MAC0329

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Bom dia !!!

00101

11010 compl. de 1

Álgebra Booleana

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Álgebra

$\mathbb{R}, +, -, *, /$

$x + y = 0$

$x + y = y + x$

Álgebra Booleana - definição axiomática

$$\langle A, +, \cdot, \bar{\ }, 0, 1 \rangle$$

A1. Comutatividade

$$x + y = y + x$$
$$x \cdot y = y \cdot x$$

A2. Distributividade

$$x \cdot (y + z) = x \cdot y + x \cdot z$$
$$x + y \cdot z = (x + y) \cdot (x + z)$$

A3. Elemento identidade

$$x + 0 = x$$
$$x \cdot 1 = x$$

A4. Complemento

$\exists \bar{x}$ tal que

$$x + \bar{x} = 1$$
$$x \cdot \bar{x} = 0$$

A1 $\begin{cases} x+y=y+x \\ x\cdot y = y\cdot x \end{cases}$ ✓

A2 $\begin{cases} x\cdot(y+z) = x\cdot y + x\cdot z \\ x + y\cdot z = (x+y)\cdot(x+z) \end{cases}$

A3 $\begin{cases} x+0=x \\ x\cdot 1 = x \end{cases}$ ✓

A4 $\begin{cases} x+\bar{x}=1 \\ x\cdot\bar{x}=0 \end{cases}$ ✓

A1 ✓

A3 ✓

A4 ✓

Exemplo

$\langle B, +, \cdot, \bar{\ }, 0, 1\rangle$

$B=\{0,1\}$

a	b	a+b	a·b	$\bar{a}$
0	0	0	0	1
0	1	1	0	1
1	0	1	0	0
1	1	1	1	0

A2 $x\cdot(y+z) = xy + xz$

$x + (y\cdot z) = (x+y)\cdot(x+z)$

$xy \qquad xz \qquad xy+xz$

x	y	z	$x\cdot(y+z)$	$xy+xz$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

$\langle B, +, -, \cdot, 0, 1 \rangle$

$B = \{0, 1\}$

$B^n = \underbrace{B \times B \times \ldots \times B}_{n \text{ vezes}}$ $\qquad B = \{0,1\} \leftarrow \underline{+, \cdot, \bar{\ }}$

$b \in B^n \iff b = (b_1, b_2, \ldots, b_n)$, $b_i \in B$

$\uparrow$

Sejam $a = (a_1, a_2, \ldots, a_n)$, $b = (b_1, b_2, \ldots, b_n) \in B^n$

Vamos definir $+$, $\cdot$, $\bar{\ }$ sobre B^n

$a + b = (a_1, a_2, \ldots, a_n) + (b_1, b_2, \ldots, b_n) = (a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n)$

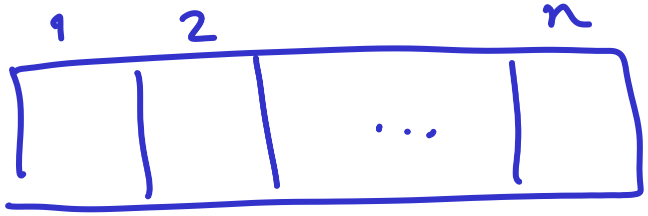
$a \cdot b = (a_1, a_2, \ldots, a_n) \cdot (b_1, b_2, \ldots, b_n) = (a_1 \cdot b_1, \ldots, a_n \cdot b_n)$

$\bar{a} = \overline{(a_1, \ldots, a_n)} = (\bar{a}_1, \bar{a}_2, \ldots, \bar{a}_n)$

$0 = \boxed{(0, \ldots, 0)}$ $\qquad 1 = \boxed{(1, \ldots, 1)}$

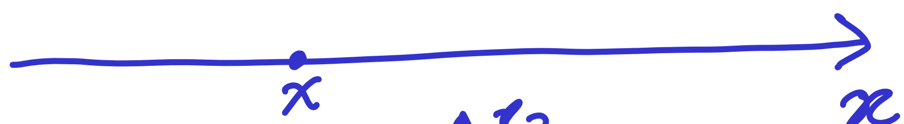
Quantos elementos há no B^n ? Resposta: $\underline{\underline{2^n}}$

$B = \{0,1\}$

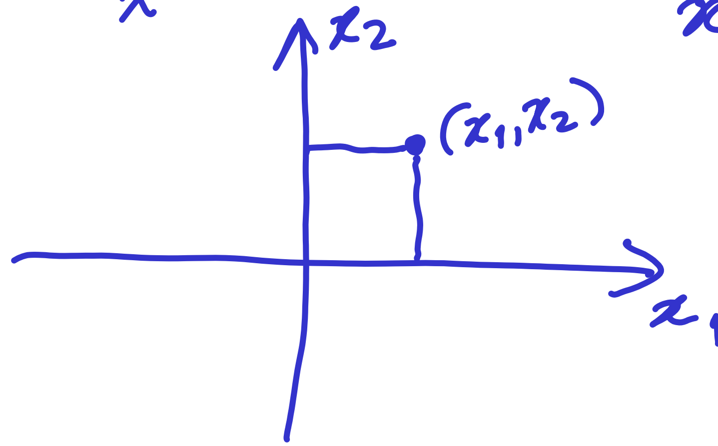


$\mathbb{Z}^2$ $\mathbb{R}^2$

$\mathbb{R}$



$\mathbb{R}^2$



$$B = \{0,1\}$$

$$B^2 = B \times B = \{ (x,y) : x,y \in \underline{\underline{B}} \}$$

$$\boxed{x \mid y}$$

$$\{ (0,0),\ (0,1),\ (1,0),\ (1,1) \}$$

$$\langle \underline{\underline{B}}^n , + , \cdot , \overline{} , 0 , 1 \rangle$$

$$\begin{cases} x+y = y+x \\ x \cdot y = y \cdot x \end{cases}$$

$n=2 \qquad B^n \qquad a=(a_1,a_2)$

$b=(b_1,b_2)$

$$\boxed{\underbrace{(a_1,a_2)+(b_1,b_2)}=(b_1,b_2)+(a_1,a_2)}$$

$$(a_1+b_1,\ a_2+b_2) = (\underline{b_1+a_1},\ \underline{b_2+a_2})$$

B B

$$= (a_1+b_1,\ a_2+b_2)$$

$$(a_1,a_2)+(b_1,b_2) = (a_1 \underline{+} b_1,\ a_2 \underline{+} b_2) = \underline{(b_1+a_1,\ b_2+a_2)}$$

$$=(b_1,b_2)+(a_1,a_2)$$

__Exemplo 3__

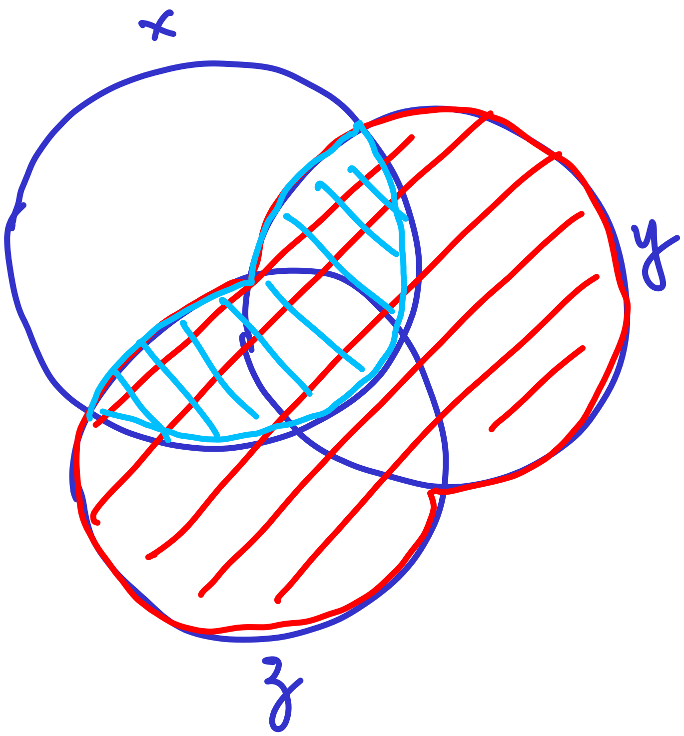
$S = \{a, b, c\}$

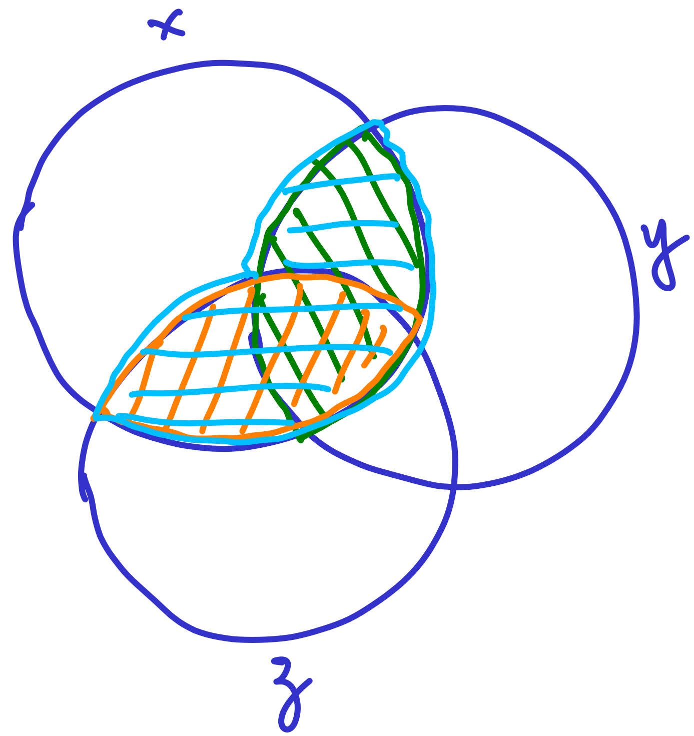
$\mathcal{P}(S) = \{ \{\}, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\} \}$

$\cup, \cap, {}^c$

$$\underbrace{\langle \mathcal{P}(S), \cup, \cap, {}^c, \{\}, S \rangle}_{\text{álgebra booleana}}$$

$$\overset{\cap}{x \cdot} (y \overset{\cup}{+} z) = x \overset{\cap}{\cdot} y \underset{\cup}{\overset{\cup}{+}} x \overset{\cap}{\cdot} z$$





$B = \{0, 1\} \longleftrightarrow P = \{F, V\}$

Proposições

E $\wedge$.

OU $\vee$.

NÃO $\neg$.

$\neg x$

$x \vee y$

$$A1 \begin{cases} x+y=y+x \\ x\cdot y=y\cdot x \end{cases}$$

← pares duais

$$A2 \begin{cases} x\cdot(y+z)=x\cdot y+x\cdot z \\ x+y\cdot z=(x+y)\cdot(x+z) \end{cases}$$

$$A3 \begin{cases} x+0=x \\ x\cdot 1=x \end{cases}$$

$$A4 \begin{cases} x+\overline{x}=1 \\ x\cdot\overline{x}=0 \end{cases}$$

Teorema de DeMorgan

$$\boxed{\overline{x+y}=\overline{x}\cdot\overline{y}}$$

dual

$$\overline{x\cdot y}=\overline{x}+\overline{y}$$