PGF5003 - Classical Electrodynamics I (2021)

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1 PS 1 - Question 6

According to the question 6, that asks the *force per unity of area over the surface of the sphere*, one way to compute this quantity is to use the idea given on Griffiths' book: `Chapter 2`:

2.5.3 ■ Surface Charge and the Force on a Conductor

Because the field inside a conductor is zero, boundary condition 2.33 requires that the field immediately *outside* is

$$\mathbf{E} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{n}}, \tag{2.48}$$

consistent with our earlier conclusion that the field is normal to the surface. In terms of potential, Eq. 2.36 yields

$$\sigma = -\epsilon_0 \frac{\partial V}{\partial n}. \tag{2.49}$$

These equations enable you to calculate the surface charge on a conductor, if you can determine $\mathbf{E}$ or V; we shall use them frequently in the next chapter.

In the presence of an electric field, a surface charge will experience a force; the force per unit area, $\mathbf{f}$, is $\sigma\mathbf{E}$. But there's a problem here, for the electric field is *discontinuous* at a surface charge, so what are we supposed to use: $\mathbf{E}_{\text{above}}$, $\mathbf{E}_{\text{below}}$, or something in between? The answer is that we should use the *average* of the two:

$$\mathbf{f} = \sigma\mathbf{E}_{\text{average}} = \frac{1}{2}\sigma(\mathbf{E}_{\text{above}} + \mathbf{E}_{\text{below}}). \tag{2.50}$$

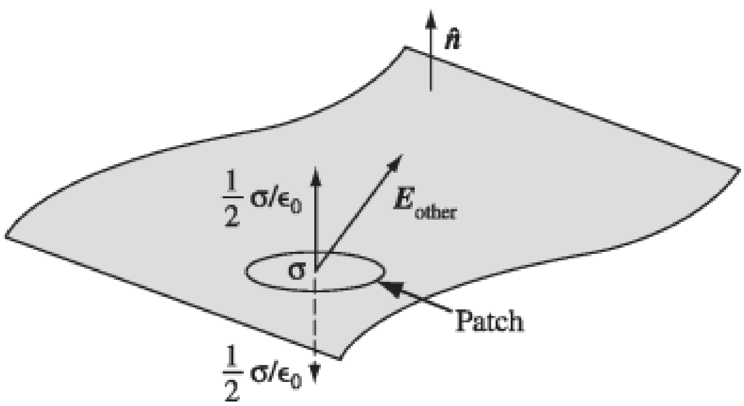

FIGURE 2.50

[11]This problem was suggested by Nelson Christensen.

[12]See M. Levin and S. G. Johnson, *Am. J. Phys.* **79**, 843 (2011).

Why the average? The reason is very simple, though the telling makes it sound complicated: Let's focus our attention on a tiny patch of surface surrounding the point in question (Fig. 2.50). (Make it small enough so it is essentially flat and the surface charge on it is essentially constant.) The *total* field consists of two parts—that attributable to the patch itself, and that due to everything else (other regions of the surface, as well as any external sources that may be present):

$$\mathbf{E} = \mathbf{E}_{\text{patch}} + \mathbf{E}_{\text{other}}.$$

Now, the patch cannot exert a force on itself, any more than you can lift yourself by standing in a basket and pulling up on the handles. The force on the patch, then, is due exclusively to $\mathbf{E}_{\text{other}}$, and *this* suffers *no* discontinuity (if we removed the patch, the field in the "hole" would be perfectly smooth). The discontinuity is due entirely to the charge on the patch, which puts out a field $(\sigma/2\epsilon_0)$ on either side, pointing away from the surface. Thus,

$$\mathbf{E}_{\text{above}} = \mathbf{E}_{\text{other}} + \frac{\sigma}{2\epsilon_0}\hat{\mathbf{n}},$$

$$\mathbf{E}_{\text{below}} = \mathbf{E}_{\text{other}} - \frac{\sigma}{2\epsilon_0}\hat{\mathbf{n}},$$

and hence

$$\mathbf{E}_{\text{other}} = \frac{1}{2}(\mathbf{E}_{\text{above}} + \mathbf{E}_{\text{below}}) = \mathbf{E}_{\text{average}}.$$

Averaging is really just a device for removing the contribution of the patch itself.

That argument applies to *any* surface charge; in the particular case of a conductor, the field is zero inside and $(\sigma/\epsilon_0)\hat{\mathbf{n}}$ outside (Eq. 2.48), so the average is $(\sigma/2\epsilon_0)\hat{\mathbf{n}}$, and the force per unit area is

$$\mathbf{f} = \frac{1}{2\epsilon_0}\sigma^2\hat{\mathbf{n}}. \tag{2.51}$$

Then, you just need to compute the eletric field due to the distribution of charge σ and consider the eletric field outside the sphere, respectivelly for above and below eletric fields, in Griffiths' notation.