$$E(\theta,\omega) = V \Rightarrow \frac{1}{2}\omega^2 - k\cos\theta = V$$

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"MONITORIA" MAT IV 19/04

P.: Seja $\Omega \subset \mathbb{R}^m$ ab., $f: \Omega \to \mathbb{R}^n$, $p \in \Omega$.

Então se $T, S: \mathbb{R}^m \to \mathbb{R}^n$ são transf. lineares tq.

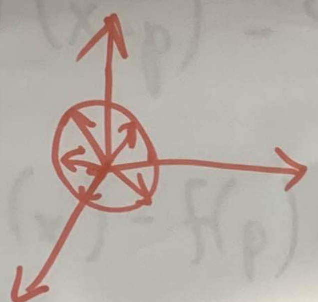
$$\lim_{x \to p} \frac{\|f(x) - f(p) - T(x-p)\|}{\|x-p\|} = \lim_{x\to p} \frac{\|f(x) - f(p) - S(x-p)\|}{\|x-p\|}$$

$$= 0$$

Então $T = S$.

Dem.

$$T=S \iff T(x)=S(x)\ \forall x\in\mathbb{R}^m \iff \|T(x)-S(x)\| = 0\ \forall x\in\mathbb{R}^m \iff \|T(x)-S(x)\| = 0\ \forall x \in \underbrace{B(0,\delta)}_{S(0,\delta)}$$ p/ algum $\delta>0$



Assumimos

Então se $x\in\mathbb{R}^m$, $y=\delta\frac{x}{\|x\|}\in S(0,\delta)$

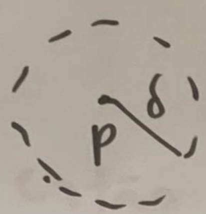
$$\Rightarrow T(y)=S(y) \Rightarrow \frac{\|x\|}{\delta}T(y) = \frac{\|x\|}{\delta}S(y)$$

$$\Rightarrow T\Big(\underbrace{\tfrac{\|x\|}{\delta}y}_{x}\Big) = S\Big(\underbrace{\tfrac{\|x\|}{\delta}y}_{x}\Big)$$ Δ

$\forall\, \varepsilon > 0$ arbit. $\exists\, \delta_1, \delta_2 > 0$ t.q.

$$\underbrace{\frac{\|f(x)-f(p)-T(x-p)\|}{\|x-p\|}}_{\text{se } \|x-p\|<\delta_1}, \quad \underbrace{\frac{\|f(x)-f(p)-S(x-p)\|}{\|x-p\|}}_{\text{se } \|x-p\|<\delta_2} < \varepsilon$$

$\delta = \min\{\delta_1, \delta_2\}$



$$\|T(x-p) - S(x-p)\| = \|-(f(x)-f(p)-T(x-p))$$

$$+ (f(x)-f(p)-S(x-p))\| \leq \|f(x)-f(p)-T(x-p)\|$$

$$+ \|f(x)-f(p)-\underset{S}{\cancel{T}}(x-p)\| < 2\varepsilon \|x-p\|$$

se $\|x-p\| < \delta$

$$\left\|\frac{x-p}{\delta}\right\| = \frac{\|x-p\|}{\delta} < \frac{\delta}{\delta} = 1$$

$\hookrightarrow B(0,1)$

$$\left\| T\left(\frac{x-p}{\delta}\right) - S\left(\frac{x-p}{\delta}\right) \right\| = \left\| \frac{1}{\delta}\left[T(x-p) - S(x-p) \right] \right\|$$

$$= \delta^{-1} \| T(x-p) - S(x-p) \| < Z\varepsilon \frac{\|x-p\|}{\delta} < Z\varepsilon$$

Se $y \in B(0,1)$ então $\frac{\delta y}{\cancel{\|\,\|}} \in B(0,\delta)$

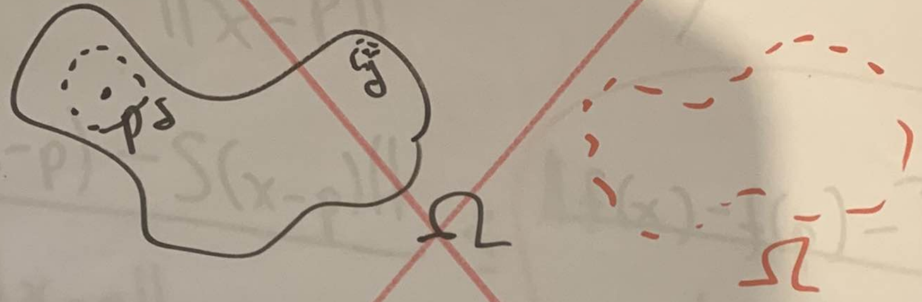
$\underbrace{p + \delta y}_{x} \in B(p,\delta)$

$\hookrightarrow \forall y \in B(0,1), \| T(y) - S(y) \| < Z\varepsilon$

$\hookrightarrow T(y) = S(y) \; \forall y \in B(0,1)$

$y \in B(0,1)$

$p \in \Omega \Rightarrow \exists\, \delta > 0$ t.q. $B(p,\delta) \subset \Omega$



$$\|T(y)-S(y)\| = \frac{\delta}{\delta}\|T(y)-S(y)\| =$$

$$= \frac{1}{\delta}\|T(\delta y)-S(\delta y)\| = \frac{1}{\delta}\|T(x-p)$$

$$\|T(x-p)-S(x-p)\| = \frac{\|T(x-p)-S(x-p)\|}{\|x-p\|}\|x-p\|$$

$$= \frac{\|[f(x)-f(p)-T(x-p)] - [f(x)-f(p)-S(x-p)]\|}{\|x-p\|}\cdot\|x-p\| \leq \|x-p\|\cdot\left(\frac{\|f(x)-f(p)-T(x-p)\|}{\|x-p\|} + \right.$$

$$\|T(x-p)-S(x-p)\| \leq \|x-p\| \left(\frac{\|f(x)-f(p)-T(x-p)\|}{\|x-p\|} + \frac{\|f(x)-f(p)-S(x-p)\|}{\|x-p\|} \right)$$

$$\frac{\|T(x-p)-S(x-p)\|}{\|x-p\|} \leq \underbrace{\frac{\|f(x)-f(p)-T(x-p)\|}{\|x-p\|} + \frac{\|f(x)-f(p)-S(x-p)\|}{\|x-p\|}}_{\alpha}$$

$$\left\| \frac{T(x-p)}{\|x-p\|} - \frac{S(x-p)}{\|x-p\|} \right\| \leq \alpha$$

$$\left\| T\left(\frac{x-p}{\|x-p\|}\right) - S\left(\frac{x-p}{\|x-p\|}\right) \right\| \leq \alpha$$

unitário

$$\|T(u)-S(u)\| \leq \alpha \quad \forall u \in S(0,1)$$

$$\alpha \to 0$$

$$\|T(u)-S(u)\| \underset{\substack{\uparrow \\ =}}{\leq} 0 \quad \forall u \in S(0,1)$$

<u>Ex.:</u> $f: \mathbb{R}^2 \to \mathbb{R}^2$, $f(x,y) = (x-y, y^2+3x^2)$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; Pra quais A que

$$\frac{\left\| f(x,y) - A\begin{pmatrix} x \\ y \end{pmatrix} \right\|^2}{\|(x,y)\|^2} \to 0 \,?$$