MAC 0329

10.06.2021

Bom dia!!

Recapitulação

1. Computadores: maq. processa dados.
2. Informações/dados devem ser codificados em binário
3. Representação de nº na base 2.
4. Processamento de dados no computador

$$f : \{0,1\}^n \longrightarrow \{0,1\}^m$$

Exemplo: $f : B^3 \to B^2$

$$(a_i, b_i, c_i) \longmapsto (s_i, c_{i+1})$$

$\downarrow$

$$f = (f_1, f_2)$$

$f_1 : B^3 \to B$

$$(a_i, b_i, c_i) \longmapsto s_i$$

$f_2 : B^3 \to B$

$$(a_i, b_i, c_i) \longmapsto c_{i+1}$$

5. $f : B^n \to B$ são booleanas

6: $f: B^n \to B$ pode ser expressa/definida por uma expressão booleana

E
OU
NÃO

7. Produtos canônicos, Somas canônicas

SOP POS

8. Álgebra booleana $+, \cdot, \bar{\ }, 0, 1$

→ Qualquer elemento de uma álgebra booleana pode ser escrito, de forma única, como soma de átomos.

$\leq$

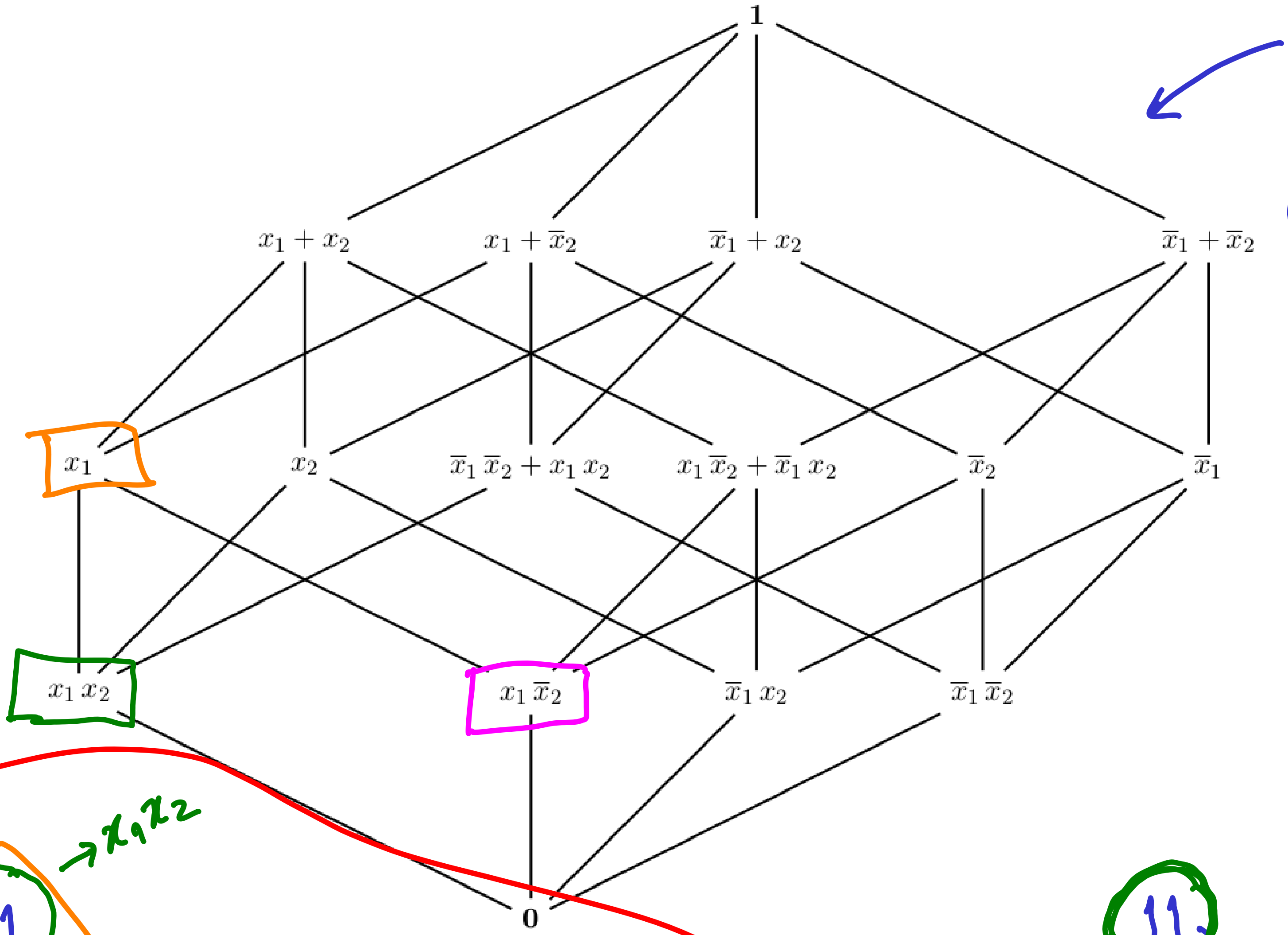
→ Conjunto de funções booleanas é uma álgebra booleana. ($f: B^n \to B$)

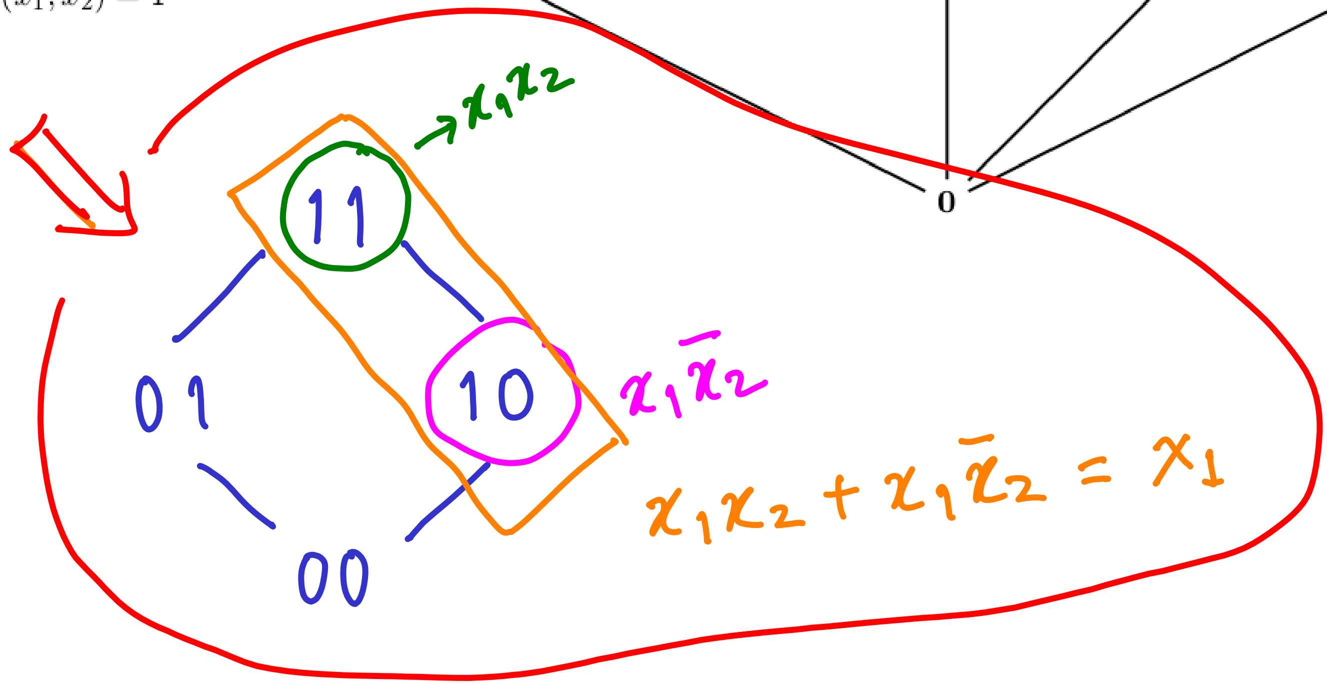
átomos $\Rightarrow$ produtos canônicos

$f_0(x_1, x_2) = 0$
$f_1(x_1, x_2) = x_1\, x_2$
$f_2(x_1, x_2) = x_1\, \overline{x}_2$
$f_3(x_1, x_2) = x_1$
$f_4(x_1, x_2) = \overline{x}_1\, x_2$
$f_5(x_1, x_2) = x_2$
$f_6(x_1, x_2) = \overline{x}_1\, x_2 + x_1\, \overline{x}_2$
$f_7(x_1, x_2) = x_1 + x_2$
$f_8(x_1, x_2) = \overline{x}_1\, \overline{x}_2$
$f_9(x_1, x_2) = \overline{x}_1\, \overline{x}_2 + x_1\, x_2$
$f_{10}(x_1, x_2) = \overline{x}_2$
$f_{11}(x_1, x_2) = x_1 + \overline{x}_2$
$f_{12}(x_1, x_2) = \overline{x}_1$
$f_{13}(x_1, x_2) = \overline{x}_1 + x_2$
$f_{14}(x_1, x_2) = \overline{x}_1 + \overline{x}_2$
$f_{15}(x_1, x_2) = 1$

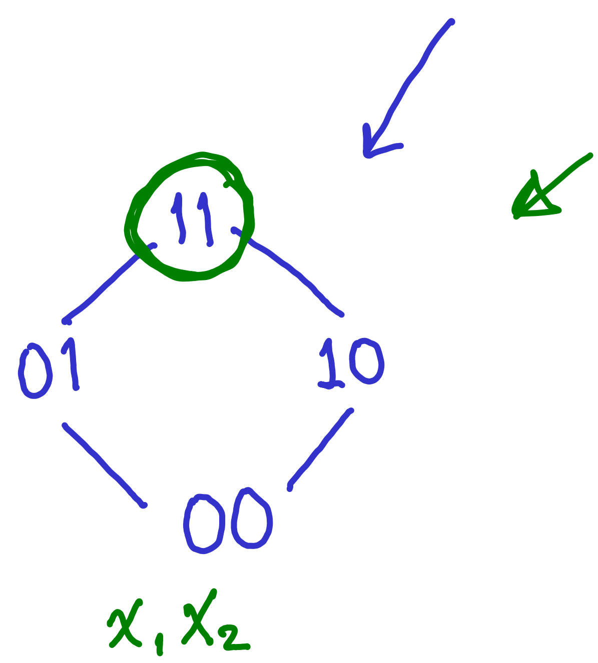
B(2)

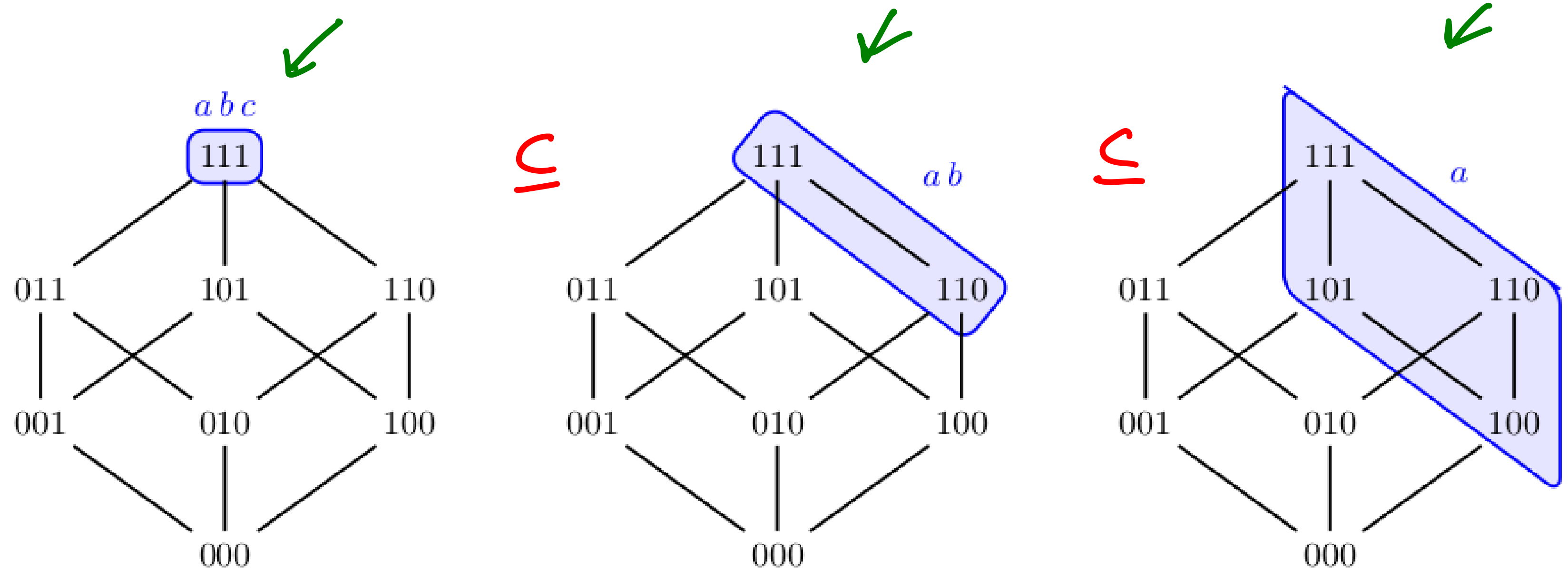
$f : B^2 \to B$





B^2





a b c	abc <	ab <	a
0 0 0	0	0	0
0 0 1	0	0	0
0 1 0	0	0	0
0 1 1	0	0	0
1 0 0	0	0	1
1 0 1	0	0	1
1 1 0	0	1	1
1 1 1	1	1	1

$abc \rightarrow 2^0 = 1$

$ab \rightarrow 2^1 = 2$

$a \rightarrow 2^2 = 4$

$1 \rightarrow 2^3 = 8$

$$f(a,b,c) = \underline{a\bar{c} + ab + bc}$$

a	b	c	f(a,b,c)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1 $\leftarrow$
1	0	0	1 $\leftarrow$
1	0	1	0
1	1	0	1 $\leftarrow$
1	1	1	1 $\leftarrow$

$$f(a,b,c) = \bar{a}bc + a\bar{b}\bar{c} + ab\bar{c} + abc$$

$$= \bar{a}bc + abc + a\bar{b}\bar{c} + ab\bar{c}$$

$$= bc(\bar{a} + a) + a\bar{c}(\bar{b} + b)$$

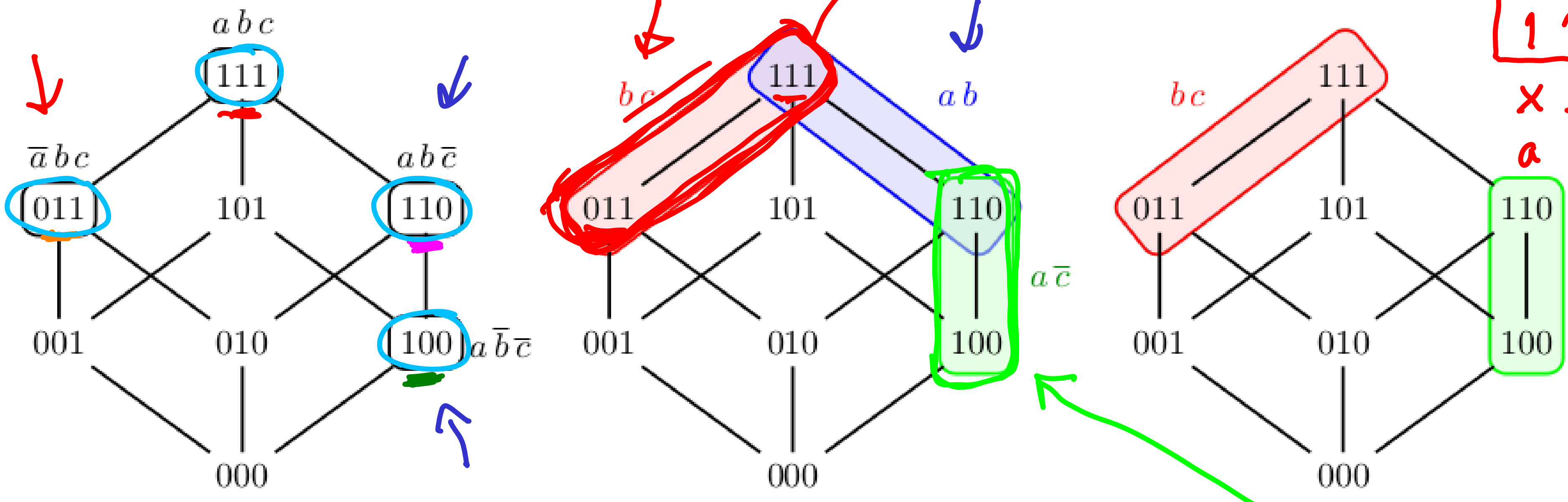
$$= \underline{bc + a\bar{c}}$$

$\bar{a}bc + abc = bc$

$$\boxed{\begin{matrix}0 & 1 & 1\\ 1 & 1 & 1\end{matrix}}$$

$x\ 1\ 1$

$a\ bc$



$\bar{a}bc + a\bar{b}\bar{c}$

$$f(a,b,c) = \bar{a}bc + a\bar{b}\bar{c} + ab\bar{c} + abc$$

$x + \bar{x} = 1$

$$\bar{a}bc + abc = bc\,\underbrace{(\bar{a} + a)}_{1} = bc$$

$$a\bar{b}\bar{c} + ab\bar{c} = a\bar{c}$$

$$f(a,b,c) = \underbrace{\bar a \bar b \bar c + \bar a \bar b c} + \underbrace{\bar a b \bar c + \bar a b c} + abc$$

$$= \underbrace{\bar a \bar b + \bar a b} + abc$$

$$= \bar a + a\underline{bc}$$

será?

$$= \underline{\bar a + bc}$$

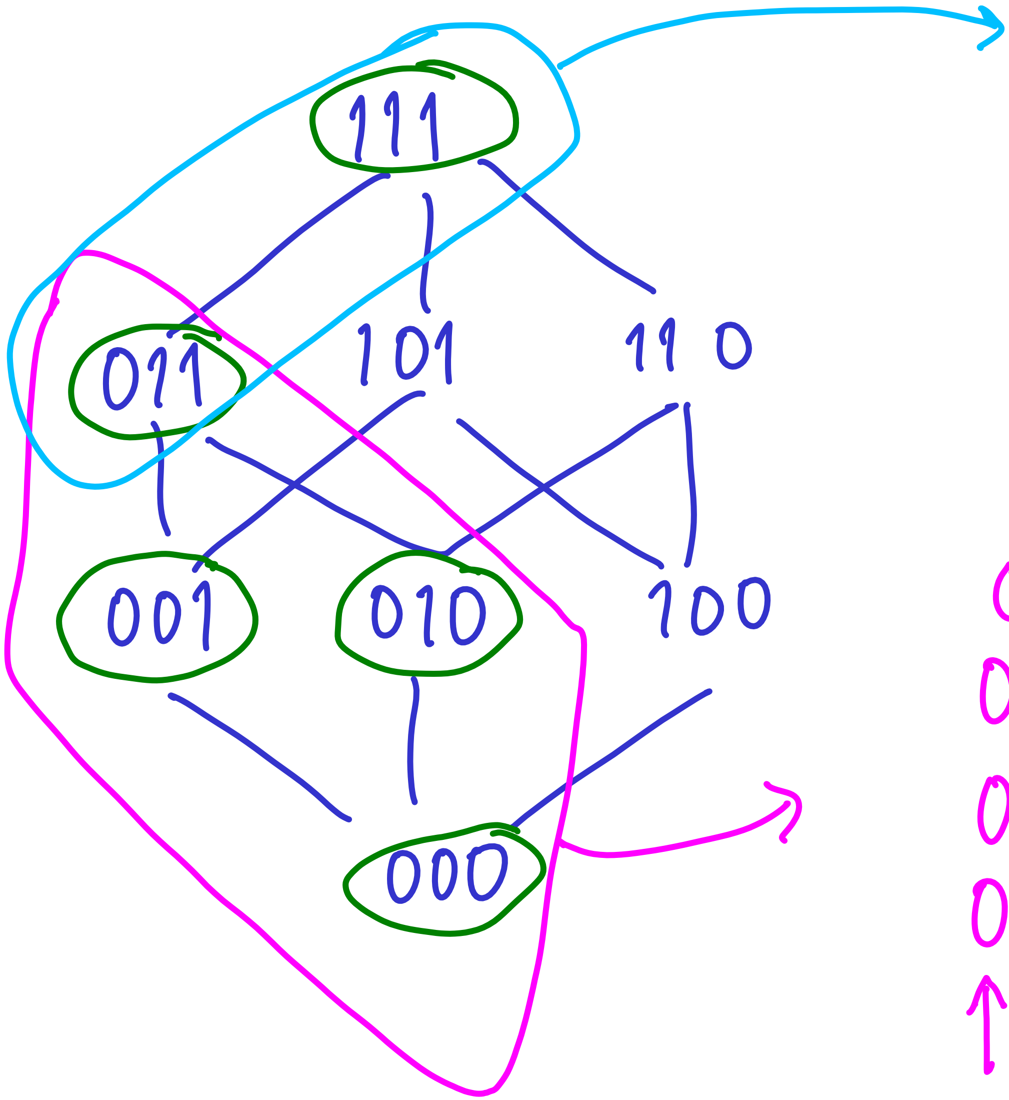
$$\bar x + xy = \bar x 1 + xy = \bar x (y + \bar y) + xy$$

$$= \bar x y + \bar x \bar y + xy = \bar x (y + \bar y) + y(x + \bar x)$$

$$= \bar x + y$$

$$f(a,b,c) = \bar a \bar b \bar c + \bar a \bar b c + \bar a b \bar c + \bar a b c + abc$$

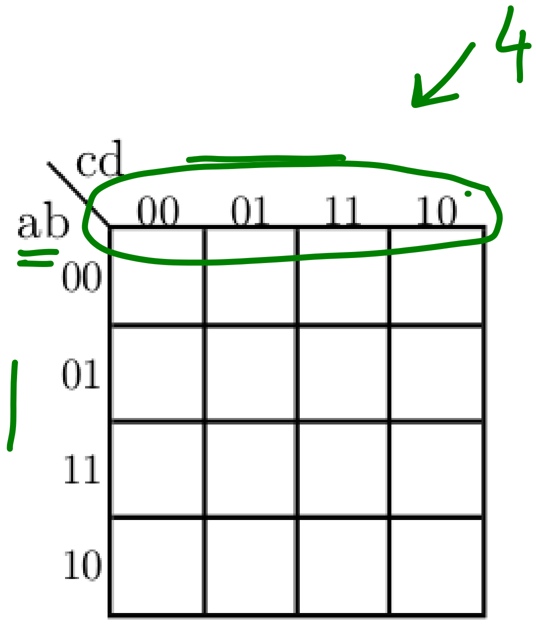
$$= \bar a + bc$$

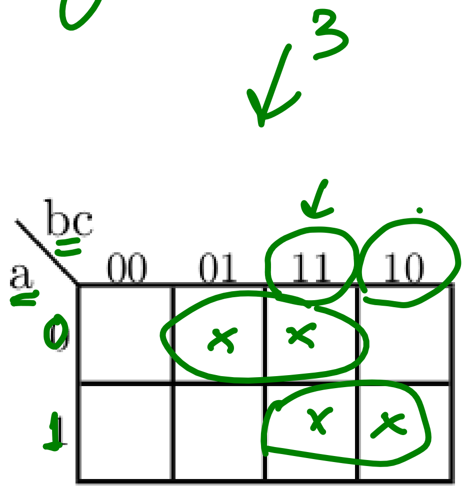


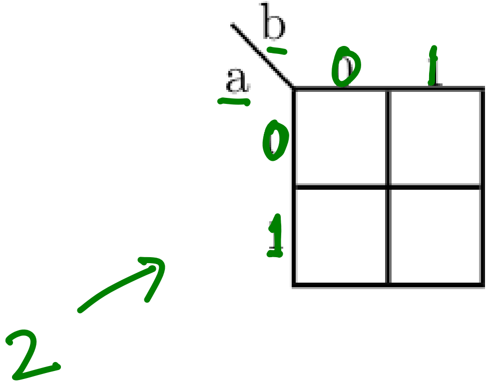
$$\left.\begin{matrix}011\\111\end{matrix}\right\} \Rightarrow x11 \Rightarrow bc$$

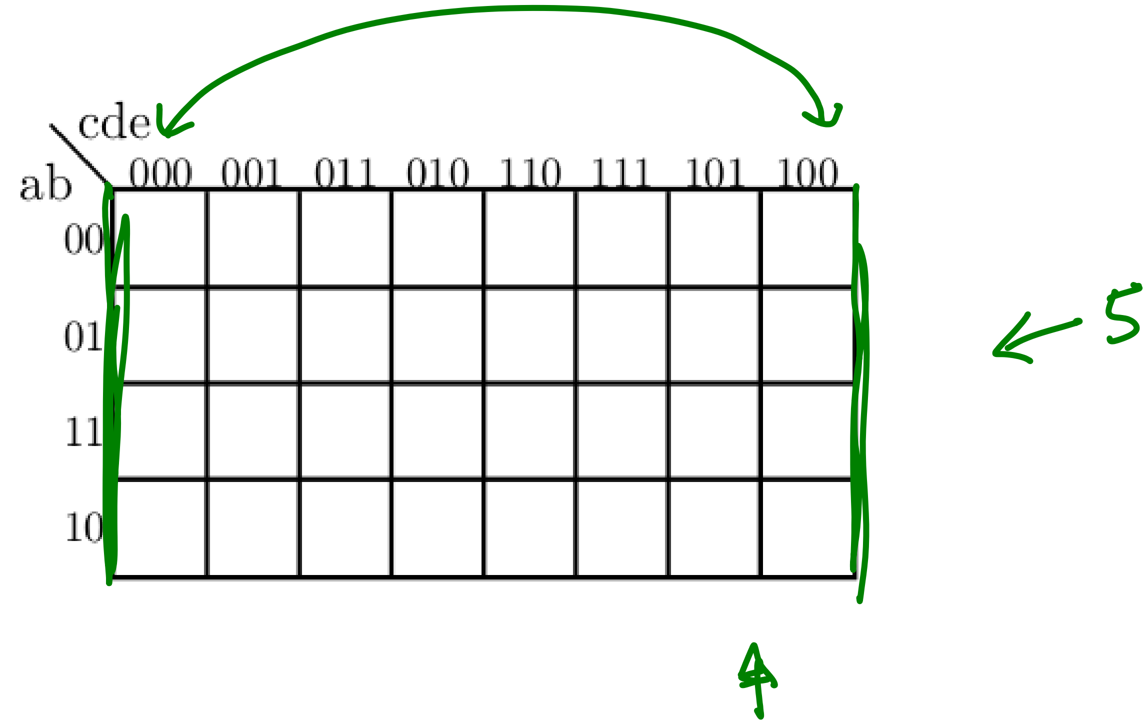
$$\left.\begin{matrix}011\\001\\010\\000\end{matrix}\right\} \Rightarrow 0XX \Rightarrow \bar a$$

Mapas de Karnaugh









01 | 10

abcde

$a \backslash bc$	00	01	11	10
0	0 000	1 001	3 011	2 010
1	4 100	5 101	7 111	6 110

$a \backslash bc$	00	01	11	10
0	1	1	1	1
1			1	

f

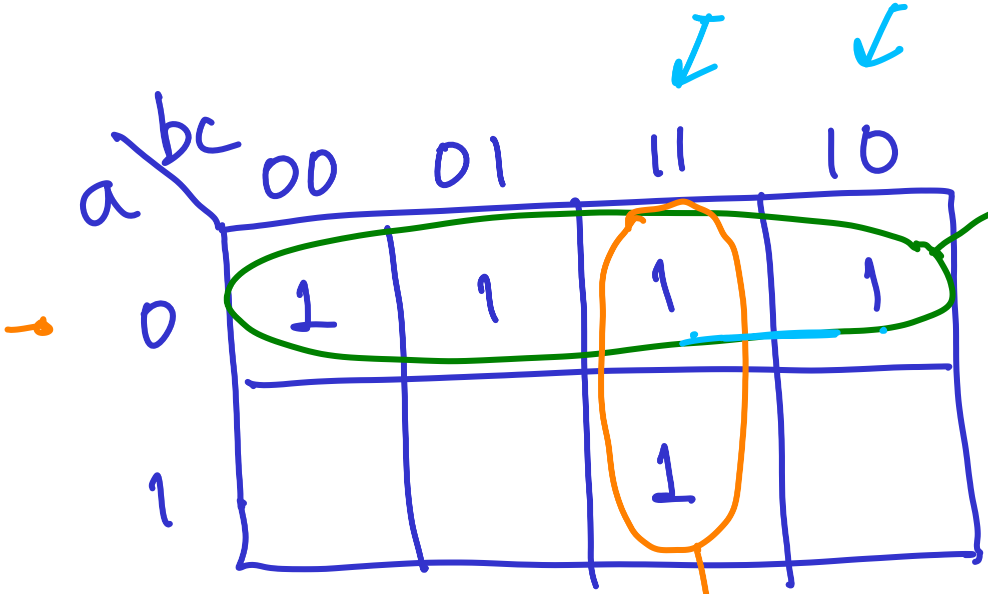
B^3

$$f(a,b,c) = \boxed{\underset{000}{\bar{a}\bar{b}\bar{c}} + \underset{001}{\bar{a}\bar{b}c} + \underset{010}{\bar{a}b\bar{c}} + \underset{011}{\bar{a}bc} + \underset{111}{abc}}$$

$$= \sum m(0,1,2,3,7)$$

forma compacta

01 10



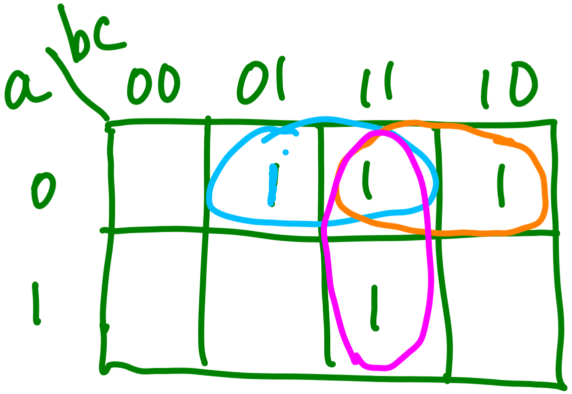
000
001
011
010

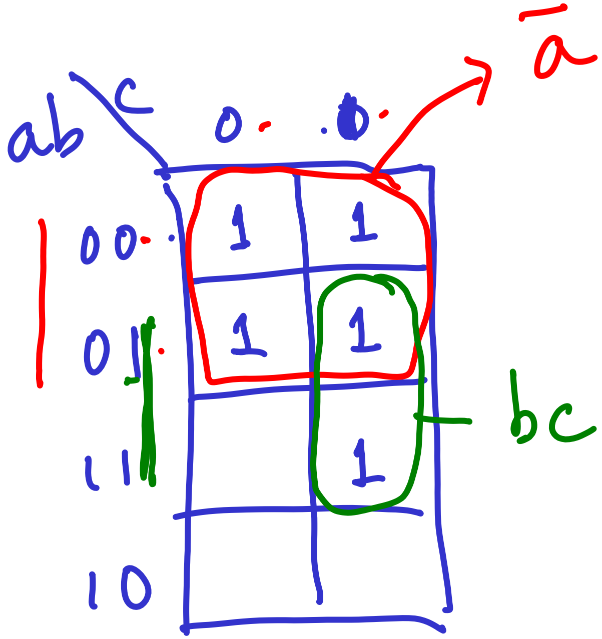
$\rightarrow \bar{a}$

011
111

$\rightarrow bc$

f





$$f(a,b,c,d) = \sum m(0,2,3,5,6,7,8,10,11,14,15)$$

0000 0010 0011 0101 0110 0111 1000 1010 1011 1110 1111

ab \ cd	00	01	11	10
00	1		1	1
01		1	1	1
11			1	1
10	1		1	1

f

$$\underbrace{\bar a b \bar c d + \bar a b c d}_{\bar a b d} + \underbrace{\bar a b c \bar d}_{\text{sobra}}$$

c

$\bar b \bar d$

$\bar a b d$

ñ devemos agrupar 3 pontos (ñ corresponde à simplif.)

$$f(a,b,c,d) = c + \bar b \bar d + \bar a b d$$

Circuitos 2-níveis

$$f(a,b,c,d) = c + \bar{b}\bar{d} + \bar{a}bd$$