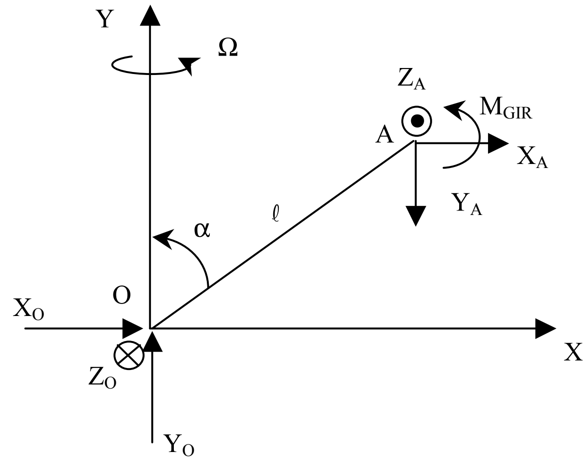
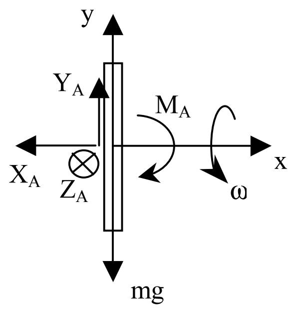
Escola Politécnica da USP – Departamento de Engenharia Mecânica
PME 2200 – Mecânica B – Prova Substitutiva: 28/06/2001 – Duração 100 min.
Gabarito

1ª Questão (3,0 pontos):





Disco isolado, TMA em A (A ≡ baricentro do disco):

$$\dot{\vec{H}}_A = \vec{M}_A$$

$$\vec{H}_A = \qquad [J]_A \begin{Bmatrix} \omega \\ \Omega \\ 0 \end{Bmatrix} = \frac{mr^2}{2}\omega\vec{i} + \frac{mr^2}{4}\Omega\vec{j} \rightarrow \dot{\vec{H}}_A = -\frac{mr^2}{2}\omega\Omega\vec{k}$$

$$\Rightarrow \vec{M}_A = -\frac{mr^2}{2}\omega\Omega\vec{k}$$

$$\boxed{\Rightarrow \vec{M}_{GIR} = \frac{mr^2}{2}\omega\Omega\vec{k}}$$

Disco isolado, TMB:

$$m\vec{a}_G = -X_A\vec{i} + (Y_A - mg)\vec{j} + Z_A\vec{k}$$

$$-m\Omega^2\ell sen\alpha\vec{i} = -X_A\vec{i} + (Y_A - mg)\vec{j} - Z_A\vec{k}$$

$$\Rightarrow \begin{cases} X_A = m\Omega^2\ell sen\alpha \\ Y_A = mg \\ Z_A = 0 \end{cases}$$

TMA na barra de massa desprezível:

$$\dot{\vec{H}}_O = \vec{0} = \vec{M}_O = \left(\frac{mr^2}{2}\omega\Omega - X_A\ell\cos\alpha - Y_A l\ell sen\alpha\right)\vec{k} \rightarrow r^2\omega\Omega - 2\Omega^2\ell^2 sen\alpha\cos\alpha - 2g\ell sen\alpha = 0$$

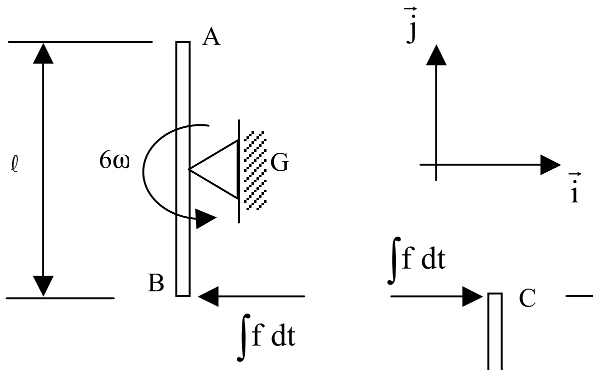
Disco + barra de massa desprezível, TMB:

$$m\vec{a}_G = X_O\vec{i} + (Y_O - mg)\vec{j} + Z_O\vec{k}$$

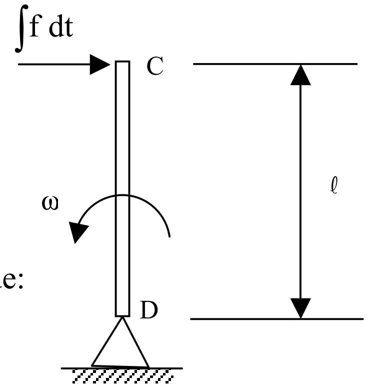
$$-m\Omega^2\ell sen\alpha\vec{i} = X_O\vec{i} + (Y_O - mg)\vec{j} + Z_O\vec{k}$$

$$\boxed{\Rightarrow \begin{cases} X_O = -m\Omega^2\ell sen\alpha \\ Y_O = mg \\ Z_O = 0 \end{cases}}$$

2ª Questão (3,5 pontos):



$$\vec{v}_C = -\omega\ell\vec{i}$$
$$\vec{v}_B = 3\omega\ell\vec{i}$$
$$\vec{v}'_C = v'_C\vec{i}$$
$$\vec{v}'_B = v'_B\vec{i}$$
$$\vec{\omega}'_{AB} = \omega'_{AB}\vec{k}$$
$$\vec{\omega}'_{CD} = -\omega'_{CD}\vec{k}$$



Relação entre as velocidades relativas antes e após o choque:

$$v'_C - v'_B = e(v_B - v_C)$$
$$v'_C - v'_B = 4\omega\ell e$$
$$\omega'_{CD}\ell - \omega'_{AB}\frac{\ell}{2} = 4\omega\ell e \quad \rightarrow \quad 2\omega'_{CD} - \omega'_{AB} = 6{,}4\omega \qquad (1)$$

Princípio do Impulso e Quantidade de Movimento para a barra AB, pólo em G:

$$\Delta\vec{H}_G = -\frac{\ell}{2}\left(\int f\,dt\right)\vec{k}$$
$$\frac{m\ell^2}{12}\left(\omega'_{AB} - 6\omega\right) = -\frac{\ell}{2}\left(\int f\,dt\right) \;\rightarrow - \int f\,dt = \frac{m\ell}{6}\left(\omega'_{AB} - 6\omega\right) \qquad (2)$$

Princípio do Impulso e Quantidade de Movimento para a barra CD, pólo em D:

$$\Delta\vec{H}_D = -\ell\left(\int f\,dt\right)\vec{k}$$
$$\frac{m\ell^2}{3}\left(-\omega'_{CD} - \omega\right) = -\ell\left(\int f\,dt\right) \quad \rightarrow - \int f\,dt = \frac{m\ell}{3}\left(-\omega'_{CD} - \omega\right) \qquad (3)$$

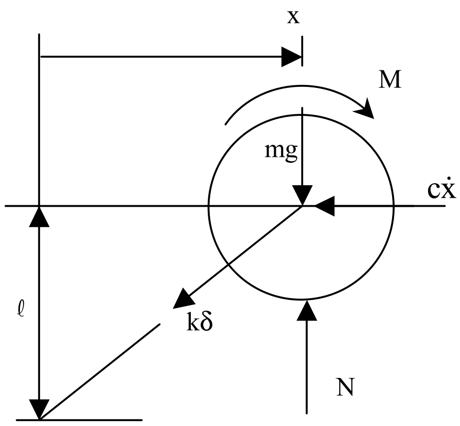
Igualando (2) e (3):

$$\omega'_{AB} + 2\omega'_{CD} = 4\omega \qquad (4)$$

Resolvendo (1) e (4):

$$\boxed{\begin{aligned}\omega'_{CD} &= 2{,}6\omega \\ \omega'_{AB} &= -1{,}2\omega\end{aligned}}$$

3ª Questão (3,5 pontos):



Deformação da mola:

$$\delta = \left(x^2 + \ell^2\right)^{1/2} - \ell$$

Energia potencial:

$$V = \frac{1}{2}k\delta^2 = \frac{1}{2}k\left[\left(x^2 + \ell^2\right) - 2\ell\left(x^2 + \ell^2\right)^{1/2} + \ell^2\right]$$

Função Dissipação de Rayleigh:

$$R_{ay} = \frac{1}{2}c\dot{x}^2$$

Energia cinética:

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}\frac{mR^2}{2}\dot{\theta}^2 = \frac{1}{2}m\dot{x}^2 + \frac{mR^2}{4}\left(\frac{\dot{x}}{R}\right)^2 = \frac{3m}{4}\dot{x}^2$$

Força generalizada:

$$\delta W = Md\theta = M\frac{dx}{R} \rightarrow Q' = \frac{M}{R}$$

Função Lagrangeana:

$$L = T - V = \frac{3m}{4}\dot{x}^2 - \frac{1}{2}k\left(x^2 + \ell^2\right) + k\ell\left(x^2 + \ell^2\right)^{1/2} - \frac{1}{2}k\ell^2$$

$$\frac{\partial L}{\partial \dot{x}} = \frac{3m}{2}\dot{x} \quad ; \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{3m}{2}\ddot{x}$$

$$\frac{\partial L}{\partial x} = -kx + k\ell x\left(x^2 + \ell^2\right)^{-1/2}$$

$$\frac{\partial R}{\partial \dot{x}} = c\dot{x}$$

$$\boxed{\frac{3m}{2}\ddot{x} + c\dot{x} + kx - k\ell x\left(x^2 + \ell^2\right)^{-1/2} = \frac{M}{R}}$$