FRIO

13/05/2021

Axiomas

$$x+y=y+x$$
$$xy=yx$$

$$x(y+z)=xy+xz$$
$$x+yz=(x+y)(x+z)$$

A3
$$x+0=x$$
$$x\cdot 1=x$$

A4.
$$\left.\begin{aligned} x+\bar{x}&=1\\ x\bar{x}&=0\end{aligned}\right\}$$

Unicidade do complemento

Suponha que $\bar{x}_1$ e $\bar{x}_2$ são tais que

$$H\ \begin{cases} x+\bar{x}_1=1\\ x\bar{x}_1=0\end{cases} \quad \text{e} \quad \begin{cases} x+\bar{x}_2=1\\ x\bar{x}_2=0\end{cases}$$

$$\bar{x}_2 \overset{A3}{=} \bar{x}_2\cdot 1 \overset{H}{=} \bar{x}_2(x+\bar{x}_1)$$

$$\overset{A2}{=} \underbrace{\bar{x}_2 x}_{x\bar{x}_2} + \bar{x}_2\bar{x}_1$$

$$\overset{H}{=} 0 + \bar{x}_2\bar{x}_1$$

$$\overset{H}{=} x\bar{x}_1 + \bar{x}_2\bar{x}_1$$

$$= \bar{x}_1(\underbrace{x+\bar{x}_2})$$

$$= \bar{x}_1\cdot 1 = \bar{x}_1$$

$$x+y=y+x$$
$$xy=yx$$

$$x(y+z)=xy+xz$$
$$x+yz=(x+y)(x+z)$$

$$x+0=x$$
$$x\cdot 1=x$$

$$\left.\begin{array}{l} x+\bar{x}=1 \\ x\bar{x}=0 \end{array}\right\}$$

Involução: $\bar{\bar{x}}=x$

$$\left\{\begin{array}{l} \bar{x}+y=1 \\ \bar{x}y=0 \end{array}\right\} \text{ } y \text{ é complemento de } \bar{x}$$

Sabemos que

$$\left\{\begin{array}{l} x+\bar{x}=1 \\ x\bar{x}=0 \end{array}\right\} \text{ } \bar{x} \text{ é complemento de } x$$

$$\Updownarrow$$

$$\left\{\begin{array}{l} \bar{x}+x=1 \\ \bar{x}x=0 \end{array}\right. \Longrightarrow \text{ } x \text{ é complemento de } \bar{x}$$

$$\bar{\bar{x}}=y=x$$

Associativa $\quad x+(y+z) = (x+y)+z$

Lema $\quad \boxed{x[(x+y)+z]} \overset{(a)}{=} [(x+y)+z]x \overset{(b)}{=} x$

a) $x[(x+y)+z] = [(x+y)+z]x$ é válido por A1

b) $x[(x+y)+z] \overset{A2}{=} \underbrace{x(x+y)}_{\text{Abs.}} + xz$

$= \underbrace{x + xz}$

$= x$

Lema : $\underline{\underline{x}}[(x+y)+z] = [(x+y)+z]\underline{\underline{x}} = x$

$Z = [(x+y)+z][x+(y+z)]$

$$\begin{array}{ccc} & Z & \\ \swarrow & & \searrow \\ \underbrace{(a)} & = & \underbrace{(b)} \end{array}$$

(a) $Z = [\underbrace{(x+y)+z}_{A}][\underbrace{x}_{B}+(\underbrace{y+z}_{C})]$

$\overset{AB+AC}{}$

$\overset{A2}{=} \boxed{[(x+y)+z]x} + [(x+y)+z](y+z)$

$\downarrow$ Lema $\qquad \downarrow$ A2

$= x + \{\underbrace{[(x+y)+z]y} + \boxed{[(x+y)+z]z}\}$

$\downarrow$ A1 $\qquad \downarrow$ Abs

$= x + \{\boxed{[(y+x)+z]y} + z\}$

$= x + (y+z)$

Lema: $\underline{\underline{x}}[(x+y)+z] = [(x+y)+z]\underline{\underline{x}} = x$

$$Z = [\underbrace{(x+y)}_{U}+\underbrace{z}_{V}]\underbrace{[x+(y+z)]}_{W}$$

$$\overset{A1+A2}{=} \underbrace{(x+y)[x+(y+z)]}_{\downarrow A2} + z[x+(\underline{y+z})]$$

$$= \{\underbrace{x[x+(y+z)]}_{\downarrow Ab} + y\underbrace{[x+(y+z)]}\} + z[x+(\underline{z+y})]$$

$$= \{x + \underbrace{[(y+z)+x]y}_{y}\} + \underbrace{[(z+y)+x]z}_{z}$$

$$= (x+y)+z$$

a) $z = x + (y + z)$

b) $z = (x + y) + z$

$$\Downarrow$$

$$x + (y + z) = (x + y) + z$$

Associativa

a	b	$a \wedge b$	$a \vee b$	$\neg a$
F	F	F	F	V
F	V	F	V	V
V	F	F	V	F
V	V	V	V	F

$\forall$

$\exists$

E lógico

OU lógico

NÃO lógico

$$a \wedge b = V \iff a = V \text{ e } b = V$$

Teorema de De Morgan

$$a + bc = (a+b)\cdot(a+c)$$

$$\boxed{\overline{x+y} = \bar{x}\,\bar{y}}$$

$$\overline{xy} = \bar{x} + \bar{y}$$

A4. $\begin{cases} x + \bar{x} = 1 \\ x\bar{x} = 0 \end{cases}$

? $\begin{cases} (x+y) + \bar{x}\,\bar{y} = 1 \\ (x+y)(\bar{x}\,\bar{y}) = 0 \end{cases}$

$$(x+y) + \bar{x}\,\bar{y} \overset{A2}{=}$$

$$= [(x+y) + \bar{x}][(x+y) + \bar{y}]$$

$$= [\underbrace{x+\bar{x}}_{1} + y][x + \underbrace{y+\bar{y}}_{1}]$$

$$= [1+y][x+1]$$

$$= 1 \cdot 1 = 1 \;//$$

$$\underbrace{(x+y)}_{a}(\bar{x}\,\bar{y}) = \bar{x}\,\bar{y}\,x + \bar{x}\,\bar{y}\,y$$

$$= \underbrace{\bar{x}x}_{0}\,\bar{y} + \bar{x}\,\underbrace{y\bar{y}}_{0}$$

$$= 0 + 0$$

$$= 0 \quad //$$

$$\left.\begin{array}{l} a + \bar{x}\,\bar{y} = 1 \\ a \cdot (\bar{x}\,\bar{y}) = 0 \end{array}\right\} \Longrightarrow \bar{a} = \bar{x}\,\bar{y}$$

$$\overline{(x+y)} = \bar{x}\,\bar{y}$$

$$\boxed{\overline{x+y} = \bar{x}\,\bar{y}}$$

$$\underbrace{+ \quad \cdot \quad \overline{}}$$

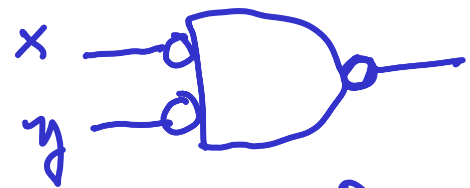
$$\overline{\overline{x+y}} = \overline{\bar{x}\,\bar{y}}$$

$$\overline{xy} = \bar{x} + \bar{y}$$

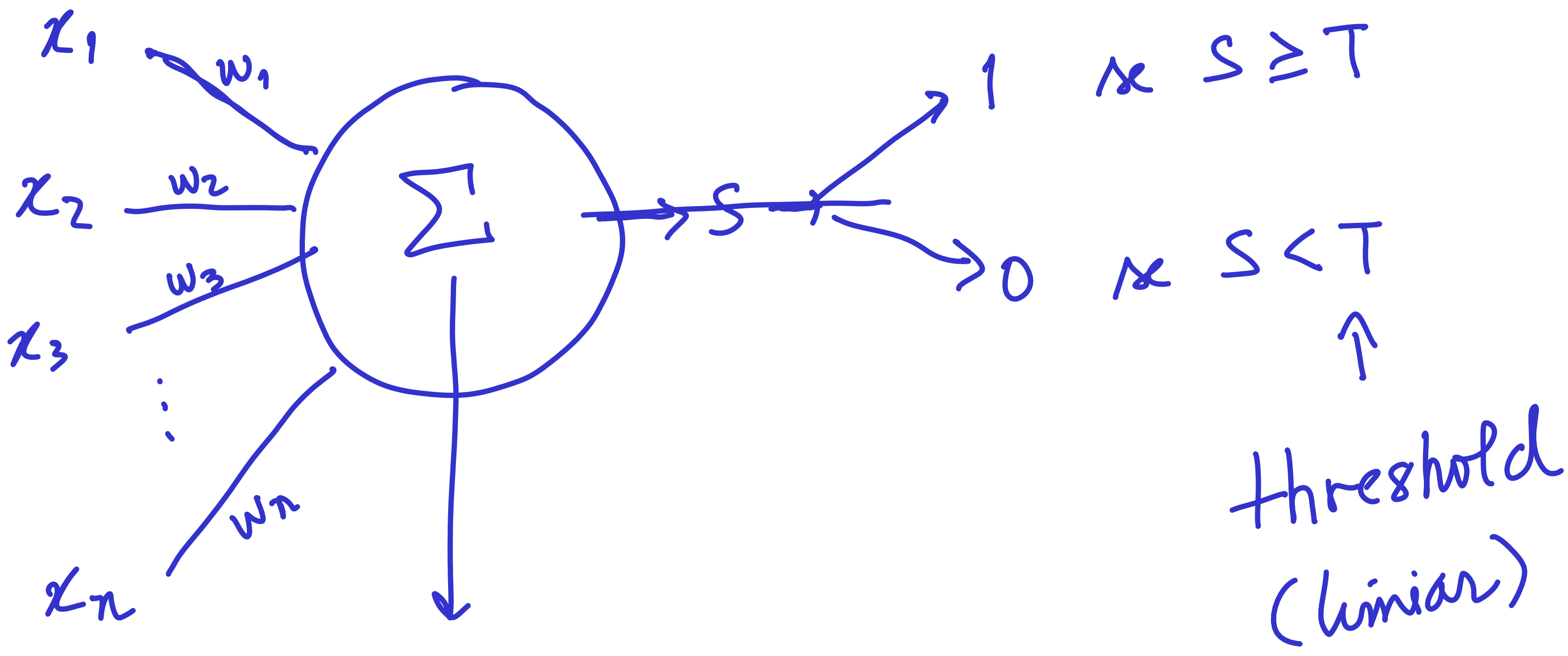
$$\boxed{x+y = \overline{\bar{x}\,\bar{y}}}$$

NAND

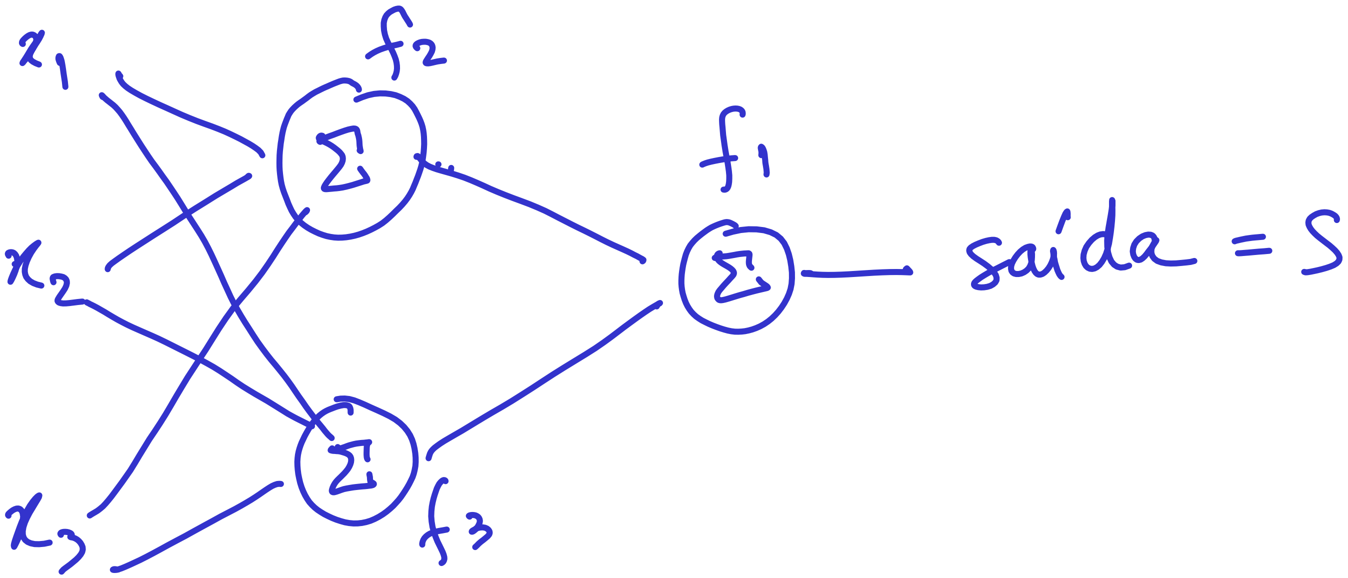
$$xy = \overline{\bar{x} + \bar{y}}$$



NAND



$$S = w_1 x_1 + w_2 x_2 + \dots + w_n x_n$$



$$S = \underline{f_1}\left(\underbrace{f_2(x_1, x_2, x_3)}, \underbrace{f_3(x_1, x_2, x_3)}\right)$$