Eletromagnetismo

$$\vec{\nabla} \cdot \vec{D} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

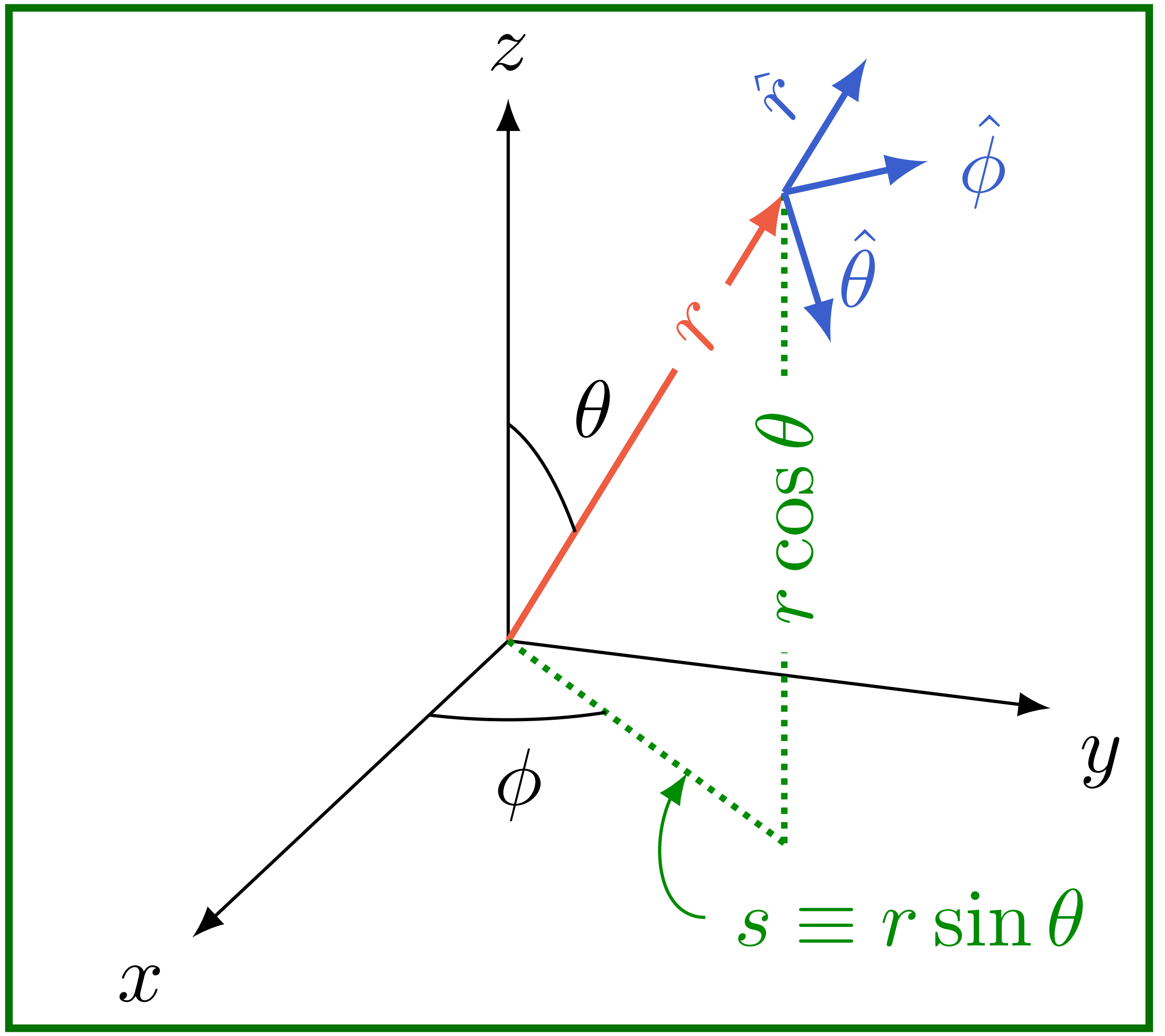
$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{j} + \epsilon_0 \frac{\partial \vec{D}}{\partial t}$$

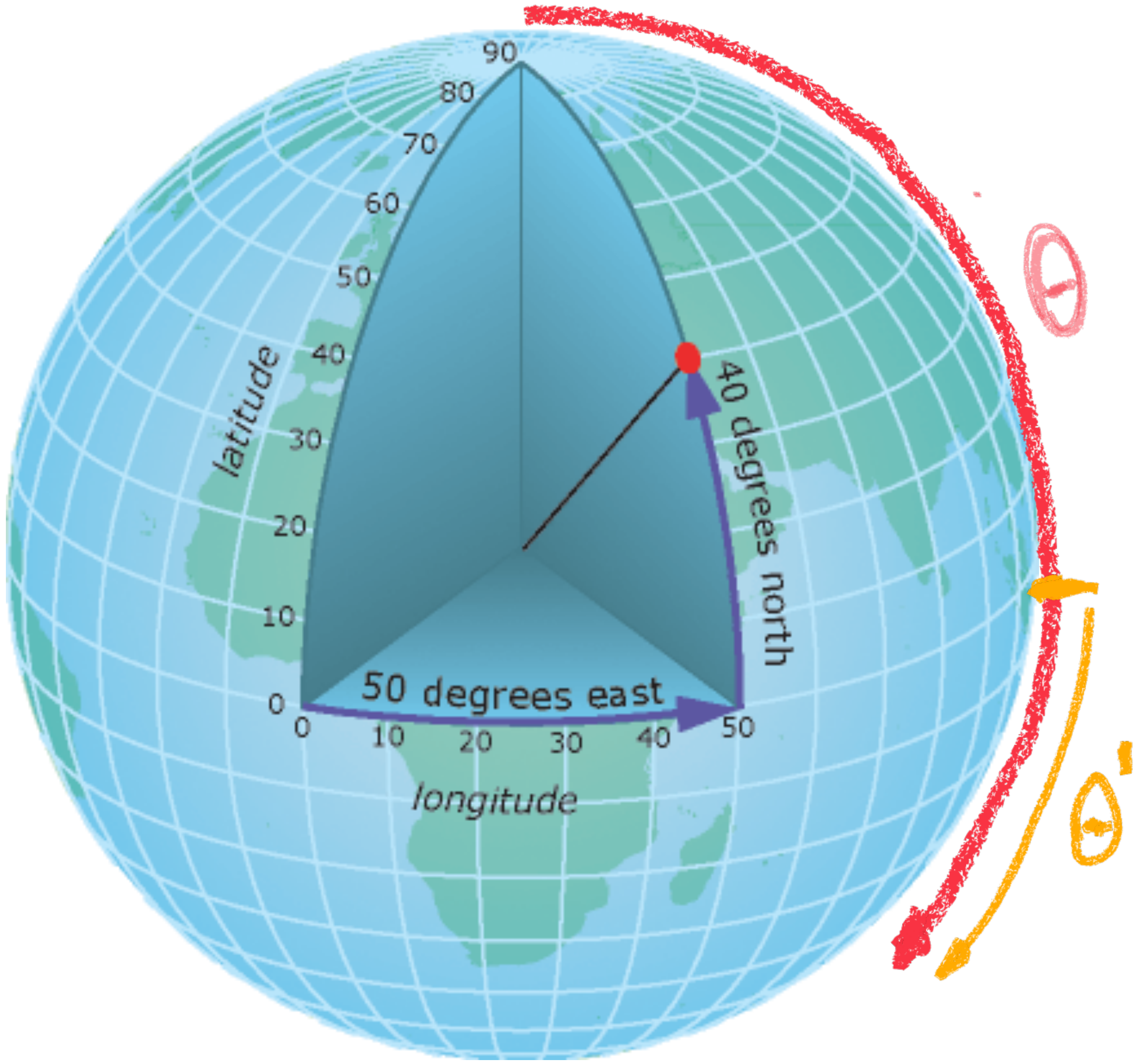
Aula de 19 de abril

Análise vetorial

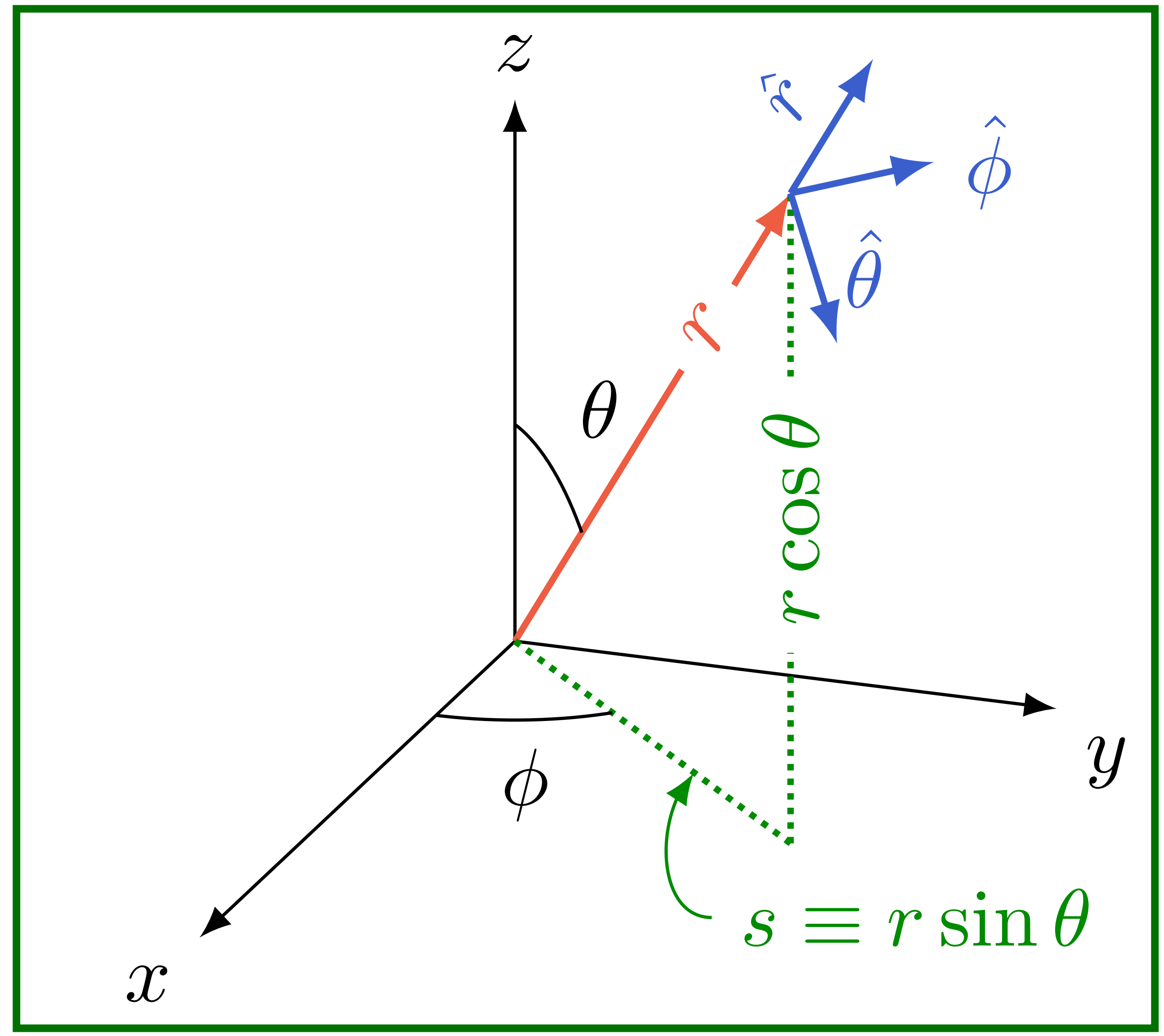
Coordenadas esféricas



Coordenadas esféricas



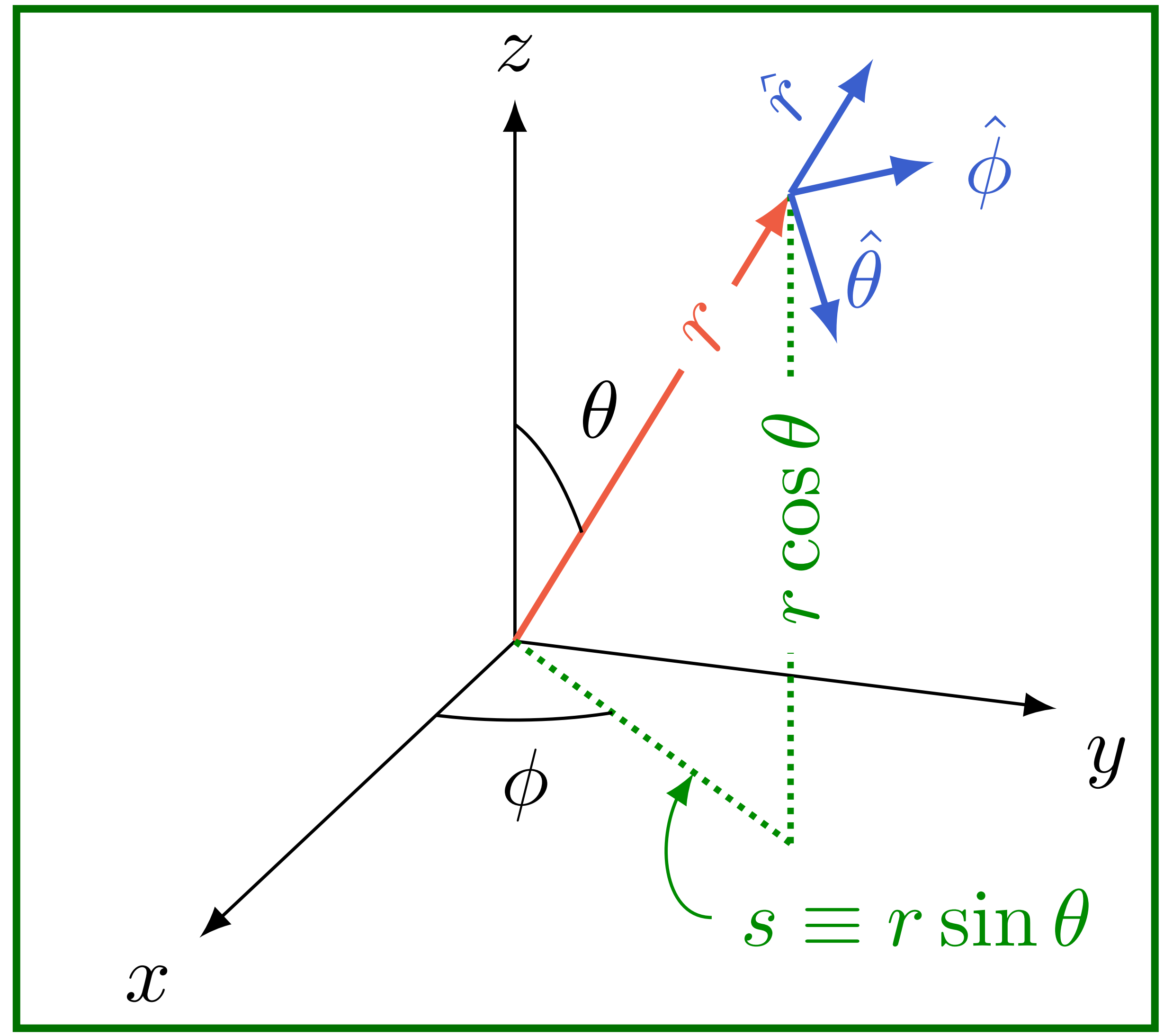
https://desktop.arcgis.com



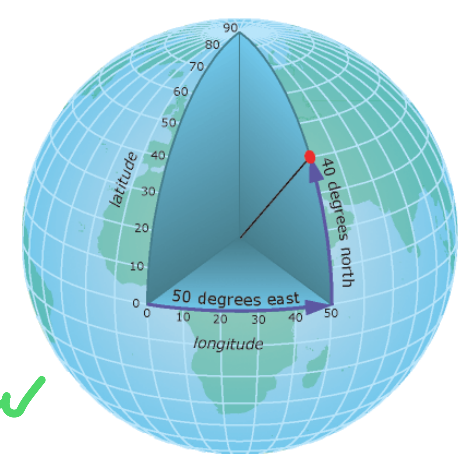
longitude $\longleftrightarrow \phi$

latitude $\longleftrightarrow \theta' = \theta - 90^\circ$

Coordenadas esféricas



27~~0~~
longitude
↳ rumo que avião toma

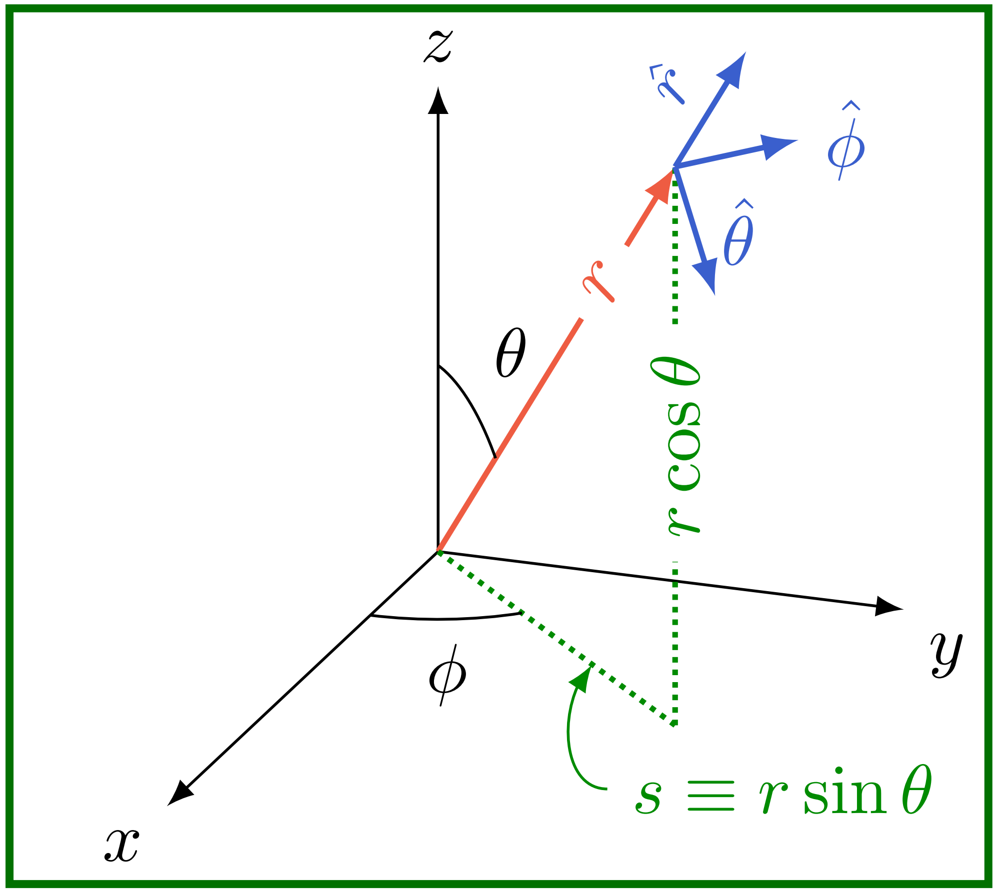


Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

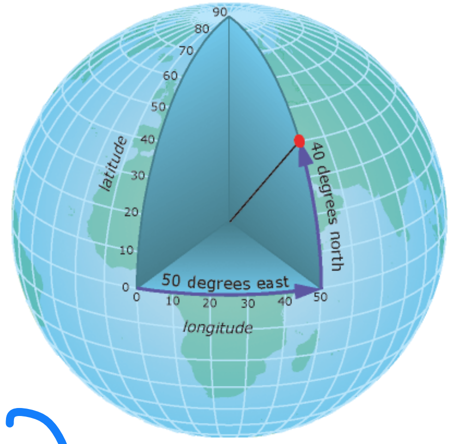
Compare com cartesianas:

$$d\vec{\ell} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$



Pratique o que aprendeu

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$



Coordenadas GPS

Trevo São Carlos: 22.01 $\theta' = 22.01$ → $\theta = 112.01$

47.90 $\phi = 47.90$

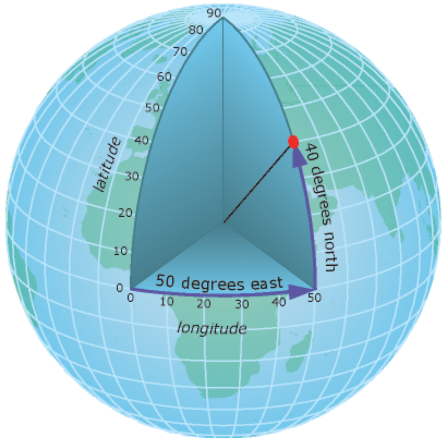
Ponte Piqueri: 23.51 $\theta' = 23.51$ → $\theta = 113.01$

46.72 $\phi = 46.72$

Qual é a distância?

Pratique o que aprendeu

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$



Coordenadas GPS

Trevo São Carlos: 22.01 $\theta' = 22.01$

47.90 $\phi = 47.90$

Ponte Piqueri: 23.51 $\theta' = 23.51$

46.72 $\phi = 46.72$

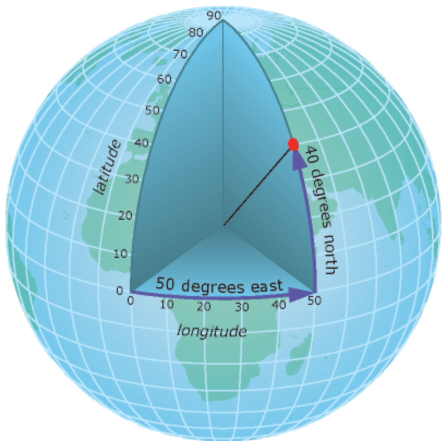
$$d\vec{\ell} = R d\theta'\,\hat{\theta} + R\cos\theta' d\phi\,\hat{\phi}$$

$d\theta' = 23.51 - 22.01 > 0 \Rightarrow$ DIREÇÃO LESTE

$d\phi = 46.72 - 47.90 < 0 \Rightarrow$ DIREÇÃO SUL

Pratique o que aprendeu

$$\mathrm{d}\vec{\ell} = \mathrm{d}r\,\hat{r} + r\,\mathrm{d}\theta\,\hat{\theta} + r\sin\theta\,\mathrm{d}\phi\,\hat{\phi}$$



Coordenadas GPS

Trevo São Carlos: 22.01 $\theta' = 22.01$

47.90 $\phi = 47.90$

Ponte Piqueri: 23.51 $\theta' = 23.51$

46.72 $\phi = 46.72$

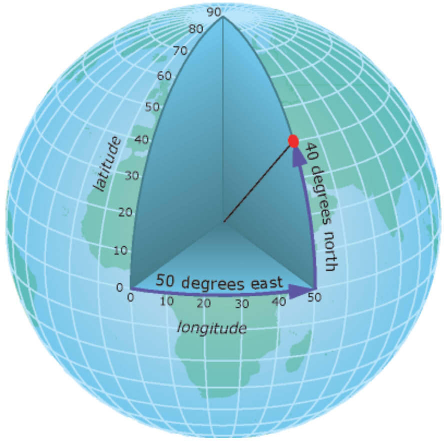
$$\mathrm{d}\vec{\ell} = R\mathrm{d}\theta'\,\hat{\theta} + R\cos\theta'\mathrm{d}\phi\,\hat{\phi}$$

$$R = 6.37 \times 10^{6}\,\mathrm{m}$$

RAIO DA TERRA

Pratique o que aprendeu

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$



Coordenadas GPS

Trevo São Carlos:

22.01 $\theta' = 22.01$

47.90 $\phi = 47.90$

Ponte Piqueri:

23.51 $\theta' = 23.51$

46.72 $\phi = 46.72$

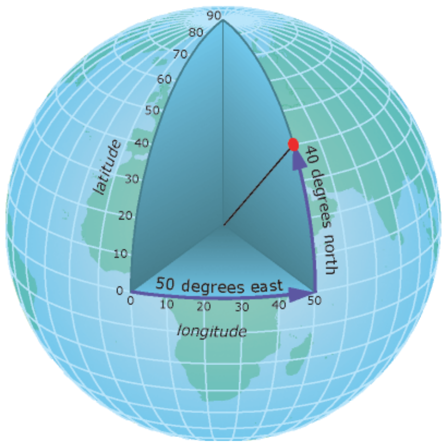
$$d\vec{\ell} = R\,d\theta'\,\hat{\theta} + R\cos\theta'\,d\phi\,\hat{\phi}$$

$$R = 6.37 \times 10^6\,\text{m}$$

$$d\ell = R\sqrt{(d\theta')^2 + \cos^2\theta'(d\phi)^2}$$

Pratique o que aprendeu

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$



Coordenadas GPS

Trevo São Carlos: 22.01 $\theta' = 22.01$

47.90 $\phi = 47.90$

Ponte Piqueri: 23.51 $\theta' = 23.51$

46.72 $\phi = 46.72$

$$d\vec{\ell} = R d\theta'\,\hat{\theta} + R\cos\theta d\phi\,\hat{\phi}$$

$$R = 6.37 \times 10^6\,\text{m}$$

$$d\ell = R\sqrt{(d\theta')^2 + \cos^2\theta'(d\phi)^2} \Longrightarrow d\ell = 206\,\text{km}$$

ESTRADA => 222 km

PQ NÃO É RETA

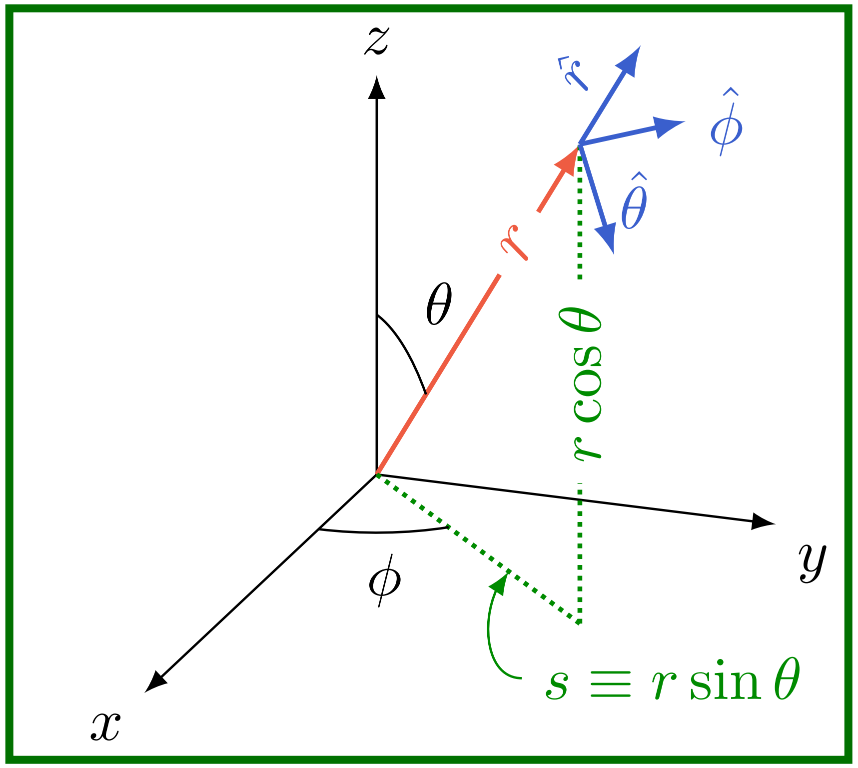
Coordenadas esféricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}r\,\hat{r} + r\,\mathrm{d}\theta\,\hat{\theta} + r\sin\theta\,\mathrm{d}\phi\,\hat{\phi}$$

$$\mathrm{d}\tau = \mathrm{d}r\,r\,\mathrm{d}\theta\,r\sin\theta\,\mathrm{d}\phi$$

ELEMENTO DE VOLUME

$$\boxed{\text{CF. CARTESIANAS} \quad d\tau = dx\,dy\,dz}$$



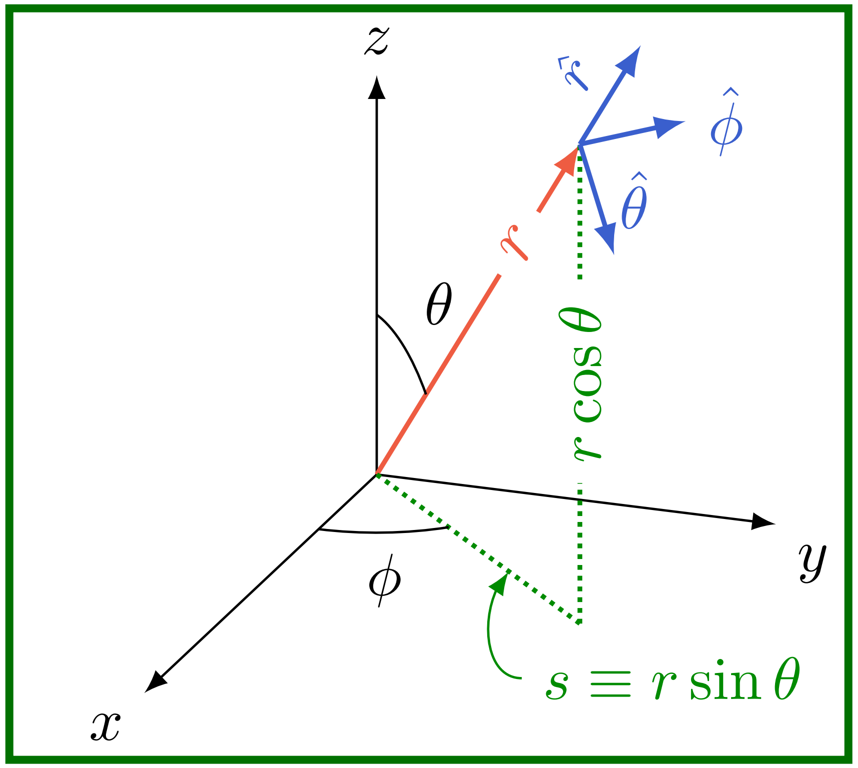
Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$\vec{\nabla} t = ?$ $\quad \vec{\nabla} t = \dfrac{\partial t}{\partial x}\hat{x} + \dfrac{\partial t}{\partial y}\hat{y} + \dfrac{\partial t}{\partial z}\hat{z}$

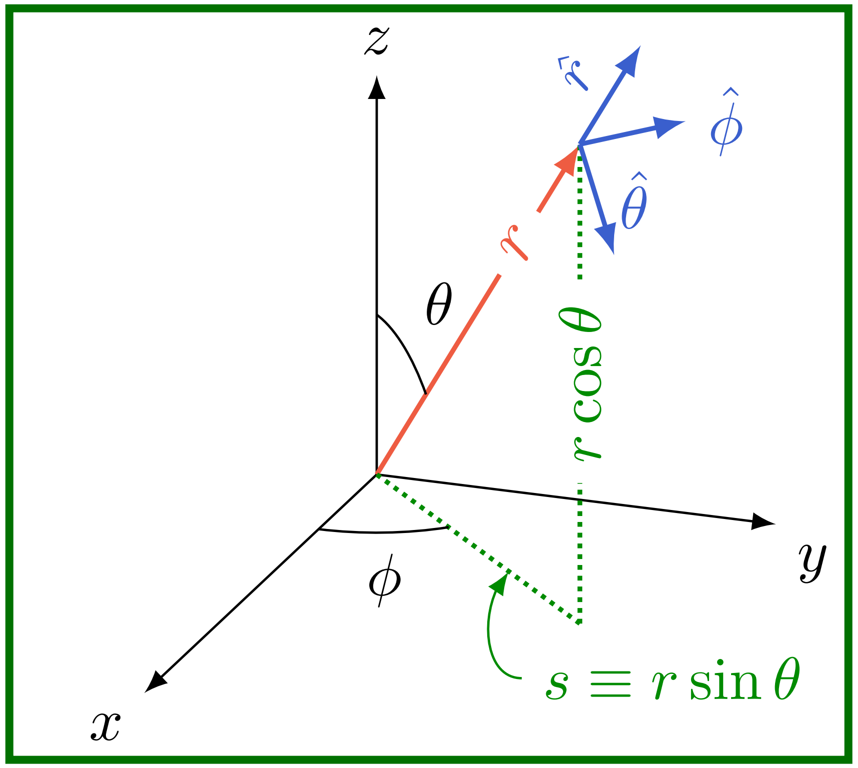
$\vec{\nabla} \cdot \vec{v} = ?$ $\quad \vec{\nabla}\cdot\vec{v} = \dfrac{\partial v_x}{\partial x} + \dfrac{\partial v_y}{\partial y} + \dfrac{\partial v_z}{\partial z}$

$\vec{\nabla} \times \vec{v} = ?$ $\quad \left[\vec{\nabla}\times\vec{v}\right]_x = \dfrac{\partial v_z}{\partial y} - \dfrac{\partial v_y}{\partial z}$



Coordenadas esféricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}r\,\hat{r} + r\,\mathrm{d}\theta\,\hat{\theta} + r\sin\theta\,\mathrm{d}\phi\,\hat{\phi}$$



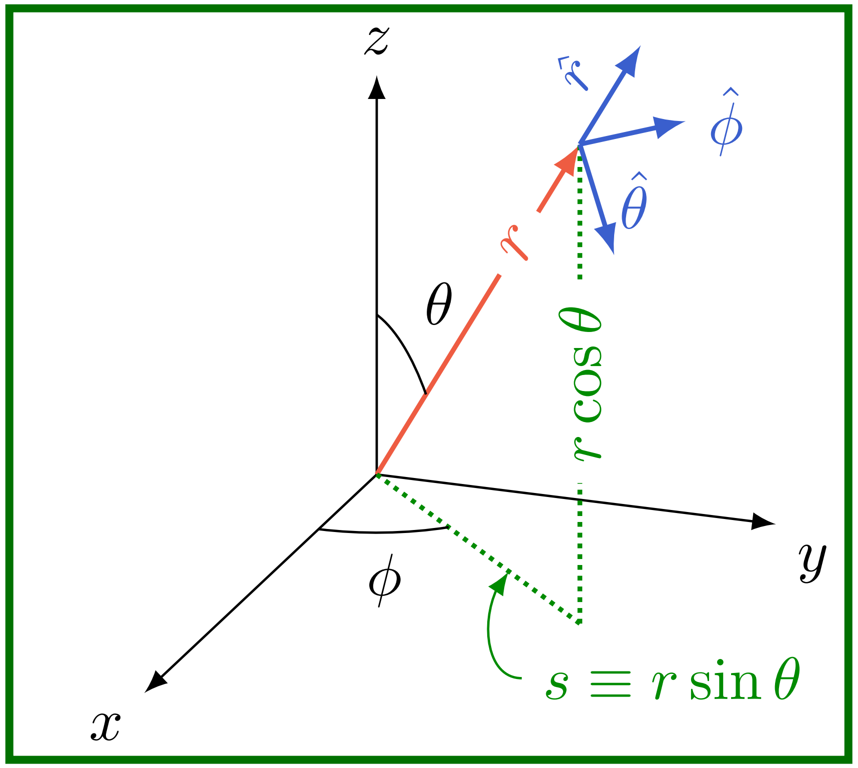
$$\vec{\nabla}\, t = \,?$$

$$\boxed{\int \vec{\nabla} t \cdot \vec{\mathrm{d}\ell} = \int \mathrm{d}t}$$

TEOREMA FUNDAMENTAL
P/ GRADIENTES

Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$



$$\vec{\nabla} t = ?$$

$$\boxed{\int \vec{\nabla} t \cdot \vec{d\ell} = \int dt}$$

$$\vec{\nabla} t \cdot d\vec{\ell} = dt$$

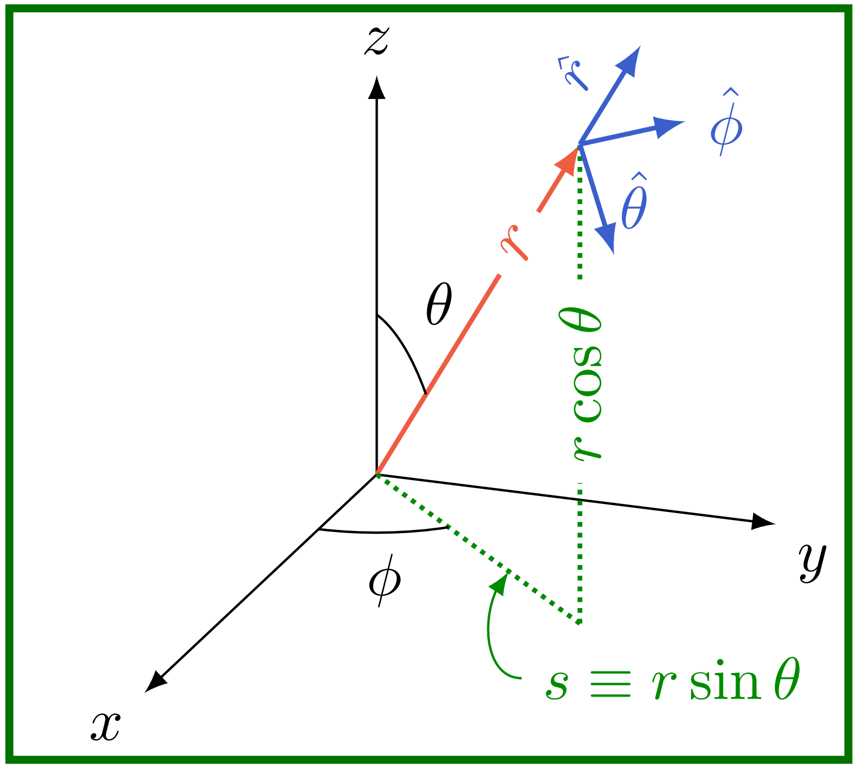
FORMA DIFERENCIAL

Coordenadas esféricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}r\,\hat{r} + r\,\mathrm{d}\theta\,\hat{\theta} + r\sin\theta\,\mathrm{d}\phi\,\hat{\phi}$$

$$\vec{\nabla} t = \,?$$

$$\vec{\nabla} t \cdot \mathrm{d}\vec{\ell} = \mathrm{d}t$$

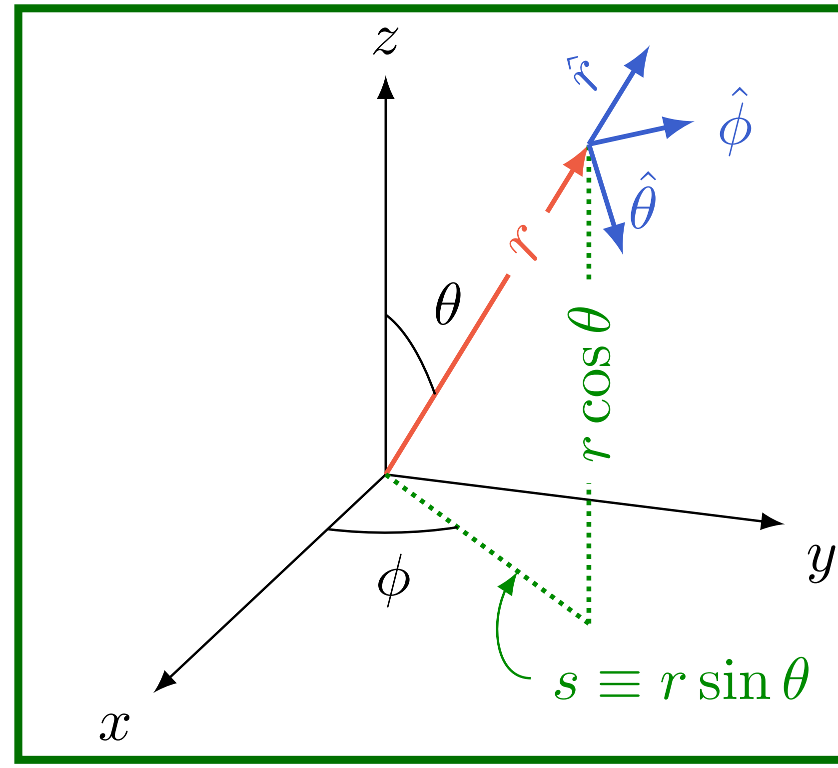


Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$\vec{\nabla} t = ?$$

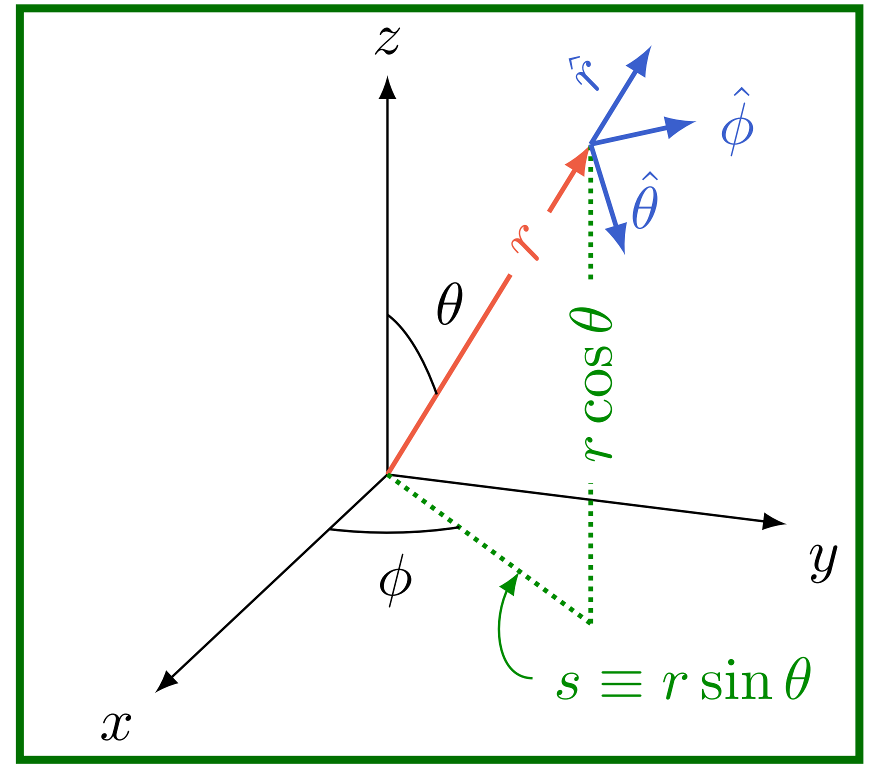
$$\vec{\nabla} t \cdot d\vec{\ell} = dt$$



$$(\vec{\nabla} t)_r dr + (\vec{\nabla} t)_\theta r\,d\theta + (\vec{\nabla} t)_\phi r\sin\theta\,d\phi = \underbrace{\frac{\partial t}{\partial r}dr + \frac{\partial t}{\partial \theta}d\theta + \frac{\partial t}{\partial \phi}d\phi}_{dt}$$

Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$



$$\vec{\nabla} t = ?$$

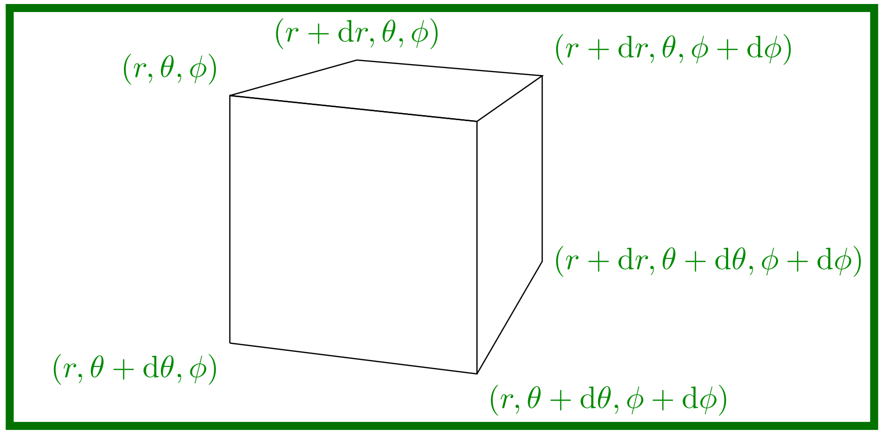
$$\vec{\nabla} t \cdot d\vec{\ell} = dt$$

$$(\vec{\nabla} t)_r dr + (\vec{\nabla} t)_\theta r\,d\theta + (\vec{\nabla} t)_\phi r\sin\theta\,d\phi = \frac{\partial t}{\partial r}dr + \frac{\partial t}{\partial \theta}d\theta + \frac{\partial t}{\partial \phi}d\phi$$

$$\Rightarrow \boldsymbol{\nabla} t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

Coordenadas esféricas

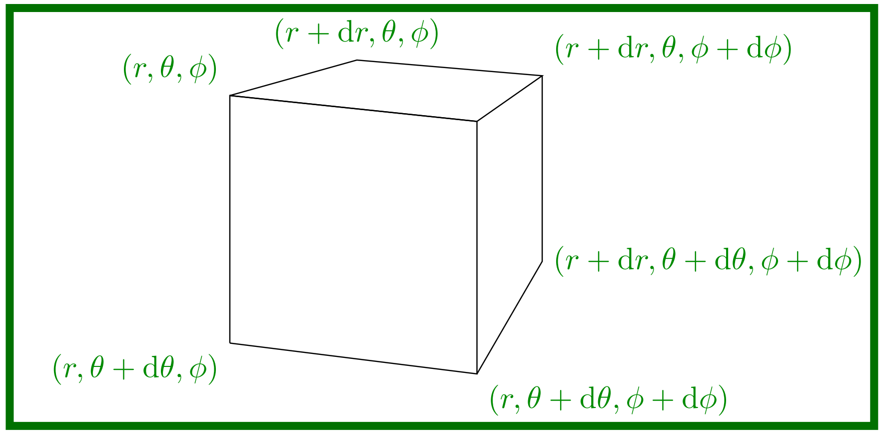
$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$



$$\vec{\nabla} \cdot \vec{v} = ?$$

MESMA ABORDAGEM

Coordenadas esféricas



$$\mathrm{d}\vec{\ell} = \mathrm{d}r\,\hat{r} + r\,\mathrm{d}\theta\,\hat{\theta} + r\sin\theta\,\mathrm{d}\phi\,\hat{\phi}$$

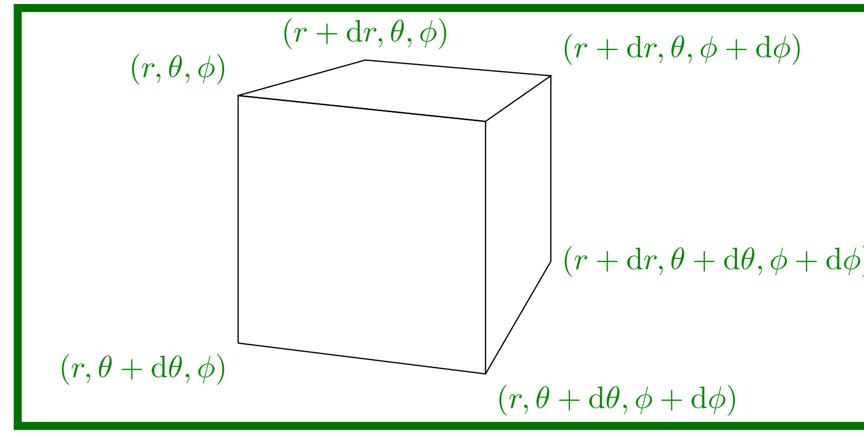
$$\vec{\nabla} \cdot \vec{v} = ?$$

$$\boxed{\int \vec{\nabla} \cdot \vec{v}\,\mathrm{d}\tau = \int \vec{v} \cdot \hat{n}\,\mathrm{d}A}$$

TEOREMA FUNDAMENTAL PARA DIVERGENTES

Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$



$$\vec{\nabla} \cdot \vec{v} = ?$$

$$\boxed{\int \vec{\boldsymbol{\nabla}} \cdot \vec{v}\, d\tau = \int \vec{v} \cdot \hat{n}\, dA}$$

$$\vec{\nabla} \cdot \vec{v}\, d\tau = \vec{v} \cdot \hat{n}\, dA$$ $\longrightarrow$ FORMA DIFERENCIAL

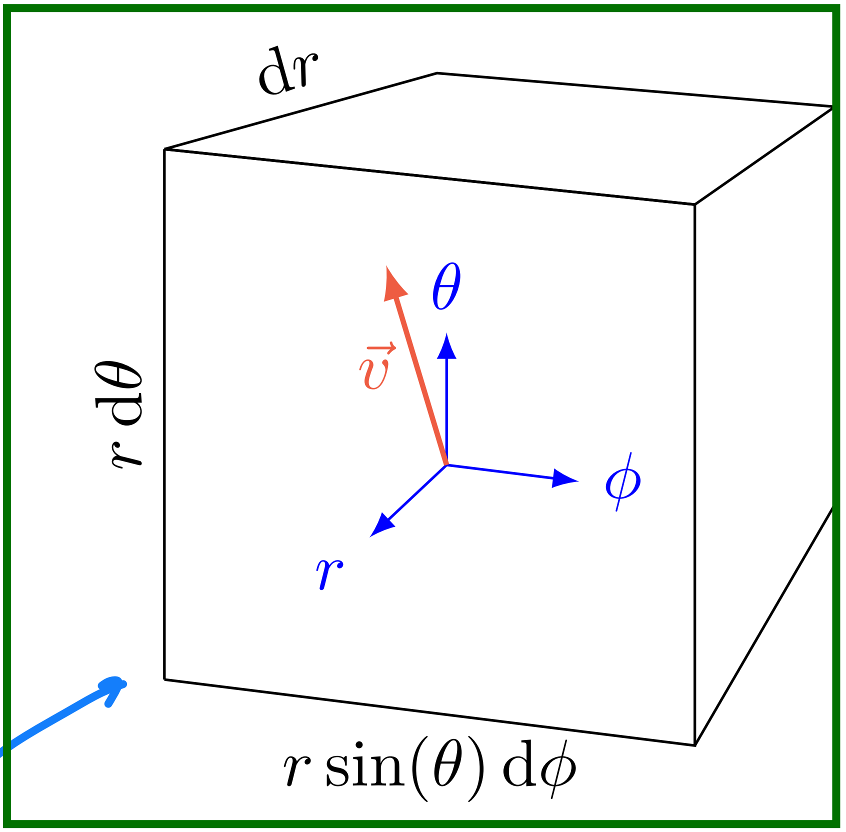
Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

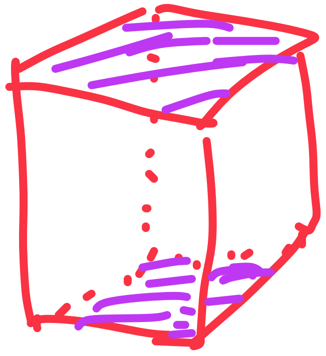
$$\vec{\nabla}\cdot\vec{v} = ?$$

$$\vec{\nabla}\cdot\vec{v}\,d\tau = \vec{v}\cdot\hat{n}\,dA$$

CALCULAR NAS SEIS FACES DO CUBO

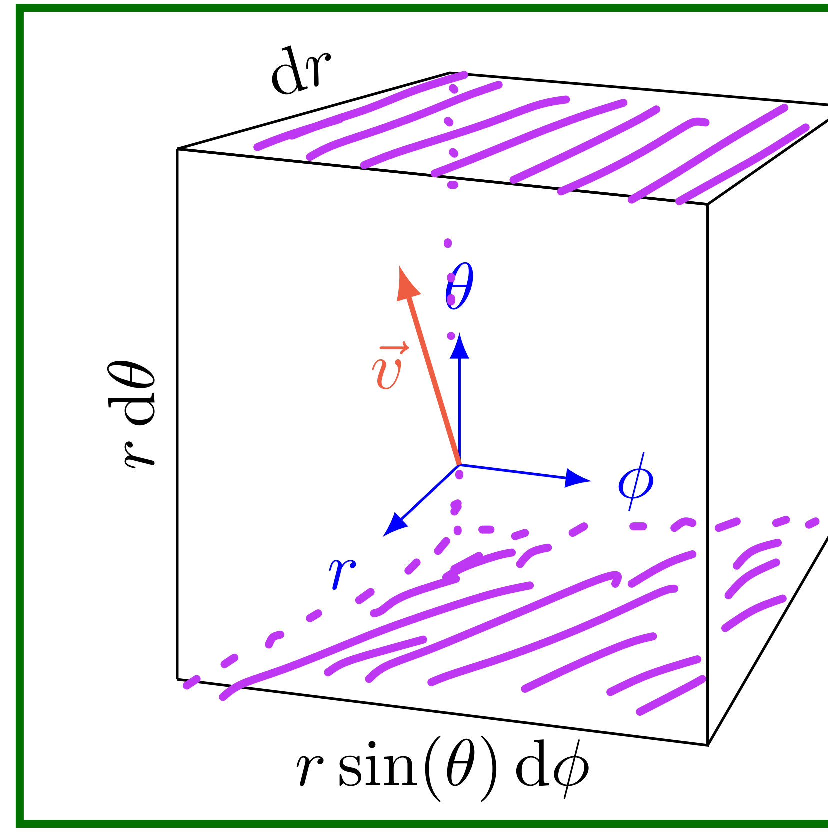


COMEÇAREMOS PELAS FACES SUPERIOR E INFERIOR



Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$



OUTRAS FACES

$$\vec{\nabla}\cdot\vec{v} = ?$$

$$\vec{\nabla}\cdot\vec{v}\,d\tau = \vec{v}\cdot\hat{n}\,dA$$

$$\vec{\nabla}\cdot\vec{v}\,d\tau = \Big(v_\theta(\theta+d\theta)\sin(\theta+d\theta) - v_\theta(\theta)\sin(\theta)\Big)\,dr\,r\,d\phi + \ldots$$

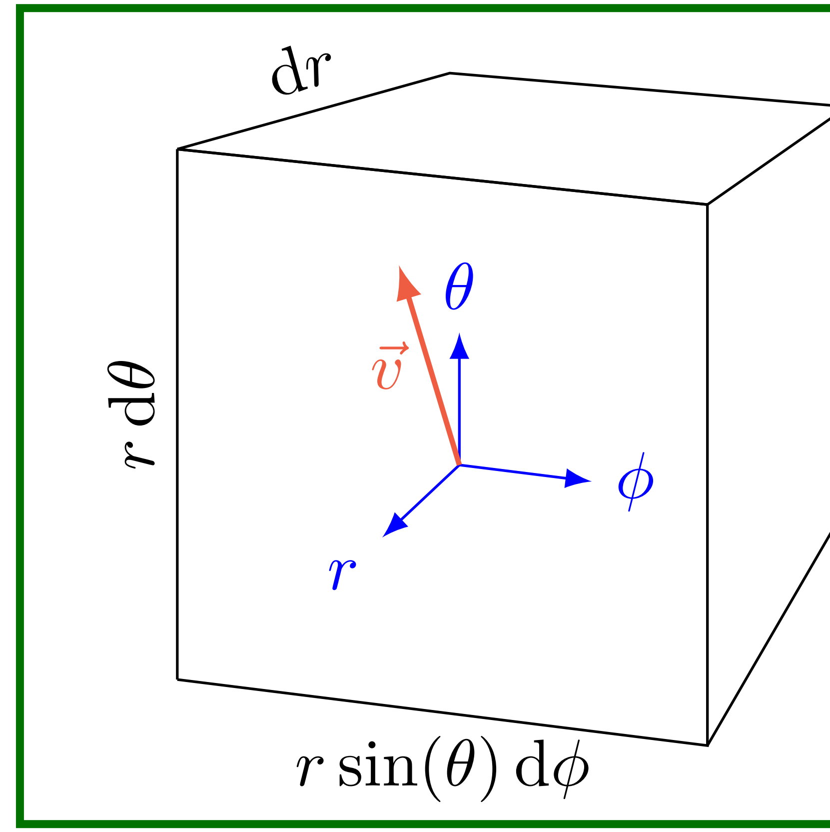
$$\vec{v}\cdot\hat{n}\,dA = \left[v_\theta\,dr\,r\,\text{sen}\,\theta\,d\phi\right](r,\theta+d\theta,\phi)$$

$$= v(\theta+d\theta)\,\text{sen}(\theta+d\theta)\,r\,dr\,d\phi$$

$$\vec{v}\cdot\hat{n}\,dA = -\left[v_\theta\,dr\,r\,\text{sen}\,\theta\,d\phi\right](r,\theta,\phi)$$

Coordenadas esféricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}r\,\hat{r} + r\,\mathrm{d}\theta\,\hat{\theta} + r\sin\theta\,\mathrm{d}\phi\,\hat{\phi}$$



$$\vec{\nabla}\cdot\vec{v} = ?$$

$$\vec{\nabla}\cdot\vec{v}\,\mathrm{d}\tau = \vec{v}\cdot\hat{n}\,\mathrm{d}A$$

$$\vec{\nabla}\cdot\vec{v}\,\mathrm{d}\tau = \Big(v_\theta(\theta+\mathrm{d}\theta)\sin(\theta+\mathrm{d}\theta) - v_\theta(\theta)\sin(\theta)\Big)\,\mathrm{d}r\,r\mathrm{d}\phi + \ldots$$

$$\vec{\nabla}\cdot\vec{v}\,\mathrm{d}\tau = \left(\frac{\partial(v_\theta\sin\theta)}{\partial\theta}\mathrm{d}\theta\right)\mathrm{d}r\,r\mathrm{d}\phi + \ldots$$

$$f(\theta+d\theta) - f(\theta) =$$

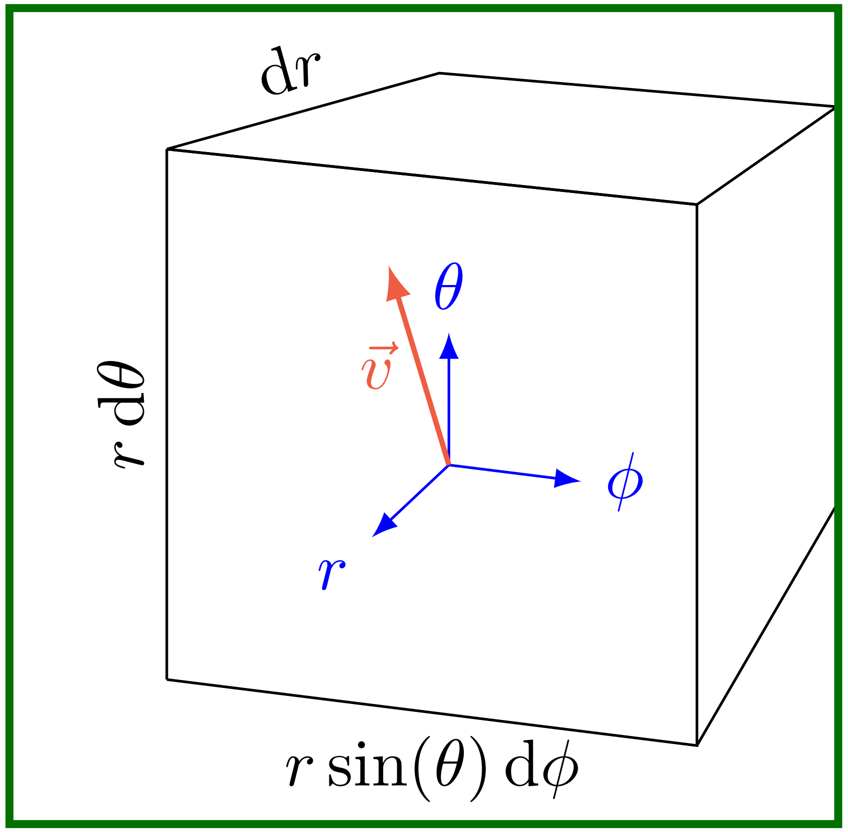
$$= \boxed{\frac{f(\theta+d\theta) - f(\theta)}{d\theta}}\,d\theta$$

$$\frac{\partial f}{\partial \theta}$$

Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$\overrightarrow{\nabla} \cdot \overrightarrow{v} = ?$$



$$\overrightarrow{\nabla} \cdot \overrightarrow{v}\,d\tau = \overrightarrow{v} \cdot \hat{n}\,dA$$

$$\overrightarrow{\nabla} \cdot \overrightarrow{v}\,d\tau = \left(\frac{\partial(v_\theta \sin\theta)}{\partial\theta} d\theta\right) dr\, r d\phi + \ldots$$

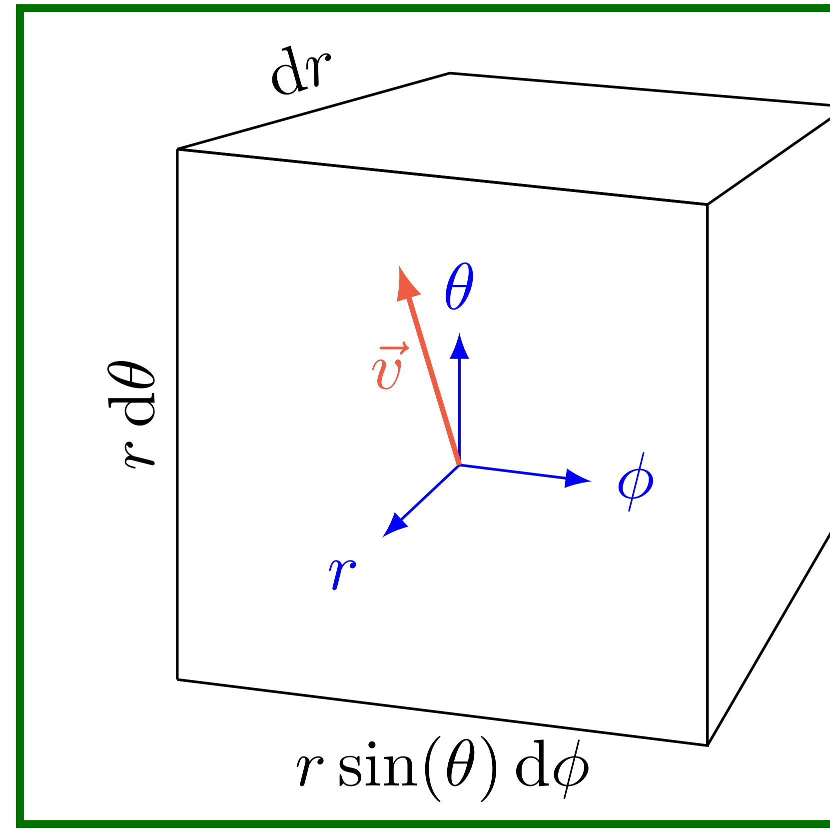
OUTRAS FACES

Coordenadas esféricas

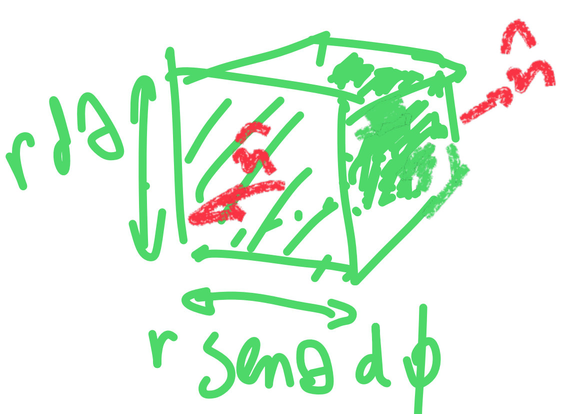
$$\mathrm{d}\vec{\ell} = \mathrm{d}r\,\hat{r} + r\,\mathrm{d}\theta\,\hat{\theta} + r\sin\theta\,\mathrm{d}\phi\,\hat{\phi}$$

$$\vec{\nabla}\cdot\vec{v} = ?$$

$$\vec{\nabla}\cdot\vec{v}\,\mathrm{d}\tau = \vec{v}\cdot\hat{n}\,\mathrm{d}A$$



$$\begin{aligned}\vec{\nabla}\cdot\vec{v}\,\mathrm{d}\tau =\; & r\frac{\partial(\sin\theta v_\theta)}{\partial\theta}\,\mathrm{d}r\mathrm{d}\theta\mathrm{d}\phi \\ & + \Big(v_r(r+\mathrm{d}r)\,(r+\mathrm{d}r)^2 - v_r(r)r^2\Big)\,\sin\theta\mathrm{d}\theta\mathrm{d}\phi \\ & + \dots\end{aligned}$$



Coordenadas esféricas

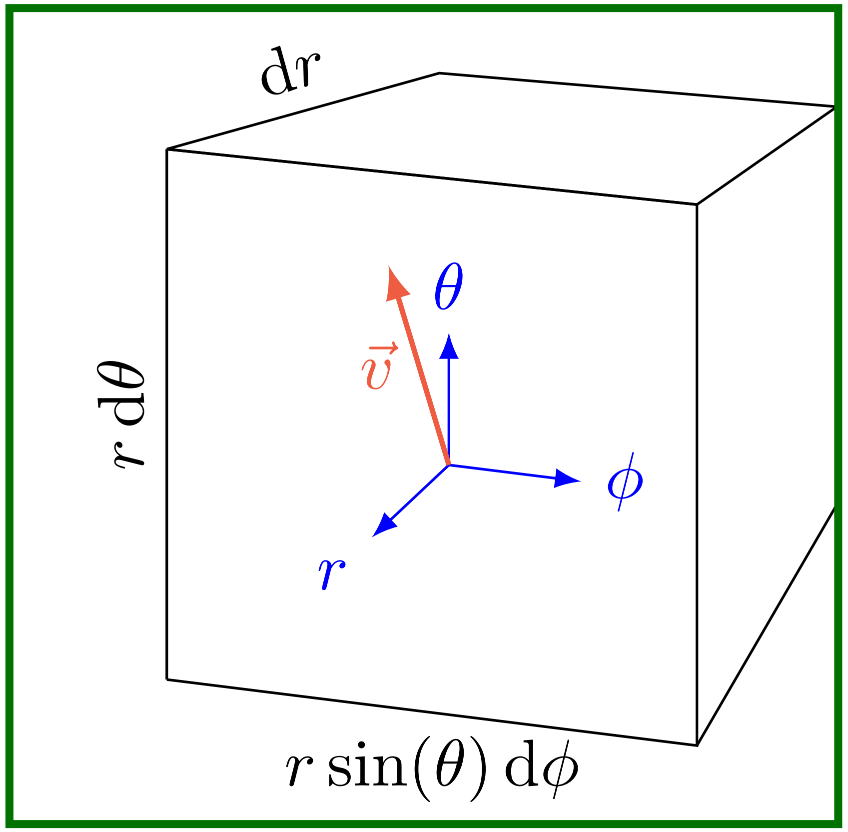
$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$\vec{\nabla}\cdot\vec{v} = ?$$

$$\vec{\nabla}\cdot\vec{v}\,d\tau = \vec{v}\cdot\hat{n}\,dA$$

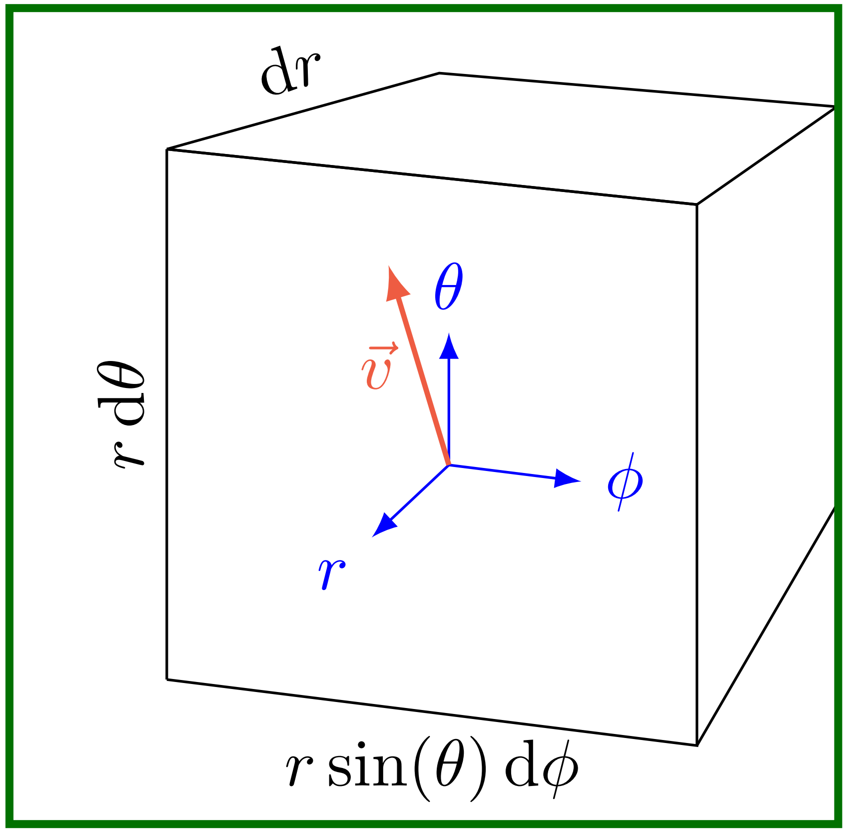
$$\vec{\boldsymbol{\nabla}}\cdot\vec{v}\,d\tau = \frac{\partial(\sin\theta v_\theta)}{\partial\theta} r\,d\theta\,d\phi + \left(\frac{\partial r^2 v_r}{\partial r}\right) dr\,d\theta\sin\theta d\phi + \dots$$

ÚLTIMAS DUAS FACES



Coordenadas esféricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}r\,\hat{r} + r\,\mathrm{d}\theta\,\hat{\theta} + r\sin\theta\,\mathrm{d}\phi\,\hat{\phi}$$



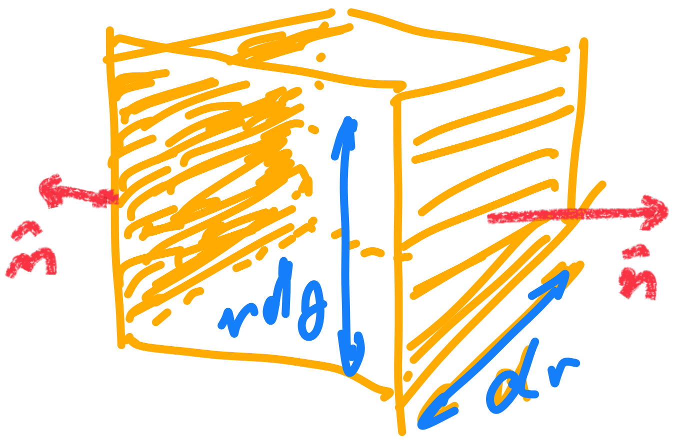
$$\overrightarrow{\nabla} \cdot \overrightarrow{v} = ?$$

$$\overrightarrow{\nabla} \cdot \overrightarrow{v}\,\mathrm{d}\tau = \overrightarrow{v} \cdot \hat{n}\,\mathrm{d}A$$

$$\vec{\boldsymbol{\nabla}} \cdot \vec{v}\,\mathrm{d}\tau = r\frac{\partial(\sin\theta v_\theta)}{\partial\theta}\,\mathrm{d}r\mathrm{d}\theta\mathrm{d}\phi$$

$$\frac{\partial(r^2 v_r)}{\partial r}\mathrm{d}r\sin\theta\mathrm{d}\theta\mathrm{d}\phi$$

$$+\Big(v_\phi(\phi+\mathrm{d}\phi) - v_\phi(\phi)\Big)\,\mathrm{d}r r\mathrm{d}\theta$$

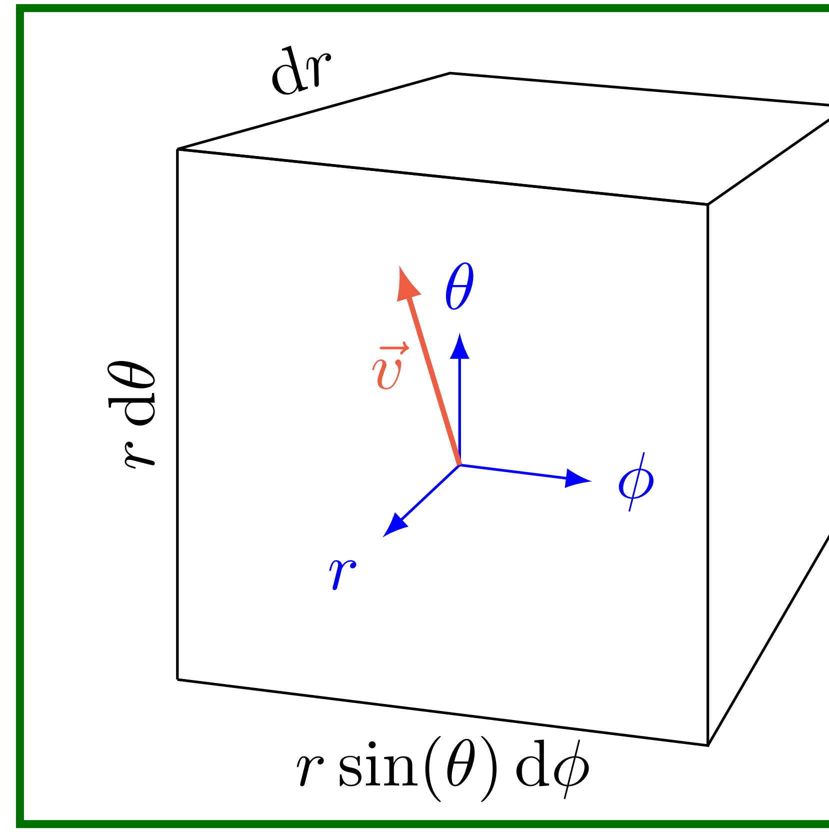


Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$\vec{\nabla}\cdot\vec{v} = ?$$

$$\vec{\nabla}\cdot\vec{v}\,d\tau = \vec{v}\cdot\hat{n}\,dA$$



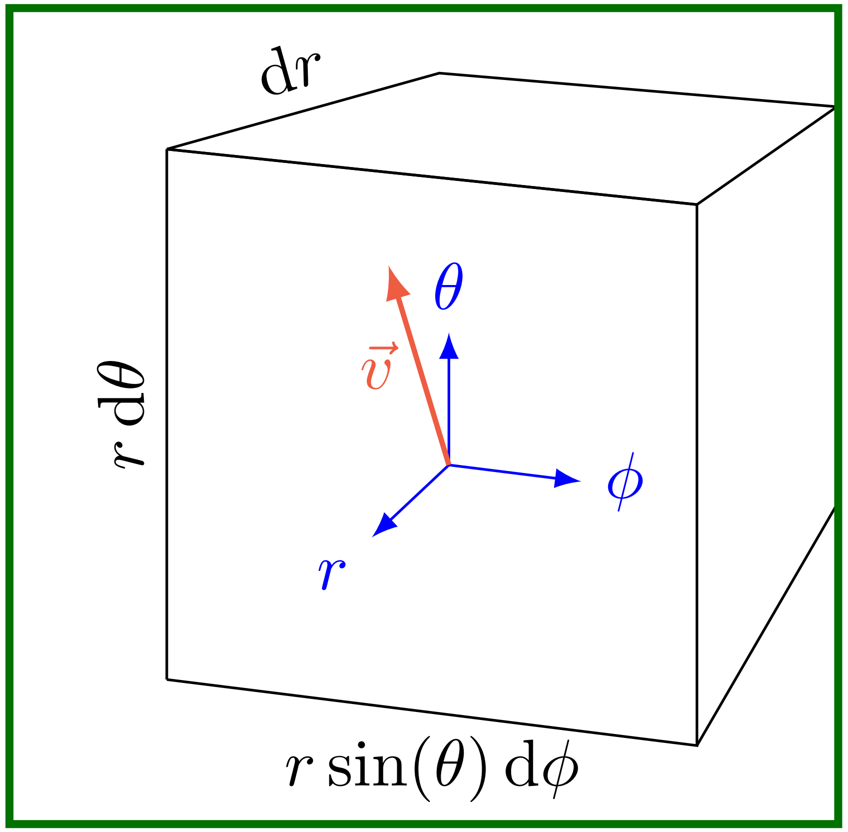
$$(\vec{\nabla}\cdot\vec{v})\,\underbrace{r^2\sin\theta\,d\cancel{r}\,d\cancel{\theta}\,d\cancel{\phi}}_{d\tau} = \frac{\partial(\sin\theta\, v_\theta)}{\partial\theta}\,d\cancel{r}\,d\cancel{\theta}\,r\,d\cancel{\phi}$$

$$\frac{\partial(r^2 v_r)}{\partial r}\,d\cancel{r}\,\sin\theta\,d\cancel{\theta}\,d\cancel{\phi}$$

$$+\left(\frac{\partial v_\phi}{\partial\phi}\right)d\cancel{\phi}\,d\cancel{r}\,r\,d\cancel{\theta}$$

Coordenadas esféricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}r\,\hat{r} + r\,\mathrm{d}\theta\,\hat{\theta} + r\sin\theta\,\mathrm{d}\phi\,\hat{\phi}$$



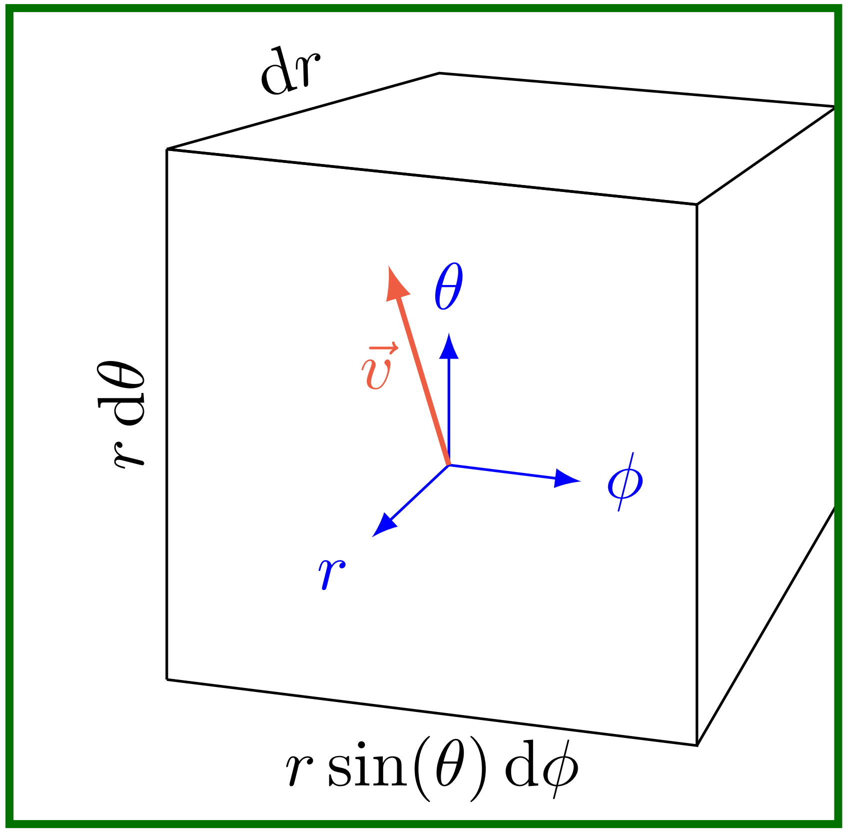
$$\overrightarrow{\nabla}\cdot\overrightarrow{v} = ?$$

$$\overrightarrow{\nabla}\cdot\overrightarrow{v}\,\mathrm{d}\tau = \overrightarrow{v}\cdot\hat{n}\,\mathrm{d}A$$

$$(\vec{\boldsymbol{\nabla}}\cdot\vec{v})\,r^2\sin\theta = \frac{\partial(\sin\theta v_\theta)}{\partial\theta}r + \frac{\partial(r^2 v_r)}{\partial r}\,\sin\theta + \frac{\partial v_\phi}{\partial\phi}\,r$$

Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$



$$\vec{\nabla}\cdot\vec{v} = ?$$

$$\vec{\nabla}\cdot\vec{v}\,d\tau = \vec{v}\cdot\hat{n}\,dA$$

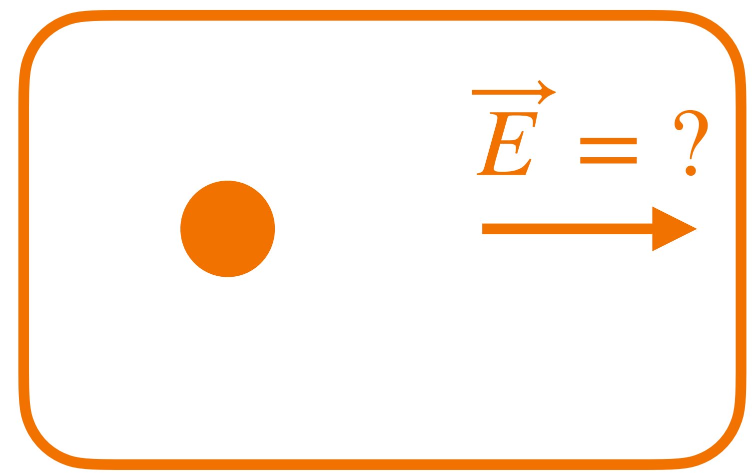
$$(\vec{\boldsymbol{\nabla}}\cdot\vec{v})\,r^2\sin\theta = \frac{\partial(\sin\theta v_\theta)}{\partial\theta}r + \frac{\partial(r^2 v_r)}{\partial r}\,\sin\theta + \frac{\partial v_\phi}{\partial\phi}\,r$$

$$\Rightarrow \vec{\boldsymbol{\nabla}}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

Pratique o que aprendeu

$$\vec{\nabla} \cdot \vec{v} = \frac{1}{r^2}\frac{\partial r^2 v_r}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

Campo elétrico de uma distribuição esférica



$$\vec{E} = E(r)\hat{r}$$

SIMETRIA ESFÉRICA DA CARGA

=> CAMPO C/ SIMETRIA ESFÉRICA

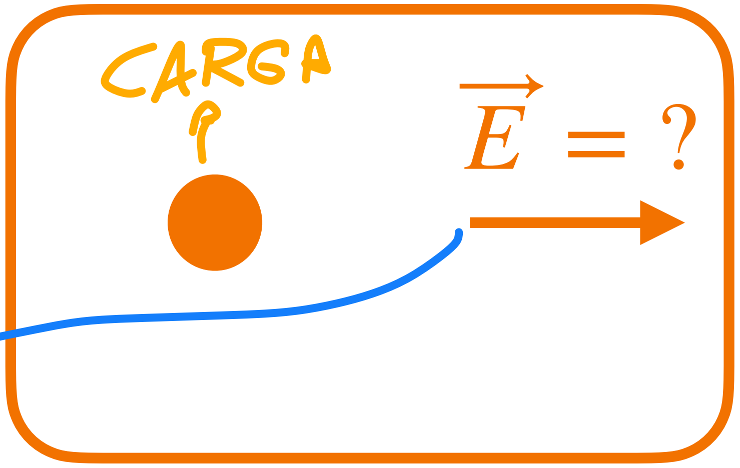
Pratique o que aprendeu

$$\vec{\nabla} \cdot \vec{v} = \frac{1}{r^2}\frac{\partial r^2 v_r}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

Campo elétrico de uma distribuição esférica

$$\vec{E} = E(r)\hat{r}$$

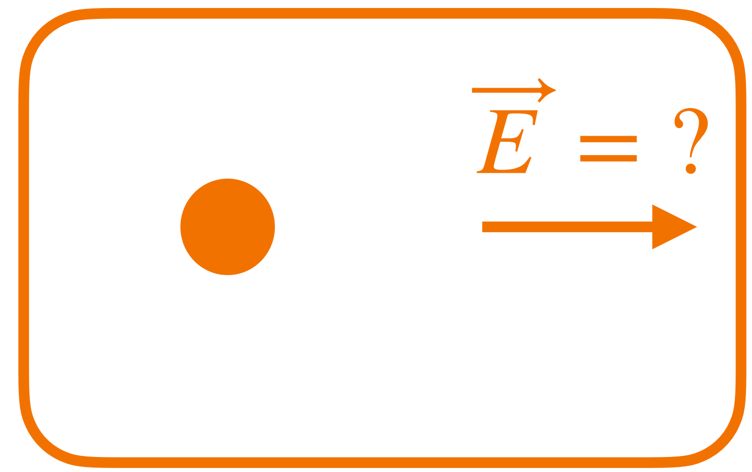
$$\vec{\nabla} \cdot \vec{E} = 0$$ ⇐ DENSIDADE DE CARGA = 0



Pratique o que aprendeu

$$\vec{\nabla} \cdot \vec{v} = \frac{1}{r^2}\frac{\partial r^2 v_r}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

Campo elétrico de uma distribuição esférica



$$\vec{E} = E(r)\hat{r}$$

$$\vec{\nabla} \cdot \vec{E} = 0$$

$E_\theta = E_\phi = 0$

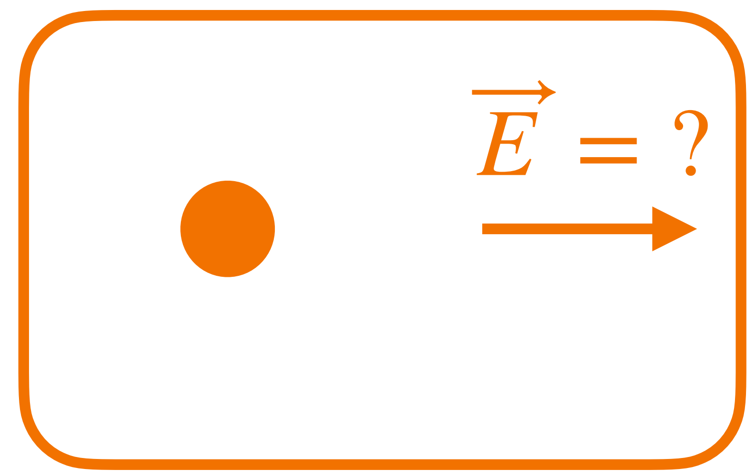
$$\frac{1}{r^2}\frac{d(r^2E)}{dr} = 0$$

$\boxed{r^2 E = \text{constante}}$

Pratique o que aprendeu

$$\vec{\nabla} \cdot \vec{v} = \frac{1}{r^2}\frac{\partial r^2 v_r}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

Campo elétrico de uma distribuição esférica



$$\vec{E} = E(r)\hat{r}$$

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\frac{1}{r^2}\frac{d(r^2 E)}{dr} = 0$$

$$\Rightarrow E(r) = \frac{A}{r^2}$$

PARA ENCONTRAR A, PRECISARÍAMOS CONHECER DISTRIBUIÇÃO DE CARGA