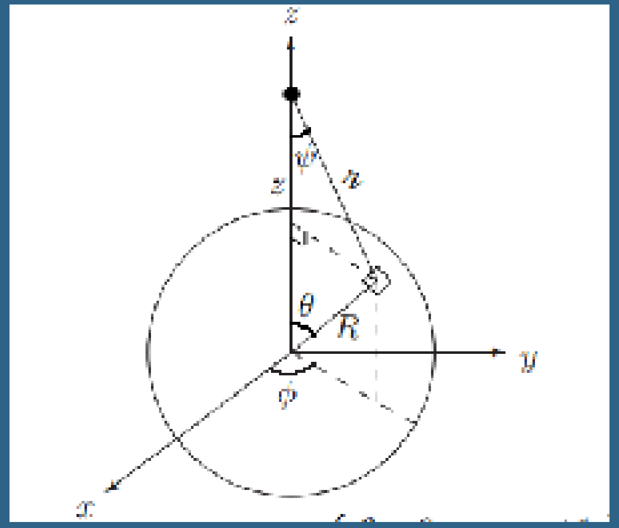
$$d\vec{E} = \frac{(z-R-u)\,du}{(R^2+z^2-2zu)^{3/2}} \times \frac{2\pi\sigma R^2}{4\pi\varepsilon_0}\vec{e_z} \qquad 0<\theta<\pi \ ; \ -1<u<1$$

$$\vec{E} = \frac{2\pi\sigma R^2}{4\pi\varepsilon_0}\int_{-1}^{1}\frac{(z-R-u)\ du\ \vec{e_z}}{(R^2+z^2-2Rzu)^{3/2}} = \frac{2\pi\sigma R^2}{4\pi\varepsilon_0}\left[\frac{1}{z^2}\,\frac{zu-R}{(R^2+z^2-2Rzu)^{1/2}}\right]_{-1}^{1}\vec{e_z}$$

$$\vec{E} = \frac{2\pi\sigma R^2}{4\pi\varepsilon_0 z^2}\left[\frac{z-R}{(R^2+z^2-2Rz)^{1/2}} - \frac{-z-R}{(R^2+z^2-Rzu)^{1/2}}\right] = \frac{2\pi\sigma R^2}{4\pi\varepsilon_0 z^2}\left(\frac{z-R}{z-R} - \frac{(-z-R)}{R+z}\right)\vec{e_z}$$

$$\vec{E} = \frac{2\pi\sigma R^2}{4\pi\varepsilon_0 z^2}\left(\frac{\cancel{z-R}}{\cancel{z-R}} + \frac{\cancel{z+R}}{\cancel{z+R}}\right)\vec{e_z} = \frac{\overbrace{4\pi\sigma R^2}^{Q}}{4\pi\varepsilon_0 z^2}\vec{e_z} = \frac{Q\ \vec{e_z}}{4\pi\varepsilon_0 z^2} \ ; \qquad z \gg R$$

$$z \ll R \ \rightarrow \ \vec{E} = \vec{0}$$



3. **2.9** Suponha que o campo elétrico numa certa região seja $\vec{E} = kr^3\hat{r}$, em coordenadas esféricas, onde k é uma constante conhecida.

(a) Encontre a densidade de carga ρ;

(b) Empregue a lei de Gauss na forma integral para encontrar a carga no interior de uma esfera de raio R centrada na origem.

a) $\vec{\nabla} \vec{E} = \frac{\rho}{\varepsilon_0}$ $\quad \vec{E} = k r^3 \hat{r}$

$\vec{\nabla} \vec{E} = \frac{\rho}{\varepsilon_0} \Rightarrow \rho = \varepsilon_0 \vec{\nabla} \cdot \vec{E}$

$\rho = \varepsilon_0 \times \frac{1}{r^2} \frac{d}{dr}\left(r^2 \times k r^3\right) = \varepsilon_0 \times \frac{1}{r^2} \frac{d}{dr}\left(k r^5\right) = \frac{\varepsilon_0}{\cancel{r^2}} \times 5 k r^{\cancel{2}4}$

$\boxed{\rho = 5 k \varepsilon_0 r^2}$

b) Lei de Gauss $\oint_S \vec{E} \cdot d\vec{S} = \frac{Q_{int}}{\varepsilon_0} \Rightarrow Q_{int} = \varepsilon_0 \oiint \vec{E} \cdot d\vec{S}$

$d\vec{S} = R^2 \operatorname{sen}\theta \, d\theta \, d\phi \, \hat{r} \Rightarrow Q_{int} = \varepsilon_0 \iint k R^3 \times R^2 \operatorname{sen}\theta \, d\theta \, d\phi$

$Q_{int} = k R^5 \varepsilon_0 \int_0^{\pi} \operatorname{sen}\theta \, d\theta \int_0^{2\pi} d\phi = 2\pi k R^5 \left[-\cos\theta\right]_0^{\pi} = \varepsilon_0 \times 2\pi k R^5 \left[+1+1\right]$

$= 4\pi k R^5 \varepsilon_0$

$\boxed{Q_{int} = 4\pi K \varepsilon_0 R^5}$

4. **2.11** Aplique a lei de Gauss para obter os campos elétricos na questão 2.

$$\oiint \vec{E}\cdot d\vec{S} = \frac{Q_{int}}{\varepsilon_0} \qquad r<R \quad Q_{int} = 0$$

$$\Rightarrow \quad \vec{E} = 0$$

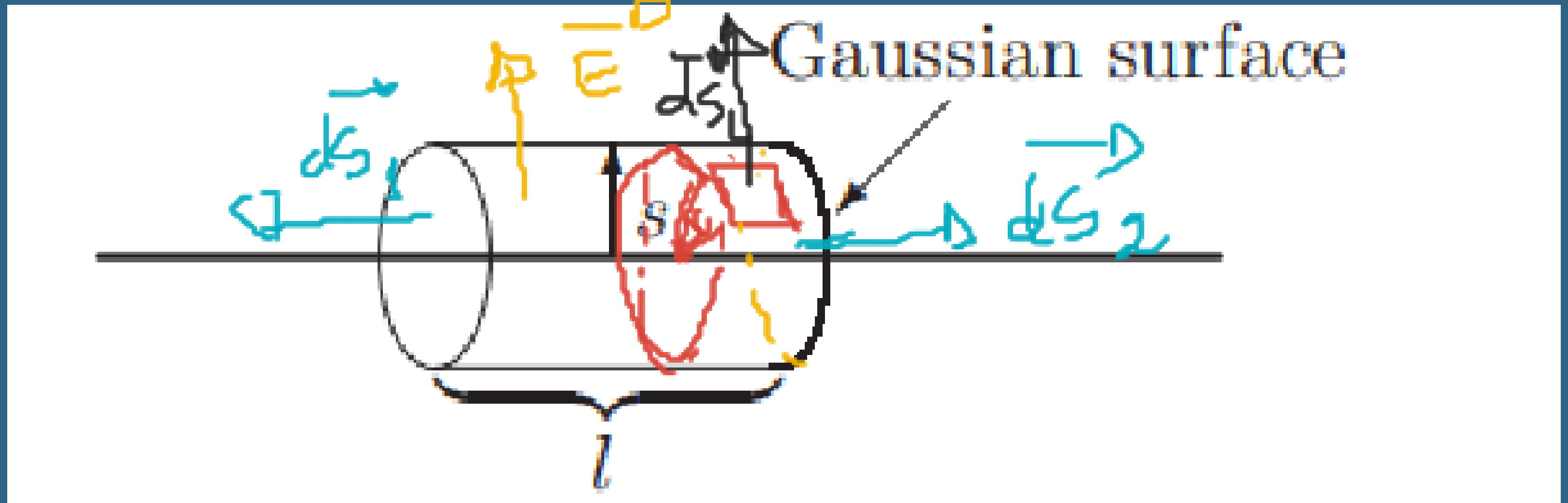
$$\text{se} \quad r > R \Rightarrow \oiint \vec{E}\cdot d\vec{S} = \frac{Q_{int}}{\varepsilon_0}$$

$$= Q$$

$$E \times 4\pi r^2 = \frac{\sigma}{\varepsilon_0} 4\pi R^2 \Rightarrow \quad E = \frac{4\pi R^2 \sigma}{\varepsilon_0 4\pi r^2} = \frac{Q}{4\pi \varepsilon_0 r^2}$$

$$\boxed{E = \frac{Q}{4\pi \varepsilon_0 r^2}}$$

5. **2.13** Encontre o campo elétrico a uma distância s de um fio retilíneo infinito carregado com densidade linear λ.



$$\oiint \vec{E}\cdot d\vec{S} = \frac{Q_{int}}{\varepsilon_0}$$

$$\oiint \vec{E}\cdot d\vec{S} = \underbrace{\iint \vec{E}\cdot d\vec{S}_1}_{=0} + \underbrace{\iint \vec{E}\cdot d\vec{S}_2}_{=0} + \iint \vec{E}\cdot d\vec{S}_L$$

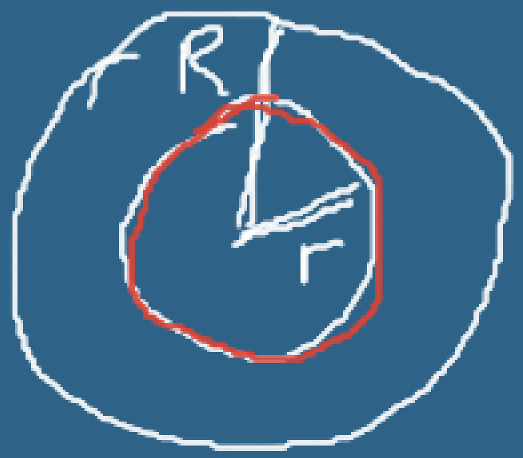
$$\oiint \vec{E}\cdot d\vec{S}_L = \oiint \vec{E}\cdot s\,d\theta\,dz\,\vec{e_r} = \frac{Q_{int}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

$$E\int_0^{2\pi} s\,d\theta \int_0^L dz = s\,(2\pi - 0)\,\Big(z\Big]_0^L = \frac{\lambda L}{\varepsilon_0} \Rightarrow E\,2\pi s L = \frac{\lambda L}{\varepsilon_0}$$

$$2\pi s \cancel{L} E = \frac{\lambda \cancel{L}}{\varepsilon_0} \Rightarrow E = \frac{\lambda}{2\pi \varepsilon_0 s} \Rightarrow \boxed{\vec{E} = \frac{\lambda\, \vec{e_s}}{2\pi \varepsilon_0 s}}$$

6. **2.14** Encontre o campo elétrico dentro de uma esfera com densidade $\rho = kr$, onde k é uma constante. É mais fácil usar a lei de Gauss na forma diferencial.

$\rho = kr$, onde k é uma constante



$$\oint \vec{E} \cdot d\vec{s} = \frac{Q_{int}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int_{\mathcal{V}} \rho \, d\tau = \iiint \frac{k}{\varepsilon_0} r' \, r'^2 \operatorname{sen}\theta \, d\theta \, dr \, d\phi$$

$$4\pi r^2 E = \frac{1}{\varepsilon_0}\int k r'^3 \, dr \int \operatorname{sen}\theta \, d\theta \int d\phi$$

$$4\pi r^2 E = \frac{k}{\varepsilon_0}\left[\frac{r'^4}{4}\right]_0^r \left[-\cos\theta\right]_0^{\pi} \times 2\pi$$

$$4\pi r^2 E = \frac{k}{\varepsilon_0}\frac{r^4}{4} \times 2 \times 2\pi \Rightarrow E = \frac{\cancel{4}\pi r^{\cancel{4}2} k}{\cancel{4} \times 4\pi \varepsilon_0 \cancel{r^2}}$$

$$E = \frac{k\pi r^2}{4\pi\varepsilon_0} \Rightarrow \vec{E} = \frac{k\pi r^2}{4\pi\varepsilon_0}\vec{e_r}$$

$$\vec{E} = \frac{kr^2}{4\varepsilon_0}\vec{e_r}$$

Forma diferencial

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} = \frac{kr}{\varepsilon_0} \Rightarrow \frac{1}{r^2}\frac{d}{dr}\left(r^2 E_r\right) = \frac{kr'}{\varepsilon_0}$$

$$d\left(r^2 E(r)\right) = \frac{kr'}{\varepsilon_0} \times r'^2 dr'$$

$$\int_0^r d\left(r^2 E_r\right) = \int_0^r \frac{k r'^3}{\varepsilon_0} dr' \Rightarrow \left(r E(r) - \underset{=0}{0 \times E(0)}\right) = \int_0^r \frac{k r'^3}{\varepsilon_0} dr'$$

$$r^2 E(r) = \frac{k}{\varepsilon_0} \int_0^r r'^3 dr$$

$$r^2 E(r) = \frac{k}{\varepsilon_0} \left[\frac{r^4}{4}\right]_0^r$$

$$E(r) = \frac{k}{\varepsilon_0} \frac{r^4}{r^2 \times 4} = \frac{k r^2}{4\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{k r^2}{4 \varepsilon_0} \vec{e_r}}$$

7. **2.16** Um cabo axial longo (Fig. 2.26) tem uma densidade de carga volumétrica positiva ρ no cilindro interno (raio a) e uma densidade superficial uniforme σ na casca cilíndrica externa (raio b). A densidade superficial é negativa e tem exatamente o valor necessário para tornar o cabo, como um todo, neutro. Encontre o campo elétrico em cada uma das três regiões:

(a) Dentro do cilindro interno ($s < a$);

(b) Entre os cilindros ($a < s < b$);

(c) Fora do cabo ($b < s$).

Mostre o campo em gráfico, em função de s.

a) $$\oiint \vec{E} \cdot d\vec{S} = E \times 2\pi s L = \frac{Q_{int}}{\varepsilon_0} = \frac{\rho \pi s^2 L}{\varepsilon_0}$$

$$E = \frac{\cancel{\pi} s^{\cancel{2}} \cancel{L} \rho}{\varepsilon_0 \times 2\cancel{\pi} \cancel{s} \cancel{L}} = \frac{\rho s}{2\varepsilon_0}$$

$$\boxed{E = \frac{\rho s}{2\varepsilon_0}}$$

$a < s < b \quad \oint \vec{E} \cdot d\vec{S} = E\, 2\pi s L = \frac{Q_{int}}{\varepsilon_0} = \int \frac{\rho\, d\tau}{\varepsilon_0} = \frac{\rho \pi a^2 L}{\varepsilon_0}$

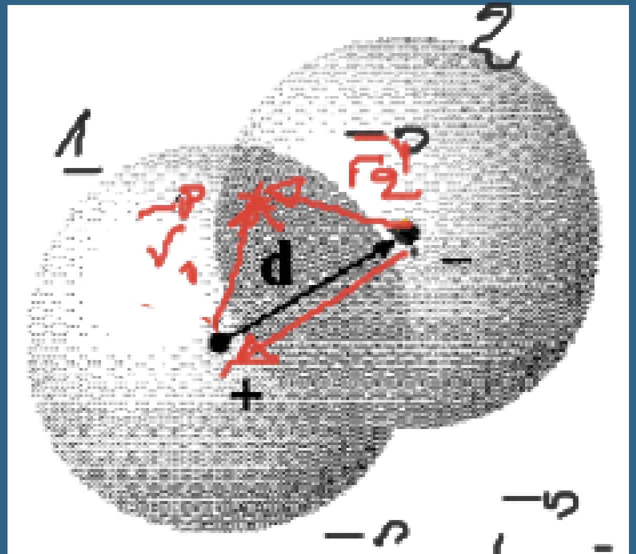
$E = \frac{\rho \cancel{\pi} a^2 \cancel{L}}{2 \cancel{\pi} s \cancel{L} \varepsilon_0} \Rightarrow E = \frac{\rho a^2}{2 s \varepsilon_0} \Rightarrow \vec{E} = \frac{\rho a^2}{2 s \varepsilon_0} \vec{e_r}$

$s > b \quad \oint \vec{E} \cdot d\vec{S} = E \times 2\pi s L = \frac{Q_{int}}{\varepsilon_0} = 0 \Rightarrow Q_{int} = 0$

$E = 0$

8. **2.18** Duas esferas de raio R estão carregadas com densidades $+\rho$ e $-\rho$ e estão posicionadas de forma a haver superposição parcial (Fig. 2.28). Chame de $\vec{d}$ o vetor que vai do centro da esfera negativa até o centro da positiva. Mostre que o campo na região de superposição é uniforme e encontre seu valor.

$$E \times 4\pi r^2 = \frac{\rho}{\varepsilon_0} \times \frac{4}{3}\pi r^3 \Rightarrow \vec{E} = \frac{\rho \vec{r}}{3\varepsilon_0}$$



$$\vec{r}_1 + \vec{d}_2 = \vec{r}_2$$

Figure 2.28

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q_{int}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int_\tau \rho \, d\tau$$

$$\vec{E}_1 = \frac{-\rho \vec{r}_1}{3\varepsilon_0}, \quad \vec{E}_2 = \frac{\rho \vec{r}_2}{3\varepsilon_0}$$

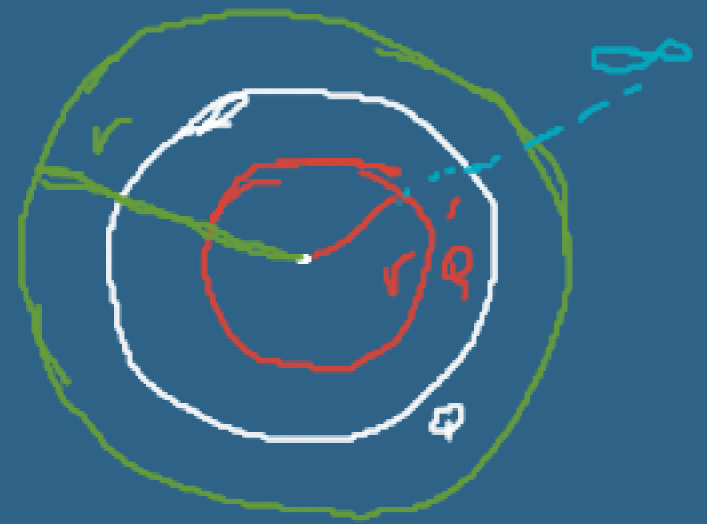
superposição: $\vec{E}_1 + \vec{E}_2 = \frac{-\rho \vec{r}_1}{3\varepsilon_0} + \frac{+\rho \vec{r}_2}{3\varepsilon_0} = \frac{\rho}{3\varepsilon_0}\left(\vec{r}_2 - \vec{r}_1\right) = \frac{\rho \vec{d}}{3\varepsilon_0}$

$$\boxed{\vec{E} = \frac{\rho \vec{d}}{3\varepsilon_0}}$$

$$E \times \cancel{4\pi} \cancel{r_1^2} = \frac{Q_{int}}{\varepsilon_0} = \frac{\cancel{4\pi}}{3} r_1^3 \times \rho$$

$$\vec{E}_1 = \frac{\rho \vec{r}_1}{3\varepsilon_0}, \quad \vec{E}_2 = \frac{-\rho \vec{r}_2}{3\varepsilon_0}$$

9. **2.21** Encontre o potencial dentro ($r < R$) e fora ($r > R$) de uma esfera de raio R carregada uniformemente com densidade ρ. Tome o infinito como ponto de referência. Tome o gradiente do potencial para verificar que ele dá o campo elétrico correto. Mostre em gráfico o potencial em função de r.



$$r > R \Rightarrow \oint \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon_0} \int \rho \, d\tau \Rightarrow$$

$$E \, 4\pi r^2 = \frac{\rho}{\varepsilon_0} \times \frac{4}{3} \pi R^3 = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{4\pi \varepsilon_0 r^2}$$

$$V = \int \vec{E} \cdot d\vec{\ell} \Rightarrow V = -\int E \cdot dr = -\int_{\infty}^{R} \frac{Q}{4\pi \varepsilon_0 r^2} dr = -\frac{Q}{4\pi \varepsilon_0} \int_{\infty}^{R} \frac{1}{r^2} dr = \frac{Q}{4\pi \varepsilon_0} \left[-\frac{1}{r} \right]_{\infty}^{R}$$

$$\boxed{V = \frac{Q}{4\pi \varepsilon_0 R}}$$

$$r < R \qquad E \times 4\pi r^2 = \frac{Q'}{\varepsilon_0} \Rightarrow E = \frac{Q'}{4\pi \varepsilon_0 r^2} ; \quad \begin{array}{l} Q' \rightarrow V' \\ Q \rightarrow V \end{array} \Rightarrow \frac{Q'}{Q} = \frac{V'}{V}$$

$$Q' = \frac{V'}{V} \times Q$$

$$Q' = \frac{Q \times \cancel{4\pi} r^3}{\cancel{3} \times \frac{\cancel{4\pi} R^3}{\cancel{3}}} = Q\frac{r^3}{R^3} \Rightarrow E = \frac{1}{4\pi\varepsilon_0 \cancel{r^2}} \times \frac{Q r^{\cancel{3}}}{R^3} \Rightarrow \boxed{E = \frac{Q}{4\pi\varepsilon_0} \frac{r}{R^3}}$$

$$r < R \; ; \quad V = -\int \vec{E} \cdot d\vec{\ell} = -\int_{\infty}^{R} E\,dr - \int_{R}^{r} E\,dr = -\int_{\infty}^{R} \frac{Q}{4\pi\varepsilon_0 r^2} dr - \int_{R}^{r} \frac{Q}{4\pi\varepsilon_0 R^3} r\,dr$$

$$V = -\frac{Q}{4\pi\varepsilon_0} \int_{\infty}^{R} \frac{dr}{r^2} - \frac{Q}{4\pi\varepsilon_0 R^3} \int_{R}^{r} r\,dr = -\frac{Q}{4\pi\varepsilon_0} \left[\left[-\frac{1}{r} \right]_{\infty}^{R} + \frac{1}{R^3} \left[\frac{r^2}{2} \right]_{R}^{r} \right]$$

$$= -\frac{Q}{4\pi\varepsilon_0} \left(-\frac{1}{R} + 0 + \frac{1}{2R^3} \left[r^2 - R^2 \right] \right)$$

$$= \frac{Q}{4\pi\varepsilon_0 R} - \frac{Q\,(r^2 - R^2)}{8\pi\varepsilon_0 R^3}$$

$$V = \frac{Q}{4\pi\varepsilon_0 R} - \frac{Q}{8\pi\varepsilon_0 R^3}(r^2 - R^2) = \frac{Q}{4\pi\varepsilon_0 R}\left(1 - \frac{(r^2 - R^2)}{2R^2}\right) = \left(1 + \frac{1}{2} - \frac{r^2}{2R^2}\right)\frac{Q}{4\pi\varepsilon_0 R}$$

$$V = \frac{Q}{4\pi\varepsilon_0 R}\left(\frac{3}{2} - \frac{r^2}{2R^2}\right)$$

$$\boxed{V = \frac{Q}{2(4\pi\varepsilon_0 R)}\left(3 - \frac{r^2}{R^2}\right)}$$

$$\vec{E} = -\vec{\text{grad}}\, V = -\vec{\nabla} V \Rightarrow \vec{E} = -\vec{e_r}\frac{d}{dr}\left(\frac{Q}{4\pi\varepsilon_0 r}\right) \qquad r > R$$

$$\vec{E} = -\vec{e_r}\frac{Q}{4\pi\varepsilon_0}\frac{d}{dr}\left(\frac{1}{r}\right) = \vec{e_r}\frac{Q}{4\pi\varepsilon_0 r^2}$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\vec{e_r}$$

$r < R$

$$\vec{E} = -\vec{\nabla} V = -\vec{e_r}\,\frac{d}{dr}\left(\frac{Q}{8\pi\varepsilon_0 R}\left(1-\frac{r^2}{R^2}\right)\right)$$

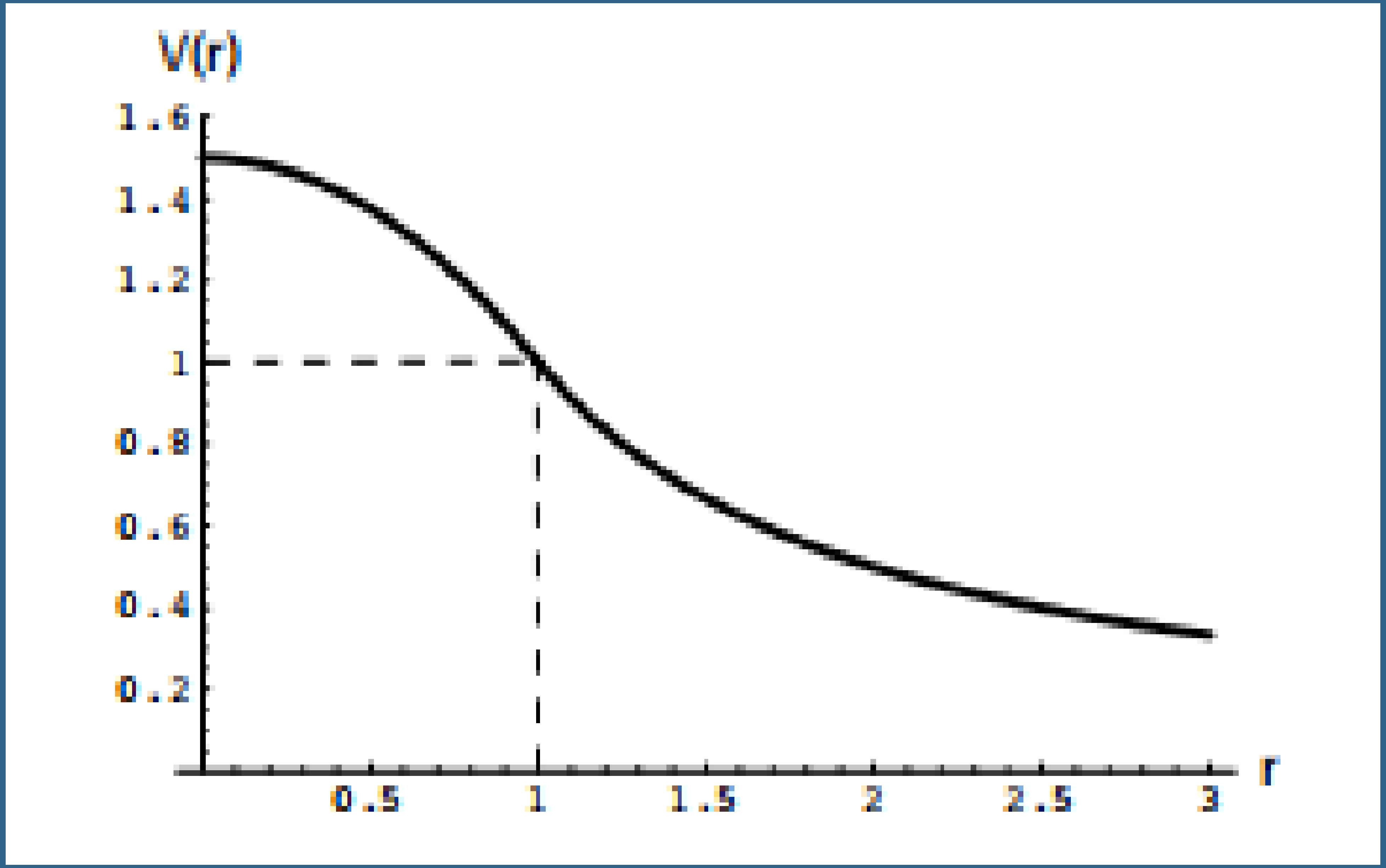
$$= \vec{e_r}\,\frac{Q}{2\times 4\pi\varepsilon_0 R}\,\frac{d}{dr}\left(1-\frac{r^2}{R^2}\right)$$

$$= \vec{e_r}\,\frac{Q}{2\times 4\pi\varepsilon_0 R}\left(0-\frac{2r}{R^2}\right) = \vec{e_r}\,\frac{Q\times\cancel{2}\,r}{\cancel{2}\times 4\pi\varepsilon_0 R^3}$$

$$\vec{E} = \vec{e_r}\,\frac{Q\,r}{4\pi\varepsilon_0 R^3}$$

$$\boxed{\vec{E} = \frac{Q\,r}{4\pi\varepsilon_0 R^3}\,\vec{e_r}}$$

Grafico



10. **2.22** Encontre o potencial elétrico a uma distância s do fio na questão 5. Calcule o gradiente e verifique que dá o campo elétrico calculado na questão 5.

questão 5: $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 s}\vec{e_s}$

$$V = -\int \vec{E}\cdot d\vec{l} = \int_a^s \vec{E}\cdot ds\,\vec{e_s} = \int_a^s \frac{\lambda}{2\pi\varepsilon_0}\frac{ds'}{s'} = \frac{\lambda}{2\pi\varepsilon_0}\int_a^s \frac{ds'}{s'} = \frac{\lambda}{2\pi\varepsilon_0}\Big[\ln s\Big]_a^s$$

$$V = \frac{\lambda}{2\pi\varepsilon_0}\left(\ln s - \ln a\right) = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{s}{a}$$

$$\boxed{V = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{s}{a}}$$

$$\vec{E} = -\vec{\nabla}V = \vec{e_s}\frac{\partial V}{\partial s} = \vec{e_s}\frac{\partial}{\partial s}\left(\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{s}{a}\right) = \vec{e_s}\frac{\lambda}{2\pi\varepsilon_0}\frac{\partial}{\partial s}\left(\ln\frac{s}{a}\right)$$

$$= \vec{e_s}\frac{\lambda}{2\pi\varepsilon_0}\frac{1/a}{s/a} = \vec{e_s}\frac{\lambda}{2\pi\varepsilon_0}\frac{1}{s}$$

$$\vec{E} = \frac{h}{2\pi \varepsilon_0 s}\vec{e_s}$$