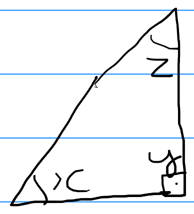
Seja x, y, z os ângulos de um triângulo. Prove que se

$$\cos x + \cos y = \operatorname{sen} z$$

então o triângulo é retângulo.

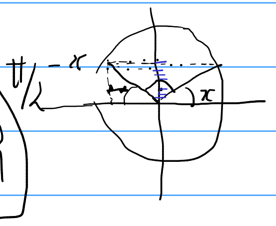
$$\cos x + \overset{=0}{\cos \pi/2} = \operatorname{sen} z \longrightarrow$$



$$\boxed{x + z = \pi/2}$$

$$\cos x = \operatorname{sen} z$$

$$\boxed{\begin{aligned}\cos x &= \operatorname{sen}(\pi/2 - x)\\ &= \cancel{\overset{1}{\operatorname{sen}\pi/2}}\ \underset{\cos x}{\underset{\parallel}{\cos -x}} + \cancel{\operatorname{sen} -x\ \overset{0}{\cos \pi/2}}\end{aligned}}$$



$$\Rightarrow x + y + z = \pi$$
$$z = \pi - (x + y)$$

$$\cos x + \cos y = \operatorname{sen} z$$

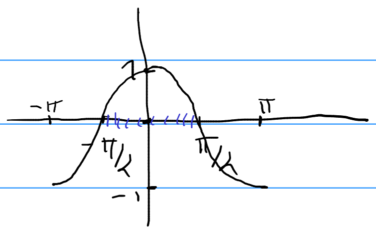
$$2\cos\frac{x+y}{2}\cos\frac{x-y}{2} = \operatorname{sen} z = \operatorname{sen}(z/2 + z/2)$$
$$= 2\operatorname{sen} z/2 \cos z/2$$

$$\operatorname{sen} z/2 = \operatorname{sen}\left(\pi/2 - \frac{x+y}{2}\right) = \cos\frac{x+y}{2}$$

$$\cancel{2\cos\frac{x+y}{2}}\cos\frac{x-y}{2} = \cancel{2\cos\frac{x+y}{2}}\cos z/2$$

$$-2\pi < x - y < 2\pi \qquad 0 < z/2 < \pi/2$$

$$\cos\frac{x-y}{2} = \cos z/2 \qquad -\pi < \frac{x-y}{2} < \pi/2$$



$$\Downarrow$$

$x + y + z = \pi$

$$x - y = z \quad \text{ou} \quad y - x = z$$
$$x = y + z \qquad\qquad \Downarrow$$
$$\Downarrow \qquad\qquad y = \pi/2$$
$$x = \pi/2$$