Slides - PEscalar

(2) $\vec{u} = (1, a, -2a-1)$

$\vec{v} = (a, a-1, 1)$, $a = ?$ tal que $\vec{u}\cdot\vec{v} = (\vec{u}+\vec{v})\cdot\vec{w}$

$\vec{w} = (a, -1, 1)$

$\vec{u}\cdot\vec{v} = (1, a, -2a-1)\cdot(a, a-1, 1) = a^2 - 2a - 1$

$\vec{u}+\vec{v} = (1, a, -2a-1) + (a, a-1, 1) = (1+a, 2a-1, -2a)$

$(\vec{u}+\vec{v})\cdot\vec{w} = (1+a, 2a-1, -2a)\cdot(a, -1, 1)$

$= a + a^2 - 2a + 1 - 2a$

$= a^2 - 3a + 1$

Igualando:

$\cancel{a^2} - 2a - 1 = \cancel{a^2} - 3a + 1$

$$a = 2$$

$x_u x_w + y_u y_w + z_u z_w$

$\vec{u}\cdot\vec{w} = (1, a, -2a-1)\cdot(a, -1, 1)$

Produto Escalar

$= 1\cdot a + a(-1) + (-2a-1)(1)$

$= \cancel{a} - \cancel{a} - 2a - 1 = -2a - 1$

PEscalar

$\ast\ast$ $\vec{u}\cdot\vec{w} = (1,2,3)\cdot(3,2,-1) = 1\cdot 3 + 2\cdot 2 + 3(-1)$

$= \cancel{3} + 4 - \cancel{3} = 4$

NÚMERO!

Soma vetorial

$\vec{u}+\vec{w} = (1,2,3) + (3,2,-1) = (1+3, 2+2, 3-1)$

$= (4, 4, -1)$

VETOR $\in \mathbb{R}^3$!

Lista 1

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$\vec{v} = (1,1,1)$

$\vec{w} = (0,1,-1)$

$\vec{t} = (2,1,-1)$

$\vec{u} = ?$

i) $|\vec{u}| = \sqrt{5}$

ii) $\vec{u} \perp \vec{t}$

iii) $\{\vec{u}, \vec{v}, \vec{w}\}$ LD

$\vec{u} = (a, b, c)$

i) $\sqrt{a^2+b^2+c^2} = \sqrt{5} \quad \therefore \quad a^2+b^2+c^2 = 5$

ii) $\vec{u} \cdot \vec{t} = 0$

$(a,b,c) \cdot (2,1,-1) = 0 \quad \therefore \quad 2a + b - c = 0$

iii) $\vec{u} = \alpha\vec{v} + \beta\vec{w}$

$(a,b,c) = \alpha(1,1,1) + \beta(0,1,-1)$

$(a,b,c) = (\alpha,\alpha,\alpha) + (0,\beta,-\beta)$

$(a,b,c) = (\alpha, \alpha+\beta, \alpha-\beta)$

$$\therefore \begin{cases} a = \alpha \\ b = \alpha + \beta \\ c = \alpha - \beta \end{cases}$$

⋮

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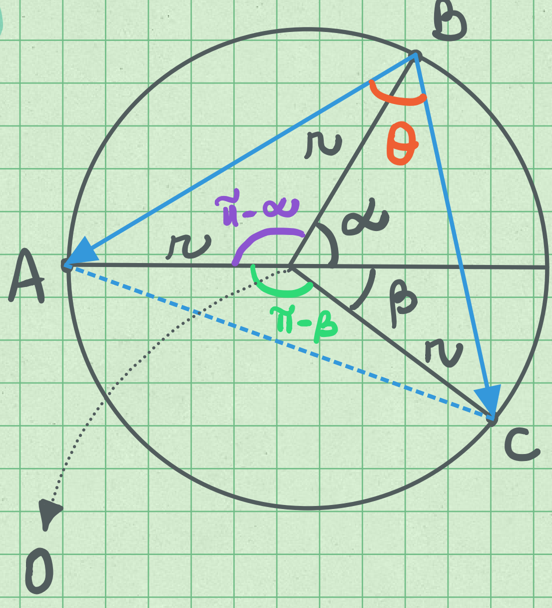

$\vec{BA} \cdot \vec{BC} = ?$

$|\vec{BA}| \neq |\vec{BC}|$

$\vec{BA} \cdot \vec{BC} = |\vec{BA}|\,|\vec{BC}| \cos\theta$

Triângulo AOB :

$$|\vec{BA}|^2 = |\vec{OA}|^2 + |\vec{OB}|^2 - 2|\vec{OA}||\vec{OB}|\cos(\pi - \alpha) \quad = -\cos\alpha$$

$$|\vec{BA}|^2 = r^2 + r^2 - 2r\cdot r(-\cos\alpha)$$

$$|\vec{BA}|^2 = 2r^2(1+\cos\alpha)$$

Triângulo BOC :

$$|\vec{BC}|^2 = |\vec{OB}|^2 + |\vec{OC}|^2 - 2|\vec{OB}||\vec{OC}|\cos(\alpha+\beta)$$

$$|\vec{BC}|^2 = r^2 + r^2 - 2rr\cos(\alpha+\beta)$$

$$|\vec{BC}|^2 = 2r^2(1-\cos(\alpha+\beta))$$

Triângulo AOC :

$$|\vec{AC}|^2 = |\vec{OA}|^2 + |\vec{OC}|^2 - 2|\vec{OA}||\vec{OC}|\cos(\pi-\beta) \quad = -\cos\beta$$

$$|\vec{AC}|^2 = r^2 + r^2 - 2rr(-\cos\beta)$$

$$|\vec{AC}|^2 = 2r^2(1+\cos\beta)$$

Triângulo ABC :

$$|\vec{AC}|^2 = |\vec{BA}|^2 + |\vec{BC}|^2 - 2|\vec{BA}||\vec{BC}|\cos\theta \quad = \vec{BA}\cdot\vec{BC}$$

$$2r^2(1+\cos\beta) = 2r^2(1+\cos\alpha) + 2r^2(1-\cos(\alpha+\beta)) - 2\,\vec{BA}\cdot\vec{BC}$$

$$-\vec{BA}\cdot\vec{BC} = r^2 - r^2\cos\beta - r^2 + r^2\cos\alpha - r^2 + r^2\cos(\alpha+\beta)$$

$$\vec{BA}\cdot\vec{BC} = r^2(1+\cos\alpha-\cos\beta-\cos(\alpha+\beta))$$

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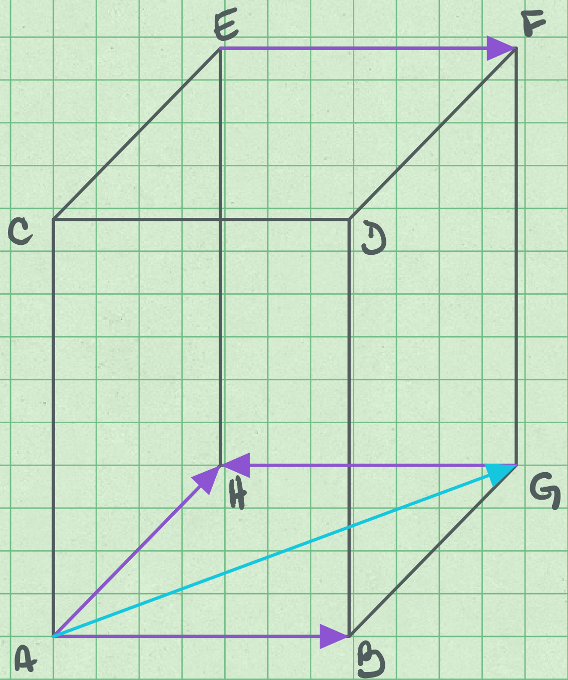


a) $\vec{x} = \vec{GH} - \vec{HE} - \vec{FE} + \vec{AE} + \vec{AB}$

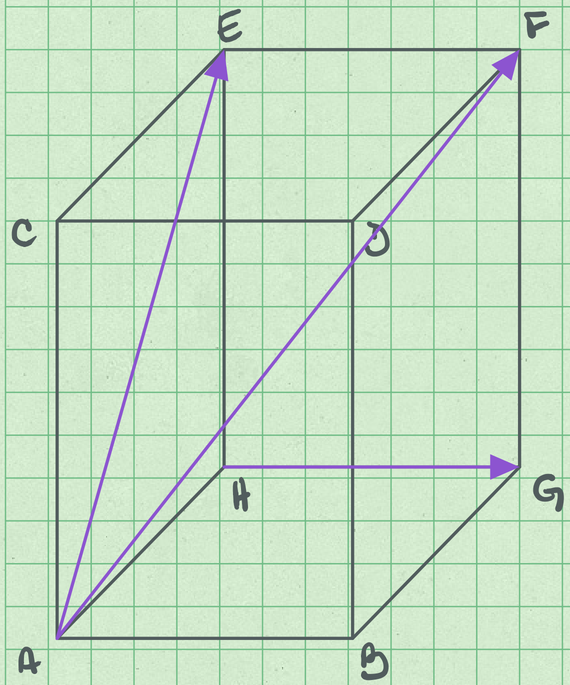
$\vec{x} = \vec{GH} + \vec{EH} + \vec{EF} + \vec{AE} + \vec{AB}$

$\vec{x} = \vec{GH} + \vec{EF} + \vec{AB} + \vec{AE} + \vec{EH}$

$\vec{x} = \underbrace{\vec{GH} + \vec{EF}}_{\vec{0}} + \vec{AB} + \vec{AH}$

$\vec{x} = \vec{AB} + \vec{AH}$

$\vec{x} = \vec{AG}$



b) $\vec{x} = \vec{AB} + \vec{HG} + \vec{AC} + \vec{DF} + \vec{CE} + \vec{BD}$

$\vec{x} = \vec{AB} + \vec{BD} + \vec{AC} + \vec{CE} + \vec{HG} + \vec{DF}$

$\vec{x} = \vec{AD} + \vec{AE} + \vec{HG} + \vec{DF}$

$\vec{x} = \vec{AF} + \vec{AE} + \vec{HG}$

$\vec{x} = \vec{AF} + \vec{AE} + \vec{EF}$

$\vec{x} = \vec{AF} + \vec{AF}$

$\vec{x} = 2\vec{AF}$

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$\{\vec{u}, \vec{v}\}$ LI

$\{\vec{u}+\vec{v}, \vec{u}-\vec{v}\}$ LI

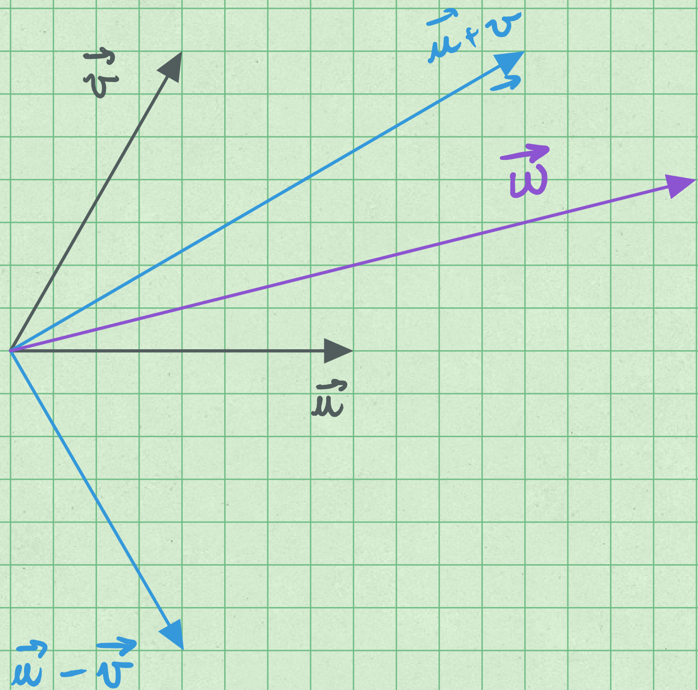
Se $\{\vec{u}, \vec{v}\}$ é base, então $\{\vec{u}+\vec{v}, \vec{u}-\vec{v}\}$ é base!

Base $\begin{cases} \text{2 vetores LI} \\ \text{gerar } \mathbb{R}^2 : \text{CL} \end{cases}$

Dado: $\{\vec{u}+\vec{v}, \vec{u}-\vec{v}\}$ é LI.

$\forall \vec{w} \in \mathbb{R}^2$, $\exists$ CL em termos de $\{\vec{u}+\vec{v}, \vec{u}-\vec{v}\}$?

Geometricamente:



Todo $\vec{w}$ coplanar com $\{\vec{u}, \vec{v}\}$ ou $\{\vec{u}+\vec{v}, \vec{u}-\vec{v}\}$ pode ser escrito como CL dos vetores do conjunto, logo, ambos os conjuntos geram o $\mathbb{R}^2$.

Algebricamente:

$\exists$ $\vec{w}$ como CL de $\{\vec{u}, \vec{v}\}$:

$\vec{w} = (x,y)$ então $\vec{w} = \alpha\vec{u} + \beta\vec{v}$

$\{\vec{u}+\vec{v}, \vec{u}-\vec{v}\}$

$\downarrow$

$\gamma(\vec{u}+\vec{v}) + \delta(\vec{u}-\vec{v})$

$(\gamma+\delta)\vec{u} + (\gamma-\delta)\vec{v}$

$$(x,y) = \alpha(xu, yu) + \beta(xv, yv)$$

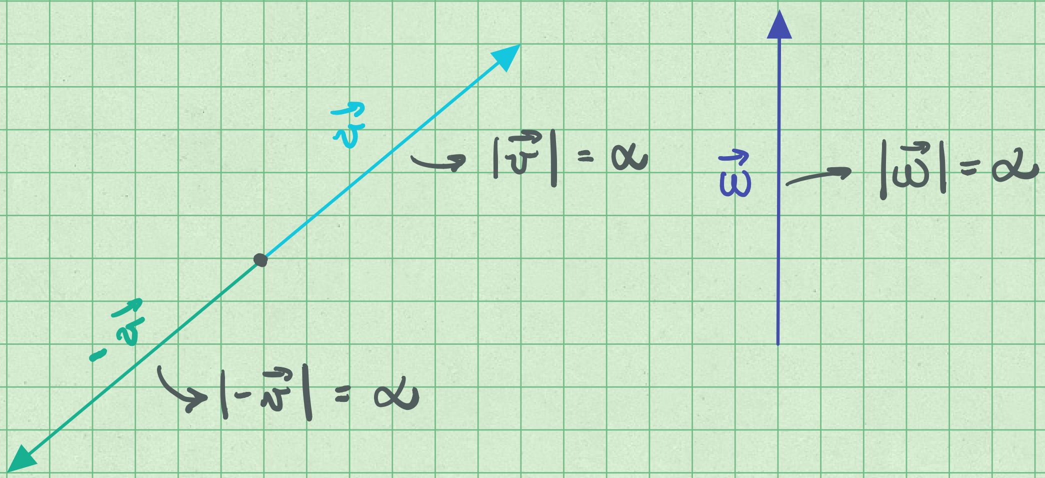
$\vdots$

$$\alpha = f(x,y)$$
$$\beta = g(x,y)$$
$\rightarrow \exists\ \alpha, \beta\ \forall x, y \in \mathbb{R}$

Logo, a CL sempre $\exists$ $\forall \vec{w} \in \mathbb{R}^2$.

② Se $|\vec{AB}| = |\vec{CD}| \Rightarrow \vec{AB} = \vec{CD}$

V ou F ?



FALSO. Vetores podem ter o mesmo módulo, independente da direção e do sentido. E para que os vetores sejam iguais, devem ter mesmos M, D e S.