Eletromagnetismo

$$\vec{\nabla} \cdot \vec{D} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

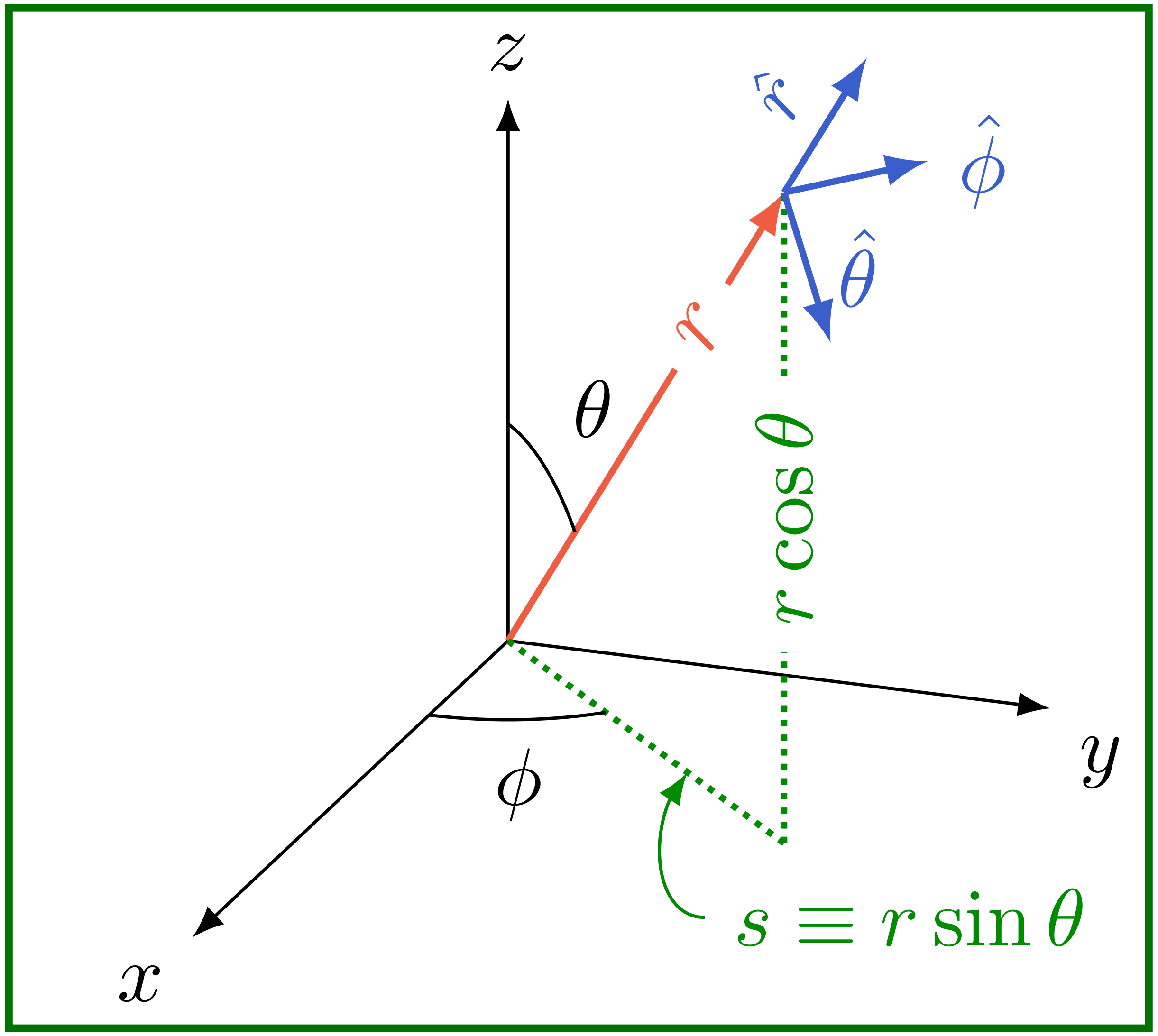
$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{j} + \epsilon_0 \frac{\partial \vec{D}}{\partial t}$$

Aula de 26 de abril

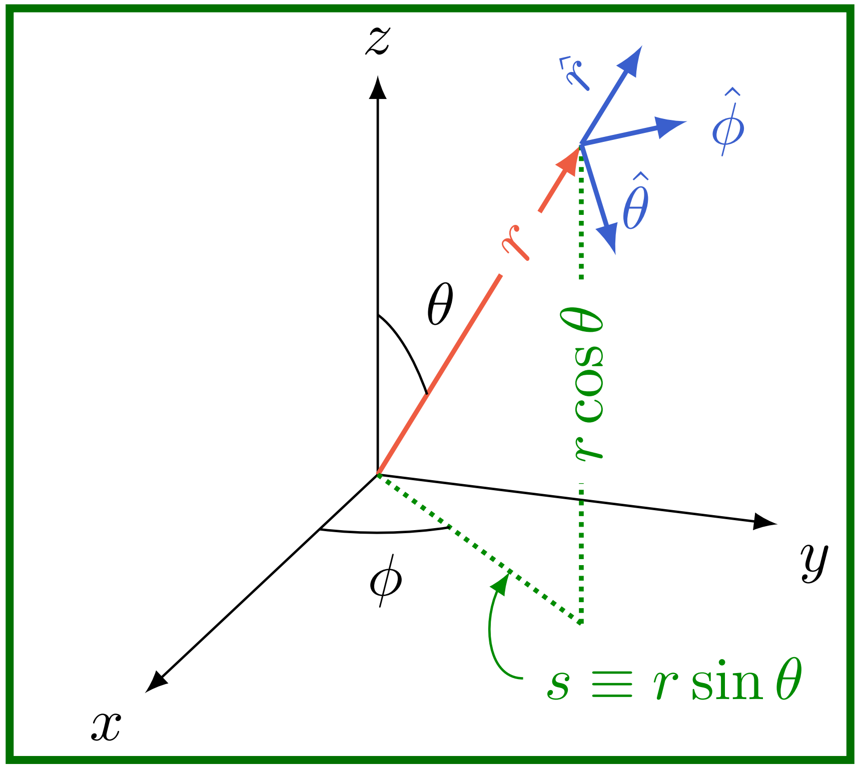
Análise vetorial

Coordenadas esféricas



Coordenadas esféricas

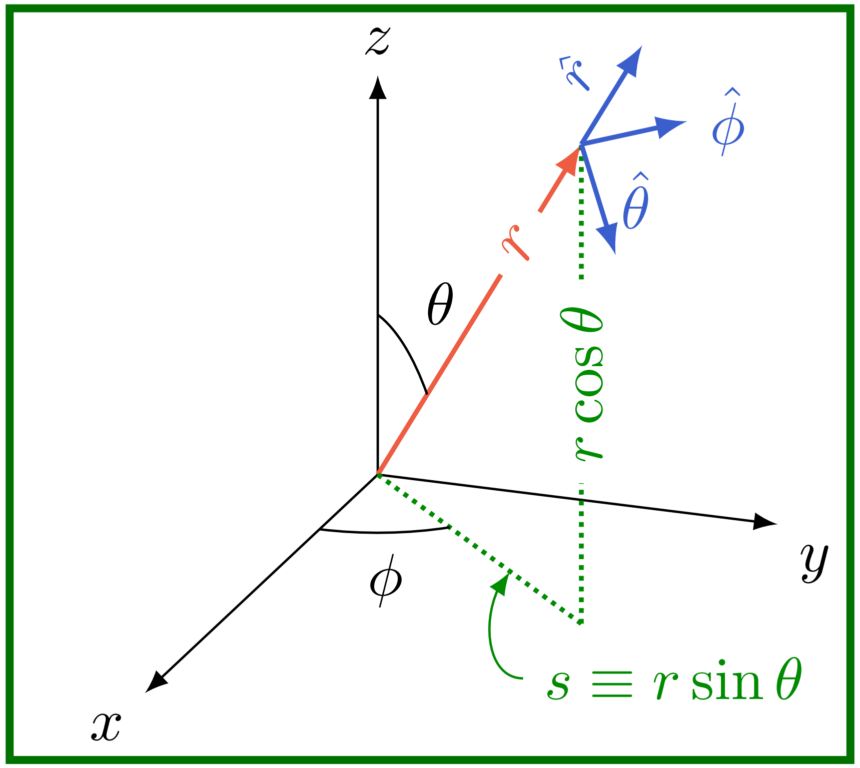
$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$



Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$$

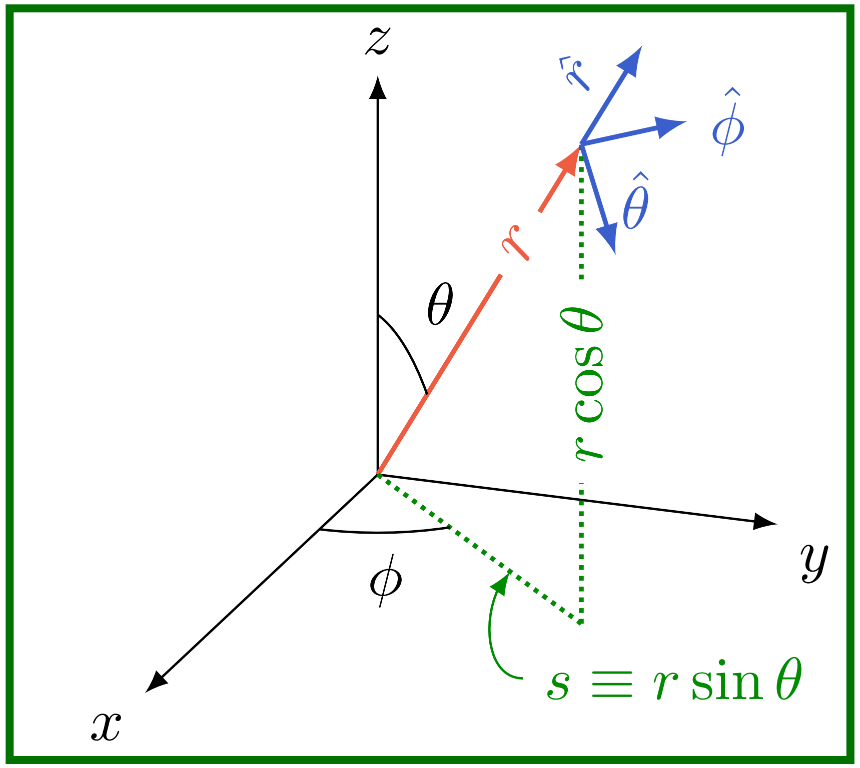


Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$$

$$\vec{\boldsymbol{\nabla}}t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$



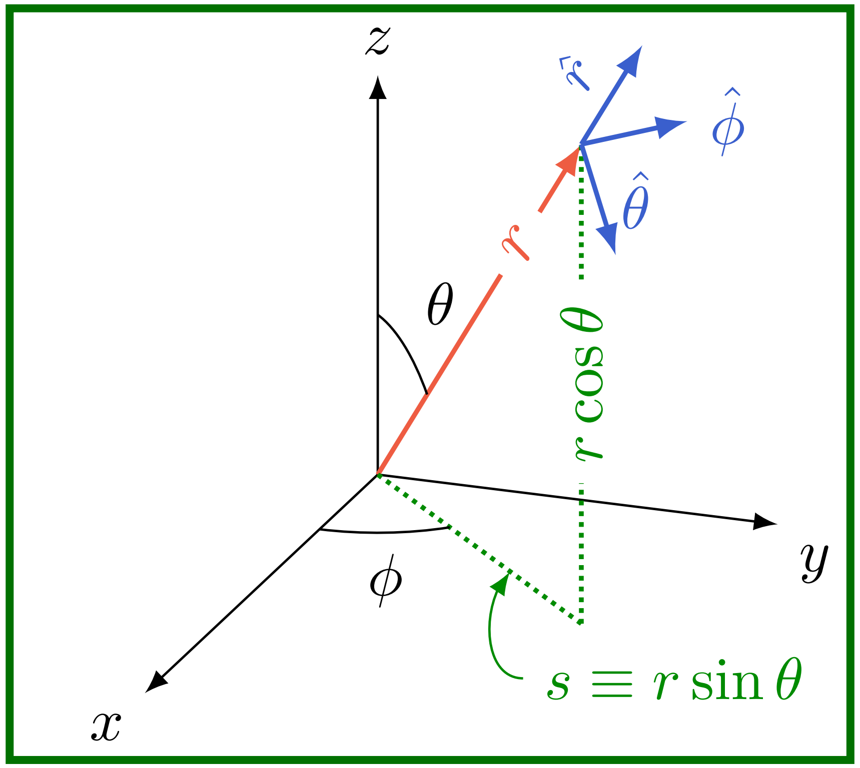
Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$d\tau = r^2 \sin\theta\,dr\,d\theta\,d\phi$$

$$\vec{\boldsymbol{\nabla}} t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

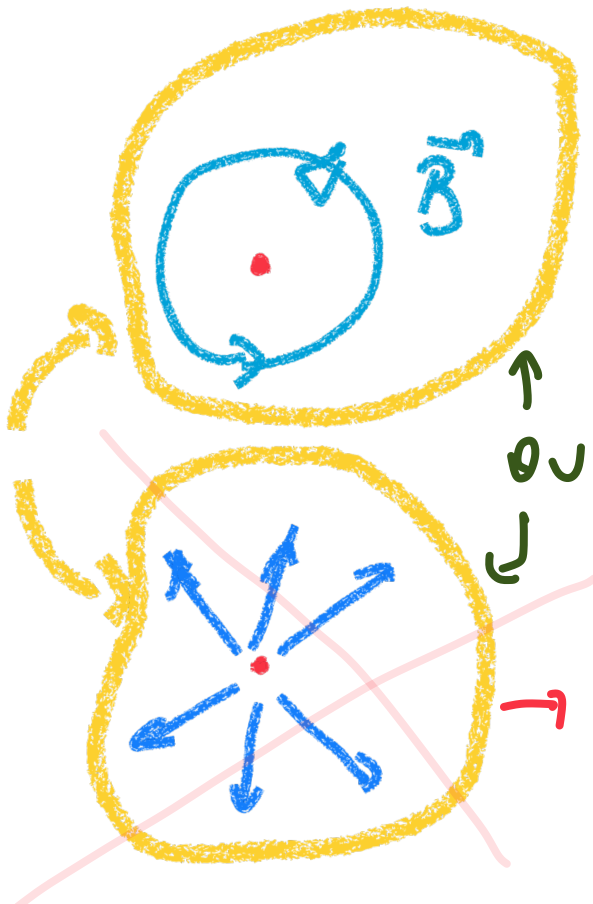
$$\vec{\boldsymbol{\nabla}} \cdot \vec{v} = \frac{1}{r^2}\frac{\partial (r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial (\sin\theta v_\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$



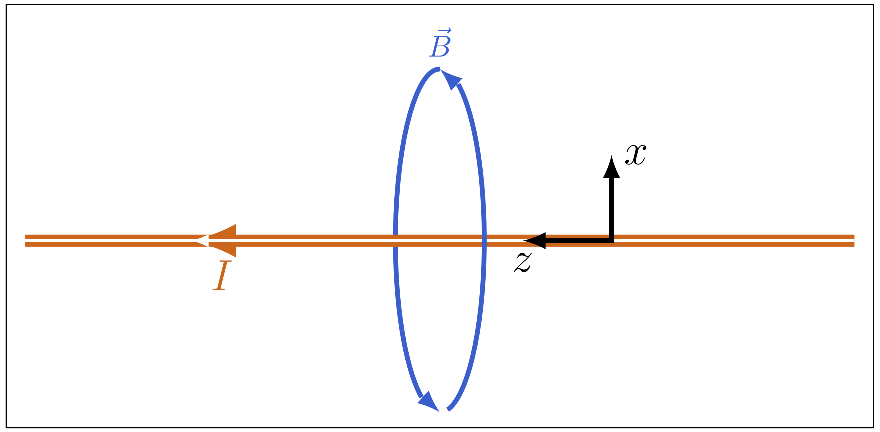
Pratique o que aprendeu

$\vec{B} = ?$

SIMETRIA ⇒

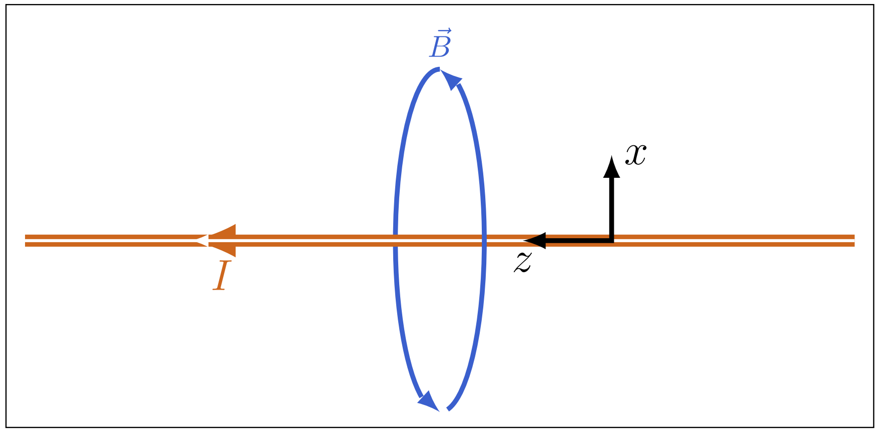


→ SÓ SE HOUVESSE MONOPOLOS MAGNÉTICOS



Pratique o que aprendeu

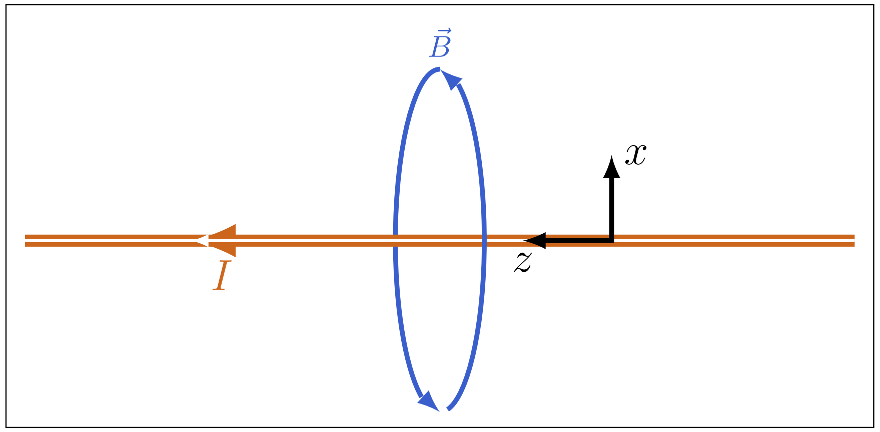
$$\overrightarrow{B} = B_\phi \hat{\phi}$$



Pratique o que aprendeu

$$\overrightarrow{B} = B_\phi \hat{\phi}$$

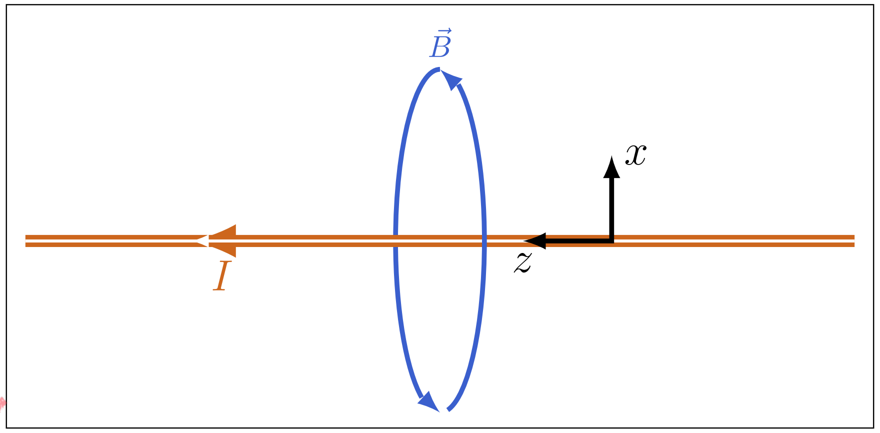
$$\overrightarrow{\nabla} \cdot \overrightarrow{B} = 0$$



Pratique o que aprendeu

$\vec{B} = B_\phi \hat{\phi}$

$\vec{\nabla} \cdot \vec{B} = 0$



$$\vec{\nabla} \cdot \vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

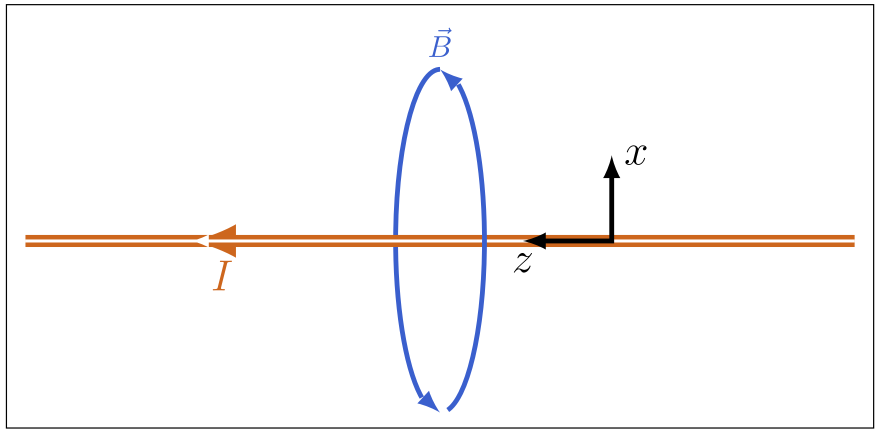
$$\vec{\nabla} \cdot \vec{B} = \frac{1}{r\sin\theta}\frac{\partial B_\phi}{\partial\phi} = 0$$

B_ϕ não depende de ϕ

Pratique o que aprendeu

$$\overrightarrow{B} = B_\phi \hat{\phi}$$

$$\overrightarrow{\nabla} \cdot \overrightarrow{B} = 0$$



$$\boldsymbol{\vec{\nabla}} \cdot \vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

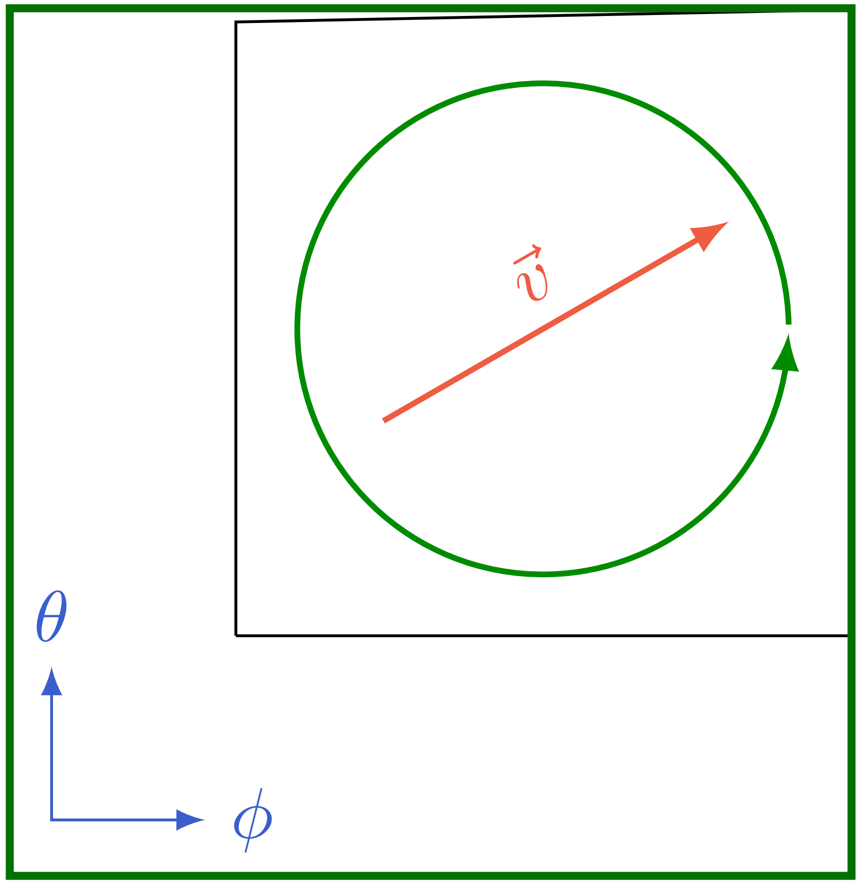
$$\overrightarrow{\nabla} \cdot \overrightarrow{B} = \frac{1}{r\sin\theta}\frac{\partial B_\phi}{\partial\phi}$$

$$B_\phi = B(r,\theta) \Rightarrow \overrightarrow{B} = B(r,\theta)\hat{\phi}$$

Coordenadas esféricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}r\,\hat{r} + r\,\mathrm{d}\theta\,\hat{\theta} + r\sin\theta\,\mathrm{d}\phi\,\hat{\phi}$$

$$\vec{\nabla} \times \vec{v} = ?$$

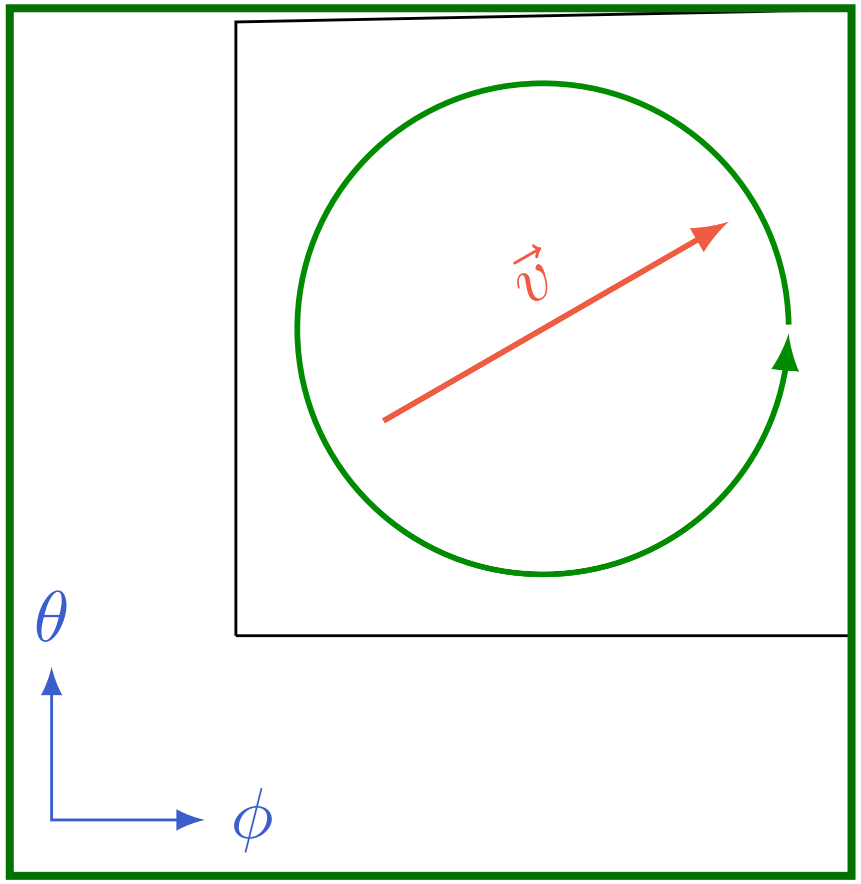


Coordenadas esféricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}r\,\hat{r} + r\,\mathrm{d}\theta\,\hat{\theta} + r\sin\theta\,\mathrm{d}\phi\,\hat{\phi}$$

$$\vec{\nabla} \times \vec{v} = ?$$

$$(\vec{\nabla} \times \vec{v})_r = ?$$



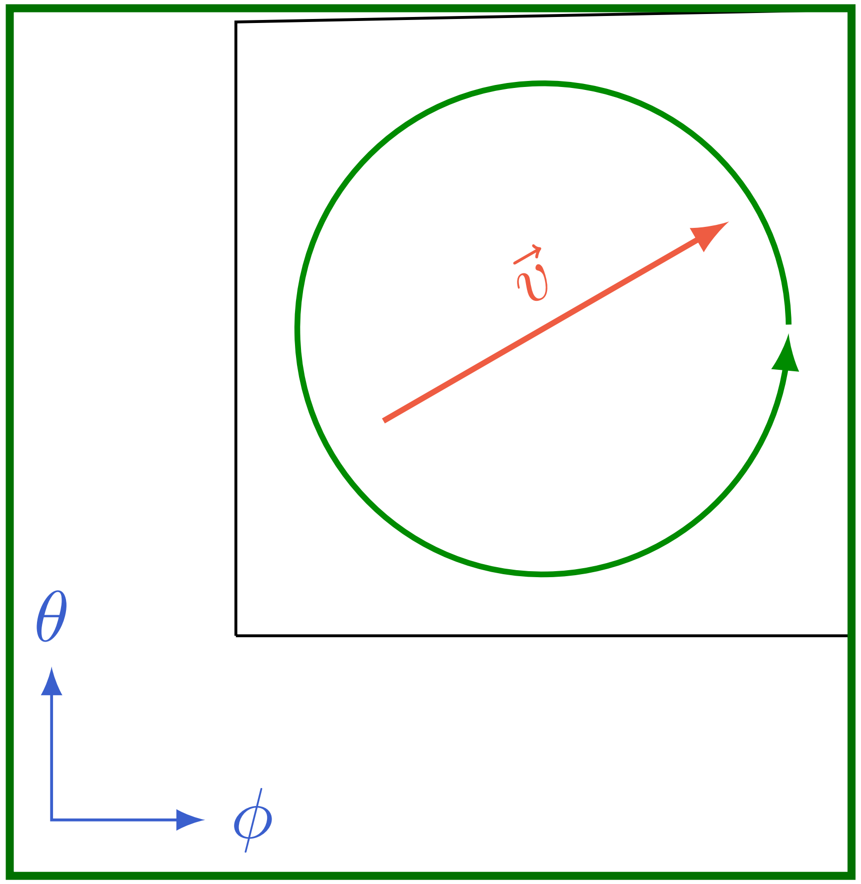
EXEMPLO.

EM LUGAR DE $(\vec{\nabla}\times\vec{v})_r$, PODERÍAMOS ACHAR $(\vec{\nabla}\times\vec{v})_\theta$ OU $(\vec{\nabla}\times\vec{v})_\phi$

Coordenadas esféricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}r\,\hat{r} + r\,\mathrm{d}\theta\,\hat{\theta} + r\sin\theta\,\mathrm{d}\phi\,\hat{\phi}$$

$$(\vec{\nabla} \times \vec{v})_r = \,?$$



Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

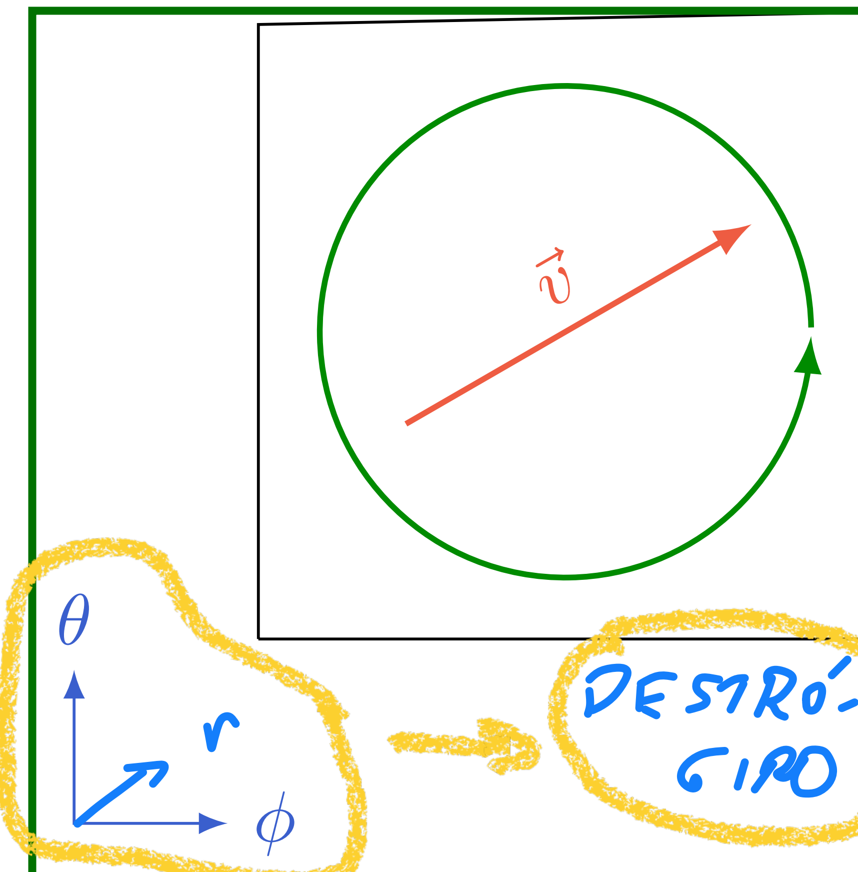
$$(\vec{\nabla} \times \vec{v})_r = ?$$

$\hat{n} = -\hat{r}$

$$\int \vec{\nabla} \times \vec{v} \cdot \hat{n}\, dA = \int \vec{v} \cdot d\vec{\ell}$$

$$-(\vec{\nabla} \times \vec{v})_r\, dA = \vec{v} \cdot d\vec{\ell}$$

$\hat{n}$ SAI DA TELA
(REGRA MÃO DIREITA)
$\odot$



DESTRÓ-GIRO

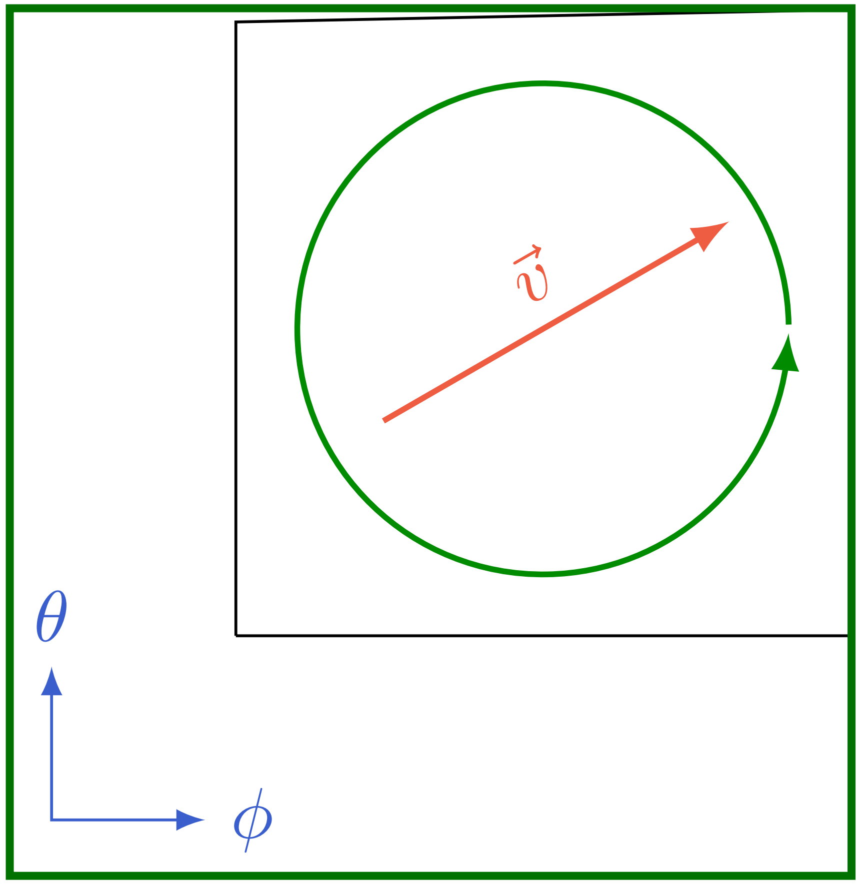
$\hat{r}$ ENTRA NA TELA

Coordenadas esféricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}r\,\hat{r} + r\,\mathrm{d}\theta\,\hat{\theta} + r\sin\theta\,\mathrm{d}\phi\,\hat{\phi}$$

$$(\overrightarrow{\nabla} \times \overrightarrow{v})_r = \,?$$

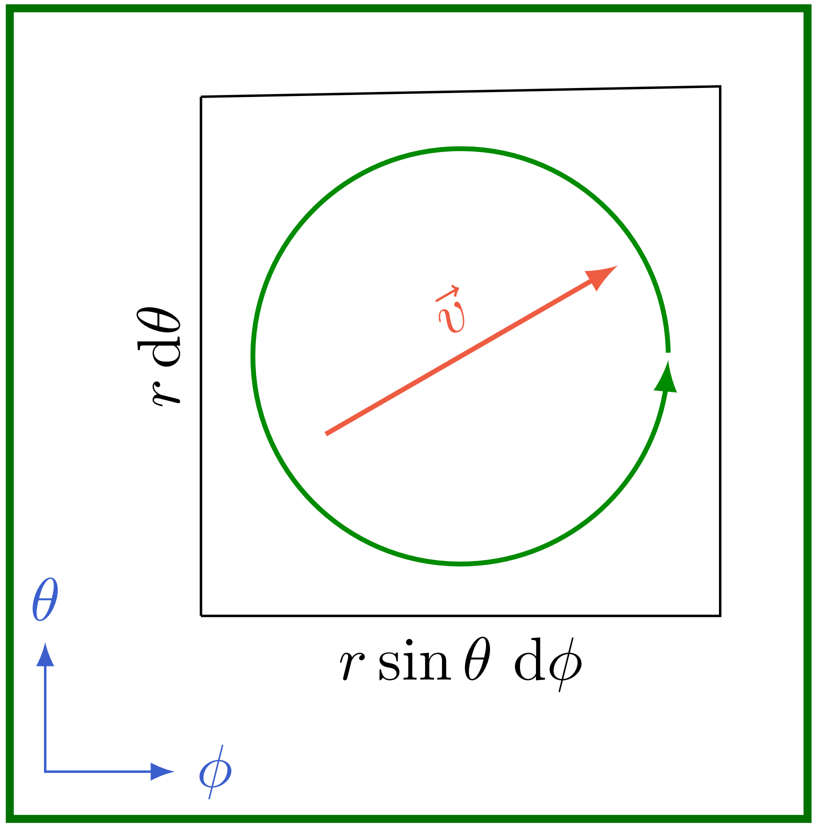
$$-(\overrightarrow{\nabla} \times \overrightarrow{v})_r\,\mathrm{d}A = \overrightarrow{v} \cdot \mathrm{d}\overrightarrow{\ell}$$



Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$(\vec{\nabla}\times\vec{v})_r = ?$$



$$-(\vec{\nabla}\times\vec{v})_r\,dA = \vec{v}\cdot d\vec{\ell}$$

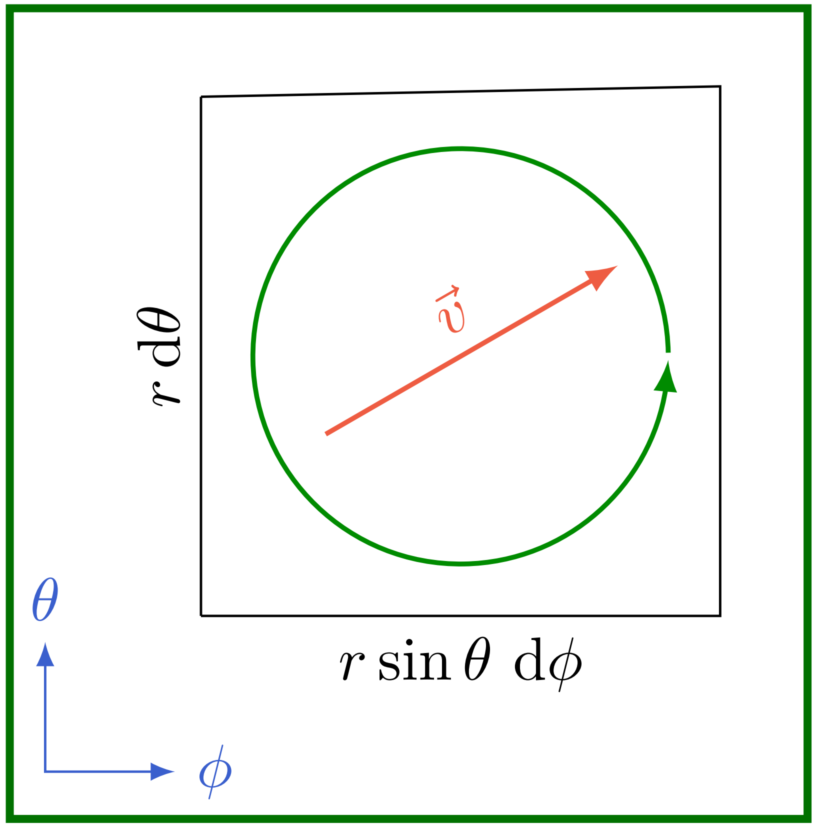
$$-(\vec{\nabla}\times\vec{v})_r\,dA = \Big(v_\theta(\phi+d\phi) - v_\theta(\phi)\Big)\,r d\theta + \ldots$$

ABREVIAÇÃO: NA VERDADE $v_\theta = v_\theta(r, \theta, \phi + d\phi)$

Coordenadas esféricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}r\,\hat{r} + r\,\mathrm{d}\theta\,\hat{\theta} + r\sin\theta\,\mathrm{d}\phi\,\hat{\phi}$$

$$(\vec{\nabla} \times \vec{v})_r = ?$$



$$-(\vec{\nabla} \times \vec{v})_r\,\mathrm{d}A = \vec{v} \cdot \mathrm{d}\vec{\ell}$$

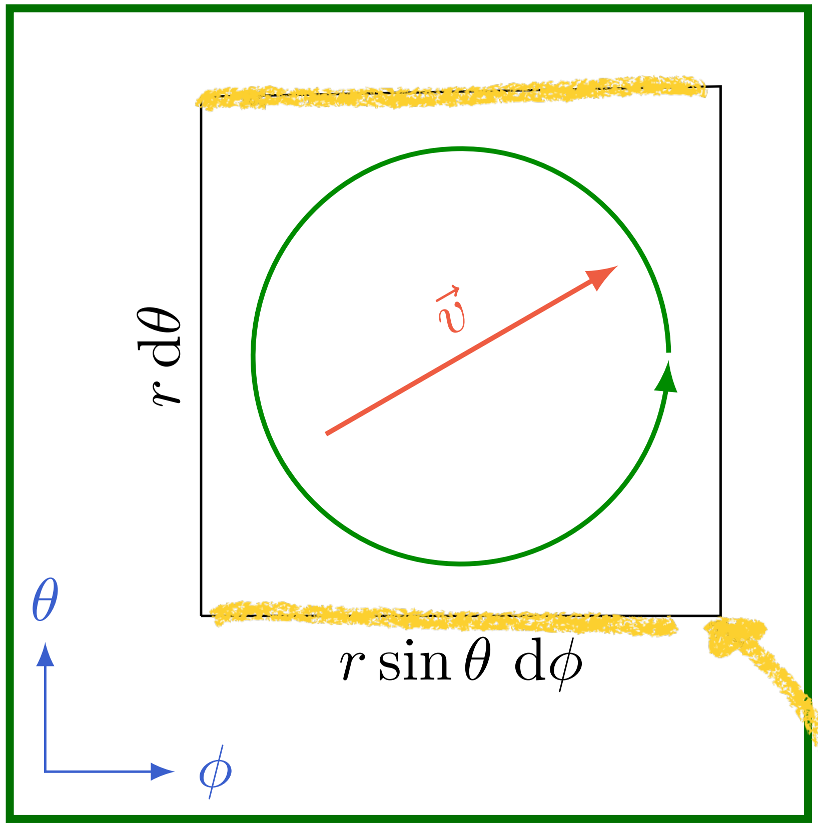
$$-(\vec{\nabla} \times \vec{v})_r\,\mathrm{d}A = \Big(v_\theta(\phi+\mathrm{d}\phi) - v_\theta(\phi)\Big)\,r\mathrm{d}\theta + \ldots$$

$$-(\vec{\nabla} \times \vec{v})_r\,\mathrm{d}A = \left(\frac{\partial v_\theta}{\partial \phi}\right)\mathrm{d}\phi\,r\mathrm{d}\theta + \ldots$$

Coordenadas esféricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}r\,\hat{r} + r\,\mathrm{d}\theta\,\hat{\theta} + r\sin\theta\,\mathrm{d}\phi\,\hat{\phi}$$

$$(\vec{\nabla} \times \vec{v})_r = \,?$$



$$-(\vec{\nabla} \times \vec{v})_r\,\mathrm{d}A = \vec{v} \cdot \mathrm{d}\vec{\ell}$$

$$-(\vec{\nabla} \times \vec{v})_r\,\mathrm{d}A = \left(\frac{\partial v_\theta}{\partial \phi}\right)\mathrm{d}\phi\, r\mathrm{d}\theta + \ldots$$

CONTRIBUIÇÃO DOS LADOS HORIZONTAIS

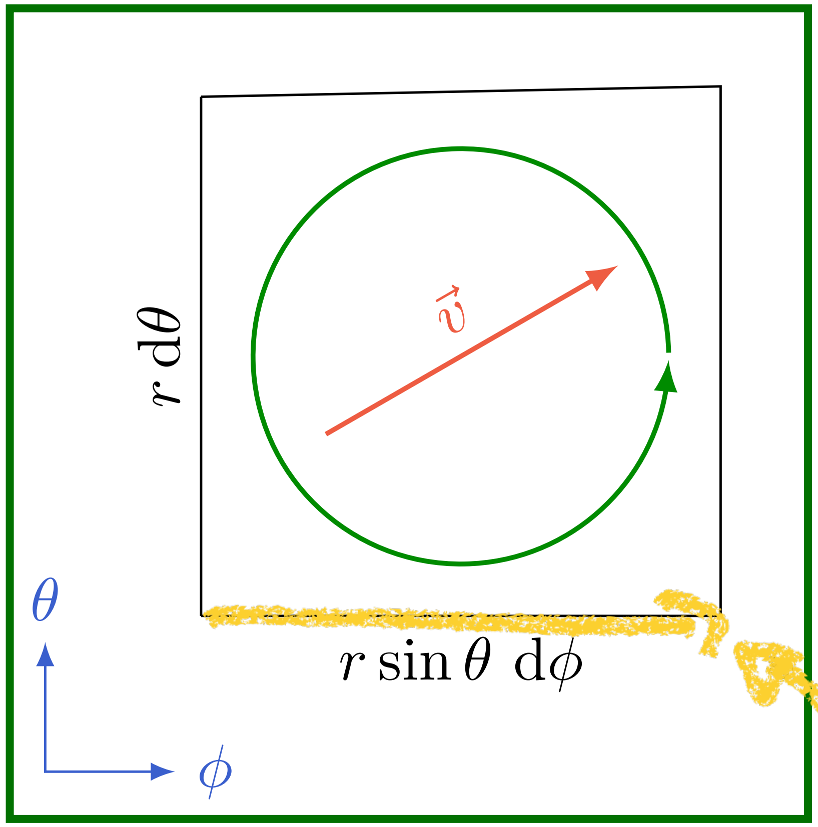
Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$(\vec{\nabla}\times\vec{v})_r = \,?$$

$$-(\vec{\nabla}\times\vec{v})_r\,dA = \vec{v}\cdot d\vec{\ell}$$

$$-(\vec{\nabla}\times\vec{v})_r\;dA = \frac{\partial v_\theta}{\partial \phi}d\phi\,r d\theta - \Big(v_\phi(\theta+d\theta)\sin(\theta+d\theta) - v_\phi(\theta)\sin(\theta)\Big)r d\phi$$

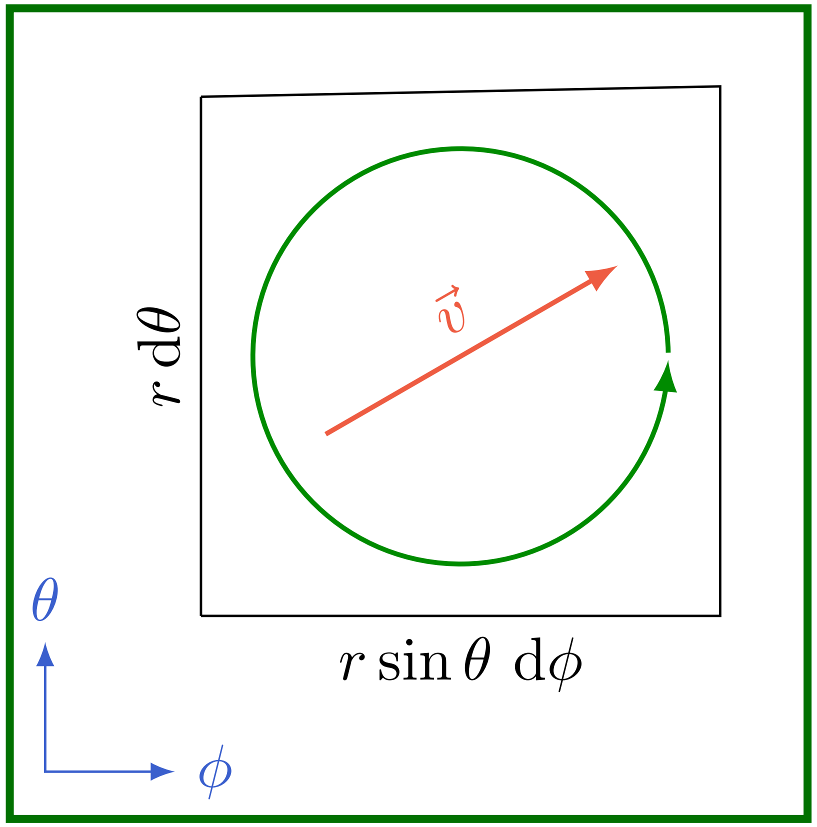


+ PORQUE DESLOCAMENTO NO SENTIDO POSITIVO →

Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$(\vec{\nabla} \times \vec{v})_r = ?$$



$$-(\vec{\nabla} \times \vec{v})_r\, dA = \vec{v} \cdot d\vec{\ell}$$

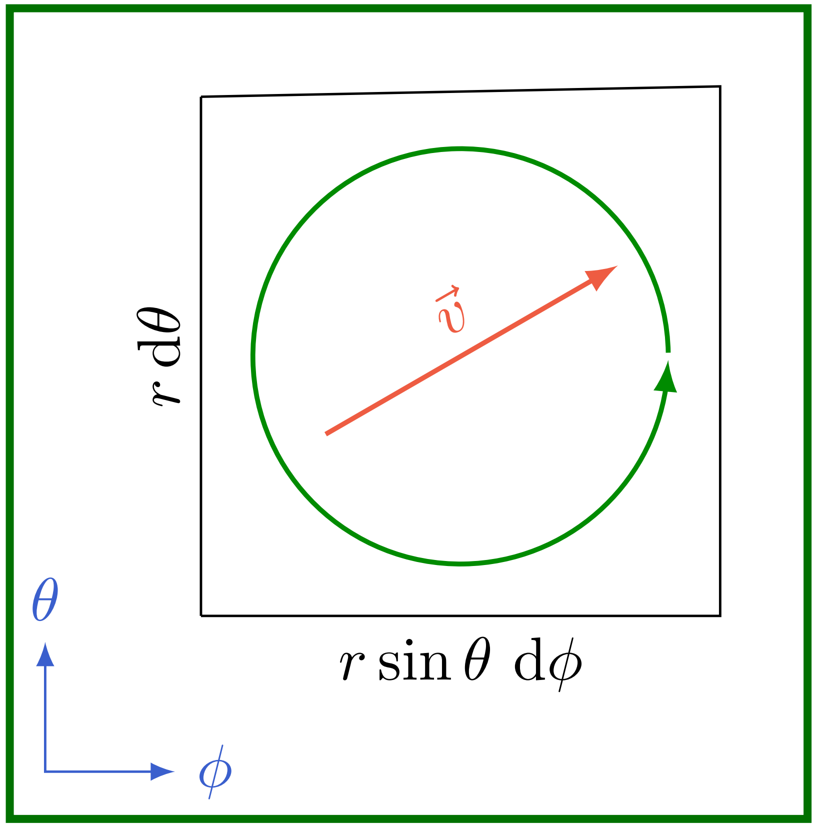
$$-(\vec{\nabla} \times \vec{v})_r\ dA = \frac{\partial v_\theta}{\partial \phi} d\phi\, r d\theta - \left(\frac{\partial (v_\phi \sin\theta)}{\partial \theta}\right) d\theta\, r d\phi$$

Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$(\vec{\nabla} \times \vec{v})_r = ?$$

$$-(\vec{\nabla} \times \vec{v})_r\, dA = \vec{v} \cdot d\vec{\ell}$$



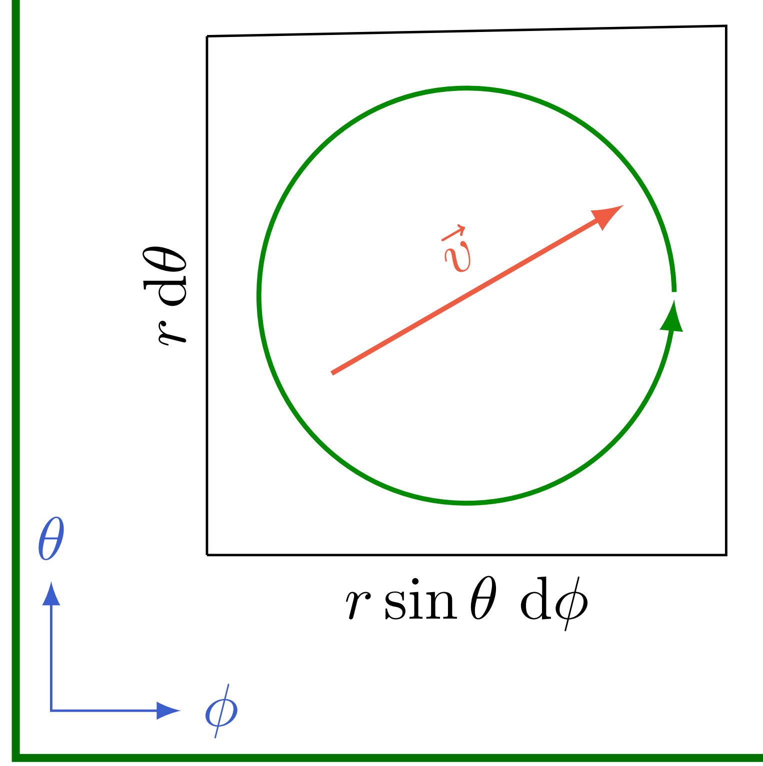
$$-(\vec{\nabla} \times \vec{v})_r\, r^2 \sin\theta\, d\theta\, d\phi = \frac{\partial v_\theta}{\partial \phi} d\phi\, r d\theta - \left(\frac{\partial(v_\phi \sin\theta)}{\partial\theta}\right) d\theta\, r d\phi$$

Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$(\vec{\nabla} \times \vec{v})_r = ?$$

$$-(\vec{\nabla} \times \vec{v})_r\, dA = \vec{v} \cdot d\vec{\ell}$$



$$(\vec{\boldsymbol{\nabla}} \times \vec{v})_r = \frac{1}{r\sin\theta}\left(\frac{\partial(v_\phi \sin\theta)}{\partial\theta} - \frac{\partial v_\theta}{\partial\phi}\right)$$

Coordenadas esféricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}r\,\hat{r} + r\,\mathrm{d}\theta\,\hat{\theta} + r\sin\theta\,\mathrm{d}\phi\,\hat{\phi}$$

$$\nabla^2 T = \overrightarrow{\nabla}\cdot\overrightarrow{\nabla}T = ?$$

$$\vec{\boldsymbol{\nabla}}t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial\phi}\hat{\phi}$$

$$\vec{\boldsymbol{\nabla}}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

Coordenadas esféricas

$$d\vec{\ell} = \mathrm{d}r\,\hat{r} + r\,\mathrm{d}\theta\,\hat{\theta} + r\sin\theta\,\mathrm{d}\phi\,\hat{\phi}$$

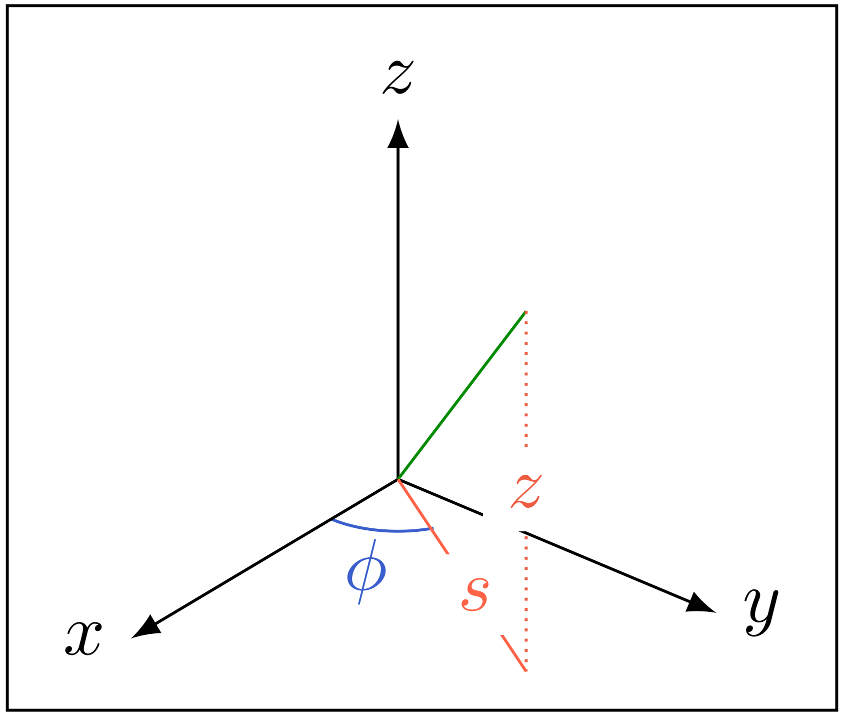
$$\nabla^2 T = \vec{\nabla}\cdot\vec{\nabla}T = ?$$

$$\vec{\nabla}t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\, v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$\nabla^2 T = \frac{1}{r^2}\frac{\partial\left(r^2\frac{\partial T}{\partial r}\right)}{\partial r} + \frac{1}{r^2\sin\theta}\frac{\partial\left(\sin\theta\frac{\partial T}{\partial\theta}\right)}{\partial\theta} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2}$$

Coordenadas cilíndricas

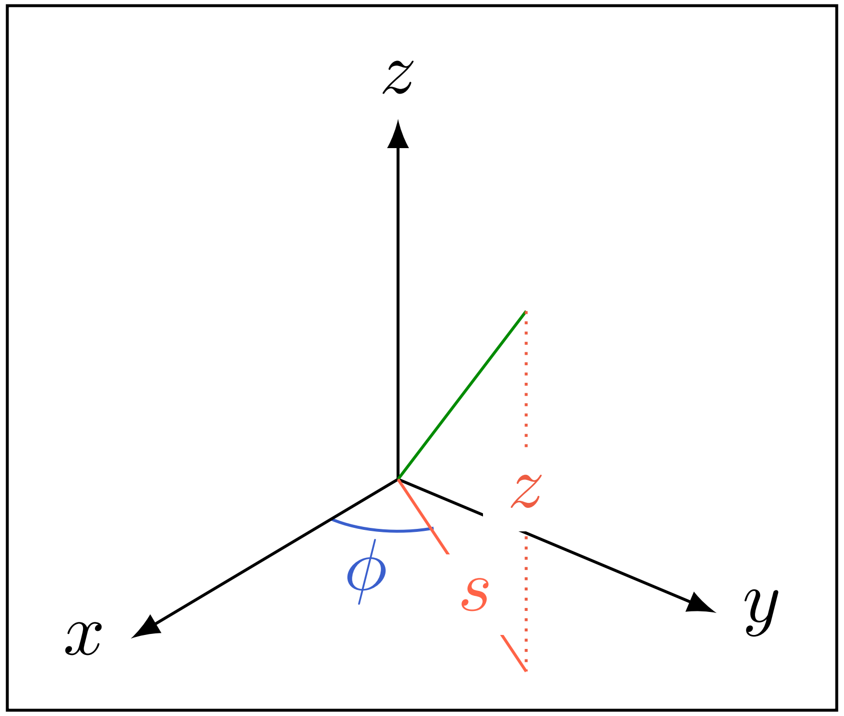


Coordenadas cilíndricas

$$x = s\cos\phi$$

$$y = s\sin\phi$$

$$z = z$$

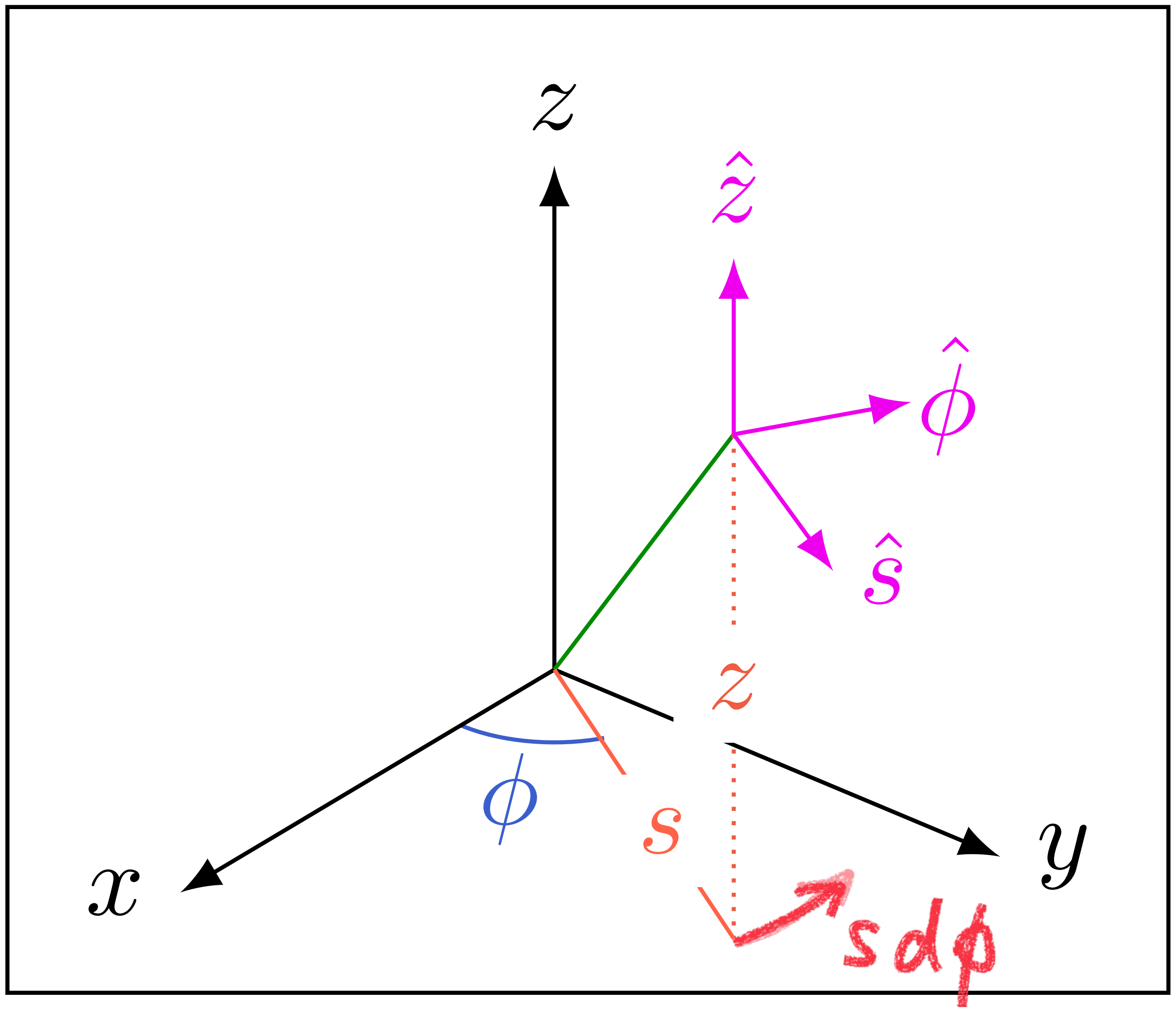


Coordenadas cilíndricas

$$x = s\cos\phi$$

$$y = s\sin\phi$$

$$\mathrm{d}\vec{\ell} = \mathrm{d}s\,\hat{s} + s\,\mathrm{d}\phi\,\hat{\phi} + \mathrm{d}z\,\hat{z}$$



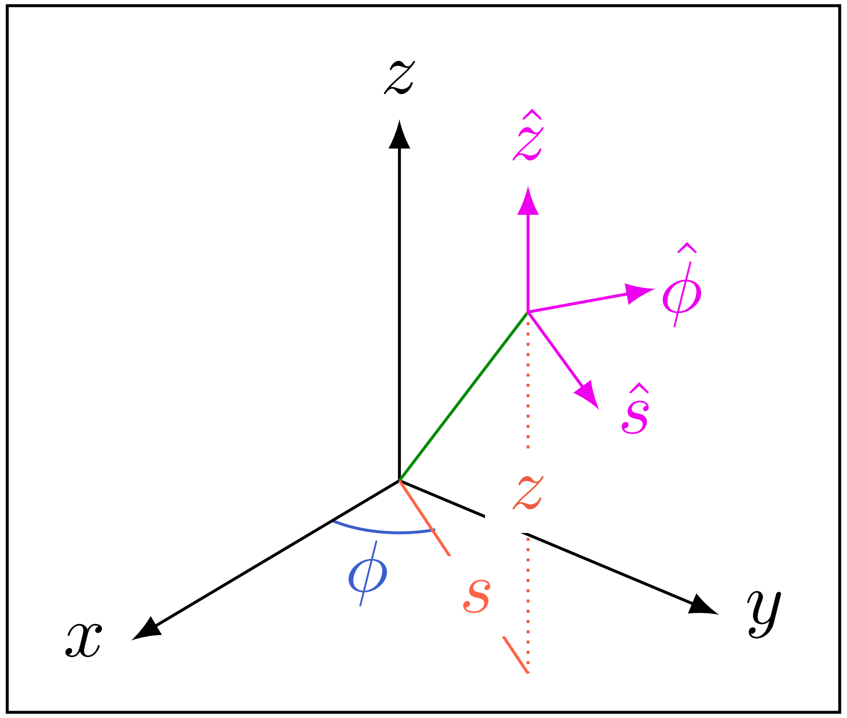
Coordenadas cilíndricas

$$x = s\cos\phi$$

$$y = s\sin\phi$$

$$\mathrm{d}\vec{\ell} = \mathrm{d}s\,\hat{s} + s\,\mathrm{d}\phi\,\hat{\phi} + \mathrm{d}z\,\hat{z}$$

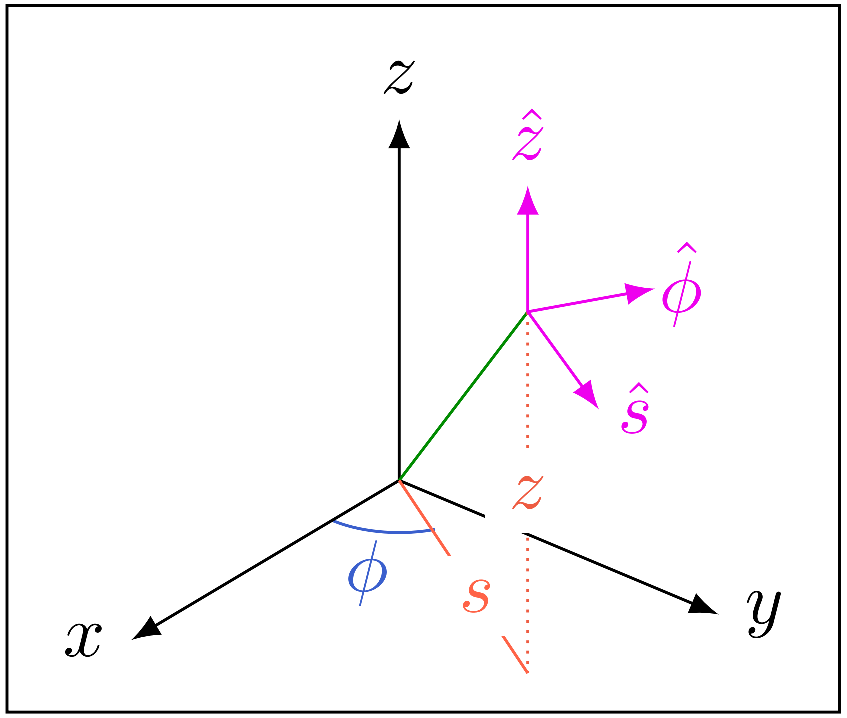
$$\vec{\boldsymbol{\nabla}}T = \frac{\partial T}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}$$



Coordenadas cilíndricas

$$x = s\cos\phi$$

$$y = s\sin\phi$$



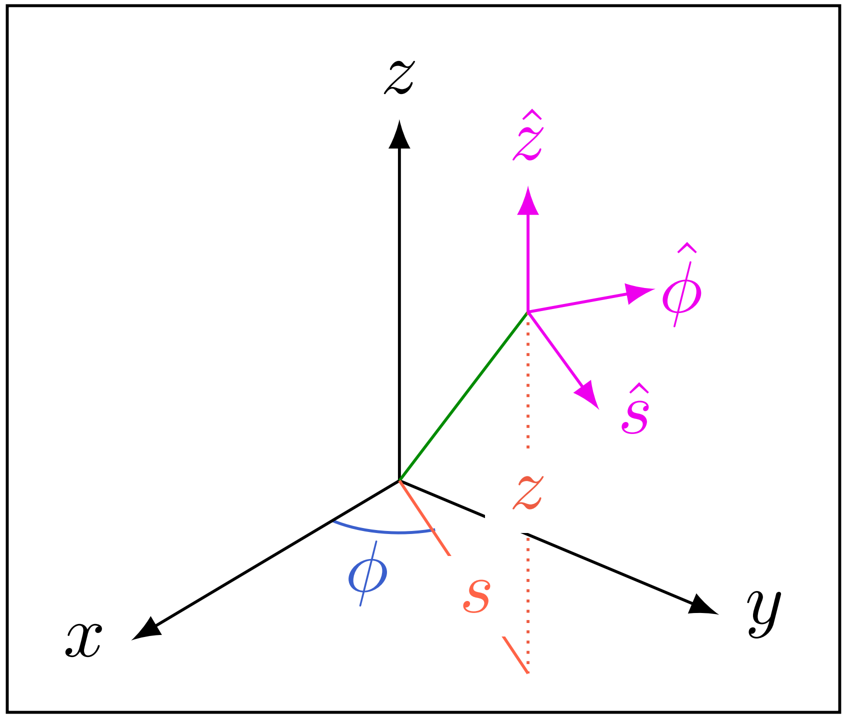
$$\mathrm{d}\vec{\ell} = \mathrm{d}s\,\hat{s} + s\,\mathrm{d}\phi\,\hat{\phi} + \mathrm{d}z\,\hat{z}$$

$$\vec{\nabla} T = \frac{\partial T}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}$$

$$\vec{\nabla} \cdot \vec{v} = \frac{1}{s}\frac{\partial (s v_s)}{\partial s} + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

Coordenadas cilíndricas

$$d\vec{\ell} = ds\,\hat{s} + s\,d\phi\,\hat{\phi} + dz\,\hat{z}$$



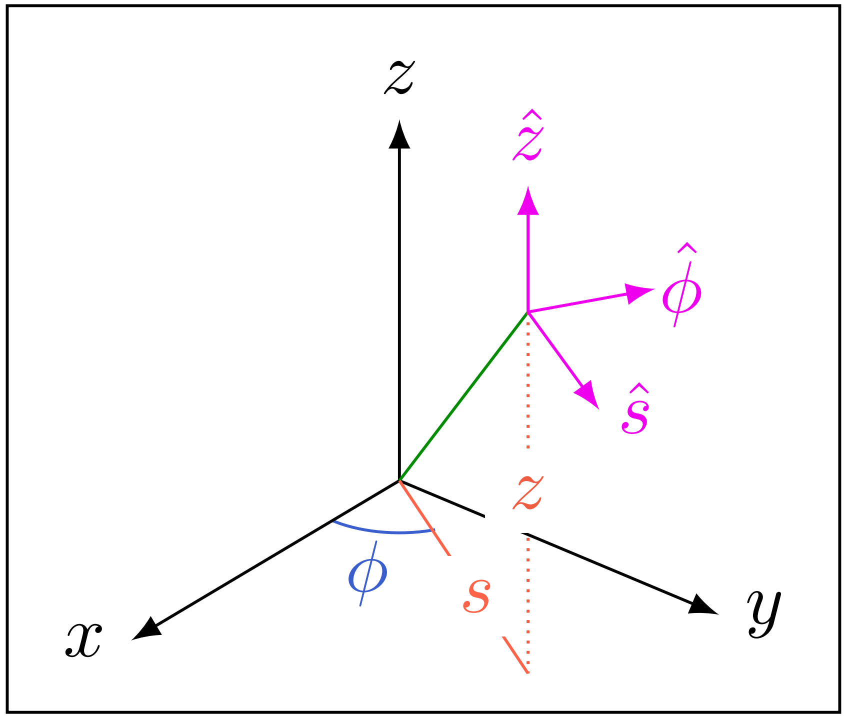
$$\vec{\nabla} T = \frac{\partial T}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}$$

$$\vec{\nabla} \cdot \vec{v} = \frac{1}{s}\frac{\partial (s v_s)}{\partial s} + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\vec{\nabla} \times \vec{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial (s v_\phi)}{\partial s} - \frac{\partial v_s}{\partial \phi}\right)\hat{z}$$

Coordenadas cilíndricas

$$d\vec{\ell} = ds\,\hat{s} + s\,d\phi\,\hat{\phi} + dz\,\hat{z}$$



$$\vec{\nabla} T = \frac{\partial T}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}$$

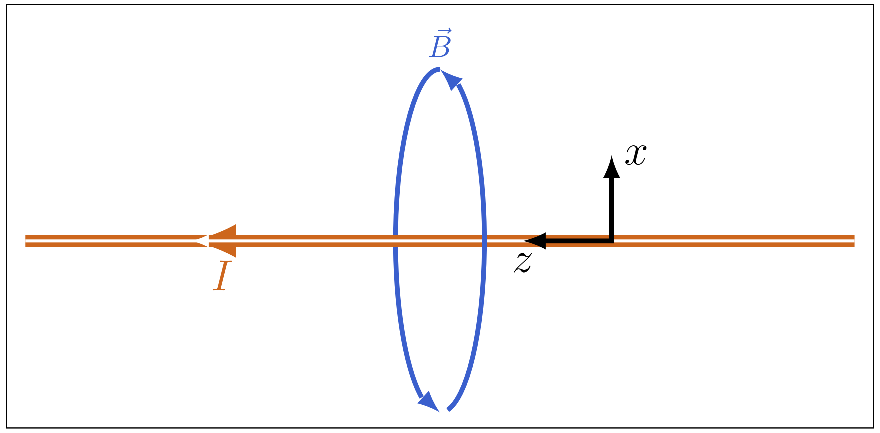
$$\vec{\nabla} \cdot \vec{v} = \frac{1}{s}\frac{\partial (s v_s)}{\partial s} + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\vec{\nabla} \times \vec{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial (s v_\phi)}{\partial s} - \frac{\partial v_s}{\partial \phi}\right)\hat{z}$$

$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$

Pratique o que aprendeu

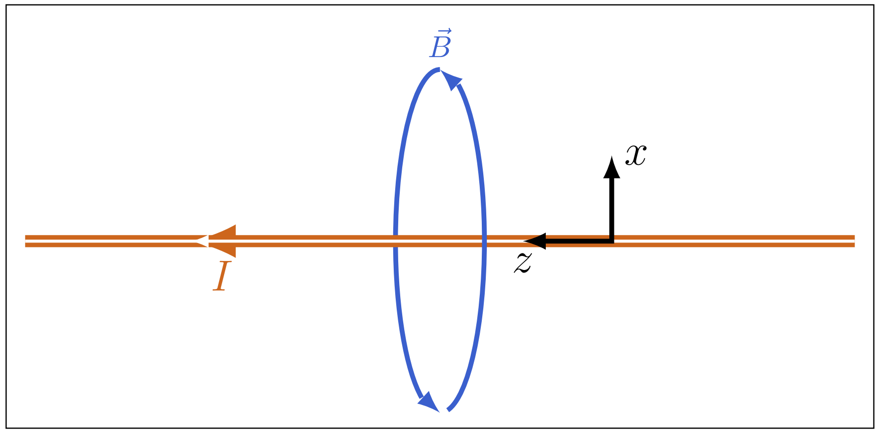
$$\overrightarrow{B} = ?$$



Pratique o que aprendeu

$$\overrightarrow{B} = B_\phi \hat{\phi}$$

$$\overrightarrow{\nabla} \cdot \overrightarrow{B} = 0$$



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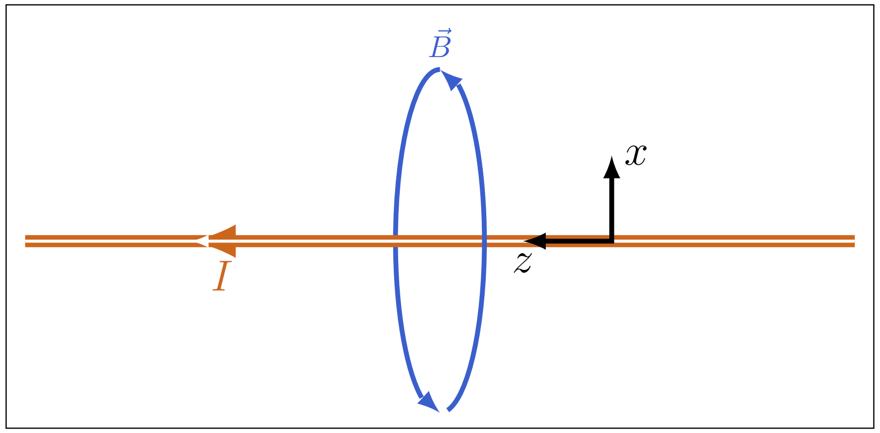
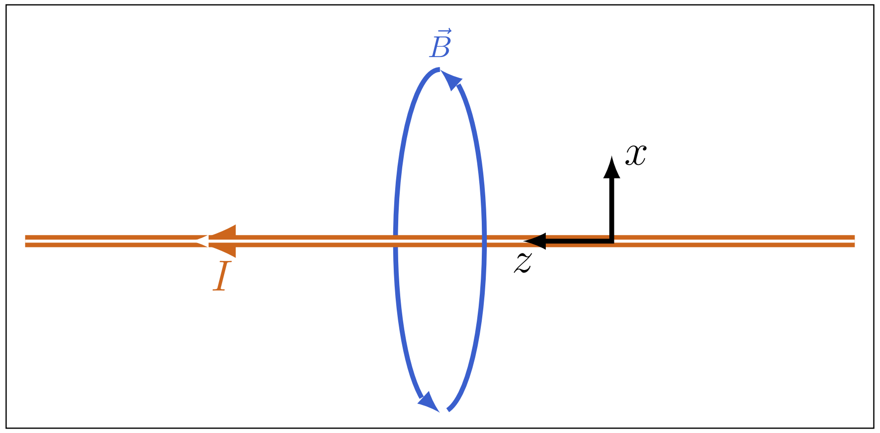


$$\vec{\nabla} \cdot \vec{v} = \frac{1}{s}\frac{\partial(s v_s)}{\partial s} + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

AGORA EM COORDENADAS CILÍNDRICAS

Pratique o que aprendeu



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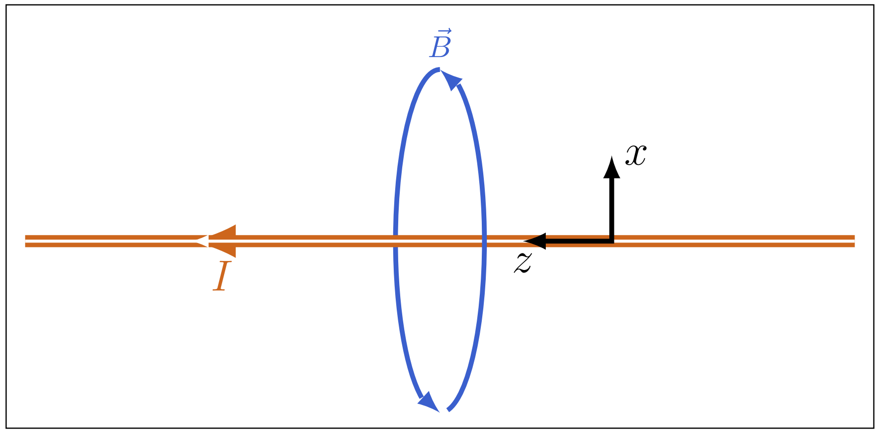
$$\overrightarrow{\nabla} \cdot \overrightarrow{B} = \frac{1}{s}\frac{\partial B_\phi}{\partial \phi}$$

Pratique o que aprendeu



$$\overrightarrow{B} = B_\phi \hat{\phi} \qquad\qquad \overrightarrow{\nabla} \cdot \overrightarrow{B} = 0$$

$$\vec{\nabla} \cdot \vec{v} = \frac{1}{s}\frac{\partial (s v_s)}{\partial s} + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\overrightarrow{\nabla} \cdot \overrightarrow{B} = \frac{1}{s}\frac{\partial B_\phi}{\partial \phi} = 0 \Rightarrow$$ B_ϕ NÃO DEPENDE DE ϕ

$$B_\phi = B(s, z) \qquad \Rightarrow B_\phi = B(s)$$

$$\vec{B} = B(s)\hat{\phi}$$

MAS B_ϕ NÃO PODE DEPENDER DE z

COORDENADAS CILÍNDRICAS ESTÃO MAIS DE ACORDO COM SIMETRIA