PTC-2640 - 1ª Lista - Gabarito

1

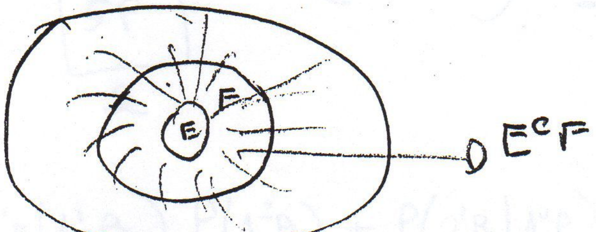
$$P(EF) = P(E) + P(F) - P(E \cup F)$$

$$P(E \cup F) \leq P(\Omega) = 1 \Rightarrow -P(E \cup F) \geq -1$$

$$\Rightarrow P(EF) \geq P(E) + P(F) - 1$$

$$P(E) = 0.9,\ P(F) = 0.8 \Rightarrow P(EF) \geq 0.9 + 0.8 - 1 = 0.7.$$

$$P(F) = P(E) + P(E^c F) \geq P(E)$$



2

a) $P(A) = p + (1-p)^2 p + (1-p)^4 p + (1-p)^6 p + \ldots$ b) $1/p$

$$P(B) = (1-p)p + (1-p)^3 p + (1-p)^5 p + \ldots$$

$$P(A) = p \sum_{i=0}^{\infty} (1-p)^{2i}, \quad P(B) = p \sum_{i=1}^{\infty} (1-p)^{2i-1}$$

$$P(A) = \frac{p}{1-(1-p)^2} = \frac{p}{\cancel{1} - (\cancel{1} - 2p + p^2)} = \frac{\cancel{p}}{\cancel{p}(2-p)} = \frac{1}{2-p}$$

$$P(B) = \frac{p(1-p)}{1-(1-p)^2} = \frac{\cancel{p}(1-p)}{\cancel{p}(2-p)} = \frac{1-p}{(2-p)}$$

$$p = 1/2 \Rightarrow \begin{cases} P(A) = 2/3 \\ P(B) = 1/3 \end{cases}$$

6) $P(2 \text{ iguais}) = P(1^\circ = 2^\circ) + P(2^\circ = 3^\circ) + P(1^\circ = 3^\circ)$

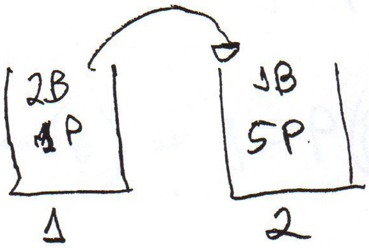
$$= 3 \times P(1^\circ = 2^\circ)$$

$$P(1^\circ = 2^\circ) = P(1^\circ = 2^\circ \neq 3^\circ) = \sum_{i=1}^{6} \sum_{j \neq i} P(1^\circ = 2^\circ = i, 3^\circ = j)$$

$$= \sum_{i=1}^{6} \sum_{j \neq i} \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{6^3} 6 \times 5 = \frac{5}{36}$$

$$P(2 \text{ iguais}) = \frac{5}{12} = \frac{15}{36}$$

7)



$$P(1^\circ B \mid 2^\circ B) = \frac{P(1^\circ B, 2^\circ B)}{P(2^\circ B)} = \frac{P(2^\circ B \mid 1^\circ B) . P(1^\circ}{P(2^\circ B)}$$

$$P(2^\circ B) = P(2^\circ B \mid 1^\circ B) . P(1^\circ B) + P(2^\circ B \mid 1^\circ P) . P(1^\circ P)$$

$$= \frac{2}{7} \times \frac{2}{3} + \frac{1}{7} \times \frac{1}{3} = \frac{5}{21}$$

$$P(1^\circ B \mid 2^\circ B) = \frac{4/21}{5/21} = \frac{4}{5}$$

3) a) $P(GG \mid 1^{\circ}G) = \dfrac{P(G).\cancel{P(G)}}{\cancel{P(G)}} = 1/2$ ③

$GG \subset \{\text{um é } G\}$

b) $P(GG \mid \text{um é } G) = \dfrac{P(GG)}{P(\text{um é } G)} = \dfrac{1/2 \times 1/2}{3/4} = \dfrac{1}{3}$

$P(\text{um é } G) = P(1^{\circ}G, 2^{\circ}G) + P(1^{\circ}G, 2^{\circ}B) + P(1^{\circ}B, 2^{\circ}G)$

$= 1 - P(BB) = 1 - \frac{1}{2} \times \frac{1}{2} = \frac{3}{4}$

6) a) $P(\text{um é } 6) = 1 - P(\{1^{\circ} \neq 6, 2^{\circ} \neq 6\}) = \dfrac{11}{36}$

$P(1^{\circ} \neq 6, 2^{\circ} \neq 6) = \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

b) $P(\text{um é } 6 \mid \text{faces diferentes}) = \dfrac{P(\text{um é } 6, \text{faces diferentes})}{P(\text{faces diferentes})} = \dfrac{10/36}{5/6} = \dfrac{1}{3}$

$P(\text{faces diferentes}) = 1 - P(\text{faces iguais}) = 1 - 6 \times \frac{1}{36} = \frac{5}{6}$

$P(\text{um é } 6, \text{faces diferentes}) = \sum_{i=1}^{5} P(\text{um é } 6, \text{outro é } i) =$

$\sum_{i=1}^{5} P(1^{\circ} \text{ é } 6, 2^{\circ} \text{ é } i) + \sum_{i=1}^{5} P(1^{\circ} \text{ é } i, 2^{\circ} \text{ é } 6) = \sum_{i=1}^{5} \underset{1/6}{P(1^{\circ} \text{ é } 6)} \, \underset{1/6}{P(2^{\circ} \text{ é } i)} + \sum_{i=1}^{5} \underset{1/6}{P(1^{\circ} \text{ é } i)} \, \underset{1/6}{P(2^{\circ} \text{ é } 6)}$

$= \frac{5}{36} + \frac{5}{36} = \frac{10}{36}$

$\frac{11}{36} - \frac{1}{36} = \frac{10}{36}$

7) A → 50 , 50% M

B → 75 , 60% M

C → 100 , 70% M

C → funcionário que se demite ser do C

$$P(C|MR) = \frac{P(MR|C) \cdot P(C)}{P(MR|C) \cdot P(C) + P(MR|A) \cdot P(A) + P(MR|B)\, P(B)}$$

$$= \frac{0.7 \times \frac{100}{\cancel{225}}}{0.7 \times \frac{100}{\cancel{225}} + 0.5 \times \frac{50}{\cancel{225}} + 0.6 \times \frac{75}{\cancel{225}}}$$

$$= \frac{70}{70+25+45} = \frac{70}{140} = \frac{1}{2}$$

5

(8) 4FB 6SB
6FG xSG

Sexo e classe eventos independentes

$$P(F,B) = P(F)P(B)$$

$$P(F,B) = \frac{4}{16+x} \quad P(F) = \frac{10}{16+x}, \; P(B) = \frac{10}{16+x}$$

$$4(16+x) = 100 \Rightarrow 16+x = 25 \Rightarrow x = 9$$

(9) a) $P(H,H,H) + P(T,T,T) = 2 \times \left(\frac{1}{2}\right)^3 = \frac{1}{4} = 0.25$

$$P\left(\left[\{H,H,H\} \cup \{T,T,T\}\right]^c\right) = 0.75$$

b) $P(H,H,H) + P(T,T,T) = \left(\frac{1}{4}\right)^3 + \left(\frac{3}{4}\right)^3 = \frac{28}{4^3} = \frac{7}{16}$

$$P\left(\left[\{H,H,H\} \cup \{T,T,T\}\right]^c\right) = \frac{9}{16}$$

(10) 5% H são daltônicos
0.25% M são daltônicas

$$P(H|D) = \frac{P(D|H)P(H)}{P(D|H)P(H) + P(D|M)P(M)}$$

$$= \frac{5 \times 1/2}{5 \times 1/2 + 0.25 \times 1/2} = \frac{20}{21}$$

14 a) $$P(F/H) = \frac{P(F,H)}{P(H)} = \frac{P(H/F)\,P(F)}{P(H/F)\,P(F) + P(H/\bar{F})\,P(\bar{F})}$$

$$= \frac{1/2 \times \cancel{1/2}}{1/2 \times \cancel{1/2} + 1 \times \cancel{1/2}} = \frac{1}{1+2} = \frac{1}{3}$$

b) $$P(F/1H,2H) = \frac{P(F,1H,2H)}{P(1H,2H)} = \frac{P(2H,1H/F)\,P(F)}{P(2H,1H/F)\,P(F) + P(1H,2H/\bar{F})\,P(\bar{F})}$$

$$= \frac{1/4 \times \cancel{1/2}}{1/4 \times \cancel{1/2} + 1 \times \cancel{1/2}} = \frac{1}{5}$$

c) $$P(F/1H,2H,3T) = 1$$

15 $$P(H/X=i) = \frac{i}{10}, \quad i = 1, \dots, 10$$

$$P(X=5/H) = \frac{P(X=5,H)}{P(H)} = \frac{P(H/X=5)\,P(X=5)}{\sum_{i=1}^{10} P(H/X=i)\,P(X=i)} = \frac{5/10 \times \cancel{1/10}}{\cancel{1/10} \sum_{i=1}^{10} \frac{i}{10}}$$

$$= \frac{5}{\frac{(1+10)}{2} 10} = \frac{1}{11}$$

8

11

: 4 FB 6 SB
6 FG ? SG

para que sexo e classe sejam independentes quando um estudante é escolhido aleatoriamente

$$P(F) = \frac{10}{16+x}, \quad P(B) = \frac{10}{16+x}, \quad P(FB) = \frac{4}{16+x}$$

$$P(S) = \frac{6+x}{16+x}, \quad P(G) = \frac{6+x}{16+x}, \quad P(SG) = \frac{x}{16+x}$$

$$P(SB) = P(S)P(G) \Leftrightarrow \frac{x}{\cancel{16+x}} = \frac{(6+x)^2}{(16+x)^{\cancel{2}}} \Leftrightarrow$$

$$x(16+x) = (6+x)^2 \Leftrightarrow 16x + \cancel{x^2} = \cancel{x^2} + 12x + 36 \Leftrightarrow$$

$$4x = 36 \Leftrightarrow \boxed{x = 9}$$

$$P(FB) = P(F)P(B) \Leftrightarrow \frac{4}{\cancel{16+x}} = \frac{100}{(16+x)^{\cancel{2}}} \Leftrightarrow 16 + x = 25 \Leftrightarrow x = 9$$

16

16

: Urna 1 : 5 W
7 B

Urna 2 : 3 W
12 B

$H \to$ Urna 1 $\quad T \to$ Urna 2

$$P(T|W) = P(W,T) / P(W) = \frac{P(W|T).P(T)}{P(W)} = \frac{P(W|T).P(T)}{P(W|T)P(T) + P(W|H)P(H)}$$

$$= \frac{\frac{3}{15}.\cancel{\frac{1}{2}}}{\frac{3}{15}.\cancel{\frac{1}{2}} + \frac{5}{12}.\cancel{\frac{1}{2}}} = \frac{\frac{1}{5}}{\frac{1}{5} + \frac{5}{12}} = \frac{1}{1 + \frac{25}{12}} = \frac{12}{37}$$

1 $\cancel{7}$: Dado é jogado 2 vezes. $X_i \to$ resultado da i-ésima jogada

(i) $Z = \max\{X_1, X_2\} \in \{1,2,3,4,5,6\}$

$$P(Z=1) = P(X_1=1 \text{ e } X_2=1) = P(X_1=1, X_2=1) = P(X_1=1)P(X_2=1) = \frac{1}{36}$$

$$P(Z=2) = P(X_1=2, X_2=1 \text{ ou } X_1=1, X_2=2 \text{ ou } X_1=2, X_2=2)$$
$$= P(X_1=2, X_2=1) + P(X_1=1, X_2=2) + P(X_1=2, X_2=2)$$
$$= P(X_1=2)P(X_2=1) + P(X_1=1)P(X_2=2) + P(X_1=2)P(X_2=2)$$
$$= \frac{3}{36} = \frac{1}{12}$$

$$P(Z=3) = P(X_1=3, X_2\le 2 \text{ ou } X_1\le 2, X_2=3 \text{ ou } X_1=3, X_2=3)$$
$$= P(X_1=3, X_2\le 2) + P(X_1\le 2, X_2=3) + P(X_1=3, X_2=3)$$
$$= P(X_1=3)P(X_2\le 2) + P(X_1\le 2).P(X_1=3) + P(X_1=3)P(X_2=3)$$
$$= \frac{1}{6}\cdot\frac{2}{6} + \frac{1}{6}\cdot\frac{2}{6} + \frac{1}{6}\cdot\frac{1}{6} = \frac{5}{36}$$

$$P(Z=6) = P(X_1=6, X_2\le 5 \text{ ou } X_1\le 5, X_2=6 \text{ ou } X_1=6, X_2=6)$$
$$= P(X_1=6, X_2\le 5) + P(X_1\le 5, X_2=6) + P(X_1=6, X_2=6)$$
$$= P(X_1=6).P(X_2\le 5) + P(X_1\le 5).P(X_2=6) + P(X_1=6)P(X_1=6)$$
$$= \frac{1}{6}\cdot\frac{5}{6} + \frac{5}{6}\cdot\frac{1}{6} + \frac{1}{6}\cdot\frac{1}{6} = \frac{11}{36}$$

$$P(Z=4) = \frac{7}{36}, \quad P(Z=5) = \frac{9}{36}$$

(ii) $Z = \min\{X_1, X_2\} \in \{1,2,3,4,5,6\}$ (9)

$$P(Z=1) = P(X_1 = 1 \text{ ou } X_2 = 1) = P(X_1=1) + P(X_2=1) - P(X_1=1, X_2=1)$$

$$= P(X_1=1) + P(X_2=1) - P(X_1=1)P(X_2=1) = \frac{1}{6} + \frac{1}{6} - \frac{1}{36} = \frac{11}{36}$$

ou

$$= P(X_1 = 1, X_2 \geq 2 \text{ ou } X_1 \geq 2, X_2 = 1 \text{ ou } X_1 = 1, X_2 = 1)$$

$$= P(X_1 = 1, X_2 \geq 2) + P(X_1 \geq 2, X_2 = 1) + P(X_1 = 1, X_2 = 1)$$

$$= P(X_1 = 1)P(X_2 \geq 2) + P(X_1 \geq 2)P(X_2 = 1) + P(X_1 = 1)P(X_2 = 1)$$

$$= \frac{1}{6} \cdot \frac{5}{6} + \frac{5}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} = \frac{11}{36}$$

(iii) $Z = X_1 + X_2 \in \{2, \dots, 12\}$

(iv) $Z = X_1 - X_2 \in \{-5, -4, \dots, 0, \dots, 4, 5\}$

18 (18)

X, Y independentes, contínuas com f. d. p. $f_X(x)$ e $f_Y(y)$ respe

a) $$P(X \leq Y) = \iint_{\{x \leq y\}} f_X(x) f_Y(y)\, dx\, dy$$

$$= \int_{-\infty}^{\infty} \underbrace{\int_{-\infty}^{y} f_X(x)\, dx}_{F_X(y)}\; f_Y(y)\, dy$$

$$= \int_{-\infty}^{\infty} F_X(y) f_Y(y)\, dy$$

$$F_X(y) = \int_{-\infty}^{y} f_X(x)\, dx$$

$$P(X \leq Y) = \int_{-\infty}^{\infty} \int_{x}^{\infty} f_Y(y)\, dy\, f_X(x)\, dx$$

$$= \int_{-\infty}^{\infty} \left(1 - F_Y(x)\right) f_X(x)\, dx$$

d) $\underset{\lambda_1}{X}, \underset{\lambda_2}{Y} \to$ exponenciais com parâmetros λ_1 e λ_2 respectivamente: $F_Y(y) = 1 - e^{-\lambda_2 y}$

$$P(X \leq Y) = \int_{0}^{\infty} e^{-\lambda_2 x} \lambda_1 e^{-\lambda_1 x}\, dx =$$

$$= \frac{\lambda_1}{\lambda_1 + \lambda_2} \underbrace{\int_{0}^{\infty} (\lambda_1 + \lambda_2)\, e^{-(\lambda_1+\lambda_2)x}\, dx}_{1} = \frac{\lambda_1}{\lambda_1 + \lambda_2} //$$

e) $X, Y \to$ uniformemente distribuídas $(0,1)$ e $(0,4)$ respectivamente

$$P(X \leq Y) = \int_{0}^{1} y \cdot \frac{1}{4}\, dy + \int_{1}^{4} 1 \cdot \frac{1}{4}\, dy = \frac{1}{4} \left.\frac{y^2}{2}\right]_0^1 + \frac{3}{4}$$

$$= \frac{1}{4} \cdot \frac{1}{2} + \frac{3}{4} = \frac{1}{4}\left(\frac{1}{2} + 3\right) = \frac{7}{8} //$$

17 : X V.A. com função densidade 11

$$f_X(x) = \begin{cases} c(4x-2x^2), & 0 < x < 2 \\ 0, & \text{caso contrário} \end{cases}$$

a) determine c

b) $P\left(\frac{1}{2} < X < \frac{3}{2}\right) = ?$

a) $\int_{-\infty}^{\infty} f_X(x)\,dx = 1 \Rightarrow \int_0^2 c(4x-2x^2)\,dx = c\left[2x^2 - \frac{2}{3}x^3\right]_0^2 =$

$c\,x^2\left[2-\frac{2}{3}x\right]_0^2 = c\,4\left(2-\frac{4}{3}\right) = c\,4\,\frac{2}{3} = c\,\frac{8}{3} = 1 \Rightarrow c = \frac{3}{8}$

b) $P\left(\frac{1}{2} < X < \frac{3}{2}\right) = F_X(3/2) - F_X(1/2) = \int_{1/2}^{3/2} \frac{3}{8}(4x-2x^2)\,dx =$

$\frac{3}{8}\,x^2\left[2-\frac{2}{3}x\right]_{1/2}^{3/2} = \frac{3}{8}\left[\frac{9}{4} - \frac{1}{4}\,\frac{5}{3}\right] = \frac{3}{32}\left(9-\frac{5}{3}\right) = \frac{27-5}{32} = \frac{22}{32} = \frac{11}{16}$

20 18 : Uma moeda com probabilidade p de dar cara é jogada até que a $r$ésima cara ocorra. Seja N o número de jogadas até que isso ocorra. Calcule $E(N)$.

$N = N_1 + N_2 + \dots + N_r$, N_i = número de jogadas entre a $(i-1)$ésima cara e a i ésima cara.

$E(N_i) = 1/p$

$E(N) = \frac{r}{p}$

17

19

$$f(x) = \begin{cases} c(4x-2x^2), & 0<x<2 \\ 0, & \text{caso contrário} \end{cases}$$

a) $\int_0^2 c(4x-2x^2)\,dx = 1 \Rightarrow c\left[\frac{4x^2}{2} - 2\frac{x^3}{3}\right]_0^2 =$

$c\left(8 - \frac{16}{3}\right) = c\left(\frac{8}{3}\right) = 1 \Rightarrow c = \frac{3}{8}$

b) $P\left(\frac{1}{2} < X < \frac{3}{2}\right) = \int_{1/2}^{3/2} \frac{3}{8}(4x-2x^2)\,dx = \frac{11}{16}$

21

18

$F_M(x) = P(\max\{X_1, \ldots, X_n\} \leq x) =$

$P(X_1 \leq x, \ldots, X_n \leq x) = P(X_1 \leq x) \ldots P(X_n \leq x) =$

$x^n, \quad x \in [0,1]$

$$f_M(x) = \frac{dF_M(x)}{dx} = \begin{cases} n x^{n-1}, & 0 \leq x \leq 1 \\ 0, & \text{caso contrário.} \end{cases}$$

19 22

sucesso = aparecer um 6 $\Rightarrow p = \frac{1}{6}$

X = número de "6's" em 3 jogadas

$$P(X \leq 1) = P(X=0) + P(X=1)$$

$$= \left(\frac{5}{6}\right)^3 + 3\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^2 = \left(\frac{5}{6}\right)^2\left(\frac{1}{2} + \frac{5}{6}\right)$$

$$= \left(\frac{5}{6}\right)^2\left(\frac{8}{6}\right)\left(\frac{4}{3}\right) = \frac{100}{108} = \frac{50}{54} = \frac{25}{27}$$

20

$$P(2C, 1O, 2N) = \frac{5!}{2!\,1!\,2!}(0.2)^2 \times (0.5) \times (0.3)^2$$

$$= 5 \times 2 \times 3 \times 0.04 + 0.5 \times 0.09 = 0.054$$

19' 21'

a) $c\int_{-1}^{1}(1-x^2)\,dx = c\left(x - \frac{x^3}{3}\right)\Big]_{-1}^{1} = c\left(2 - \frac{2}{3}\right) = c\frac{4}{3} = 1$

$c = \frac{3}{4}$

b) $F(x) = \begin{cases} 0, & x < -1 \\ \frac{3}{4}\left((x+1) - \frac{1}{3}(x^3+1)\right), & -1 \leq x \leq 1 \\ 1, & x \geq 1 \end{cases}$

21

S → 1/2

F → 1/2

X → número de acertos - binomial (10, 1/2)

$$P(X \geq 7) = \sum_{i=7}^{10} \binom{10}{i} \left(\frac{1}{2}\right)^{10} = \left(\frac{10 \times 9 \times 8}{3 \times 2 \times 1} + \frac{10 \times 9}{2} + 10 + 1 \right) \frac{1}{2^{10}}$$

$$= \frac{5}{2^{10}} \left(24 + 9 + 2 + \frac{1}{5} \right) = \frac{176}{2^{10}} = 0,171875$$

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X → nº de passageiros que se apresentam → binomial (52, 0.95)

S → 0.95 (passageiro se apresenta)

F → 0.05 (" ñ " ")

$$P(X \geq 51) = P(\text{alguém não tem lugar}) =$$

$$= \binom{52}{51} (0.95)^{51} (0.05) + \binom{52}{52} (0.95)^{52}$$

$$= (0.95)^{51} (52 \times 0.05 + 0.95)$$

$$= (0.95)^{51} \times 3.55 = 0.259$$

$$P(\text{todos terem lugar}) = 1 - 0.259 = 0.741$$

$r_1 = 10\%,\ r_2 = 18\%,\ \sigma_1 = 15\%,\ \sigma_2 = 30\%\quad \rho = 0,1$

15

21

a) $$\Sigma = \begin{bmatrix} 0,15^2 & 0,1\times 0,15\times 0,3 \\ 0,1\times 0,15\times 0,3 & 0,3^2 \end{bmatrix} =$$

$$= \begin{bmatrix} 0,0225 & 0,0045 \\ 0,0045 & 0,09 \end{bmatrix}$$

$$\sigma^2 = \begin{bmatrix} w_1 & 1-w_1 \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} \begin{bmatrix} w_1 \\ 1-w_1 \end{bmatrix}$$

$$= (w_1\sigma_1)^2 + ((1-w_1)\sigma_2)^2 + 2w_1(1-w_1)\rho\sigma_1\sigma_2$$

$$\frac{d\sigma^2}{dw_1} = \cancel{2}\, w_1\sigma_1^2 - \cancel{2}(1-w_1)\sigma_2^2 + \cancel{2}\rho\sigma_1\sigma_2(1-2w_1) = 0$$

$$\Rightarrow (\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)\,w_1 = \sigma_2^2 - \rho\sigma_1\sigma_2 \Rightarrow$$

$$\Rightarrow w_1 = \frac{\sigma_2^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} = \frac{0,09 - 0,0045}{0,0225 + 0,09 - 2\times 0,0045}$$

$$= \frac{0,0855}{0,1035} = 82,61\%$$

$$\sigma_{min}^2 = (0,8261\times 0,15)^2 + (0,1739\times 0,3)^2 +$$

$$2\times 0,8261\times 0,1739\times 0,1\times 0,15\times 0,3$$

$$= (0,1239)^2 + (0,05217)^2 + 0,001293$$

$$= 0,01535 + 0,0027217 + 0,001293$$

$$= 0,01936$$

$$\Rightarrow \sigma_{min} = 13,91\%$$

$$\mu_{min} = \underbrace{0,8261\times 0,1}_{0,08261} + \underbrace{0,1739\times 0,18}_{0,0313} = 11,39\%$$

c) $$\begin{cases} \sigma^2 = (w_1\sigma_1)^2 + ((1-w_1)\sigma_2)^2 + 2\rho\sigma_1\sigma_2 w_1(1-w_1) \\ \mu = w_1 r_1 + (1-w_1) r_2 = w_1(r_1 - r_2) + r_2 \end{cases}$$

$$\sigma^2 = (0,2)^2, \; \mu = 0,15$$

$$(0,2)^2 = (w_1 0,15)^2 + ((1-w_1)0,3)^2 + 2\rho(0,15\times 0,3)\, w_1(1-w$$

$$0,15 = w_1(0,1-0,18) + 0,18 = -0,08w_1 + 0,18$$

$$w_1 = \frac{0,03}{0,08} = \frac{3}{8}$$

$$(0,2)^2 = \left(\frac{3}{8}\times 0,15\right)^2 + \left(\frac{5}{8}\times 0,3\right)^2 + 0,09\times\frac{3}{8}\times\frac{5}{8}\times\rho$$

$$\rho = \frac{0,04 - 0,003164 - 0,03515}{0,09\times\frac{15}{64}} = \frac{0,001686}{0,0211}$$

$$= 0,079$$

$$b)\ \sigma^2 = w_1^2(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2) + w_1(2(\rho\sigma_1\sigma_2 - \sigma_2)) + \sigma_2^2$$

$$= w_1^2(0,1125 - \overbrace{2\times 0,15\times 0,3\times 0,1}^{0,009}) - 2w_1(0,09 - 0,1\times 0,15\times 0,3) +$$

$$0,09$$

$$a^2x^2 - 2ab\,x + b^2$$

$$= (0,3217w_1)^2 - \overbrace{0,171w_1}^{2\times 0,3217\times u = 0,171} + 0,09$$

$$= (0,3217w_1 - 0,2658)^2 + 0,09 - \underbrace{(0,2658)^2}_{0,0706}$$

$$= (0,3217w_1 - 0,2658)^2 + 0,01935$$

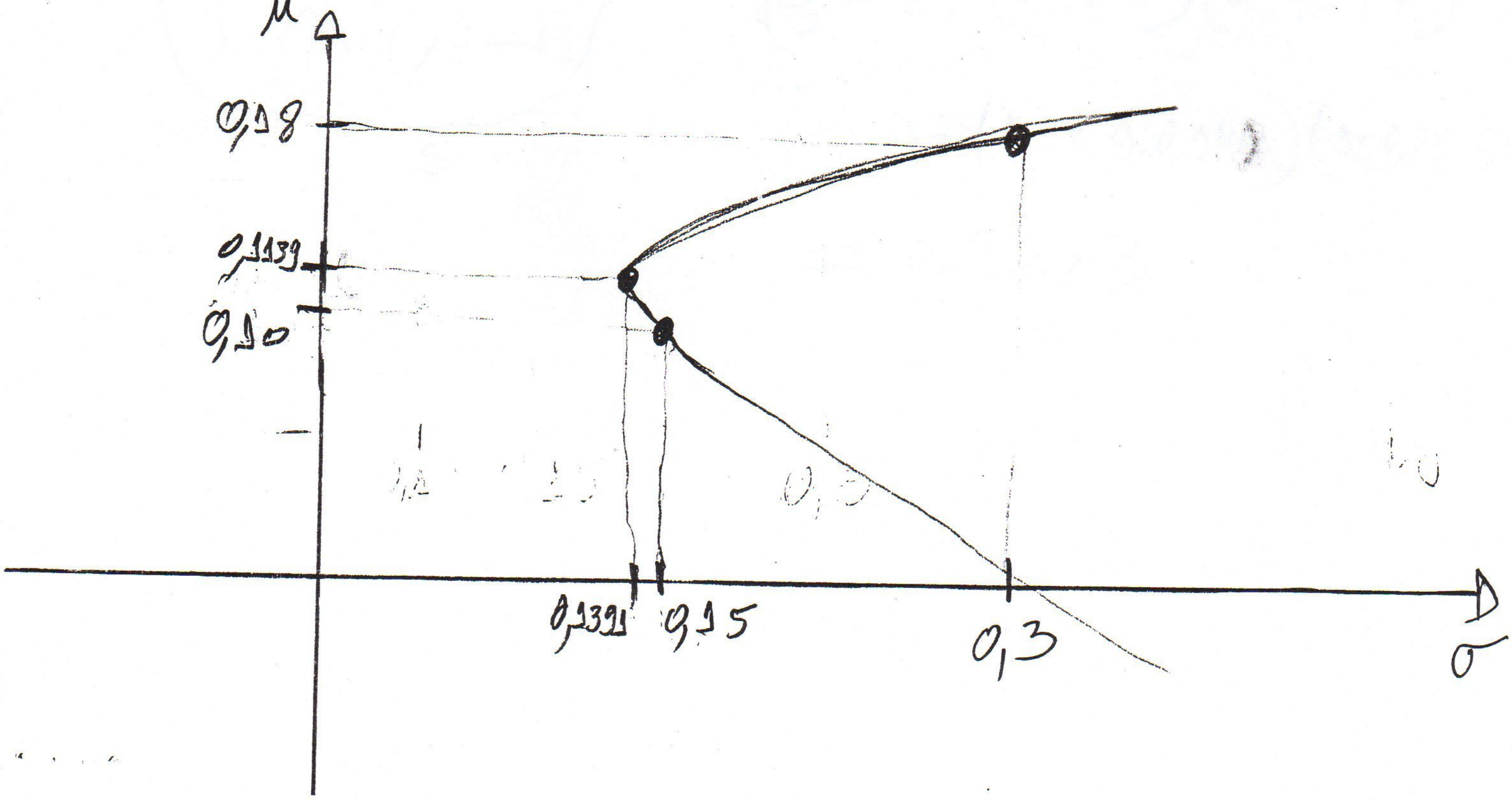
$$w_1 = \frac{\mu - r_2}{r_1 - r_2} = \frac{\mu - 0,18}{0,1 - 0,18} = \frac{0,18 - \mu}{0,08}$$

$$\sigma^2 = \left(0,3217 \times \left(\frac{0,18-\mu}{0,08}\right) - 0,2658\right)^2 + 0,01935$$

$$= (0,458 - 4,02\mu)^2 + 0,01935$$

$$\frac{\sigma^2}{0,01935} - \frac{(4,02)^2}{0,01935}\left(\mu - \frac{0,458}{4,02}\right)^2 = 1$$

$$\left(\frac{\sigma}{0,1391}\right)^2 - \left(\frac{\mu - 0,1139}{0,0346}\right)^2 = 1$$



24

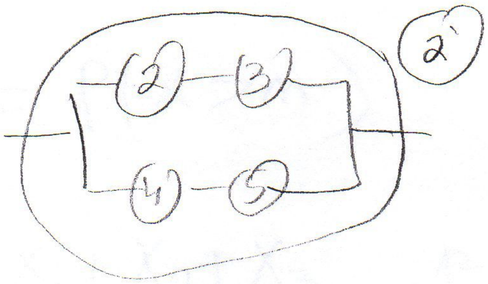
27

$$P(S|A) = \frac{P(A|S)\,P(S)}{P(A|S)\,P(S) + P(A|C)\,P(C)}$$

$$= \frac{1 \cdot 0{,}3}{1 \cdot 0{,}3 + 0{,}25 \cdot 0{,}7}$$

$$= \frac{0{,}3}{0{,}475} = 0{,}63$$

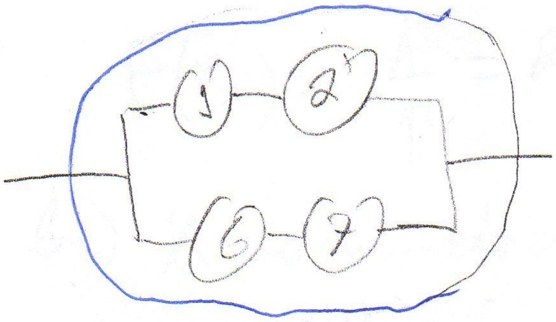
30



$$P_{2'} = 1 - (1 - P_2 P_3)(1 - P_4 P_5)$$

$$= 1 - (1 - 0{,}8 \cdot 0{,}9)(1 - 0{,}95 \cdot 0{,}85)$$

$$= 0{,}9461$$



S

$$P_S = 1 - (1 - P_1 P_{2'})(1 - P_6 P_7)$$

$$= 1 - (1 - 0{,}9 \cdot 0{,}9461)(1 - 0{,}75 \cdot 0{,}9$$

$$= 0{,}9573$$

(26) T = instante de falha

$$P(T \geq 30) = e^{-\lambda 30} = e^{-0{,}005 \cdot 30} = 0{,}86$$

$p_A = p_B = 0{,}86$ $E_A = \{$unidade A funcionar os 30 dias$\}$

$p_A = P(E_A)$, $p_B = P(E_B)$ $E_B = \{$unidade B funcionar os 30 dias$\}$

$E = \{$cidade abastecida 30 dias$\}$

$S_A = \{$unidade A atende se B falhar$\}$

$S_B = \{$unidade B atende se A falha$\}$

$$P(S_A \mid E_B^c) = 0{,}75 = p_S$$

$$P(S_B \mid E_A^c) = 0{,}75 = p_S$$

$$E^c = \{E_A^c E_B^c\} \cup \{E_A^c E_B S_B^c\} \cup \{E_A E_B^c S_A^c\}$$

$$P(E^c) = P(E_A^c E_B^c) + P(E_A^c E_B S_B^c) + P(E_A E_B^c S_A^c)$$

$$P(E_A^c E_B^c) = P(E_A^c)P(E_B^c) = (1-p_A)(1-p_B) = 0{,}14^2 = 0{,}01\ldots$$

$$P(E_A^c E_B S_B^c) = P(E_B)P(S_B^c \mid E_A^c)P(E_A^c) =$$

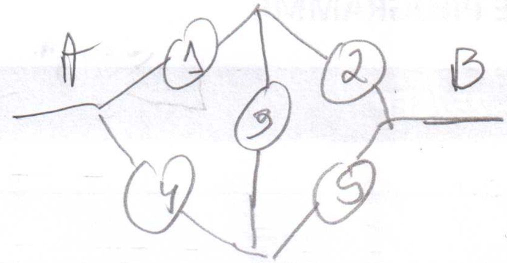
$$p_B(1-p_S)(1-p_A) = 0{,}86 \cdot 0{,}25 \cdot 0{,}14 = 0{,}0301$$

$$P(E_A E_B^c S_A^c) = 0{,}0301$$

$$P(E^c) = 0{,}0196 + 2 \cdot 0{,}0301 = 0{,}0798$$

$$P(E) = 1 - 0{,}0798 = 0{,}9202$$

(27)

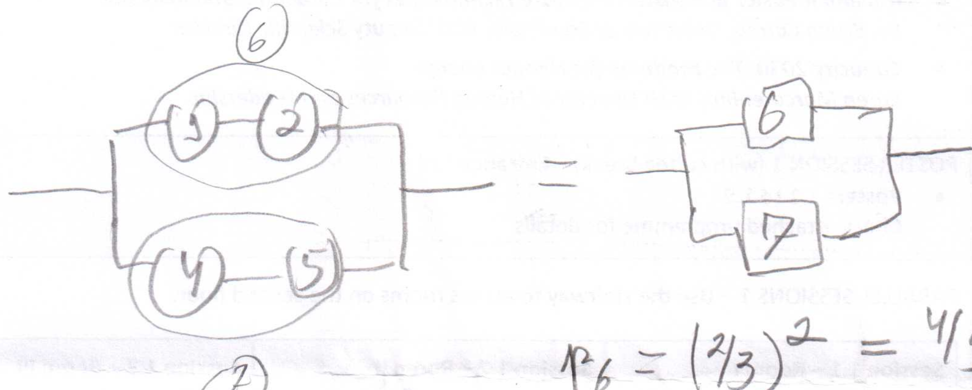


$E = \{\text{sistema funziona}\}$

$E_i = \{\text{componente } i \text{ funziona}\}$

$$P(E) = P(E|E_3)P(E_3) + P(E|E_3^c)P(E_3^c)$$

E_3^c



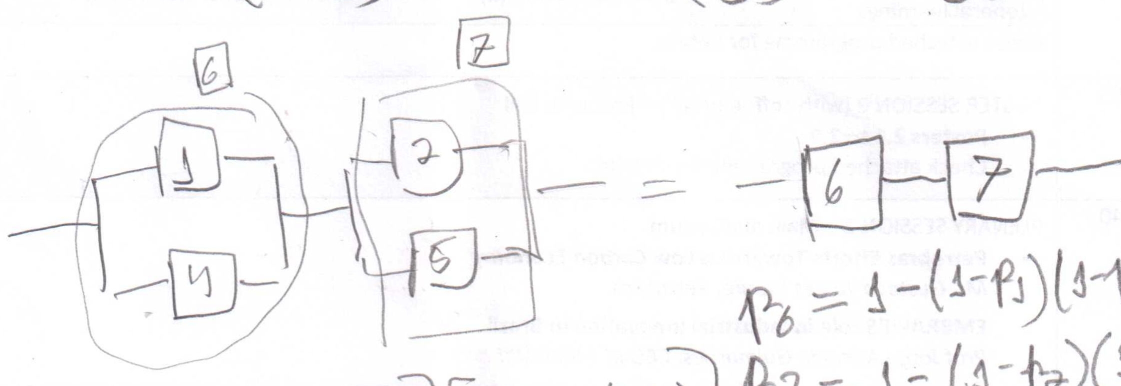
$$p_6 = (2/3)^2 = 4/9$$

$$p_7 = (2/3)^2 = 4/9$$

$$P(E|E_3^c) = 1-(1-p_6)(1-p_7)$$

$$= 1-\left(1-\frac{4}{9}\right)^2 = 1-\left(\frac{5}{9}\right)^2 = \frac{81-25}{81} = \frac{56}{81}$$

E_3



$$p_6 = 1-(1-p_1)(1-p_4)$$

$$p_7 = 1-(1-p_2)(1-p_5)$$

$$P(E|E_3) = \left[1-(1-p_1)(1-p_4)\right]\left[1-(1-p_2)(1-p_5)\right]$$

$$= \left(1-\left(\frac{1}{3}\right)^2\right)^2 = \left(\frac{8}{9}\right)^2 = \frac{64}{81}$$

$$\boxed{P(E) = \frac{64}{81}\cdot\frac{2}{3} + \frac{56}{81}\cdot\frac{1}{3} = \frac{184}{243} = 0{,}7572}$$

(1) X = moeda escolhida (1)

$$P(N=n) = E\big(P(N=n|X)\big)$$

$$P(N=n|X=i) = \left(1-\frac{i}{10}\right)^{n-1}\frac{i}{10}$$

$$P(N=n) = \sum_{i=1}^{10}\left(1-\frac{i}{10}\right)^{n-1}\frac{i}{10}\times\frac{1}{10}$$ → não é geométrica

Para ser geométrica, moeda teria de ser re-escolhida a cada jogada.

$$E\big(P(N=n|X_1,\dots,X_n)\big)$$

$$P(N=n) = \sum_{\{j_i\}} P(N=n \mid X_i=j_i,\ i=1,\dots,n)\,\underbrace{P(X_1=j_1)\dots P(X_n=j_n)}_{\frac{1}{10}}$$

$$= \sum_{j_1=1}^{10}\dots\sum_{j_n=1}^{10}\prod_{i=1}^{n-1}\left(1-\frac{j_i}{10}\right)\left(\frac{j_n}{10}\right)\frac{1}{10}\times\dots\times\frac{1}{10}$$

$$= \sum_{j_2=1}^{10}\dots\sum_{j_n=1}^{10}\underbrace{\left(\sum_{j_1=1}^{10}\left(\frac{10-j_1}{10}\right)\right)}_{\frac{\left(\frac{1}{10}+\frac{9}{10}\right)9}{2}=\frac{9}{2}}\prod_{i=2}^{n-1}\left(1-\frac{j_i}{10}\right)\left(\frac{j_n}{10}\right)\times\frac{1}{10^{10}}$$

$$\sum_{j_n=1}^{10}\frac{j_n}{10} =$$

$$= \left(\frac{9}{20}\right)^{n-1}\left(\frac{11}{20}\right) = (1-p)^{n-1}p$$

$$p = \frac{11}{20}$$

(2)

(2) $f(x,y) = \dfrac{(y^2-x^2)e^{-y}}{8}$, $0 < y < \infty$, $-y \le x \le y$

$$f_Y(y) = \int_{-y}^{y} \frac{(y^2-x^2)e^{-y}}{8}\,dx$$

$$= \frac{y^2 e^{-y}(\cancel{2}y)}{\cancel{8}\,4} - \frac{e^{-y}}{8}\left(\frac{x^3}{3}\right]_{-y}^{y}$$

$$= \frac{y^3 e^{-y}}{4} - \frac{e^{-y}\,\cancel{2}y^3}{4\,\cancel{8}\,3}$$

$$= \frac{y^3 e^{-y}}{4}\left(1-\frac{1}{3}\right) = \frac{y^3 e^{-y}}{6}$$

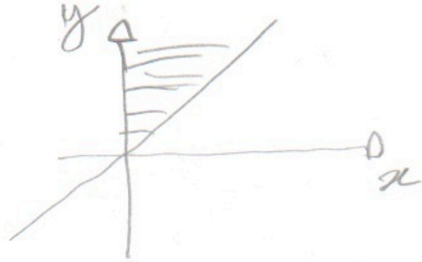
$$f_{X|Y}(x|y) = \frac{\frac{y^2-x^2}{8}\cancel{e^{-y}}}{\frac{y^3\cancel{e^{-y}}}{6}} = \frac{y^2-x^2}{y^3}\left(\frac{3}{4}\right), \quad -y \le x \le y$$

$$E(X|Y=y) = \int_{-y}^{y} x\frac{(y^2-x^2)}{y^3}\left(\frac{3}{4}\right)dx$$

$$= \left(\frac{1}{y}\right)\left(\frac{3}{4}\right)\frac{x^2}{2}\Big]_{-y}^{y} - \frac{1}{y^3}\left(\frac{3}{4}\right)\frac{x^4}{4}\Big]_{-y}^{y}$$

0 0

$$= 0$$



(3) $f_{XY}(x,y) = \frac{e^{-y}}{y}, \quad 0 < x < y$

$E(X^2 | Y=y) = ?$

$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_{-\infty}^{\infty} \frac{e^{-y}}{y} 1_{\{0<x<y\}}\,dx =$

$= \int_0^y \frac{e^{-y}}{y}\,dx = \frac{e^{-y}}{y}\int_0^y dx = e^{-y}$

$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} = \frac{\cancel{e^{-y}}}{y} \cdot \frac{1}{\cancel{e^{-y}}} = \frac{1}{y}, \quad 0<x<$

uniforme $(0,y)$

$E(X^2|Y=y) = \int_0^y \frac{1}{y} x^2\,dx = \frac{x^3}{3y}\Big]_0^y = \frac{y^{\cancel{3}2}}{3\cancel{y}} = \frac{y^2}{3}$

(4)

2D V	3D V	L
1ª P	2ª P	3ª P
0.5	0.3	0.2

a) N → número de dias até liberdade
Y → 1ª porta escolhida

$$E(N) = E(E(N|Y))$$

$$E(N|Y=1) = 2 + E(N)$$

$$E(N|Y=2) = 3 + E(N)$$

$$E(N|Y=3) = 0$$

$$E(N) = 0.5(2+E(N)) + 0.3(3+E(N)) + 0.2(0) = 1.9 + 0.8E(N)$$

$$E(N) = \frac{1.9}{0.2} = \frac{19}{2} = 9.5$$

b) $E(N) = E(E(N|Y))$

$$E(N|Y=1) = 2 + \tfrac{1}{2}3 + \tfrac{1}{2}0 = \frac{7}{2}$$

$$E(N|Y=2) = 3 + \tfrac{1}{2}2 + \tfrac{1}{2}0 = 4$$

$$E(N|Y=3) = 0$$

$$E(N) = \frac{1}{3} \times \frac{7}{2} + \frac{1}{3} \times 4 + 0 = \frac{1}{3}\left(4 + \frac{7}{2}\right) = \frac{5}{2} = 2.5$$

c) a) $E(N^2|Y=1) = E((2+N)^2) = 4 + 4E(\overset{19/2}{N}) + E(N^2) =$

$$= 4 + 38 + E(N^2) = 42 + E(N^2)$$

$$E(N^2|Y=2) = E((3+N)^2) = 9 + \cancel{2} \times 3 \times \frac{19}{\cancel{2}} + E(N^2) =$$

$$= 9 + 57 + E(N^2) = 66 + E(N^2)$$

$$E(N^2|Y=3) = 0$$

$$E(N^2) = 0.5\left(42 + E(N^2)\right) + 0.3\left(66 + E(N^2)\right) + 0$$

$$= 21 + 19.8 + 0.8\,E(N^2)$$

$$E(N^2) = \frac{40.8}{0.2} = \frac{20.4}{0.1} = 204$$

$$Var(N) = E(N^2) - E(N)^2 = 204 - \left(\frac{19}{2}\right)^2 = 113{,}75$$

$\frac{1}{2}3 + \frac{1}{2}0 = 3/2$

b) $E(N^2 \mid Y=1) = E\left((2+N_1)^2 \mid Y=1\right) = 4 + 4E(N_1 \mid Y=1) +$

$E(N_1^2 \mid Y=1) = 4 + \cancel{4}^{2} \times \frac{3}{\cancel{2}} + \frac{1}{2}9 + \cancel{\frac{1}{2}0}\ 0 = 10 + 4.5 = 14.5$

$\frac{1}{2}\times 2 + \frac{1}{2}\times 0 = 1$

$E(N^2 \mid Y=2) = E\left((3+N_2)^2 \mid Y=2\right) = 9 + 6E(N_2 \mid Y=2) +$

$E(N_2^2 \mid Y=2) = 9 + 6 + \frac{1}{2}\times 4 + \cancel{\frac{1}{2}\times 0}\ 0 = 9 + 6 + 2 = 17$

$$E(N^2) = \frac{1}{3}(14.5 + 17) = 10.5$$

$$Var(N) = 10.5 - (2.5)^2 = 4{,}25$$

(5) $p_1 = 0.3$ (6)

$p_2 = 0.5$ X = moeda escolhida

$p_3 = 0.7$

(a) $P(N=n) = E\left(P(N=n \mid X)\right)$

$$P(N=n \mid X=i) = \binom{10}{n}(1-p_i)^{10-n} p_i^n$$

$$P(N=n) = \sum_{i=1}^{n} \frac{1}{3}\binom{10}{n} p_i^n (1-p_i)^{10-n}$$

$$= \frac{1}{3}\binom{10}{n}\left[0.3^n \times 0.7^{10-n} + 0.5^n \times 0.5^{10-n} + 0.7^n \cdot 0.3^{10-n}\right]$$

(b) não é binomial; se a cada jogada a moeda fosse recolocada, teríamos

$S_n = \{l_1, \dots, l_{10};\ l_i = 0 \text{ ou } 1,\ \sum_{i=1}^{10} l_i = n\}$

$V = \{i_1, \dots, i_{10};\ i_j = 1, 2, \text{ou } 3,\ j = 1, \dots, 10\}$

$$P(N=n) = E\left(P(X=n \mid X_1, \dots, X_{10})\right)$$

$$P(X=n \mid X_1=i_1, \dots, X_{10}=i_{10}) = \sum_{\{l_1, \dots, l_{10} \in S_n\}} (1-p_{i_1})^{1-l_1} \dots (1-p_{i_{10}})^{1-l_{10}}\, p_{i_1}^{l_1} \dots p_{i_{10}}^{l_{10}}$$

$$P(X_1=i_1, \dots, X_{10}=i_{10}) = P(X_1=i_1) \dots P(X_{10}=i_{10}) = \left(\frac{1}{3}\right)^{10}$$

$$P(N=n) = \sum_{\{i_1, \dots, i_{10} \in V\}} \left(\frac{1}{3}\right)^{10} \sum_{\{l_1, \dots, l_{10} \in S_n\}} (1-p_{i_1})^{1-l_1} \dots (1-p_{i_{10}})^{1-l_{10}}\, p_{i_1}^{l_1} \dots p_{i_{10}}^{l_{10}}$$

b

$$= \left(\frac{1}{3}\right)^{10} \sum_{\{l_1,\dots,l_{10} \in S_n\}} \sum_{i_1=1,2,3} \cdots \sum_{i_{10}=1,2,3} (1-p_{i_1})^{1-l_1} \cdot p_{i_1}^{l_1} \cdots (1-p_{i_{10}})^{1-l_{10}} p_{i_{10}}^{l_{10}}$$

$$= \left(\frac{1}{3}\right)^{10} \sum_{\{l_1,\dots,l_{10} \in S_n\}} \left(\sum_{i_1=1,2,3} (1-p_{i_1})^{1-l_1} \cdot p_{i_1}^{l_1} \right) \cdots \left(\sum_{i_{10}=1,2,3} (1-p_{i_{10}})^{1-l_{10}} \cdot p_{i_{10}}^{l_{10}} \right)$$

Como $l_j = 0$ ou 1, segue que

$$\sum_{i_j=1,2,3} (1-p_{i_j})^{1-l_j} p_{i_j}^{l_j} = 3\left(1 - \frac{\overbrace{p_1+p_2+p_3}^{0,3+0,5+0,7 \,=\, 1,5}}{3}\right)^{1-l_j} \left(\frac{\overbrace{p_1+p_2+p_3}^{0,3+0,5+0,7 \,=\, 1,5}}{3}\right)^{l_j}$$

$$= 3(1-0,5)^{1-l_j} \, 0,5^{l_j} = 3 \cdot \frac{1}{2}$$

Logo,

$$P(N=n) = \cancel{\left(\frac{1}{3}\right)^{10}} \sum_{\{l_1,\dots,l_{10} \in S_n\}} \cancel{3^{10}} \left(\frac{1}{2}\right)^{10} = \binom{10}{n}\left(\frac{1}{2}\right)^{n}\left(\frac{1}{2}\right)^{10-n}$$

pois $\# S_n = \binom{10}{n}$. Logo é N é binomial com parâmetros 10 e 1/2.

(c) Y = valor ganho $\Rightarrow E(Y) = E(E(Y/X)) =$

$$\frac{1}{3}\left(E(Y|X=1) + E(Y|X=2) + E(Y|X=3)\right) = \frac{10}{3}\left(2\cdot(\overbrace{p_1+p_2+p_3}^{1,5}) - 3\right) = 0$$

jogo justo

$$E(Y|X=i) = 10 \cdot p_i - 10(1-p_i) = 10(2p_i - 1)$$

6 $X \to$ Poisson com média λ

$\lambda \to$ exponencial com média 1.

$$P(X=n) = E(P(X=n \mid \lambda))$$

$$P(X=n \mid \lambda=z) = e^{-z}\frac{z^n}{n!},$$

$$E(P(X=n \mid \lambda=z)) = \int_0^\infty e^{-z}\frac{z^n}{n!}e^{-z}\,dz =$$

$$\int_0^\infty \frac{z^n}{n!}e^{-2z}\,dz = \frac{1}{2^{n+1}}\underbrace{\left(\int_0^\infty 2(2z)^n e^{-2z}\,dz\right)\frac{1}{n!}}_{=1 \text{ distribuição gama com parâmetros } n+1 \text{ e } 2} = \frac{1}{2^{n+1}}$$

Logo,

$$P(X=n) = \frac{1}{2^{n+1}},\ n = 0,1,2,\ldots$$

(34)

6 D
44 B

$\to n=5$

X = número de peças defeituosas em $n=5$

hipergeométrica $a=6$ $n=5$
$b=44$

$$P(X=i) = \frac{\binom{6}{i}\binom{44}{n-i}}{\binom{50}{n}}, \quad i=0,1,2,3,4,5.$$

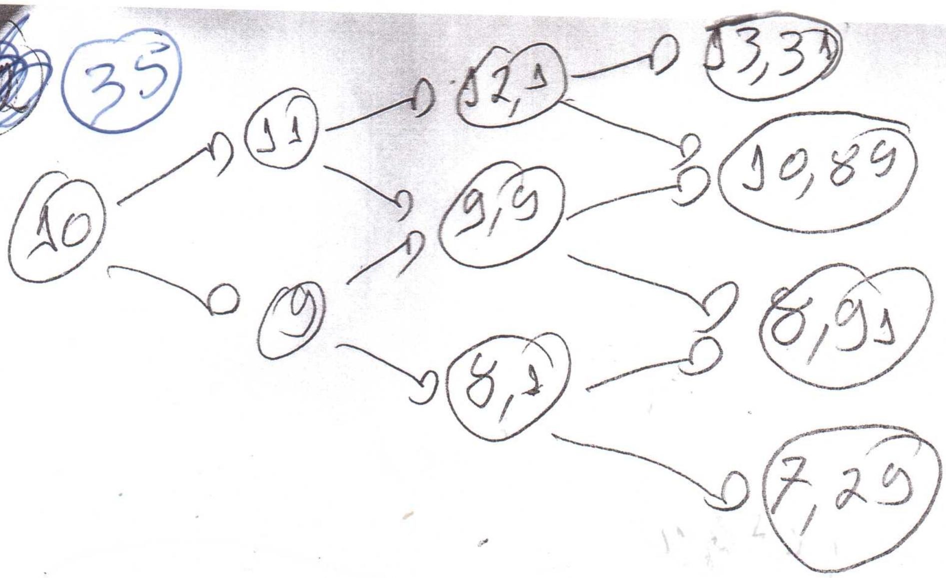
$$P(X=0) = \frac{\binom{6}{0}\binom{44}{5}}{\binom{50}{5}} = \frac{\frac{44!}{\cancel{5!}\,39!}}{\frac{50!}{\cancel{5!}\,45!}},$$

$$\frac{44.43.42.41.40}{50.49.48.47.46} = 0,5125$$

$E = \{\text{lote aceito diretamente}\} = \{X=0\}$

$P(E) = 0,5125.$

35



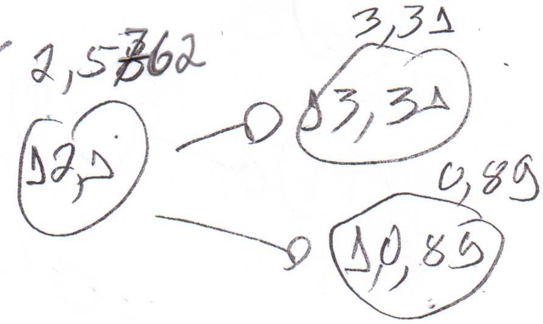
$\mu - d = 0,$

$\Delta = \frac{C_u - C_d}{(\mu - d)\cdot}$

$B = \frac{\mu C_d - d C}{(\mu-d)(1+r}$

$t=0$ $t=1$ $t=2$ $t=3$

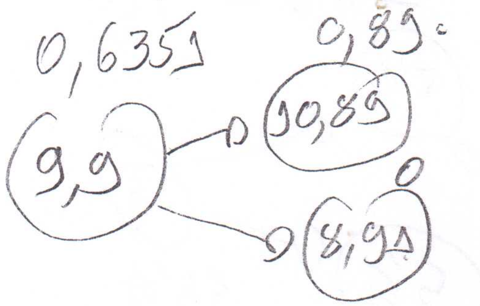
$t=2$



$\Delta = \frac{3,31 - 0,89}{0,2 \cdot 12,1} = 1$

$B = \frac{-2}{0,2 \cdot 1,05} = -9,52$

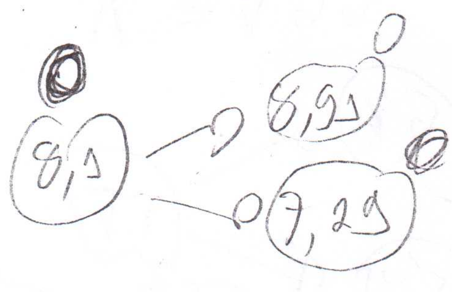
$V_{uu} = B + \Delta S = -9,52 + 12,1$
$= 2,5\cancel{7}62$



$\Delta = \frac{0,89}{0,2 \cdot 9,9} = 0,449$

$B = -\frac{0,9 \cdot 0,89}{0,2 \cdot 1,05} = -3,81$

$V_{ud} = 0,635\cancel{7}$



$\Delta = 0$
$B = 0$
$V_{dd} = 0$

$t=1$

1,99 — 11 → 12,1 (2,5762); 9,9 (0,6357)

$$\Delta = \frac{2{,}58-0{,}6351}{0{,}2\cdot 11} = 0{,}8$$

$$B = \frac{-1{,}623}{0{,}2\cdot 1{,}05} = -7{,}$$

$$V_u = 1{,}99^{15} = 9{,}7025-7{,}711$$

0,45 — 9 → 9,9 (0,6357); 8,1 (0)

$$\Delta = \frac{0{,}6357}{0{,}2\cdot 9} = 0{,}352$$

$$B = \frac{-0{,}57}{0{,}2\cdot 1{,}05} = -2{,}7$$

$$V_d = 0{,}4540 = 3{,}1785-2{,}7244$$

$t=0$

10 → 11 (1,9915); 9 (0,45407)

$$\Delta = \frac{1{,}99-0{,}45}{0{,}2\cdot 10} = 0{,}77$$

$$B = \frac{-1{,}291}{0{,}2\cdot 1{,}05} = -6{,}17$$

$$V(0) = 1{,}53 = 7{,}68715-6{,}1565$$

Probabilidade Neutra ao Risco

$$q = \frac{1+r-d}{u-d} = \frac{1,05-0,9}{1,1-0,9} = \frac{0,15}{0,2} = \frac{3}{4}$$

$13,31 - 10 = 3,31$ $\quad \left(\frac{3}{4}\right)^3 = \frac{27}{64}$

$10,89 - 10 = 0,89$ $\quad 3\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right) = \frac{27}{64}$

0

0

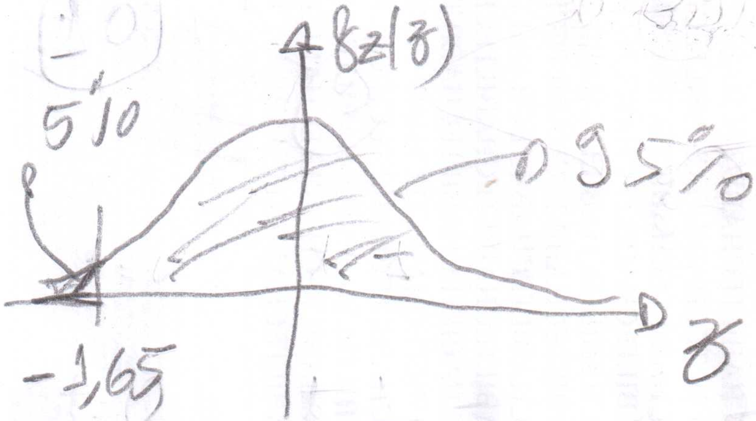
$$C = \frac{1}{1,05^3}\left\{3,31 \cdot \frac{27}{64} + 0,89 \cdot \frac{27}{64}\right\} =$$

$$= \left(\frac{1}{1,05^3}\right)\left(\frac{27}{64}\right)\left(3,31 + 0,89\right)$$

$$\cong 1,531.$$

36

$R \sim N(3{,}75,\ 0{,}05)$



US\$ 200 000,00

$\Downarrow$

R\$ 750.000,00
(valor inicial em real)

$$Z = \frac{R - \mu}{\sigma} = \frac{R - 3{,}75}{0{,}05}$$

$$P(Z \geq -1{,}65) = 0{,}95$$

$$\frac{r_{min} - 3{,}75}{0{,}05} = -1{,}65 \Rightarrow r_{min} = 3{,}75 - 0{,}05 \cdot 1{,}65$$

$\Rightarrow r_{min} = 3{,}6675$ com 95% de chance

Nesse caso, $200000{,}00 \times 3{,}6675 =$ R\$ 733500,00

Perda Máxima com 95% de chance

$$\begin{array}{r} 750000{,}00 \\ -\ 733500{,}00 \\ \hline \boxed{\text{R\$ } 16.500} \end{array}$$