PGF5003: Classical Electrodynamics I
Problem Set 2

Professor: Luis Raul Weber Abramo
Monitor: Natalí Soler Matubaro de Santi
(Due to April 27, 2021)

Guideline: *in this PS you will need to use different math relations (for instance, for Legendre Polynomials, Bessel functions of first and second kind). Use the Jackson's book or your preferred reference to look for them. Write every equation that you have used in the solution. Moreover, in every Laplacian solution, write the differential equation and general solution in each exercise.* **Remember:** *the PS is written in CGS system of units, so you are strongly recommended to use it. If not, please, write in the top of your solution: "I am using MKS system of units".*

1 Question (1 point)

Compute the energy of a sphere uniformly charged, building the sphere from shells of charge dq brought from infinity until some radius r, in the way that the result will be a sphere of a uniform density charge distribution.

2 Question (1 point)

Consider two cylinders (that are both coaxial and conductive), with the same length L and with radius a and b, in the way that $a < b$, $L \gg a$ and $L \gg b$. Supposing that both cylinders are uniformly charged, the inner one with $+q$ and the exterior with $-q$, find:

a) *the capacitance of this system;*

b) *the energy per unit of length (this result is in terms of the capacitance and density of charge);*

3 Question (1 point)

The two-dimensional region: $\rho \geq a$, $0 \leq \phi \leq \beta$ is bounded by conducting surfaces at $\phi = 0$, $\rho = a$ and $\phi = \beta$ held at zero potential, as indicated in the figure. At large ρ, the potential is determined by some configuration of charges and/or conductors at fixed potentials.

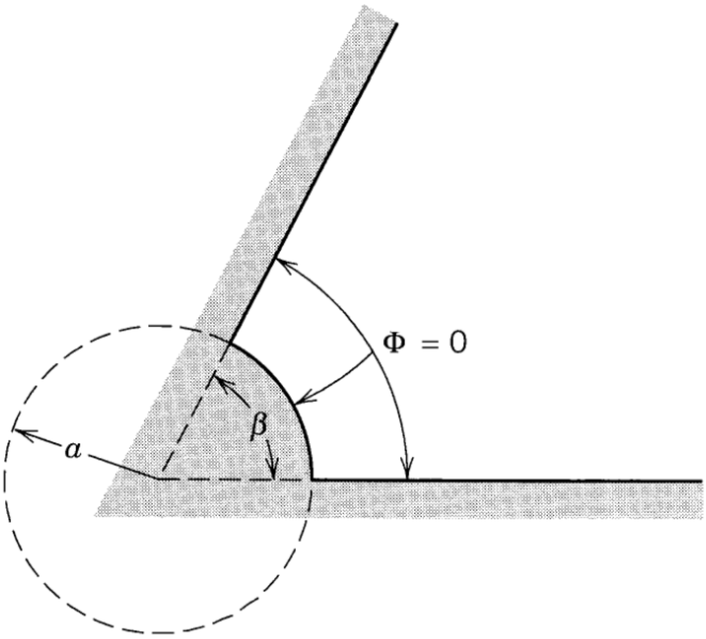

Figure 1: Figure for the question 3.

a) *Write down a solution for the potential $\Phi(\rho, \phi)$ that satisfies the boundary conditions for finite ρ.*

b) *Keeping only the lowest non vanishing terms, calculate the electric field components E_ρ and E_ϕ and also the surface-charge densities $\sigma(\rho, 0)$, $\sigma(\rho, \beta)$ and $\sigma(a, \phi)$ on the three boundary surfaces.*

---

4 Question (1 point)

Consider a cube with conductive faces and side a. If the face on $z = a$ has constant potential Φ_0 and the other faces have zero potential, what is the electric potential inside the cube?

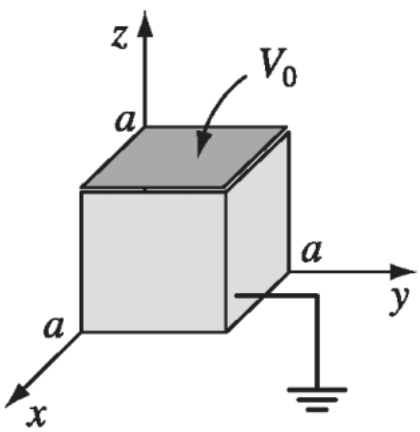

Figure 2: Figure for the question 4.

5 Question (2 points)

A spherical surface of radius R has charge uniformly distributed over its surface with a density $Q/4\pi R^2$, except for a spherical cap at the north pole, defined by the cone $\theta = \alpha$.

a) *Show that the potential inside the spherical surface can be expressed as*

$$\Phi = \frac{Q}{2}\sum_{\ell=0}^{\infty}\frac{1}{(2\ell+1)}\left[P_{\ell+1}(\cos\alpha) - P_{\ell-1}(\cos\alpha)\right]\frac{r^\ell}{R^{\ell+1}}P_\ell(\cos\theta), \tag{1}$$

where, for $\ell = 0$, $P_{\ell-1}(\cos\alpha) = -1$. What is the electric potential outside?

b) *Find the magnitude and the direction of the electric field at the origin.*

---

6 Question (2 points)

Consider a half-infinite cylinder grounded, with a electric potential ϕ_0 at its closed end.

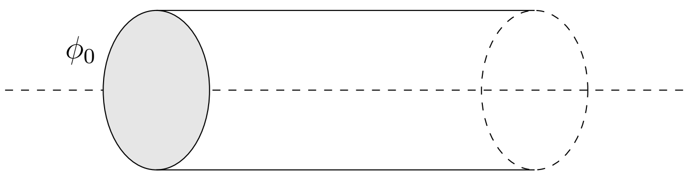

Figure 3: Semi-infinite cylinder.

Show that the electric potential in the inner parts of the cylinder is given by:

$$\phi(\rho,\theta,z) = 2\phi_0\sum_k \frac{e^{-kz}}{ka}\frac{J_0(k\rho)}{J_1(ka)}, \tag{2}$$

where $J_0(ka) = 0$ and, of course, J_i is the Bessel function of first kind and order i.

---

7 Question (2 points)

Two concentric conducting spheres of inner and outer radii a and b, respectively, carry charges $\pm Q$. The empty space btween the spheres is half-filled by a hemispherical shell of dieletric (of dieletric constant ϵ), as shown in the Figure.

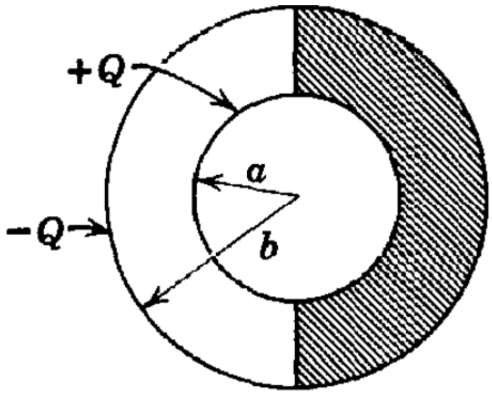

Figure 4: Figure for the question 7.

a) *Find the eletric field everywhere beteeen the spheres.*

b) *Calculate the surface-charge distribution on the inner sphere.*