MAC 0329

29/04/21

Bom dia!

$n=4$

Representação binária	Valor em decimal de acordo com as interpretações			
$d_3\,d_2\,d_1\,d_0$	sem sinal	sinal-magnitude	complemento de 1	complemento de 2
0000	0	0	0	0
0001	1	1	1	1
0010	2	2	2	2
0011	3	3	3	3
0100	4	4	4	4
0101	5	5	5	5
0110	6	6	6	6
0111	7	7	7	7
1000	8	-0	-7	-8
1001	9	-1	-6	-7
1010	10	-2	-5	-6
1011	11	-3	-4	-5
1100	12	-4	-3	-4
1101	13	-5	-2	-3
1110	14	-6	-1	-2
1111	15	-7	-0	-1

Adição $n=2$

00	00	00	00
00	01	10	11
00	01	10	11
0+0=0	0+1=1	0+2=2	0+3=3

2 1 0

	1		$\boxed{1}$1
01	01	01	01
00	01	10	11
01	10	11	00
1+0=1	1+1=2	1+2=3	1+3=0

		$\boxed{1}$	$\boxed{1}$
10	10	10	10
00	01	10	11
10	11	00	01
2+0=2	2+1=3	2+2=0	2+3=1

	$\boxed{1}$1	$\boxed{1}$	$\boxed{1}$1
11	11	11	11
00	01	10	11
11	00	01	10
3+0 =3	3+1=0	3+2=1	3+3=2

00	00	00	00
00	01	10	11
00	01	10	11
0+0=0	0+1=1	0+(-2)=-2	0+(-1)=-1

	$\boxed{1}$		1 1
01	01	01	01
00	01	10	11
01	10	11	00
1+0=1	1+1=-2	1+(-2)=-1	1+(-1)=0

		$\boxed{1}$	$\boxed{1}$
10	10	10	10
00	01	10	11
10	11	00	01
(-2)+0=-2	(-2)+1=-1	(-2)+(-2)=0	(-2)+(-1)=1

	1 1	$\boxed{1}$	1 1
11	11	11	11
00	01	10	11
11	00	01	10
(-1)+0=-1	(-1)+1=0	(-1)+(-2)=1	(-1)+(-1)=-2

Subtraction

$$A - B = \underline{A + (-B)}$$

$$\downarrow$$

$$-B = ?$$

$$-B = \bar{B} + 1$$

Exemplo $(n=4)$

$$\boxed{B = 0111} \rightsquigarrow 7_{(10)}$$

$$-B = \bar{B} + 1 = 1000 + 1 = \boxed{\boxed{1}\underbrace{001} = -B}$$

1: negativo

$$\begin{array}{r} 110 \\ 1 \\ \hline 111 \end{array} \rightsquigarrow 7_{(10)}$$

$-7_{(-10)}$

$$A = 1001 \rightsquigarrow -7_{(10)}$$

$$-A = \bar{A} + 1 = 0110 + 1 = 0111 \rightsquigarrow 7_{(10)}$$

$$B \qquad -B = ?$$

$$B - B = 0$$

$$B + (-B) = 0$$

$$B = 0111 \rightsquigarrow \overline{B} = 1000$$

$$B + \overline{B} = 1111$$

$$\overline{B} + 1 \rightsquigarrow -B$$

$$\underbrace{B} + \underbrace{\overline{B} + 1} = 1\boxed{0000}$$

sobrando

Base 10 $\quad n = 2 \quad 0 \rightsquigarrow 9$

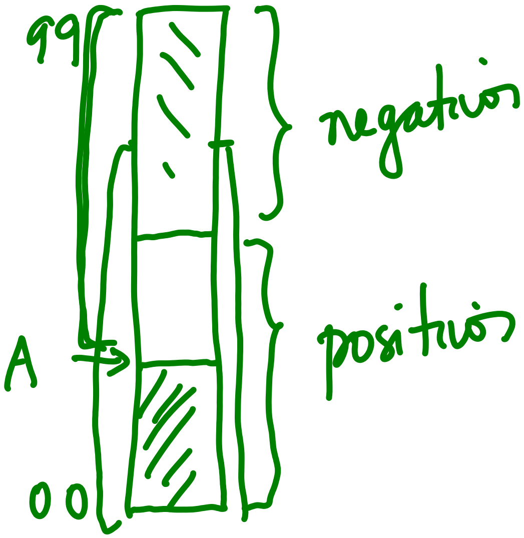
$\longrightarrow 00$ a 99

$A = 34 \qquad -A = ?$

$\bar{A} = 65$

$A + \bar{A} = 99$

$\underbrace{A} + \underbrace{\bar{A} + 1} = 1\underbrace{00}_{0} \Longrightarrow \bar{A} + 1 \rightsquigarrow -A$



Subtração

$$A - B = A + (-B)$$

$$= A + (\overline{B} + 1)$$

vai-um na coluna 0

$$\begin{array}{r} {}^{1} \\ A \\ + \ \overline{B} \\ \hline \end{array}$$

Subtração $n=2$

$A - B = A + (-B) = A + (\bar{B} + 1)$

$00 \to 0$
$01 \to 1$
$10 \to -2$
$11 \to -1$

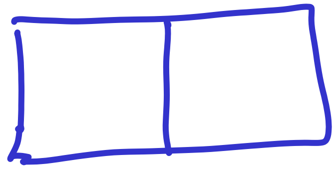
111 00 11 00 $0+(-0)=0$	1 00 10 11 $0+(-1)=3$	11 00 01 10 $0+(-2)=2$	1 00 00 01 $0+(-3)=1$
111 01 11 01 $1+(-0)=1$	111 01 10 00 $1+(-1)=0$	11 01 01 11 $1+(-2)=3$	11 01 00 10 $1+(-3)=2$
111 10 11 10 $2+(-0)=2$	1 1 10 10 01 $2+(-1)=1$	111 10 01 00 $2+(-2)=0$	1 10 00 11 $2+(-3)=3$
111 11 11 11 $3+(-0)=3$	111 11 10 10 $3+(-1)=2$	111 11 01 01 $3+(-2)=1$	111 11 00 00 $3+(-3)=0$

sem sinal

111 00 11 00 $0+(-0)=0$ 0-0	1 00 10 11 $0+(-1)=-1$ 0-1	11 00 01 10 $0+(-(-2))\neq-2$ 0-(-2)	1 00 00 01 $0+(-(-1))=1$ 0-(-1) 0+(-(-1))
111 01 11 01 $1+(-0)=1$ 1-0	111 01 10 00 $1+(-1)=0$ 1-1	11 01 01 11 $1+(-(-2))\neq-1$	11 01 00 10 $1+(-(-1))\neq-2$
111 10 11 10 $(-2)+(-0)=-2$ -2-0	1 1 10 10 01 $(-2)+(-1)\neq1$ -2-1	111 10 01 00 $(-2)+(-(-2))=0$	1 10 00 11 $(-2)+(-(-1))=-1$
111 11 11 11 $(-1)+(-0)=-1$ -1-0	111 11 10 10 $(-1)+(-1)=-2$ -1-1	111 11 01 01 $(-1)+(-(-2))=1$	111 11 00 00 $(-1)+(-(-1))=0$

0, 1, -2, -1

compl. 2



2^2

0 0	$\longrightarrow$	0	0
0 1	$\longrightarrow$	1	1
1 0	$\longrightarrow$	2	-2
1 1	—	3	-1

0 a 3

-2 a 1

$$A - B = A + (-B)$$

$$\boxed{A - B = A + (\overline{B} + 1)}$$

	compl. 2
00	0 ✓
01	1
10	-2 ✓
11	-1

$$A = -2$$

$$B = -2$$

$$A - B = \underline{(-2) - (-2)}$$

$$= -2 + 2 = 0$$

$$A + (-B) \longrightarrow$$

$$\begin{array}{rr} & 1\ \\ A \rightsquigarrow & 10 \\ -B \rightsquigarrow & 10 \\ \hline & 1\boxed{00} \end{array}$$

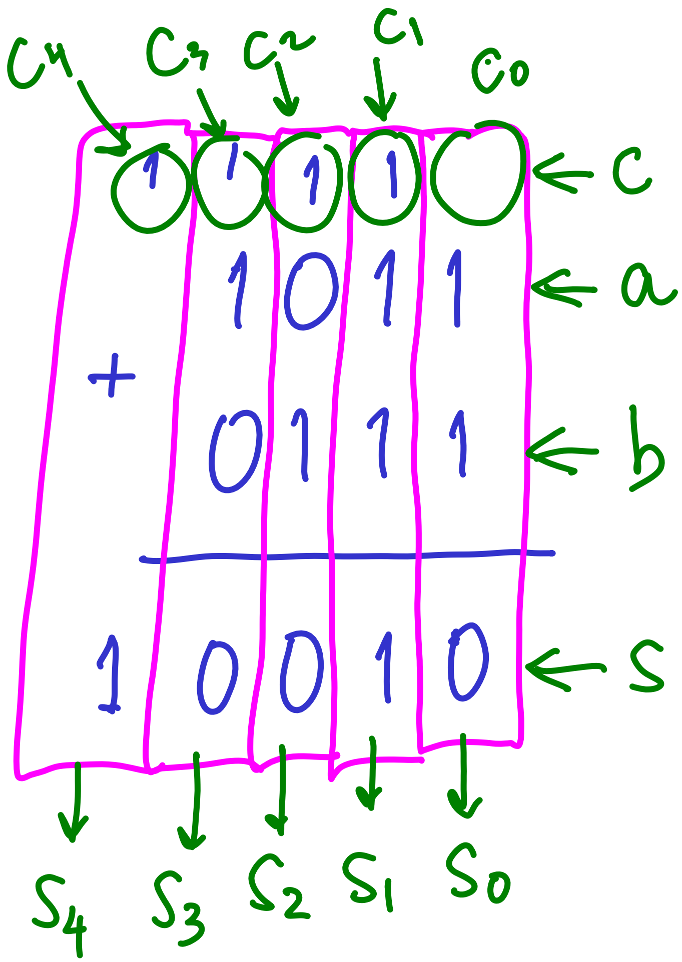
$$0$$

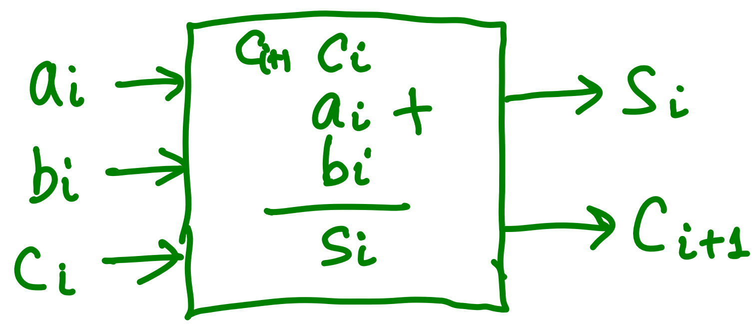
$$A + (\overline{B} + 1)$$

$$\begin{array}{rr} & 111 \\ A \rightsquigarrow & 10 \\ \overline{B} & 01 \\ \hline & 1\boxed{00} \end{array}$$

$$-B = \overline{B} + 1 = 01 + 1 = 10$$

$$\begin{array}{r} 1\ \\ 01 \\ 1 \\ \hline 10 \end{array}$$





Somador de bits.

Como implementar isso?

a_i	b_i	c_i	s_i	c_{i+1}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$s_i = f_1(a_i, b_i, c_i)$$

$$c_{i+1} = f_2(a_i, b_i, c_i)$$

$$f_1, f_2 \in \{0,1\}$$

a_i	b_i	c_i	s_i	c_{i+1}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$S_i = f_1(a_i, b_i, c_i)$$

$$\boxed{S_i = (a_i + b_i + c_i) \% 2}$$

resto da divisão

$$c_{i+1} = (a_i + b_i + c_i) // 2$$

parte inteira da divisão.

a_i	b_i	c_i	s_i	c_{i+1}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$S_i = 1 \iff$

$(a=0 \text{ e } b=0 \text{ e } c=1)$

ou

$(a=0 \text{ e } b=1 \text{ e } c=0)$

ou

$(a=1 \text{ e } b=0 \text{ e } c=0)$

ou

$(a=1 \text{ e } b=1 \text{ e } c=1)$

a_i	b_i	c_i	S_i	C_{i+1}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$\boxed{\underset{1}{\bar{a}}\ E\ \underset{1}{\bar{b}}\ E\ \underset{1}{c}}$ ← Valor 1

$a = 0$ e $b = 0$ e $c = 1$

x	y	x E y	x OU y	NÃO x ($\bar{x}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

a_i	b_i	c_i	s_i	c_{i+1}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$S_i = 1 \iff$ ( ) ou ( ) ou ( ) ou ( )

$(\bar{a} \text{ E } \bar{b} \text{ E } c)$

ou $\rightarrow$ +

$(\bar{a} \text{ E } b \text{ E } \bar{c})$

ou

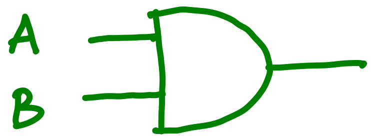
$(a \text{ E } \bar{b} \text{ E } \bar{c})$

ou

$(a \text{ E } b \text{ E } c)$

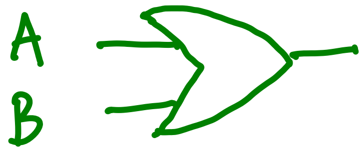
$\bar{a} \cdot \bar{b} \cdot c + \bar{a} b \bar{c} + a \bar{b} \bar{c} + abc$

E



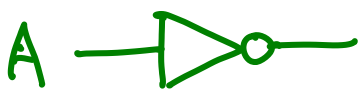
AB (A E B)

OU



$A+B$ (A OU B)

NÃO



$\overline{A}$ (NÃO A)