Eletromagnetismo

Aula de 14 de abril

Análise vetorial

$$\vec{\nabla} \cdot \vec{D} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{j} + \epsilon_0 \frac{\partial \vec{D}}{\partial t}$$

Análise vetorial

Teorema fundamental para o gradiente

$$\int_{\vec{a}}^{\vec{b}} \vec{\nabla} T \cdot \mathrm{d}\vec{\ell} = T(\vec{b}) - T(\vec{a})$$

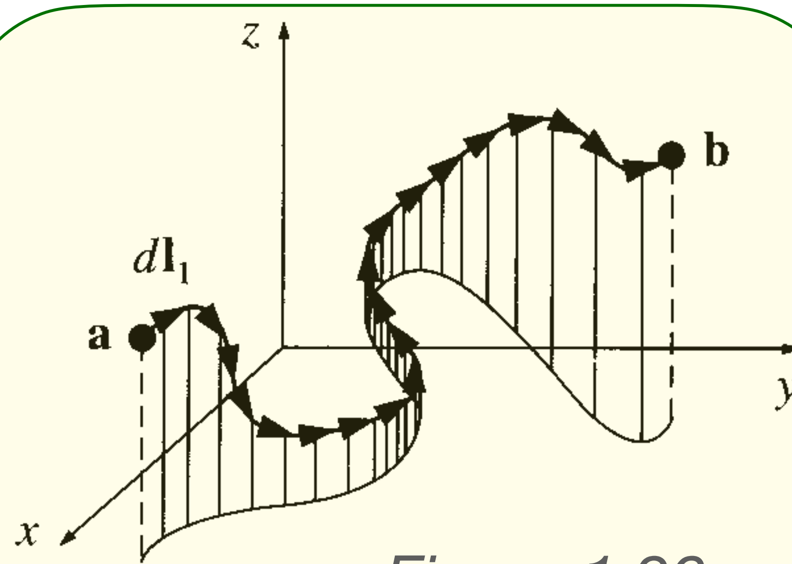


Figura 1.26

Compare com o teorema fundamental do cálculo:

$$\int_a^b \frac{dT}{dx} dx = T(b) - T(a)$$

Análise vetorial

Teorema fundamental para o gradiente

$$\int_{\vec{a}}^{\vec{b}} \vec{\nabla} T \cdot \mathrm{d}\vec{\ell} = T(\vec{b}) - T(\vec{a})$$

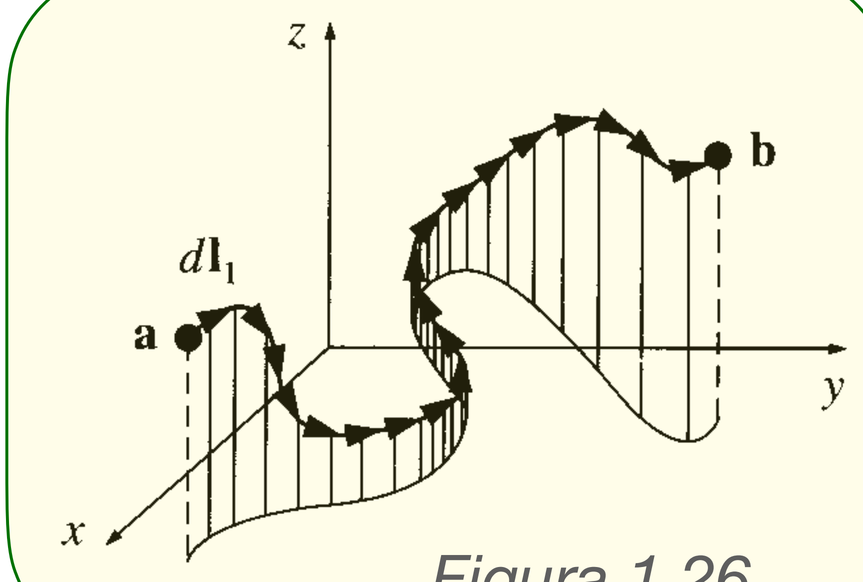


Figura 1.26

$$\int \vec{\nabla} T \cdot \mathrm{d}\vec{\ell} = \int \frac{\partial T}{\partial x} dx + \frac{\partial T}{\partial y} dy + \frac{\partial T}{\partial z} dz$$

Análise vetorial

Teorema fundamental para o gradiente

$$\int_{\vec{a}}^{\vec{b}} \vec{\nabla} T \cdot \mathrm{d}\vec{\ell} = T(\vec{b}) - T(\vec{a})$$

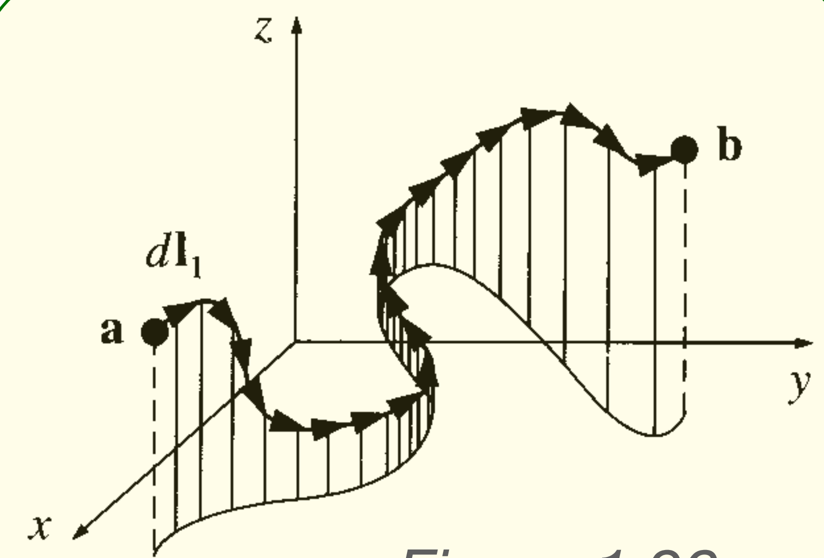


Figura 1.26

$$\int \vec{\nabla} T \cdot \mathrm{d}\vec{\ell} = \int \frac{\partial T}{\partial x} dx + \frac{\partial T}{\partial y} dy + \frac{\partial T}{\partial z} dz$$

$$\int_{\vec{a}}^{\vec{b}} \vec{\nabla} T \cdot \mathrm{d}\vec{\ell} = \int_{\vec{a}}^{\vec{b}} \mathrm{d}T$$

Análise vetorial

Teorema fundamental para o gradiente

$$\int_{\vec{a}}^{\vec{b}} \vec{\nabla} T \cdot \mathrm{d}\vec{\ell} = T(\vec{b}) - T(\vec{a})$$

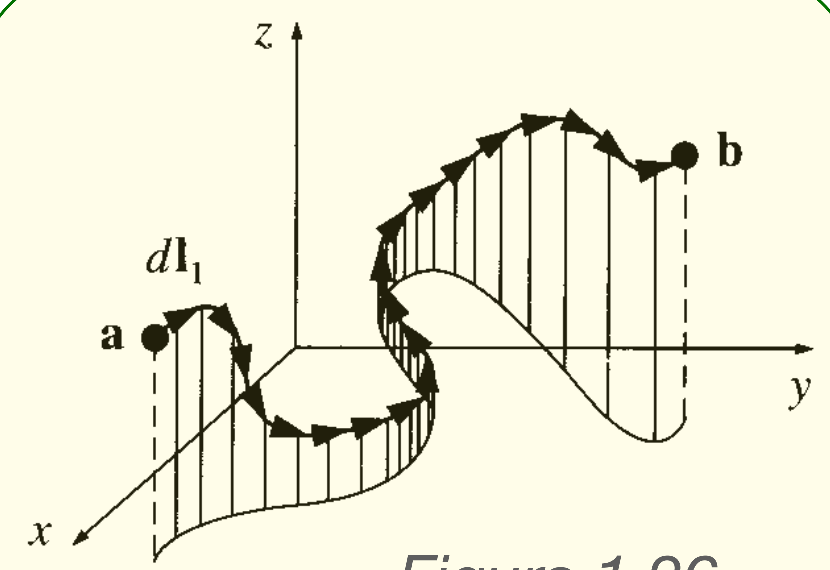


Figura 1.26

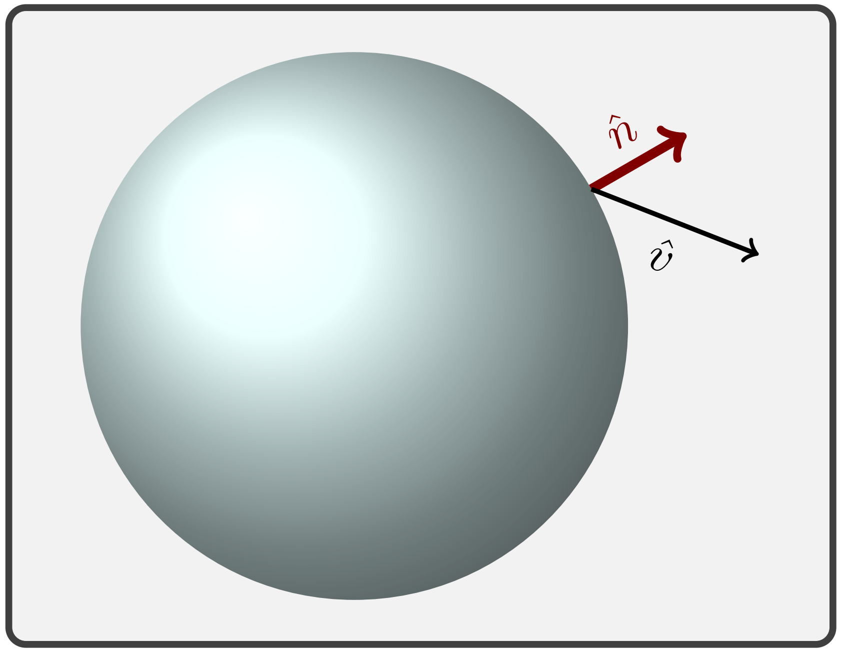
$$\int \vec{\nabla} T \cdot \mathrm{d}\vec{\ell} = \int \frac{\partial T}{\partial x}dx + \frac{\partial T}{\partial y}dy + \frac{\partial T}{\partial z}dz$$

$$\int_{\vec{a}}^{\vec{b}} \vec{\nabla} T \cdot \mathrm{d}\vec{\ell} = \int_{\vec{a}}^{\vec{b}} \mathrm{d}T$$

$$\int_{\vec{a}}^{\vec{b}} \vec{\nabla} T \cdot \mathrm{d}\vec{\ell} = T(\vec{b}) - T(\vec{a})$$

Análise vetorial

Teorema fundamental para o divergente (Gauss)



$$\int_{\mathcal{V}} (\vec{\nabla} \cdot \vec{v})\, \mathrm{d}\tau = \int_{\mathcal{S}} \vec{v} \cdot \hat{n}\, \mathrm{d}A$$

Volume

Superfície

Análise vetorial

Teorema fundamental para o rotacional (Stokes)

$$\int_{\mathcal{S}} (\vec{\nabla} \times \vec{v}) \cdot \hat{n} \, \mathrm{d}A = \int_{\mathcal{P}} \vec{v} \cdot \mathrm{d}\vec{\ell}$$

superfície   perímetro

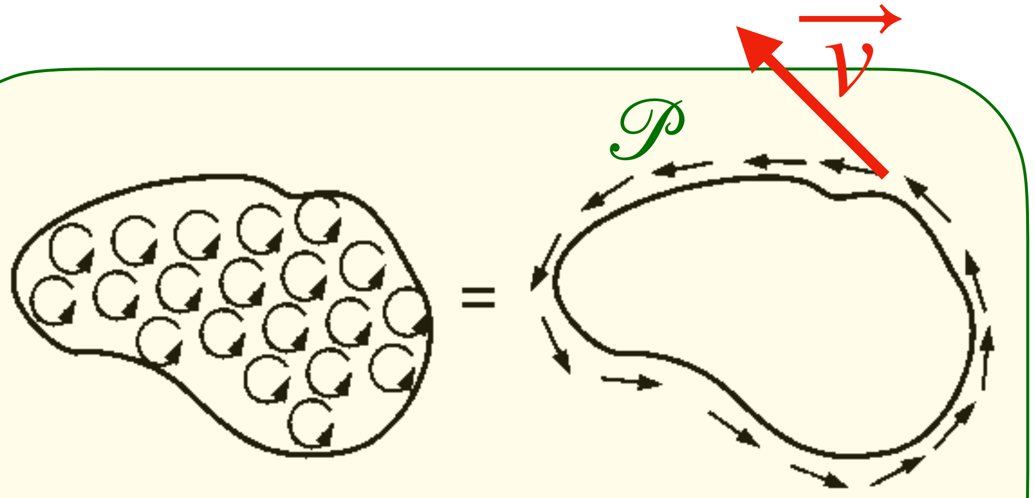


Figura 1.31

Análise vetorial

Teorema fundamental para o rotacional (Stokes)

$$\int_{\mathcal{S}}(\vec{\nabla}\times\vec{v})\cdot\hat{n}\,\mathrm{d}A=\int_{\mathcal{P}}\vec{v}\cdot\mathrm{d}\vec{\ell}$$

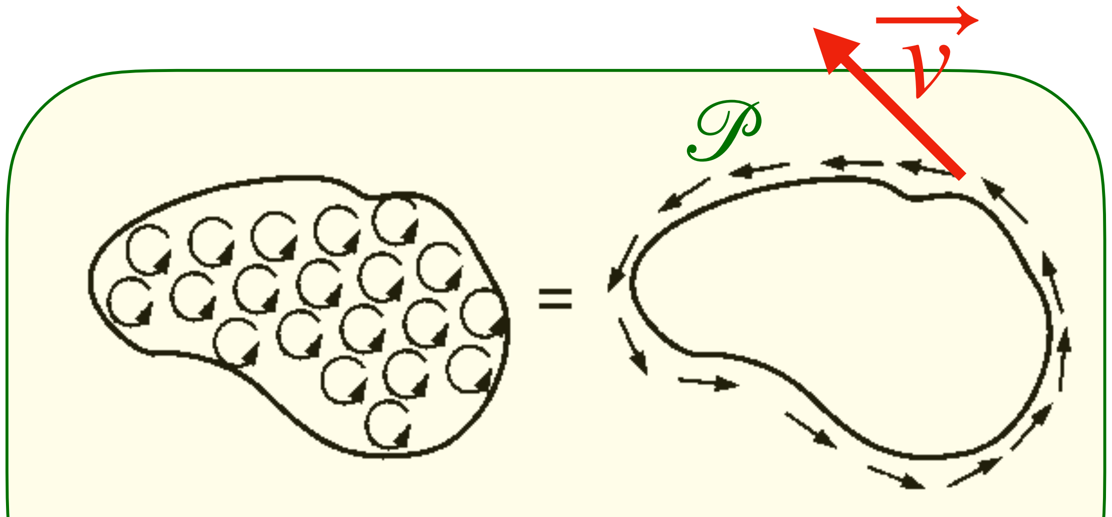
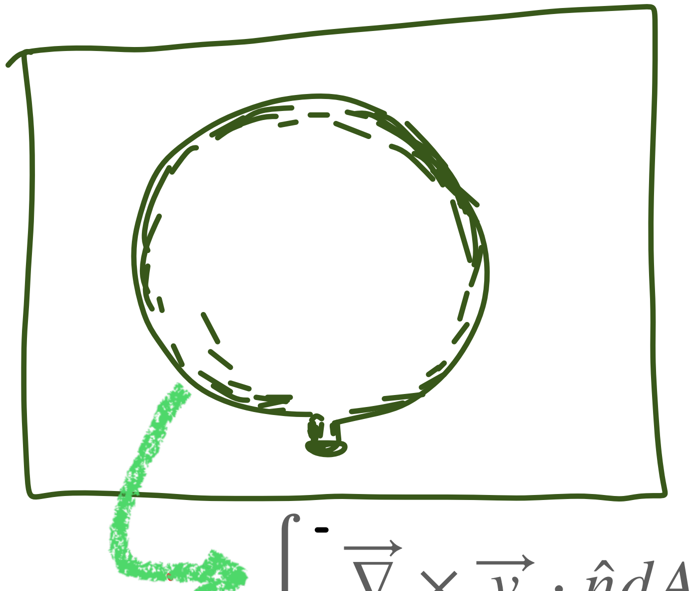


Figura 1.31

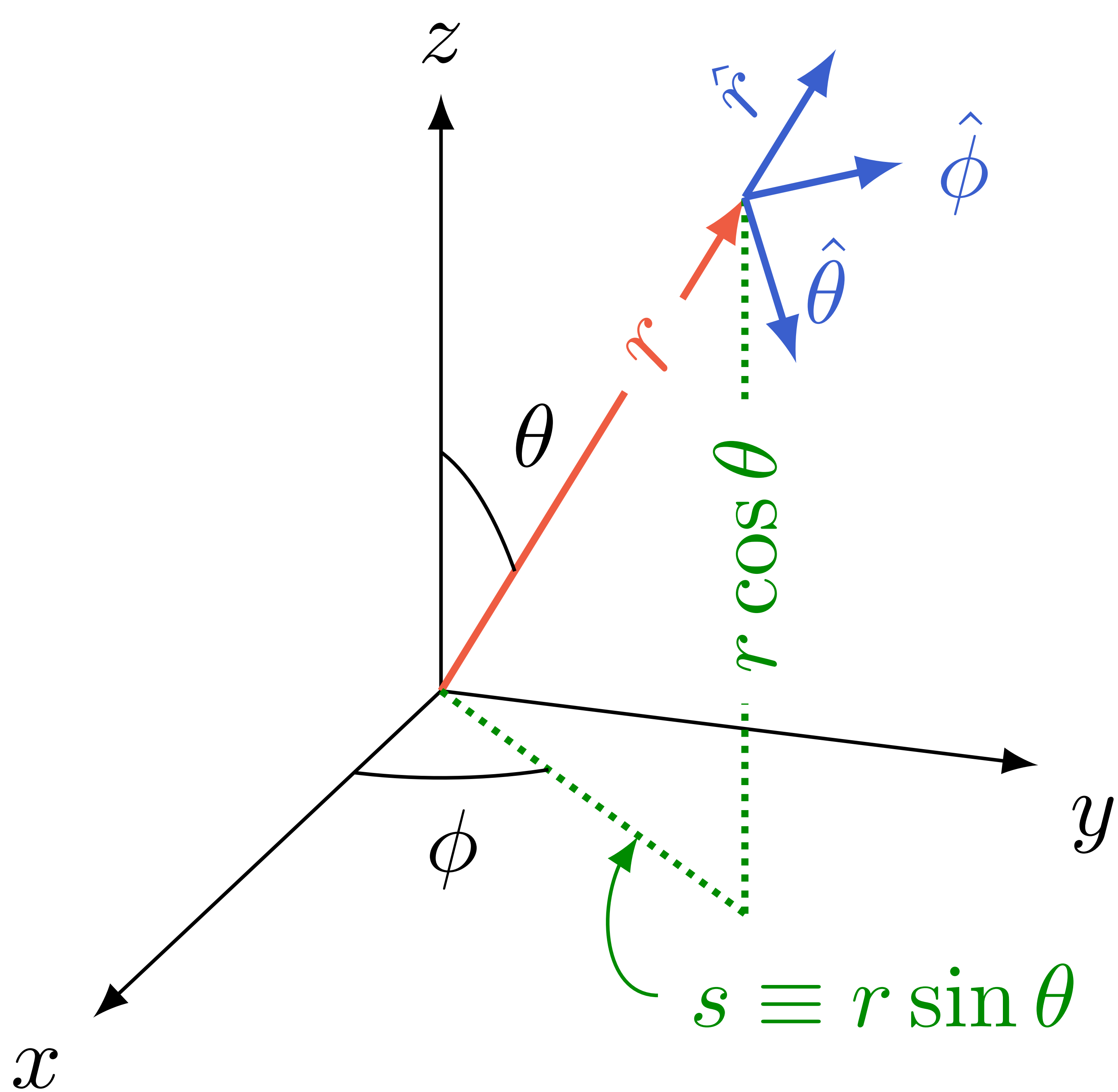


$$\int_{\mathcal{S}}\vec{\nabla}\times\vec{v}\cdot\hat{n}dA=0$$

perímetro

Análise vetorial

Coordenadas esféricas



Análise vetorial

Coordenadas esféricas

$$x = r\sin\theta\cos\phi$$

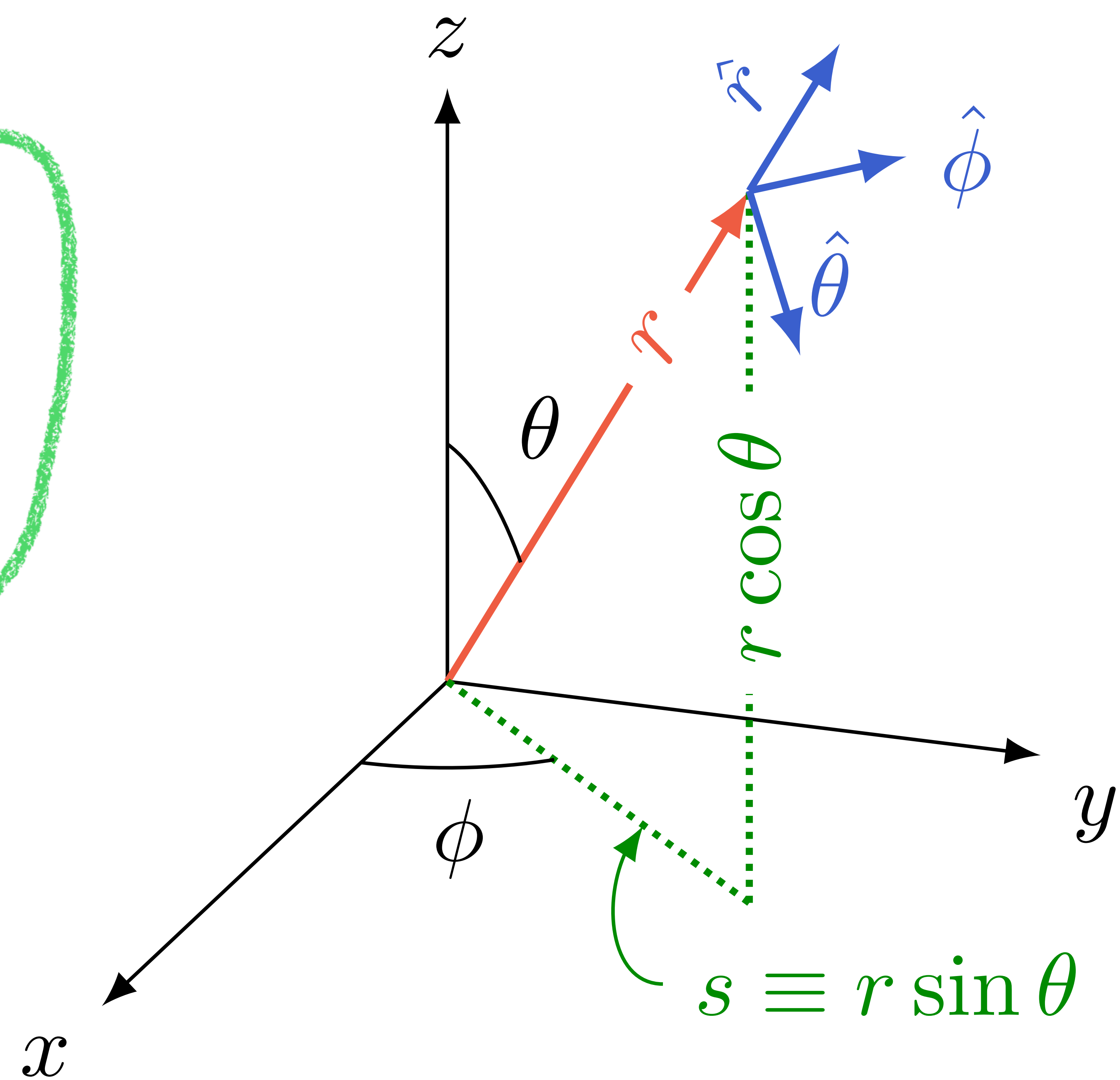
$$y = r\sin\theta\sin\phi$$

$$z = r\cos\theta$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\cos\theta = \frac{z}{r}$$

$$\tan\phi = \frac{y}{x}$$

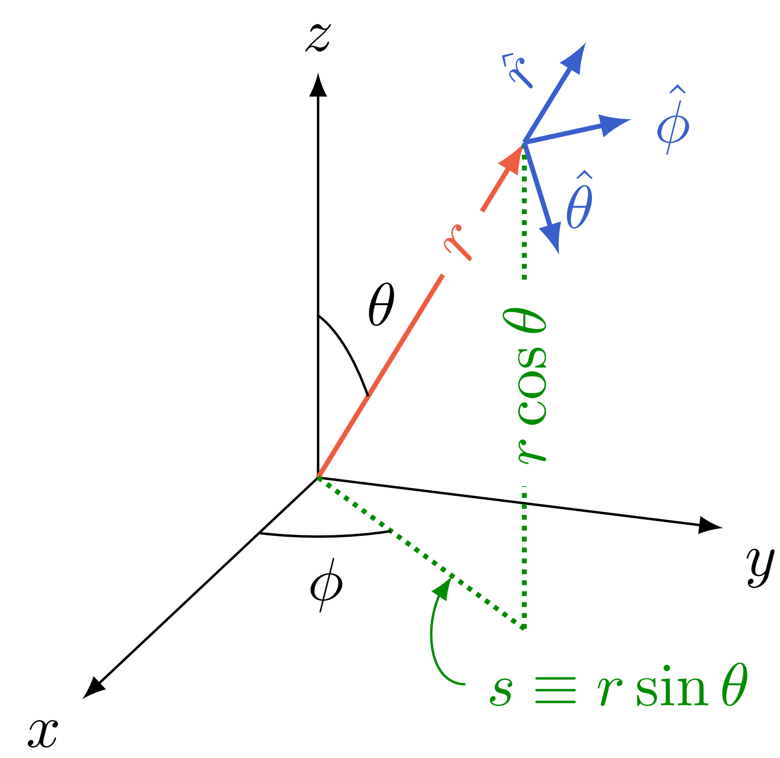


Análise vetorial

Coordenadas esféricas

$$d\overrightarrow{\ell} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$

$$d\overrightarrow{\ell}(dr, d\theta, d\phi) = \,?$$



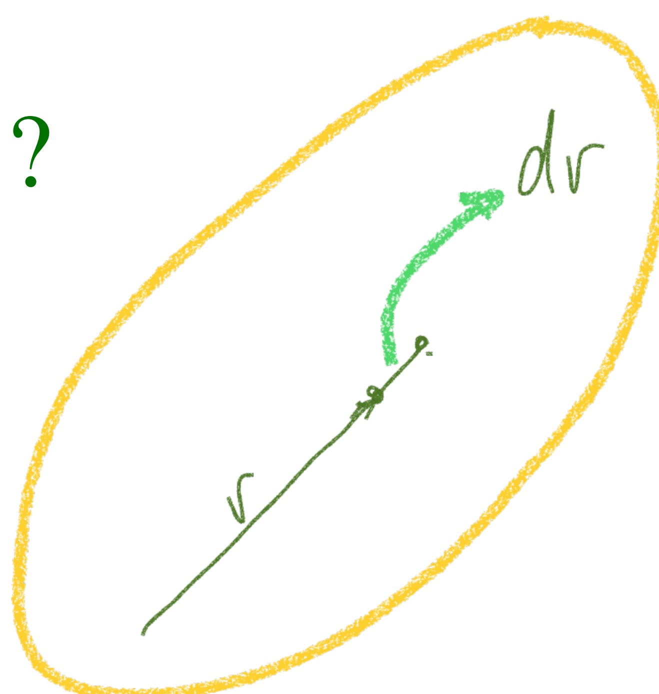
Análise vetorial

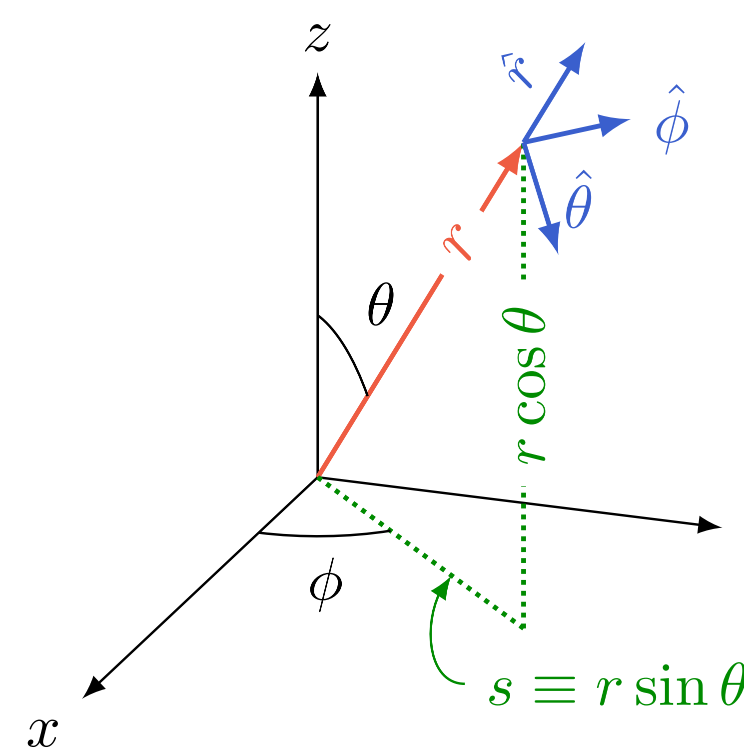
Coordenadas esféricas

$$d\vec{\ell} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$

$$d\vec{\ell}(dr, d\theta, d\phi) = \,?$$

$$d\vec{\ell} = dr\,\hat{r} + ?$$





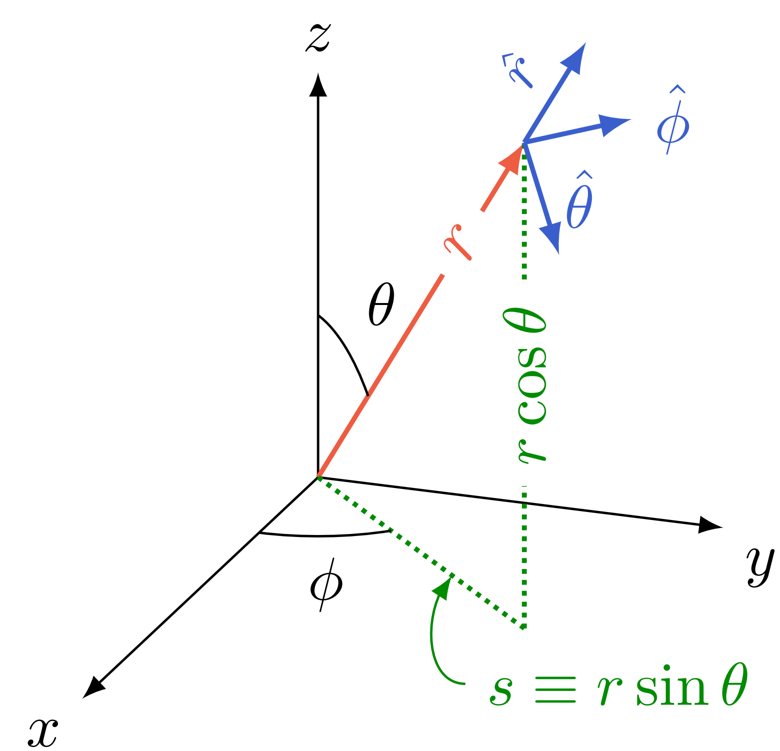
Análise vetorial

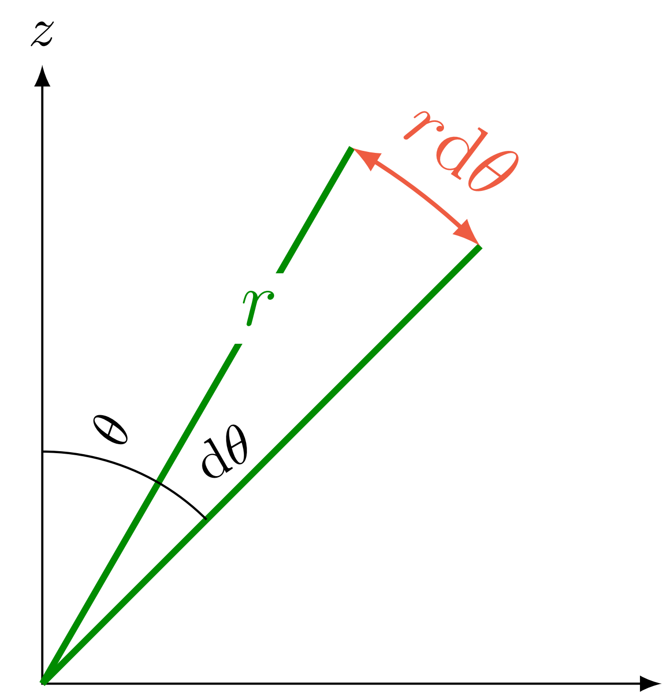
Coordenadas esféricas

$$d\vec{\ell} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$

$$d\vec{\ell}(dr, d\theta, d\phi) = ?$$

$$d\vec{\ell} = dr\,\hat{r} + ?$$





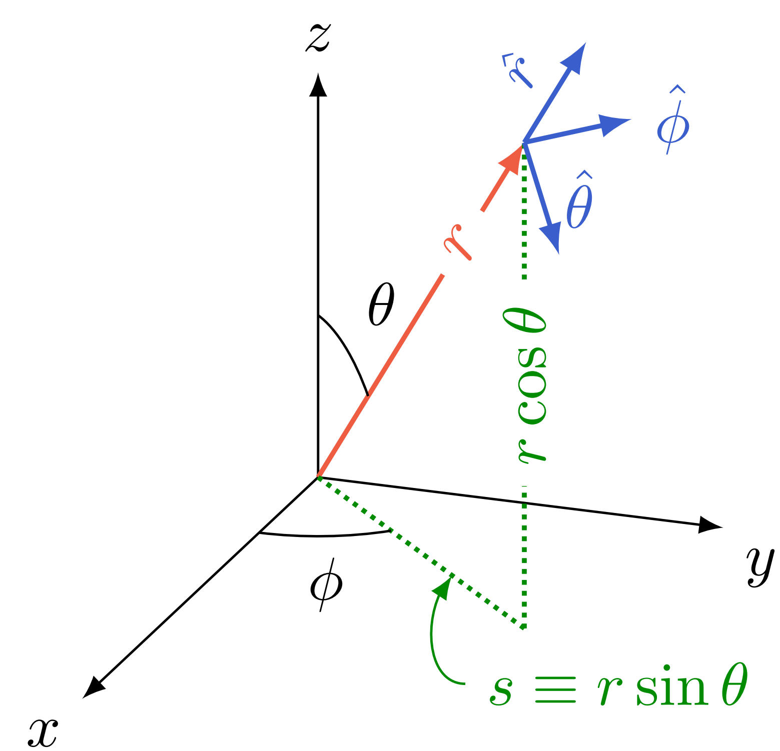
Análise vetorial

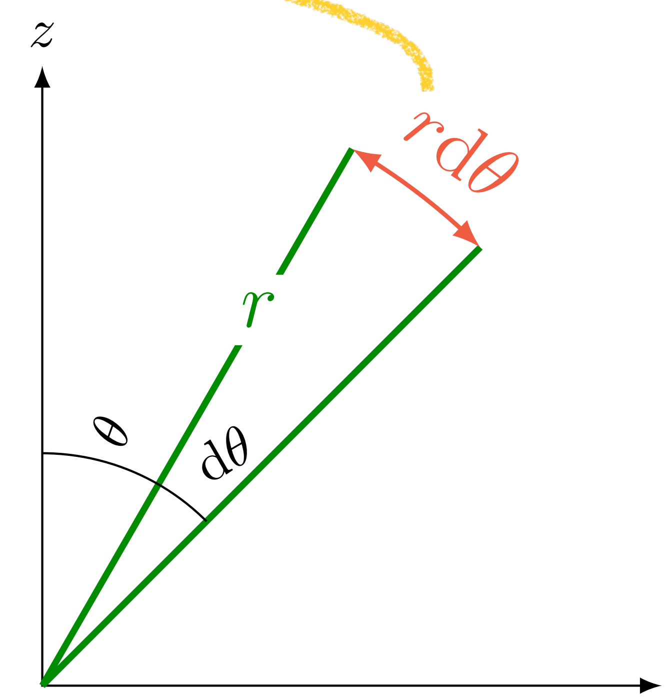
Coordenadas esféricas

$$d\overrightarrow{\ell} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$

$$d\overrightarrow{\ell}(dr, d\theta, d\phi) = ?$$

$$d\overrightarrow{\ell} = dr\,\hat{r} + rd\theta\,\hat{\theta} + ?$$





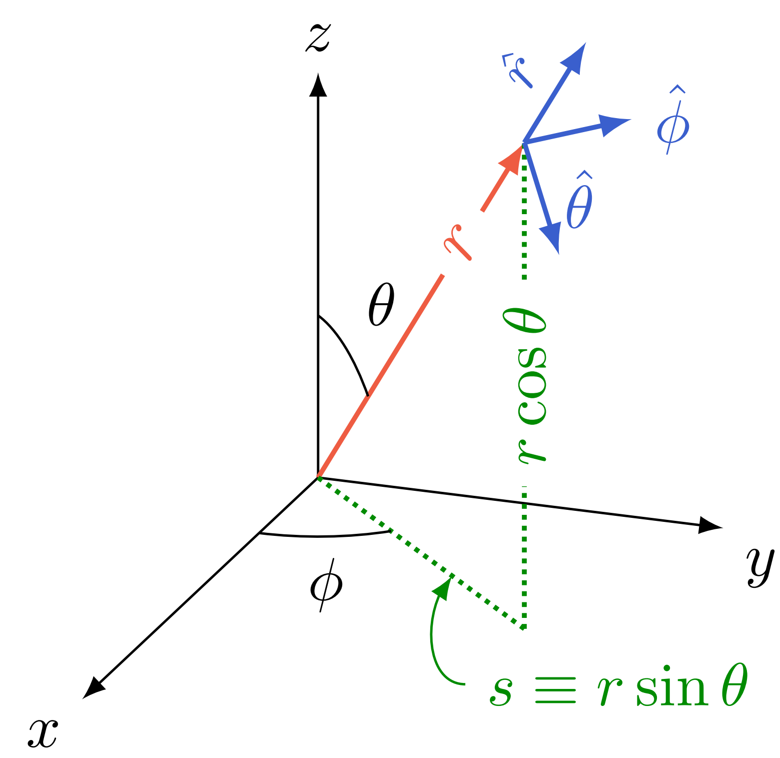
Análise vetorial

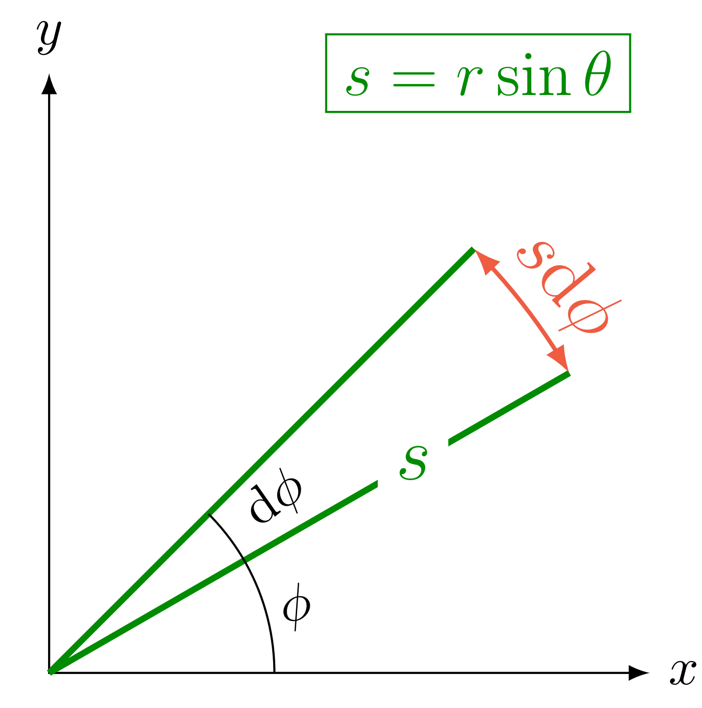
Coordenadas esféricas

$$d\overrightarrow{\ell} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$

$$d\overrightarrow{\ell}(dr, d\theta, d\phi) = ?$$

$$d\overrightarrow{\ell} = dr\,\hat{r} + rd\theta\,\hat{\theta} + ?$$





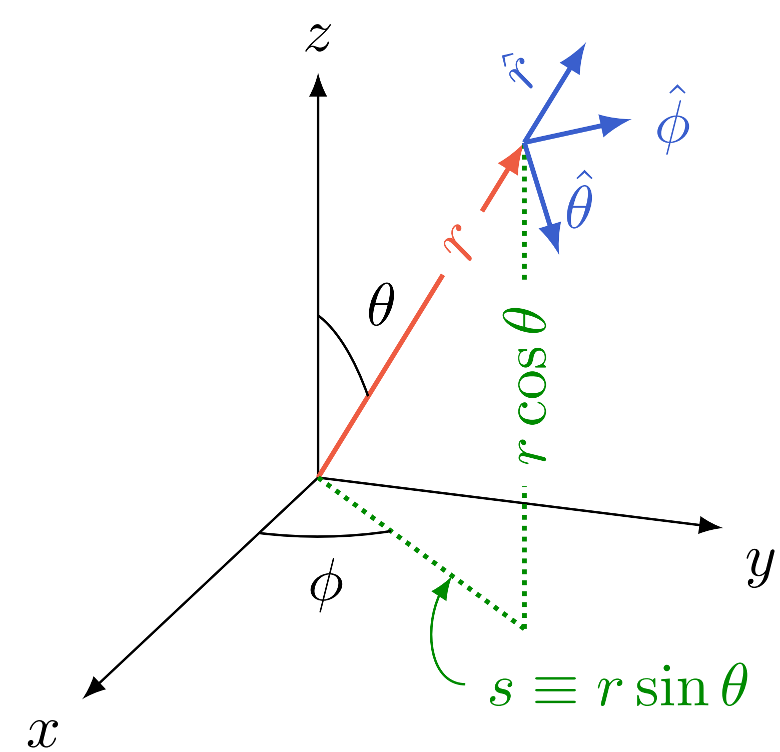
Análise vetorial

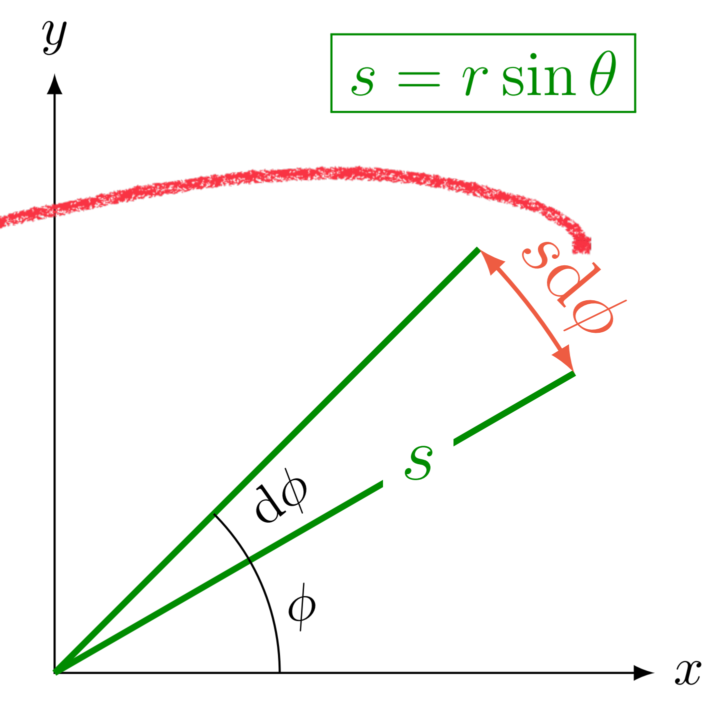
Coordenadas esféricas

$$d\vec{\ell} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$

$$d\vec{\ell}(dr, d\theta, d\phi) = ?$$

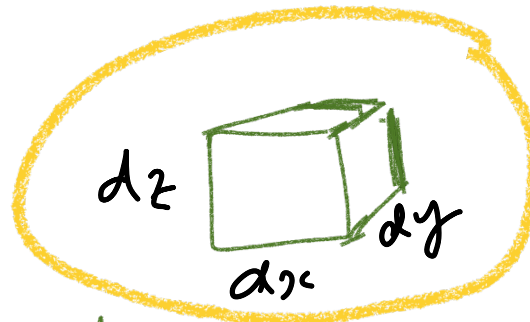
$$d\vec{\ell} = dr\,\hat{r} + rd\theta\,\hat{\theta} + r\sin\theta d\phi\,\hat{\phi}$$



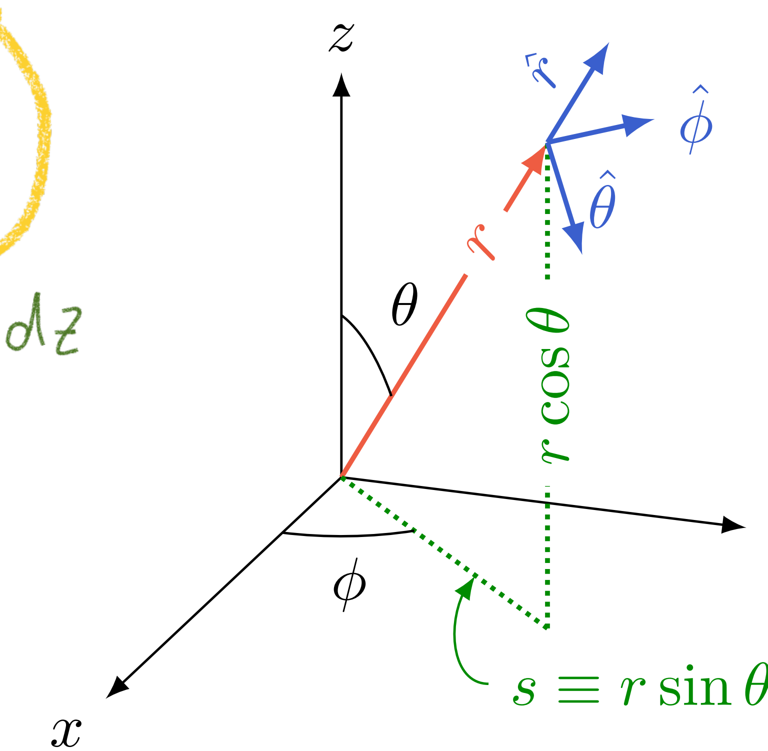


Análise vetorial

Coordenadas esféricas



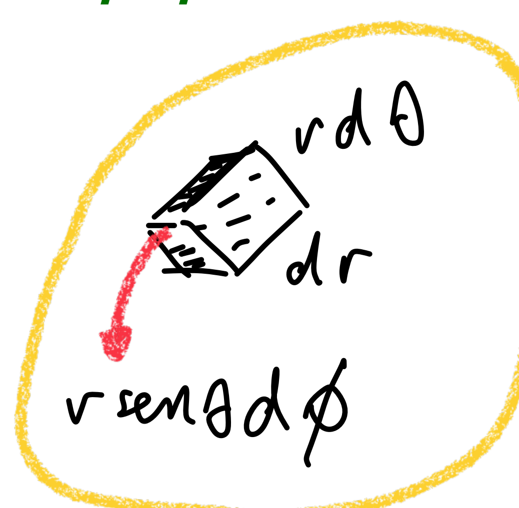
$$d\vec{\ell} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z} \Rightarrow d\tau = dx\,dy\,dz$$



$$d\vec{\ell}\,(dr, d\theta, d\phi) = \,?$$

$$d\vec{\ell} = dr\,\hat{r} + r d\theta\,\hat{\theta} + r\sin\theta d\phi\,\hat{\phi}$$

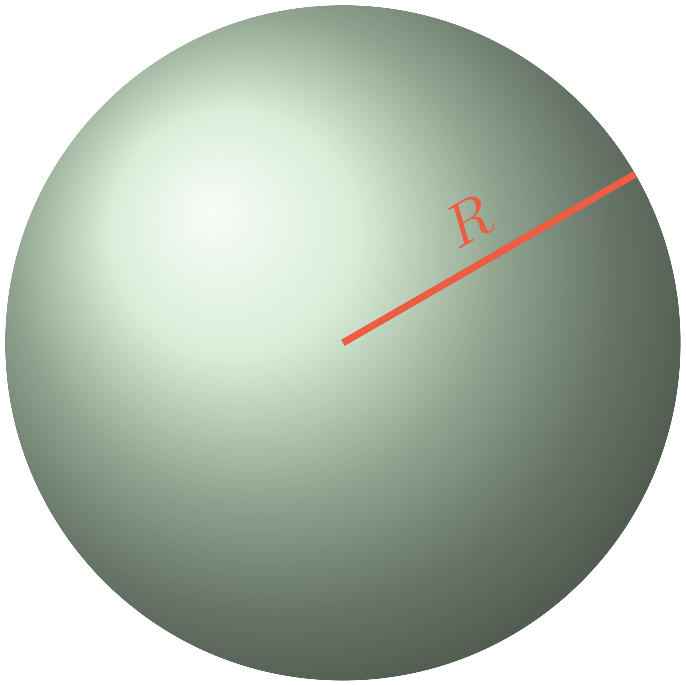
$$\mathrm{d}\tau = r^2 \sin\theta \mathrm{d}r\,\mathrm{d}\theta\; d\phi$$



Pratique o que aprendeu

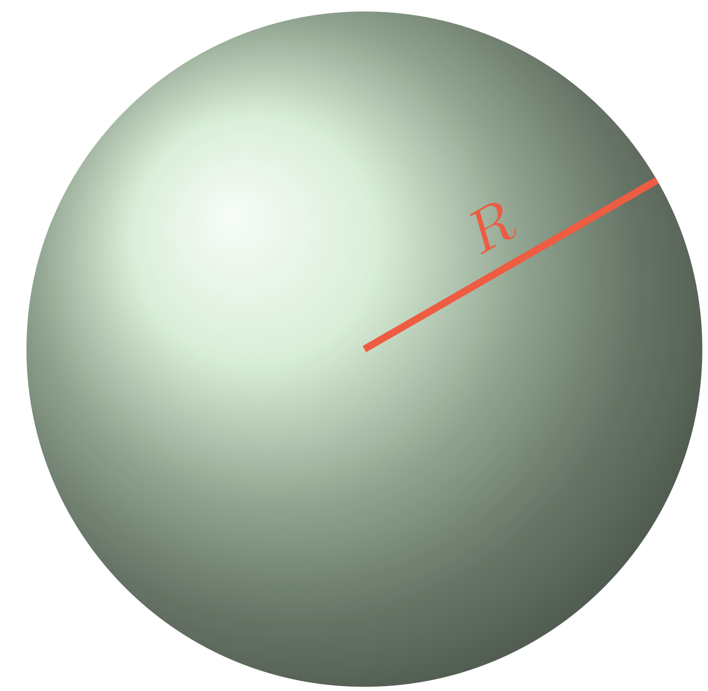
Volume da esfera

$$V = \int_{\mathscr{V}} d\tau$$

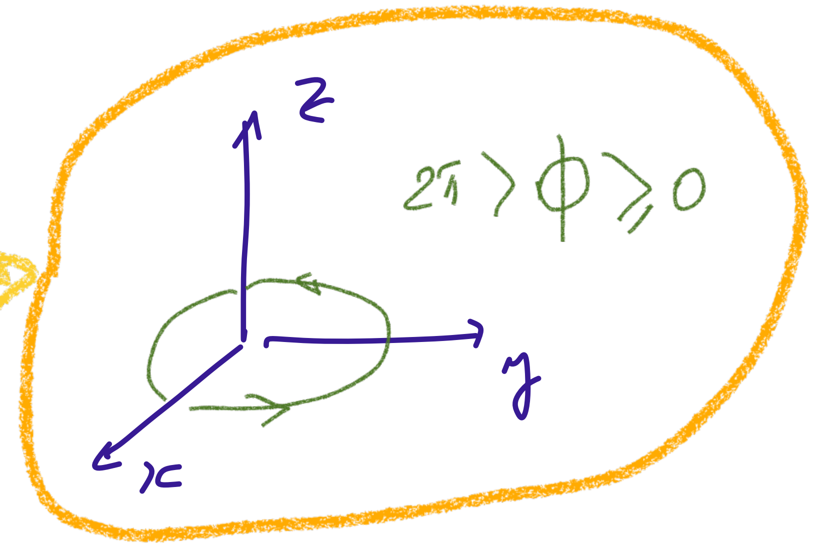


Pratique o que aprendeu

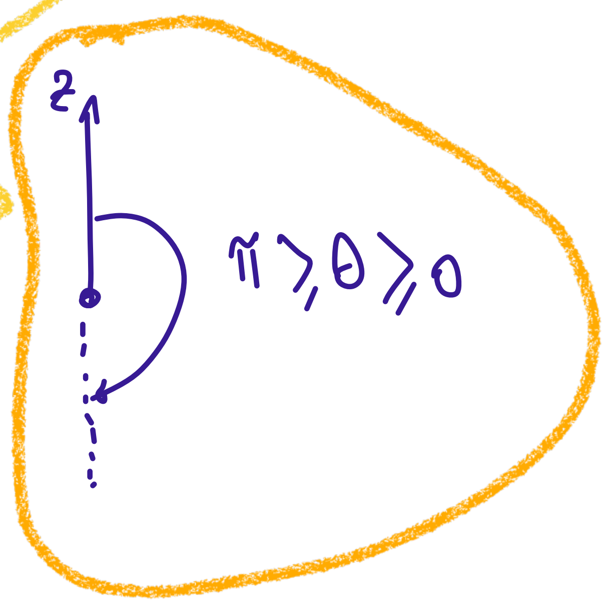
Volume da esfera



$$V = \int_{\mathscr{V}} d\tau$$

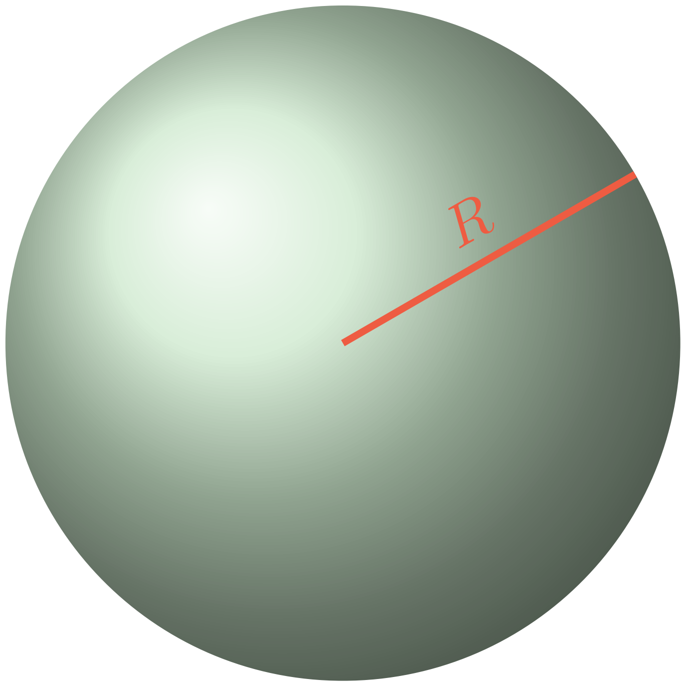


$$V = \int_0^R \int_0^\pi \int_0^{2\pi} r^2 \sin\theta d\phi d\theta dr$$



Pratique o que aprendeu

Volume da esfera



$$V = \int_{\mathscr{V}} d\tau$$

$$V = \int_0^R \int_0^\pi \int_0^{2\pi} r^2 \sin\theta d\phi d\theta dr$$

$$\int_0^\pi \Big(\Big) \sin\theta d\theta \rightarrow \int_0^\pi \Big(\Big)(-d\cos\theta)$$

$$\int_0^\pi \Big(\Big) \sin\theta d\theta \rightarrow \int_{-1}^1 \Big(\Big)\, \mathrm{d}u$$

$$\int_0^\pi (\,)\sin\theta d\theta \rightarrow \int_0^\pi (\,)(-d\cos\theta)$$

$$\int_0^\pi (\,)\sin\theta d\theta \rightarrow \int_{-1}^1 (\,)\,\mathrm{d}u$$

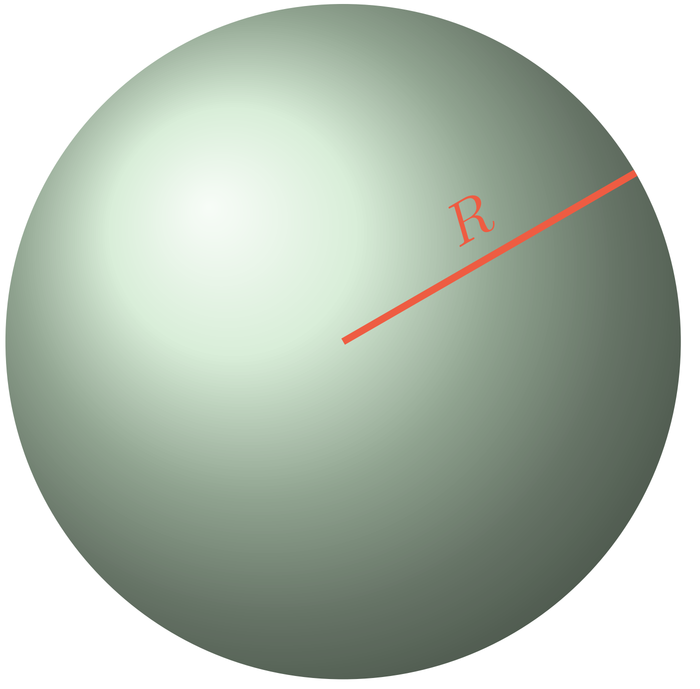
Pratique o que aprendeu

Volume da esfera

$$V = \int_{\mathscr{V}} d\tau$$

$$V = \int_0^R \int_0^\pi \int_0^{2\pi} r^2 \sin\theta d\phi d\theta dr$$

$$V = \int_0^R \int_1^1 \int_0^{2\pi} r^2 d\phi du dr$$

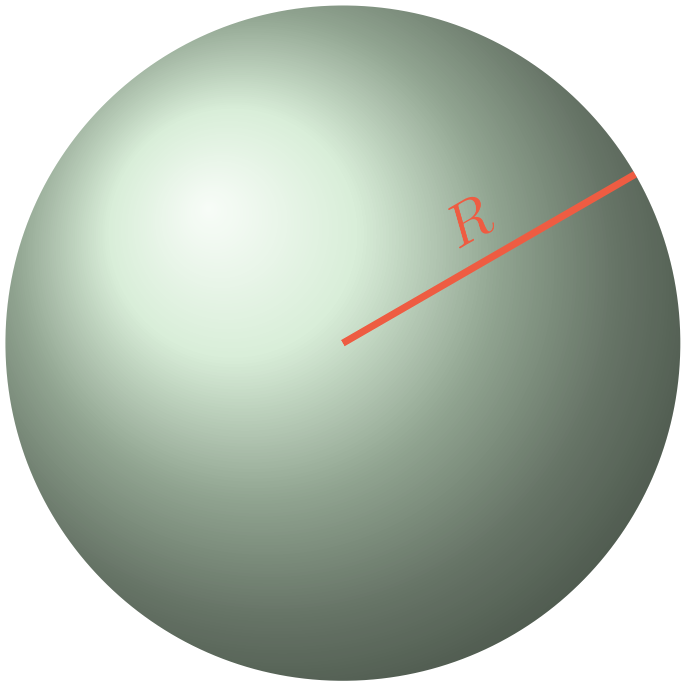


$$\int_0^{\pi} \left(\right) \sin\theta d\theta \rightarrow \int_0^{\pi} \left(\right) (-d\cos\theta)$$

$$\int_0^{\pi} \left(\right) \sin\theta d\theta \rightarrow \int_{-1}^{1} \left(\right) \mathrm{d}u$$

Pratique o que aprendeu

Volume da esfera



$$V = \int_{\mathscr{V}} d\tau$$

$$V = \int_0^R \int_0^{\pi} \int_0^{2\pi} r^2 \sin\theta d\phi d\theta dr$$

$$V = \int_0^R \int_1^1 \int_0^{2\pi} r^2 d\phi du dr$$

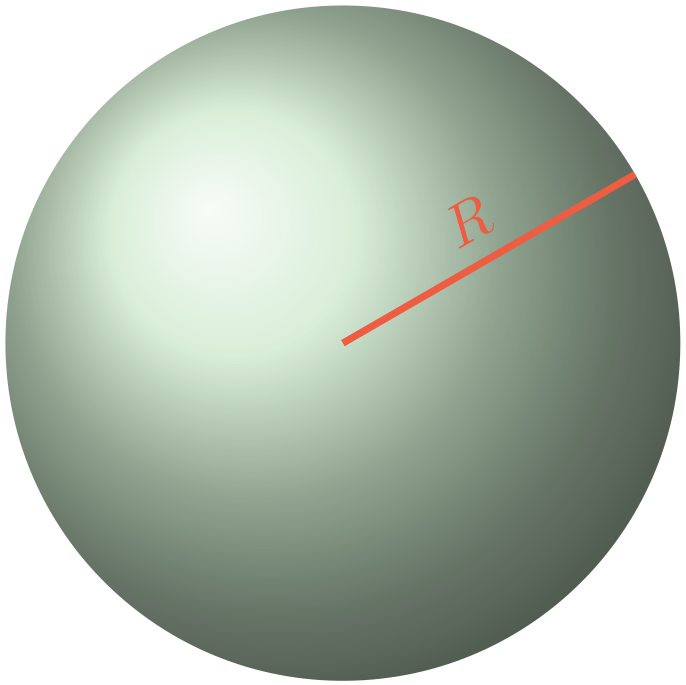
$$V = 4\pi \int_0^R r^2 dr$$

$$\int_0^{\pi}\Big(\Big)\sin\theta d\theta \rightarrow \int_0^{\pi}\Big(\Big)(-d\cos\theta)$$

$$\int_0^{\pi}\Big(\Big)\sin\theta d\theta \rightarrow \int_{-1}^{1}\Big(\Big)\,\mathrm{d}u$$

Pratique o que aprendeu

Volume da esfera



$$V = \int_{\mathscr{V}} d\tau$$

$$V = \int_0^R \int_0^{\pi} \int_0^{2\pi} r^2 \sin\theta d\phi d\theta dr$$

$$V = \int_0^R \int_1^1 \int_0^{2\pi} r^2 d\phi du dr$$

$$V = 4\pi \int_0^R r^2 dr = \frac{4\pi}{3} R^3$$

Análise vetorial

Gradiente em coordenadas esféricas

$$d\overrightarrow{\ell} = dr\,\hat{r} + rd\theta\,\hat{\theta} + r\sin\theta d\phi\,\hat{\phi}$$

Análise vetorial

Gradiente em coordenadas esféricas

$$d\overrightarrow{\ell} = dr\,\hat{r} + rd\theta\,\hat{\theta} + r\sin\theta d\phi\,\hat{\phi}$$

$$dt = \frac{\partial t}{\partial r}dr + \frac{\partial t}{\partial \theta}d\theta + \frac{\partial t}{\partial \phi}d\phi$$

Análise vetorial

Gradiente em coordenadas esféricas

$$d\overrightarrow{\ell} = dr\,\hat{r} + rd\theta\,\hat{\theta} + r\sin\theta d\phi\,\hat{\phi}$$

$$dt = \frac{\partial t}{\partial r}dr + \frac{\partial t}{\partial \theta}d\theta + \frac{\partial t}{\partial \phi}d\phi$$

$$dt = \overrightarrow{\nabla}t \cdot d\overrightarrow{\ell}$$

teorema fundamental

$$\int_a^b \vec{\nabla}t \cdot d\vec{\ell} = \int_a^b dt$$

Análise vetorial

Gradiente em coordenadas esféricas

$$d\overrightarrow{\ell} = dr\,\hat{r} + rd\theta\,\hat{\theta} + r\sin\theta d\phi\,\hat{\phi}$$

$$dt = \frac{\partial t}{\partial r}dr + \frac{\partial t}{\partial \theta}d\theta + \frac{\partial t}{\partial \phi}d\phi$$

$$dt = \overrightarrow{\nabla}t \cdot d\overrightarrow{\ell}$$

$$dt = (\overrightarrow{\nabla}t)_r\,dr + (\overrightarrow{\nabla}t)_\theta\,r\,d\theta + (\overrightarrow{\nabla}t)_\phi\,r\sin\theta d\phi$$

Análise vetorial

Gradiente em coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + rd\theta\,\hat{\theta} + r\sin\theta d\phi\,\hat{\phi}$$

$$dt = \frac{\partial t}{\partial r}dr + \frac{\partial t}{\partial \theta}d\theta + \frac{\partial t}{\partial \phi}d\phi$$

$$dt = \vec{\nabla}\,t \cdot d\vec{\ell}$$

$$dt = (\vec{\nabla}\,t)_r\,dr + (\vec{\nabla}\,t)_\theta\,r\,d\theta + (\vec{\nabla}\,t)_\phi\,r\sin\theta d\phi$$

$$\vec{\nabla}\,t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$