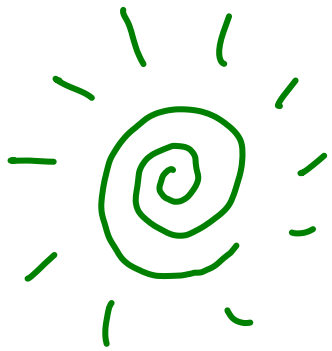


MAC 0329

11 / 05 / 2021



BOM DIA!

Álgebra Booleana

$\langle A, +, \cdot, \bar{}, 0, 1 \rangle$

$$A1. \quad \begin{aligned} x + y &= y + x \\ x \cdot y &= y \cdot x \end{aligned}$$

$$A2. \quad \begin{aligned} x(y + z) &= x \cdot y + x \cdot z \\ x + y \cdot z &= (x + y) \cdot (x + z) \end{aligned}$$

$$A3. \quad \begin{aligned} x + 0 &= x \\ x \cdot 1 &= x \end{aligned}$$

$$A4. \quad \begin{aligned} x + \bar{x} &= 1 \\ x \cdot \bar{x} &= 0 \end{aligned}$$

$$\underline{A1}$$

$$\begin{cases} x+y = y+x \\ xy = yx \end{cases}$$

$$\underline{A2}$$

$$\begin{cases} x(y+z) = xy+xz \\ x+yz = (x+y) \cdot (x+z) \end{cases}$$

$$\underline{A3}$$

$$\begin{cases} x+0 = x \quad \leftarrow \\ x \cdot 1 = x \quad \leftarrow \end{cases}$$

$$\underline{A4}$$

$$\begin{cases} x+\bar{x} = 1 \\ x\bar{x} = 0 \end{cases}$$

1) Unicidade dos 0 e 1

Suponha que 0_1 e 0_2 são tais que

$$\begin{aligned} \text{i) } x+0_1 &= x \\ \text{ii) } x+0_2 &= x \end{aligned} \quad \forall x \in A$$

De (i), $0_2+0_1 = 0_2$ tomando $x=0_2$

De (ii), $0_1+0_2 = 0_1$ tomando $x=0_1$

$$0_2 \stackrel{(i)}{=} 0_2 + 0_1 \stackrel{A1}{=} 0_1 + 0_2 \stackrel{(ii)}{=} 0_1$$

↑ ↑

$$\text{i) } x \cdot 1_1 = x$$

$$\text{ii) } x \cdot 1_2 = x$$

$$\left. \begin{aligned} \Rightarrow 1_2 \cdot 1_1 &= 1_2 \\ \Rightarrow 1_1 \cdot 1_2 &= 1_1 \end{aligned} \right\} 1_2 = 1_2 \cdot 1_1 = 1_1 \cdot 1_2 = 1_1$$

↑ ↑

$$\underline{A1}$$

$$\begin{cases} x+y = y+x \\ xy = yx \end{cases}$$

$$\underline{A2}$$

$$x(y+z) = xy + xz$$

$$x + yz = (x+y) \cdot (x+z)$$

$$\underline{A3}$$

$$\begin{cases} x+0 = x \leftarrow \\ x \cdot 1 = x \leftarrow \end{cases}$$

$$\underline{A4}$$

$$\begin{cases} x + \bar{x} = 1 \leftarrow \\ x \bar{x} = 0 \leftarrow \end{cases}$$

2) Idempotência

$$i) \boxed{x+x = x} \leftarrow$$

$$ii) x \cdot x = x$$

$$x+x = \text{-----} = x$$

$$\underline{x+x} \stackrel{A3}{=} (x+x) \cdot \underline{1}$$

$$\stackrel{A4}{=} (x+x) \cdot (x + \bar{x}) \leftarrow$$

$$\stackrel{A2}{=} x + x\bar{x}$$

$$\stackrel{A4}{=} x + 0$$

$$\stackrel{A3}{=} \underline{x}$$

$$B = \{0, 1\}$$

$$|P(S)| = 2^{|S|}$$

↓

$x + x = x$

←

$$(x+x)\bar{x} = x\bar{x}$$

$$x\bar{x} + x\bar{x} = 0$$

$$0 + 0 = 0$$

$$0 = 0 \quad \vee$$

$x + x =$ → x

$$\underline{A1}$$

$$\begin{cases} x+y = y+x \\ xy = yx \end{cases}$$

$$\underline{A2}$$

$$\begin{aligned} x(y+z) &= xy+xz \\ x+yz &= (x+y) \cdot (x+z) \end{aligned}$$

$$\underline{A3}$$

$$\begin{cases} x+0 = x \quad \leftarrow \\ x \cdot 1 = x \quad \leftarrow \end{cases}$$

$$\underline{A4}$$

$$\begin{cases} x+\bar{x} = 1 \\ x\bar{x} = 0 \end{cases}$$

3) Elemento anuladores

i) $x+1 = 1$

ii) $x \cdot 0 = 0$

$$\Rightarrow x+1 = 1$$

$$\begin{aligned} x+1 &= x+x+\bar{x} \\ &\uparrow \\ &= \underbrace{x+\bar{x}} \\ &= 1 \end{aligned}$$

Associativa
(\bar{x} provamos
ainda)

$$\begin{aligned} x+1 &= x+(x+\bar{x}) \\ &= (x+x)+\bar{x} \end{aligned}$$

A1

$$\downarrow \begin{cases} x+y = y+x \\ xy = yx \end{cases}$$

A2

$$\downarrow \begin{cases} x(y+z) = xy+xz \\ x+yz = (x+y) \cdot (x+z) \end{cases}$$

A3

$$\downarrow \begin{cases} x+0 = x \quad \leftarrow \\ x \cdot 1 = x \quad \leftarrow \end{cases}$$

A4

$$\downarrow \begin{cases} x+\bar{x} = 1 \\ x\bar{x} = 0 \end{cases}$$

$$\underline{x+1 = 1}$$

$$x+1 \stackrel{A3}{=} (x+1) \cdot 1$$

$$\stackrel{A4}{=} (x+1)(x+\bar{x})$$

$$\stackrel{A2}{=} x+1\bar{x}$$

$$\stackrel{A1}{=} x+\bar{x}$$

$$\stackrel{A4}{=} 1$$

$$\underline{x0 = 0}$$

$$\begin{aligned}
 \underline{x+1} & \stackrel{A3}{=} (x+1) \cdot 1 \\
 & \stackrel{A4}{=} (x+1) \cdot (x+\bar{x}) \\
 & \stackrel{A2}{=} x + 1 \cdot \bar{x} \\
 A1+ & \stackrel{A3}{=} x + \bar{x} \\
 & \stackrel{A4}{=} 1
 \end{aligned}$$

$$\begin{aligned}
 \underline{x \cdot 0} & \stackrel{A3}{=} (x \cdot 0) + 0 \\
 & \stackrel{A4}{=} (x \cdot 0) + (x \cdot \bar{x}) \\
 & \stackrel{A2}{=} x \cdot (0 + \bar{x}) \\
 A1+A3 & \stackrel{A3}{=} x \cdot \bar{x} \\
 & \stackrel{A4}{=} 0
 \end{aligned}$$

$$\underline{x+1=1}$$

$$\underline{x \cdot 0 = 0}$$

$$\begin{array}{l} \underline{A1} \\ \left\{ \begin{array}{l} \overline{x+y} = y+x \\ \overline{xy} = yx \end{array} \right. \end{array}$$

$$\begin{array}{l} \underline{A2} \\ \left\{ \begin{array}{l} \overline{x(y+z)} = xy + xz \\ \overline{x + yz} = (x+y) \cdot (x+z) \end{array} \right. \end{array}$$

$$\begin{array}{l} \underline{A3} \\ \left\{ \begin{array}{l} x + 0 = x \quad \leftarrow \\ x \cdot 1 = x \quad \leftarrow \end{array} \right. \end{array}$$

$$\begin{array}{l} \underline{A4} \\ \left\{ \begin{array}{l} x + \bar{x} = 1 \\ x \bar{x} = 0 \end{array} \right. \end{array}$$

4) Complemento de 0 e de 1

$$\begin{array}{l} \text{i) } \overline{\overline{0}} = 1 \\ \text{ii) } \overline{\overline{1}} = 0 \end{array}$$

$$\text{(i) } \overline{\overline{0}} \stackrel{A3}{=} \overline{0} + 0 \stackrel{A4}{=} 1$$

$$\text{(ii) } \overline{\overline{1}} \stackrel{A3}{=} \overline{1} \cdot 1 \stackrel{A4}{=} 0$$

$$\overline{A1} \begin{cases} x+y = y+x \\ xy = yx \end{cases}$$

$$\overline{A2} \begin{cases} x(y+z) = xy+xz \\ x+yz = (x+y) \cdot (x+z) \end{cases}$$

$$\overline{A3} \begin{cases} x+0 = x \quad \leftarrow \\ x \cdot 1 = x \quad \leftarrow \end{cases}$$

$$\overline{A4} \begin{cases} x+\bar{x} = 1 \\ x\bar{x} = 0 \end{cases}$$

5) Absorção

$$(i) x+xy = x$$

$$(ii) x(x+y) = x$$

$$(i) \quad \begin{aligned} x+xy &\stackrel{A3}{=} x \cdot 1 + xy \\ &= \end{aligned}$$

$$\stackrel{A2}{=} x(1+y)$$

$$\stackrel{A1+P3}{=} x \cdot 1 \rightarrow \underline{\text{anulador}}$$

$$\stackrel{A3}{=} x$$

(ii) trivial.