MAC 0329

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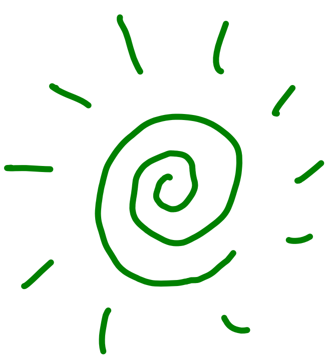

BOM DIA!

Álgebra Booleana

$$\langle A, +, \cdot, \bar{\ }, 0, 1 \rangle$$

A1. $x + y = y + x$

$x \cdot y = y \cdot x$

A2. $x(y+z) = x \cdot y + x \cdot z$

$x + y \cdot z = (x+y) \cdot (x+z)$

A3. $x + 0 = x$

$x \cdot 1 = x$

A4. $x + \bar{x} = 1$

$x \cdot \bar{x} = 0$

A1

$$\begin{cases} x+y=y+x \\ xy=yx \end{cases}$$

A2

$$\begin{cases} x(y+z)=xy+xz \\ x+yz=(x+y)\cdot(x+z) \end{cases}$$

A3

$$\begin{cases} x+0=x \\ x\cdot 1=x \end{cases}$$

A4

$$\begin{cases} x+\bar{x}=1 \\ x\bar{x}=0 \end{cases}$$

1) Unicidade do 0 e 1

Suponha que 0_1 e 0_2 são tais que

i) $x+0_1=x$
ii) $x+0_2=x$ $\quad \forall x \in A$

De (i), $\underline{0_2+0_1=0_2}$ tomando $x=0_2$

De (ii), $\underline{0_1+0_2}=0_1$ tomando $x=0_1$

$$0_2 \overset{(i)}{=} 0_2+0_1 \overset{A1}{=} 0_1+0_2 \overset{(ii)}{=} 0_1$$

i) $x\cdot 1_1=x$
ii) $x\cdot 1_2=x$

$$\left.\begin{aligned} &\Rightarrow 1_2\cdot 1_1=1_2 \\ &\Rightarrow 1_1\cdot 1_2=1_1 \end{aligned}\right\} \quad 1_2=1_2\cdot 1_1=1_1\cdot 1_2=1_1$$

A1

$$\begin{cases} x+y=y+x \\ xy=yx \end{cases}$$

A2

$$x(y+z)=xy+xz$$

$$x+yz=(x+y)\cdot(x+z)$$

A3

$$\begin{cases} x+0=x \\ x\cdot 1=x \end{cases}$$

A4

$$\begin{cases} x+\bar{x}=1 \\ x\bar{x}=0 \end{cases}$$

2) Idempotência

i) $\boxed{x+x=x}$

ii) $xx=x$

$$x+x= \longrightarrow =x$$

$$\underline{x+x} \overset{A3}{=} (x+x)\cdot \underline{1}$$

$$\overset{A4}{=} (x+x)\cdot(x+\bar{x})$$

$$\overset{A2}{=} x+x\bar{x}$$

$$\overset{A4}{=} x+0$$

$$\overset{A3}{=} \underline{x}$$

$$B=\{0,1\}$$

$$|\mathcal{P}(S)|=2^{|S|}$$

$$\boxed{x + x = x}$$

$$(x + x)\bar{x} = x\bar{x}$$

$$x\bar{x} + x\bar{x} = 0$$

$$0 + 0 = 0$$

$$0 = 0 \quad \checkmark$$

$$x + x = x$$

A1

$$\begin{cases} x+y=y+x \\ xy=yx \end{cases}$$

A2

$$x(y+z)=xy+xz$$

$$x+yz=(x+y)\cdot(x+z)$$

A3

$$\begin{cases} x+0=x \\ x\cdot 1=x \end{cases}$$

A4

$$\begin{cases} x+\bar{x}=1 \\ x\bar{x}=0 \end{cases}$$

3) Elementos anuladores

i) $x+1=1$

ii) $x\cdot 0=0$

$\Rightarrow x+1=1$

$$x+1 = \underbrace{x+x}+\bar{x}$$

$$= x+\bar{x}$$

$$= 1$$

Associativa (ñ provamos ainda)

$$x+1 = x+(x+\bar{x})$$

$$= (x+x)+\bar{x}$$

A1

$$\begin{cases} x+y = y+x \\ xy = yx \end{cases}$$

A2

$$\begin{cases} x(y+z) = xy+xz \\ x + yz = (x+y)\cdot(x+z) \end{cases}$$

A3

$$\begin{cases} x+0 = x \\ x \cdot 1 = x \end{cases}$$

A4

$$\begin{cases} x+\bar{x} = 1 \\ x\bar{x} = 0 \end{cases}$$

$$\underline{x+1 = 1}$$

$$\boxed{\begin{aligned} x+1 &\overset{A3}{=} (x+1)\cdot 1 \\ &\overset{A4}{=} (x+1)(x+\bar{x}) \\ &\overset{A2}{=} x+1\bar{x} \\ {\scriptstyle A1+}\ &\overset{A3}{=} x+\bar{x} \\ &\overset{A4}{=} 1 \end{aligned}}$$

$$\underline{x0 = 0}$$

$$x+1 \overset{A3}{=} (x+1)\cdot 1 \overset{A4}{=} (x+1)\cdot(x+\bar{x}) \overset{A2}{=} x + 1\cdot\bar{x} \overset{A1+A3}{=} x+\bar{x} \overset{A4}{=} 1$$

$$x+1=1$$

$$x\cdot 0 \overset{A3}{=} (x\cdot 0)+0 \overset{A4}{=} (x\cdot 0)+(x\cdot\bar{x}) \overset{A2}{=} x\cdot(0+\bar{x}) \overset{A1+A3}{=} x\cdot\bar{x} \overset{A4}{=} 0$$

$$x\cdot 0=0$$

A1

$$\begin{cases} x+y = y+x \\ xy = yx \end{cases}$$

A2

$$\begin{cases} x(y+z) = xy+xz \\ x+yz = (x+y)\cdot(x+z) \end{cases}$$

A3

$$\begin{cases} x+0 = x \\ x\cdot 1 = x \end{cases}$$

A4

$$\begin{cases} x+\bar{x} = 1 \\ x\bar{x} = 0 \end{cases}$$

4) Complemento do 0 e do 1

i) $\bar{0} = 1$

ii) $\bar{1} = 0$

(i) $\bar{0} \overset{A3}{=} \bar{0}+0 \overset{A4}{=} 1$

(ii) $\bar{1} \overset{A3}{=} \bar{1}\cdot 1 \overset{A4}{=} 0$

A1

$$\begin{cases} x+y=y+x \\ xy=yx \end{cases}$$

A2

$$\begin{cases} x(y+z)=xy+xz \\ x+yz=(x+y)\cdot(x+z) \end{cases}$$

A3

$$\begin{cases} x+0=x \\ x\cdot 1=x \end{cases}$$

A4

$$\begin{cases} x+\bar{x}=1 \\ x\bar{x}=0 \end{cases}$$

5) Absorção

(i) $x+xy=x$

(ii) $x(x+y)=x$

(i) $x+xy \overset{A3}{=} x\cdot 1+xy$

$\overset{A2}{=} x(1+y)$

$\overset{A1+P3}{=} x\cdot 1$ → anulador

$\overset{A3}{=} x$

(ii) trivial.