Set-Theoretic Methods for the Social Sciences

A Guide to Qualitative Comparative Analysis

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Qualitative Comparative Analysis (QCA) and other set-theoretic methods distinguish themselves from other approaches to the study of social phenomena by using sets and the search for set relations. In virtually all social science fields, statements about social phenomena can be framed in terms of set relations, and using set-theoretic methods to investigate these statements is therefore highly valuable. This book guides readers through the basic principles of set-theory and then on to the applied practices of QCA. It provides a thorough understanding of basic and advanced issues in set-theoretic methods together with tricks of the trade, software handling, and exercises. Most arguments are introduced using examples from existing research. The use of QCA is increasing rapidly and the application of set theory is both fruitful and still widely misunderstood in current empirical comparative social research. This book provides the comprehensive guide to these methods for researchers across the social sciences.

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Set-Theoretic Methods for the Social Sciences

A Guide to Qualitative Comparative Analysis

Carsten Q. Schneider
and
Claudius Wagemann

In honor of Philippe Schmitter, wise advisor, generous colleague, and good friend.

Dedicated to Sheila, Giulia, and Leo, without whom this book would have been finished much sooner.

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Acknowledgements

Last but not least, we owe deep gratitude to our families and friends for their patience and for listening (or at least pretending to) when diverting dinner conversations into discussions on the pros and cons of set relations, untenable pregnant men, and the asymmetries of the world we live in.

Introduction

Easy reading guide

The Introduction presents an overview of the book. We spell out what this book contains, what it is good for – and what it is not! Rather than starting with technical details of set-theoretic methods, we put the content of the book into a broader context of current methodological debates. The Introduction will help the reader to find out whether, in general, this book might be interesting and, if so, which chapters in particular are most relevant for him/her.

In a first step, we show that notions of sets and their relations are more common in the social sciences than might probably be known. Then we describe Qualitative Comparative Analysis (QCA) as the most developed form of set-theoretic method. We spell out the defining features of QCA and how they differentiate it from other set-theoretic methods. In the next section, we explain the differences and similarities among the various forms of QCA. In the following section, we not only explain the structure of the book, but also provide details on how to use the book by addressing some of its features, such as the Easy reading guides, At-a-glance boxes, the Glossary, or the online material which contains chapter-by-chapter "how-to" sections and exercises.

In short, by reading this Introduction, readers should get a better understanding of what to expect from this book and how to use it in order to maximize its utility.

Set-theoretic approaches in the social sciences

Arguments about set relations are pervasive in the social sciences, but this is not always obvious. Take, for example, Brady's (2010) intriguing deconstruction of the widely debated claim that, in the 2000 US Presidential Election, George W. Bush lost about 10,000 votes because Al Gore had been declared the winner before the closure of the polling stations in those western counties of Florida that are on Central Standard Time (i.e., the Panhandle). This claim is made by Lott (2000), who arrived at this inference by estimating a "'difference-in-differences' form of regression analysis, based on data-set

observations" (Brady 2010: 238). Using causal-process observations, Brady cogently shows that this inference is "highly implausible" (241) and that, instead of 10,000 lost voters, a more adequate estimate would be a maximum of 224 or, even more realistically, 28 to 56 voters (NB: total voters, not percentage!). Brady successfully frames his debate of Lott as an argument in favor of causal-process observations – "diagnostic 'nuggets' of data that make a strong contribution to causal inference" (Brady 2010: 237).

Brady's argument is set-theoretic in nature (Goertz and Mahoney 2012). In essence, he claims that the set of *voters not voting for Bush due to the premature announcement of Gore as the winner* (Y) can only be very small because membership in this set requires simultaneous membership in several other sets. Such allegedly lost Bush voters must, of course, also be members of the set of *registered voters in the Panhandle counties* (P), who are also members of the set of voters who had *not yet voted* (V), and the set of voters who had *received the news through the media* (M). Using plausible arguments about the rough percentage of voters that tend to vote late and the percentage of voters listening to the media, Brady shows that the sets of P, V, and M are small and that, as a direct consequence of this, the set of Y must be even smaller. This is because membership in each of the three sets P, V, and M is necessary in order to be a member of set Y (Goertz and Mahoney 2012: 54–56).

This example illustrates that many arguments in the social sciences can be (re-)framed in terms of relations between sets. The notion of sets is not explicitly invoked in Brady's original analysis, and there is nothing wrong with this. We do claim, however, that an explicit framing of arguments in terms of set relations is often adequate and that, once set relations are invoked, set-theoretic methods provide a powerful toolkit for such analyses.

Different mathematical sub-disciplines provide the underpinnings for the vast majority of social science methods and techniques. Most of the well-known and commonly applied statistical methods in the social sciences are applications of probability calculus or matrix algebra to social science data. While most of these mathematical sub-disciplines might be remembered from school, set theory is less familiar to most people. Although formal logic, a close relative of set theory, is a well-studied system of thought in disciplines such as philosophy and mathematics, it currently plays only a marginal role in school education and social science methods training in many parts of the world. This is unfortunate, because, as shown, set-theoretic notions are invoked in social science research more often than is usually recognized. The notion of sets and their relations is almost unavoidably invoked when

forming concepts or when verbally formulating (causal) relations between social phenomena. This book is motivated by the belief that the study of set-theoretic relationships provides an important perspective on social science research problems, thus adding to the currently predominant correlational approaches.

What are set-theoretic methods? Implicitly or explicitly, they all share three features: first, they work with membership scores of cases in sets; second, they perceive relations between social phenomena as set relations; third, these set relations are interpreted in terms of sufficiency and necessity, as well as forms of causes that can be derived from them, such as so-called INUS and SUIN conditions (section 3.3.2). Let us discuss these three points individually.

First, the data on which set-theoretic methods operate are membership scores of cases in sets which represent social science concepts. For instance, France is an element of the set of *European Countries* whereas the USA is not. France's set membership score in this set is therefore 1, while that of the USA is 0. When we invoke the notion of sets, it might seem unavoidable that we perceive them as dichotomies. This is not the case, though. Even an apparently straightforward dichotomous concept such as the set of *European Countries* might not be clearly dichotomous at all – just think of the case of Turkey and the discussion it triggers about where the (geographic, cultural, economic, military, etc.) boundaries of Europe are.[1] In fact, for many social science concepts, it is difficult to perceive them as clear dichotomies, or *crisp sets*, in which cases can be assigned full (non-)membership scores. Luckily, set theory can go beyond crisp sets. In its *fuzzy set* version, it also allows for partial set membership. Cases are not forced to be either full members of the set of European countries, or full non-members of it, but can also be partial members. A case like Turkey would receive a partial (or fuzzy-set) member-ship score lower than 1 and higher than 0 in the set European countries. This fuzziness does not derive from imprecise *empirical* information about the case of Turkey – we can gather very detailed information of its geographical location, economic structure, etc. Instead, fuzziness stems from non-sharp *conceptual* boundaries inherent in the notion of European country. Virtually

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[1] Even concepts which most clearly seem to be dichotomous can be problematic. Just think about EU membership, about which we would think that it is clearly dichotomous. However, on closer examin-ation we see differences on some of the aspects that we would use to determine crisp set membership; for example, the UK is neither a member of the Schengen Protocol nor uses the euro. As such we might want to see the UK as a qualitatively different type of member than say, Luxembourg or Germany. Likewise, Switzerland is not a formal member of the EU and yet it adopts a huge share of European legislation, frequently word-for-word (Kux and Sverdrup 2000: 251), something which other non-members (such as India, the Ivory Coast, or Samoa) do not do.

all social science concepts have fuzzy boundaries, and fuzzy sets are a tool for numerically expressing that.

The second trait shared by all set-theoretic methods in the social sciences is that relations between social phenomena are perceived of as set relations. Take, for example, the empirical observation that all NATO members are democracies. Although it might not be obvious, this is a clear-cut example of a set-theoretic statement. The verbal descriptions "NATO members" and "democratic countries" both represent sets in which different cases have different membership scores. If we observe further that all NATO members are democracies, but that not all democracies are NATO members (think of Sweden or Japan), then the set of NATO members is a subset of the set of democratic countries. This, in turn, implies that the set of democratic countries is a superset of the set of NATO members.

This simple recasting of social science phenomena in terms of set relations might not seem very inspiring on its own, and it might rather come across as a simple play on words. This rephrasing, however, gains great analytic potential once we understand that subset relations are intimately linked to the ideas of sufficiency and necessity. This is the third aspect of set-theoretic methods: set relations are usually interpreted in terms of sufficient or necessary conditions, or of their more complex modifications INUS and SUIN,[2] either in a causal or a descriptive manner. Applied to our example, we can conclude that being a democracy is a necessary condition for being a NATO member, for the latter is a subset of the former. Statements about conditions being either necessary or sufficient abound in the social sciences. Gary Goertz, one of the pioneers in the empirical study of necessary conditions, counts not fewer than 150 hypotheses about necessary conditions in the field of international relations alone (Goertz 2003). Hypotheses about sufficient conditions are at least as widespread (Ragin 2000). However, often we do not recognize these claims immediately, since they are frequently hidden in verbal formulations that do not explicitly use the terms *necessity* or *sufficiency* (Mahoney 2004).

Suppose we claim that "Citizens of small, rural towns in the USA vote for the Republican Party." This relationship denotes a subset relation. The set of all small-town, rural voters (X) is a subset of all Republican voters (Y). This means that all cases which exhibit X (i.e., voters living in small, rural towns)

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[2] INUS stands for Insufficient but Necessary part of a condition which is itself Unnecessary but Sufficient for the result (Mackie 1965: 246). SUIN instead stands for Sufficient but Unnecessary part of a factor that is Insufficient but Necessary for the result (Mahoney, Kimball, and Koivu 2009: 126). As we will describe in this book, both forms of causal factors represent advanced forms of causal complexity and refer to components that do not count as necessary or sufficient conditions when taken alone, but which play a subtle causal role in intricate combinations with other factors.
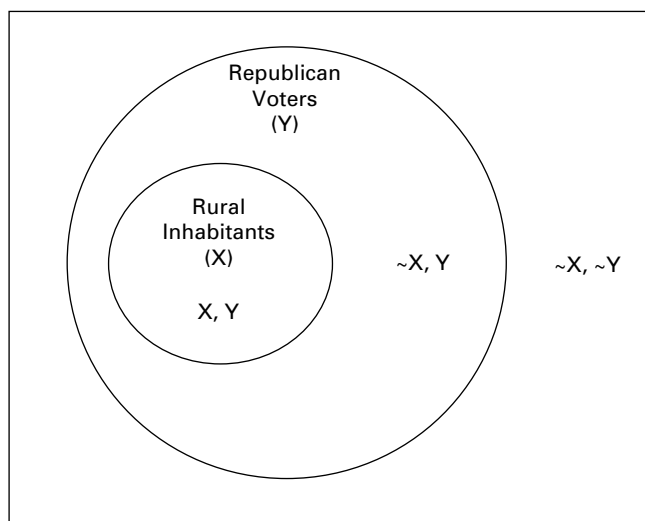
Figure 0.1   Venn diagram for relation of sufficiency

also exhibit Y (they cast their ballot for the Republican Party). This denotes the inner circle (X, Y) in the Venn diagram in Figure 0.1.

As we will learn throughout this book, such a pattern in the data can be interpreted to mean that X is sufficient for Y. Note that this statement does not tell us anything about the voting behavior of citizens *not* living in small, rural towns in the USA. They might be Republican voters (area ~X, Y) or they might not (~X, ~Y). Nor does the sufficiency claim entail that all voters for the Republican Party are living in small, rural towns. There are, of course, many non-rural voters of the Republican Party, as indicated by area ~X, Y of Figure 0.1. The point is, however, that such voters are irrelevant when it comes to corroborating the claim that living in a rural town is sufficient for voting for the Republican Party. The fact that there are other types of voters for the Republican Party simply indicates that there are other sufficient conditions for voting for the Republicans.

The intimate link between subset relations and the notions of necessity and sufficiency triggers several analytic consequences. For instance, saying that there is a sufficient (but not necessary) condition generally requires the existence of other sufficient conditions for the same outcome. This, in turn, means that by embracing a set-theoretic perspective on social science phenomena one unavoidably recognizes the existence of *equifinality*, i.e., a scenario in which alternative factors can produce the same outcome. Also, more often than not, in order to find perfect set relationships, one might

need to refer to combinations of various sets, where single conditions do not display their effect on their own, but only together with other conditions. For instance, it might be that only the set of young male inhabitants of rural towns vote Republican. Set theory is therefore also closely linked to the notion of *conjunctural causation*. Further, combining equifinality and conjunctural causation automatically implies the existence, and causal relevance, of the much-discussed INUS and SUIN conditions.

Yet another aspect of set theory consists of the *asymmetry* of concepts and causal relations. A set-theoretic perspective on concepts requires two separate definitions and operationalizations of concepts that in non-set-theoretic approaches are often not distinguished (Goertz and Mahoney 2012: chs. 9–13). For instance, an autocracy is not simply the opposite of a democracy. Richness is not simply the opposite of poverty. Consider, for instance, college students who are usually not "rich," but their non-membership in the set of rich persons does not imply that they are "poor." From this follows that we need two different sets to capture the two qualitatively different states of being rich and being poor. In most social science approaches, however, only one indicator is used – say, monthly disposable income – and the degree of richness (high or low, with low values on the richness scale being equal to poorness) inferred from this. The causal interpretation of asymmetry is that the explanation for the non-occurrence of the outcome cannot automatically be derived from the explanation for the occurrence of the outcome. For example, when trying to explain the conditions for successful democratization, we most likely will need to consider quite different conditions than a study that tries to understand failed democratization. In set-theoretic methods, there usually is no symmetry between the combinations of conditions for the occurrence of the outcome and its non-occurrence. This is a major difference from standard correlational methods (see also 3.3.3). We thus define set-theoretic methods as follows:

Set-theoretic methods are approaches to analyzing social reality in which (a) the data consists of set membership scores; (b) relations between social phenomena are modeled in terms of set relations; and (c) the results point to sufficient and necessary conditions and emphasize causal complexity in terms of INUS und SUIN causes.

Set-theoretic methods often come under different labels. They are sometimes called "Boolean methods" (Caramani 2009) or "logical methods" (Mill 1843). Rihoux and Ragin (2009) have coined the term "Configurational Comparative Methods" (CCM) in an attempt to find a name for a group of similar methods. By choosing the acronym CCM, they emphasize a feature that is shared

by all set-theoretic methods: they all understand the world in terms of configurations of conditions. We prefer the term set-theoretic methods because it is more encompassing and emphasizes the core analytic fact that all of them model social reality in terms of set-theoretic relations. It is the set-theoretic foundation from which all other features of this family of methods derive.

The use of set theory in the social sciences is not as new as it might seem. A closer look reveals that it provides the underlying logic for many, mostly qualitative approaches in the social sciences. As Mahoney notes, many comparative case-study approaches apply a set-theoretic reasoning in an informal and intuitive manner (Mahoney 2007: 135). One example for this is concept formation. If, for instance, we define a concept as the simultaneous presence of several phenomena – say, the concept of democracy being defined as the simultaneous presence of free elections and civil liberties – then we make use of set-theoretic logic: the set of all democracies is represented by the intersection of the set of countries that display free and fair elections with the set of countries that display civil liberties. Put differently, these are individually necessary and jointly sufficient elements of democracy. As Goertz (2006a) shows, adopting a set-theoretic perspective on concept definitions is often more in line with the underlying linguistic meaning conveyed by those definitions and also triggers important consequences for the data aggregation procedure. Rather than adding or averaging information across different dimensions of a concept, a set-theoretic perspective looks at necessary and sufficient components of a concept in order to maintain a strong link between the verbal meaning of a concept and its numerical representation. Ignoring this can lead to a severe misfit between the meaning of a concept and its operationalization. In our example, averaging the two indicators of free elections and civil liberties would mean that a totally illiberal country that happens to hold free elections would count as a half-democracy, whereas the set-theoretic approach would classify it as a non-democracy.

Set theory also provides a fruitful perspective on the creation of typologies (Elman 2005; George and Bennett 2005: ch. 11). Typologies can be seen as concepts for which information is not aggregated into a unidimensional scale of set membership (e.g., all countries being ranked in a way that represents their degree of membership in the concept of democracy), but where cases are classified on multiple dimensions. The example of the welfare state can help us to illustrate this point: countries differ not only in the (unidimensional) *degree* to which they provide welfare to their citizens but also in the (multidimensional) *type* of welfare state they have developed for this purpose. If, for the sake of illustration, we postulate that welfare states vary along two

dimensions – labor market protection and transfer payments – then there are four different ideal-typical forms of the welfare state: high labor market protection with high transfer payments; high labor market protection with low transfer payments; low labor market protection with high transfer payments; and low labor market protection with low transfer payments. As Kvist (2006) shows, a set-theoretic approach to forming and arguing about typologies can be very helpful, especially if we – as Kvist does – go beyond dichotomous (crisp) sets and work with fuzzy sets in which cases can have degrees of membership in each dimension.

Notions of set theory are also useful for those more ambitious social science practices that are designed to give a causal interpretation to patterns found in the data. Prominent examples are John Stuart Mill's methods (see, e.g., Mahoney 2003). The possibility of interpreting them in a set-theoretic manner is an aspect that has not received enough attention so far (Mahoney 2007: 134).

At-a-glance: set-theoretic approaches in the social sciences

Set-theoretic methods operate on membership scores of elements in sets; causal relations are modeled as subset or superset relations; **necessity**[3] and **sufficiency** or **INUS** and **SUIN** conditions are at the center of attention.

The use of set theory focuses attention on unraveling causally complex patterns in terms of **equifinality**, **conjunctural causation**, and **asymmetry**.

Set theory can be useful for concept formation, the creation of typologies, and causal analysis.

Qualitative Comparative Analysis as a set-theoretic approach and technique

Qualitative Comparative Analysis, commonly known under its acronym QCA, is the methodological tool that is perhaps most directly associated with set theory. QCA distinguishes itself from other set-theoretic approaches by the combined presence of the following features. First, it aims at a causal interpretation. This is not necessarily true for other set-theoretic approaches – just think of concept formation or the creation of typologies, which typically do not include any reference to an outcome (for two exceptions, Elman 2005 and George and Bennett 2005). Second, QCA makes use of so-called truth tables.

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[3] All the terms that are further defined in the Glossary are printed in bold in the At-a-glance boxes.

This allows researchers to visualize and analyze central features of causal complexity, such as equifinality or conjunctural causation and the presence of INUS or SUIN conditions. Other set-theory based methods, such as Mill's methods or set-theory-based historical explanations (Mahoney, Kimball, and Koivu 2009), do not employ truth tables. Third, QCA approaches make use of the principles of logical minimization, a process by which the empirical information is expressed in a more parsimonious yet logically equivalent manner by looking for commonalities and differences among cases that share the same outcome. With a few exceptions (see Elman 2005), logical minimization does not play a role in the set-theoretic literature on typological theories (George and Bennett 2005: ch. 11); if it does, it is usually performed in an intuitive rather than formalized manner.

Large sections of this book are dedicated to explaining QCA, for it is arguably the most formalized and complete set-theoretic method. It requires more of a proper and systematic introduction in basic concepts from formal logic, set theory, and Boolean algebra than other set-theoretic methods. In addition, QCA can, and should, be performed with the help of specialized computer software. Related to this is the fact that most, if not all, other set-theoretic approaches can be interpreted as either specializations or extensions of specific elements of QCA. For instance, the use of set theory for classifying cases in multidimensional typologies can be interpreted as a specialized QCA without an outcome and thus without any causal interpretation. Yet other set-theoretic approaches are extensions of QCA. For instance, standard QCA has only indirect ways of including time as a causally relevant dimension into the analysis. Partially in response to this, Mahoney, Kimball, and Koivu (2009) have elaborated the conceptual foundations for combining historical explanations and set-theoretic reasoning. Similarly, Caren and Panofsky (2005) and Ragin and Strand (2008) have made specific suggestions for extending the QCA algorithm by allowing the order of events to matter causally. In short, by learning about the principles and practice of QCA, readers will learn about set-theoretic methods at large.

Figure 0.2 provides a graphical overview of our understanding of the different set-theoretic approaches in the social sciences and their relation to some other empirical comparative approaches. It shows that the umbrella term of set-theoretic methods covers several prominent and less prominent approaches to studying social reality. And QCA is just one of them.

The idea of making use of set theory for the interpretation and analysis of social science data in QCA has been put forward by the American social scientist Charles C. Ragin (1987, 2000, 2008). Interest in QCA has grown in
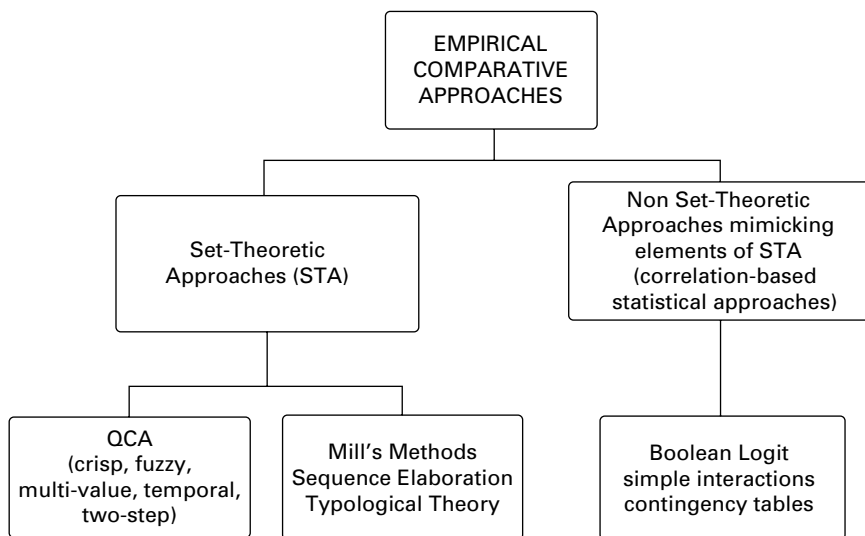
Figure 0.2 Set-theoretic approaches in the social sciences

recent years as comparative social science has revived fundamental debates on empirical social science methodology (e.g., King, Keohane, and Verba 1994; Gerring 2001, 2007; Brady and Collier 2004, 2010; or George and Bennett 2005; Gerring 2012; Goertz and Mahoney 2012). In this debate, QCA is often presented as a third way between quantitative statistical techniques and case-study methodology. By putting so much emphasis on QCA as a hybrid method that would, supposedly, combine the best of two worlds, and by focusing on the related claim that QCA is a method designed for analyzing mid-sized (that is, medium-N) datasets, its distinct characteristic as a set-theoretic method is often less widely recognized than it should be. As a matter of fact, in the early days, QCA's set-theoretic foundation was downplayed even by its inventor itself: Ragin's 1987 book, widely seen as the foundational work for QCA, does not mention set theory at all. All his later books have the term "set" in the title, though. Approaching QCA from a set-theoretic perspective has the double advantage of being able to explain its analytic features in a succinct manner and to unravel the fact that, contrary to widely held beliefs, QCA is not really a method invented *ex novo*, but makes use both of an established subfield in mathematics and of principles and practices well known in social science methodology.

Set-theoretic methods have a close affinity to case-oriented comparative approaches. As such, they cannot be seen only as data analysis techniques.

Rather, the process of data gathering and generating is an integral part of set-theoretic approaches. This is also true for QCA: it is not only a data analysis technique, but also a research approach (Rihoux and Ragin 2009).[4] QCA as a research approach refers to the processes before and after the analysis of the data, such as the (re-)collection of data, (re-)definition of the case selection criteria, or (re-)specification of concepts, often based on preliminary insights gained through QCA-based data analysis. Ragin (1987) refers to this process as the back-and-forth between ideas and evidence (see also Rihoux and Lobe 2009). In fact, most of the time and energy in good QCA-based research is devoted to issues related to QCA as an approach. QCA as a technique – sometimes also labeled the "analytic moment" (Ragin 2000) – is, in contrast, considerably less time-consuming. QCA as a data analysis technique refers to the data analysis based on standardized algorithms and the appropriate software. In essence, this analysis consists of finding (combinations of) conditions that are subsets or supersets of the outcome and thus to arrive at sufficient and necessary (or INUS or SUIN) conditions. In order to find such set relations, QCA relies on so-called truth tables and straightforward rules of logical minimization.

It might sound obvious, perhaps even trivial, to underline that social science method competence should mean more than to know which buttons on the computer keyboard need to be pressed in order to generate "results." Deeper methodological awareness gains particular importance in the case of QCA (and set-theoretic methods in general) not least since these methods, by and large, operate outside the statistical template. One consequence of this is that measures of uncertainty tend to be less standardized and more closely related to judgments by the individual researcher than in statistical methods. The researchers' insights, and the knowledge they have acquired before and after the analytic moment, are therefore crucial for making the results of the analysis both robust and plausible. Such knowledge is particularly needed if the aim consists of drawing causal inferences.

We very much believe in the importance of perceiving QCA as both an approach and a technique and highlight this importance throughout the book. However, as is almost unavoidable for a textbook, our focus will be more on the technical aspects of QCA. It is here where QCA deviates most from the predominant practices of analyzing data in the social sciences. In contrast to the technical aspects, the requirements for performing QCA as

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[4] On the difference between the notion of research design and data analysis, see also Gerring (2012: 78ff.).

an approach are less QCA-specific, but rather coincide largely with what is preached (and practiced) in good case-oriented comparative approaches (Mahoney and Rueschemeyer 2003; George and Bennett 2005; Gerring 2007; Rohlfing 2012). In Chapter 11 we explicitly deal with issues that arise when QCA as a technique meets QCA as an approach, such as the question of how to select cases for in-depth case studies after a QCA or how to evaluate empirical results obtained with QCA in the light of existing theoretical knowledge.

Before moving on to distinguishing between different variants of QCA, we wish to address the issue of how researchers should justify the use of QCA. In short, we believe that the decision to apply set-theoretic or other methods should be guided by the goal of achieving a good fit between theories and research aims on the one hand, and the method-specific assumptions on the other. Or, as Hall (2003) puts it, the aim should be to achieve a good fit between ontology and methodology. If there are good reasons to believe that the phenomenon of interest is best understood in terms of set relations, then this represents a strong argument for the use of set-theoretic methods such as QCA. If no such reasons exist, non-set-theoretic methods are more appropriate. Somewhat surprisingly, however, in applied QCA, researchers usually use an empirical argument for justifying the use of QCA by pointing out that it is better suited than either standard statistical approaches or comparative case studies for datasets with an intermediate number of cases. Such a mid-sized N is usually defined as being somewhere between 10 and 50 cases (Ragin 2000: 25). We certainly acknowledge that the mid-sized N argument also has its merits. For one, QCA does work in such settings and populations of interest with an N between 10 and 50 cases occur very frequently in comparative social sciences.[5] But the empirical argument must be subordinated to the theoretical argument. Even if researchers are confronted with a medium-N dataset, the use of QCA would not be appropriate if there are no explicit expectations about set relations. Likewise, the use of QCA would be appropriate even if the N is large if, and only if, researchers are interested in set relations rather than correlations.

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[5] Think, for instance, of the more than 30 OECD member countries, the 50 US states, the 28 NATO members, or the 27 EU countries. Even more examples come to mind if we move away from countries as units of comparison and think about studies that might compare 12 civil wars, the social science departments from 20 universities in the American Midwest, 25 urban grassroots organizations opposing the Iraq War, 10 European left parties, 30 local operatives of the Republican Party in Texas, 25 Chinese villages, 40 members of the British Parliament, and so on.

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At-a-glance: Qualitative Comparative Analysis as a set-theoretic approach and technique

QCA is both a research approach and a data analysis technique. The plausibility of findings from a QCA **as a technique** much depends on the quality of the work done before and after the analysis, i.e., QCA **as a research approach**.

   QCA aims at a causal analysis, operates with truth tables, and makes use of logical minimization procedures. In this way, it can be distinguished from other set-theoretic methods, all of which employ some, but never all, defining features of QCA.

   The motivation for using QCA should be the researcher's interest in set relations rather than the number of cases under investigation.

   QCA can be applied to the analysis of a mid-sized number of cases without violating any of its assumptions. It can also be used for analyzing large-N data.

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Variants of QCA

Ragin's writings, together with several other book publications (De Meur and Rihoux 2002; Goertz and Starr 2003; Rihoux and Grimm 2006, Schneider and Wagemann 2007; Rihoux and Ragin 2009), many articles (for a comprehensive list, see www.compasss.org), and other contributions (such as adequate software) have contributed to the recognition of QCA as a methodological tool with a potential added value.[6] However, despite all this, notable confusion still exists as to what QCA exactly is and what it does. For quite a few people, this perplexity starts with the name: what does QCA stand for? Most users and consumers know that it is the acronym for Qualitative Comparative Analysis (though the authors of this book have also more than once encountered the claim that it stands for *Quantitative* Comparative Analysis). What is less commonly known is that QCA denotes a whole *family* of techniques, or, perhaps more accurately, that QCA entails different versions.

   QCA's two main variants are crisp-set QCA (csQCA) and fuzzy-set QCA (fsQCA). They differ in the type of sets on which they operate. csQCA operates exclusively on conventional sets where cases can either be members or non-members in the set. Their set membership score is either 0 or 1. In fsQCA, by contrast, cases are allowed to have gradations of their set membership. A case does not necessarily have to be a full member or a full non-member of a set, but can also be a partial member. The membership scores can fall anywhere between the two extremes of full membership value of 1

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[6]   For a detailed mapping of QCA applications over the past 25 years, see Rihoux *et al.* (in press).

and full non-membership value of 0. A country can be a partial member of the set of democracies as indicated by a fuzzy-set membership score of, say, 0.8. This value indicates that this case can be seen as more of a democracy than a non-democracy, but that it falls short of fulfilling all the criteria for a full-fledged democracy. Such a differentiation is useful for many, if not most social science concepts.

Fuzzy sets were neither invented by Ragin, nor is their application limited to the social sciences. Quite the contrary. Fuzzy sets were introduced by the Azerbaijan-born mathematician Lotfi Zadeh in the 1960s (1965, 1968). Since then, this system of thought has triggered volumes of literature in disciplines as different as mathematics, philosophy, engineering, and computer science (see, e.g., Kosko 1993, 1996; Zimmermann 2001; Zadeh 2002; Seising 2007). Modern elevators make use of them, as do washing machines. There have also been early attempts to make them an innovative and fruitful tool for the analysis of social science data (Smithson 1987). From a marketing point of view, labeling this type of sets with the adjective "fuzzy" might not have been the most successful strategy, though. It too easily evokes negative connotations, and it seems to make people think that fuzzy means imprecise, superficial, unclear, or the like. As will become clear, fuzzy sets are really none of these things. Although better names could probably be found, in this book we stick to the name fuzzy set because it stems from a broad body of literature and has a well-defined meaning.

Clearly, the introduction of fuzzy-set QCA (fsQCA) mitigates many of the problems and concerns about crisp-set QCA's (csQCA) insistence on dividing the world into black and non-black. With fuzzy sets, different shades of grey can be empirically captured and inform the analysis and interpretation of results. The flexibility of fsQCA becomes greater by the fact that in fsQCA both fuzzy sets and crisp sets can be used. An important point to keep in mind, though, is that fuzzy sets do maintain a *qualitative* distinction between cases. In other words, fuzzy sets establish differences in kind, just as crisp sets do, but they add differences in degree to this. Indeed, fuzzy sets incorporate the insight that many social science concepts are dichotomous *in principle*, but that their empirical manifestations occur in degrees. For example, we might have an idea, or definition, of what a developed democracy looks like and how it differs from a non-developed democracy, but cases empirically correspond to this ideal-typical democracy to different degrees. Nevertheless, fuzzy sets remain committed to a qualitative differentiation between types of cases (e.g., democracies vs. non-democracies). Not even interval scales – usually considered to represent one of the highest levels of measurement – can help in

identifying these qualitative differences between types of cases. Many scholars might correctly note that they are not interested in establishing qualitative differences between cases and prefer to focus on their differences in degree. This is perfectly fine, as long as one is not using interval variables to draw conclusions about types of cases. Put differently, in order to say, for instance, that being a rich country is sufficient for avoiding repugnant inequality, one must also indicate where on the variables that measure richness and levels of inequality countries begin to count as being rich and as repugnantly unequal, respectively. This, in turn, is nothing other than imposing a threshold – a qualitative anchor in the terminology of set-theoretic methods – and thus turning a variable like GDP into a set of rich countries in which cases have (fuzzy or crisp) membership scores. Unless one transforms raw data into set membership scores, it seems difficult, if not impossible, to formulate statements about the set relations of social phenomena.

It is quite common for fsQCA to be interpreted as an extension of csQCA, probably because the latter was introduced prior to the former. We hold, however, that the opposite perspective is more adequate. A crisp set is nothing else than a very special case of a fuzzy set, one that only allows for full membership and full non-membership. The intimate similarities between csQCA and fsQCA will become more apparent once we introduce the operations, algorithms, and principles that guide both versions of QCA. Because of its greater generality, we think that one should use fsQCA whenever possible. This does not imply that the use of crisp sets should or needs to be completely avoided. If a concept happens to present itself as a pure dichotomy, it can be integrated into a fsQCA without any problems.

Because of the shared analytic features between csQCA and fsQCA we also deem inaccurate and unproductive another often-encountered tendency in the literature: the interpretation of csQCA and fuzzy-set analysis as two very separate forms of analyzing social science data. The huge similarities in principles and practices between csQCA and fsQCA are also the reason why, in this book, we will always treat them together. Most of the methodological arguments will be first introduced for csQCA and then generalized to fsQCA for didactic reasons only, since crisp sets are still more in line with everyday thinking than fuzzy sets.

Apart from these two main versions, further variants of QCA exist. In multi-value QCA (mvQCA: Cronqvist and Berg-Schlosser 2008), multinomial categorical data can be processed. For instance, rather than classifying countries as either two-party systems (1) or non-two-party system (0), mvQCA allows for multiple values in the category of 'type of party system', such as, for

instance, the one-party system (1); the two-party system (2); the multi-party system (3); and the dominant party system (4). With mvQCA, the aim is still to make statements of necessity and/or sufficiency, and it is also based on an analytic device that resembles a truth table. However, by allowing for multiple categories, the set-theoretic grounding of mvQCA is less straightforward, i.e., mvQCA does not require the data to represent set membership scores. This triggers several analytic consequences that we discuss in section 10.2.

Another sub-type is temporal-QCA (tQCA: Caren and Panofsky 2005; Ragin and Strand 2008). It is firmly grounded in set theory, operates on sets, uses truth tables as an analytic device, and aims at making statements of necessity and/or sufficiency. It can even be applied to both crisp and fuzzy data. It is a distinct form of QCA because it allows for specific ways of formally incorporating the temporal ordering of conditions as causally relevant information. The reason why we do not treat tQCA in as much detail as csQCA and fsQCA is simple: it shares the great majority of its features with these core versions of QCA. Hence, in order to understand tQCA, one first needs to understand QCA in general, and so our main focus is on csQCA and fsQCA. Because of this, and because of its degree of recognition, we use the acronym QCA in this book when discussing properties of all members of the methodological family. Whenever a given argument holds only for a specific type of QCA, we refer to it as csQCA, fsQCA, mvQCA, or tQCA.

At-a-glance: variants of QCA

In **crisp-set QCA** (csQCA), only the membership values of 1 and 0, indicating perfect membership and perfect non-membership in a set (respectively) can be used. In **fuzzy-set QCA** (fsQCA), differentiations between 0 and 1, expressing the degree of presence or absence of the concept in a specific case, can be made. **Fuzzy sets** take into account the fact that most social science concepts establish qualitative differences between cases in principle, but that cases manifest adherence to these criteria in various degrees.

Multi-value QCA (mvQCA), which deals with multinomial conditions, and **temporal QCA** (tQCA), which aims at including the temporal order in which conditions occur as potentially causally relevant, are further types of QCA which, however, share many aspects with the two main variants of csQCA and fsQCA.

Plan of the book

The challenge in understanding set-theoretic methods is not so much in grasping the math that is behind them. In fact, in terms of standard mathematical

operations, not much more is required than simple subtraction and division of natural numbers. It is not even required to delve too deeply into the more complex intricacies of formal logic and set theory. The three rather simple logical operators (AND, OR, and NOT) and the notion of subsets and super-sets suffice for denoting any possible result that can be obtained using QCA. Yet understanding and correctly using set-theoretic methods is challenging. Our experience from teaching students with a wide range of different disciplinary and methodological backgrounds has revealed that the biggest challenge rests in capturing the far-reaching consequences that are triggered when shifting the aim of social research to identifying set relations rather than correlations.

Our book consists of four main parts, each subdivided in several chapters. In Part I, we lay out the basic principles that are needed in order to understand QCA. In Part II, we introduce measures that have to be taken when neat formal logical logic meets noisy and often lousy social science data. In Part III, we provide several critiques at the current standards of good QCA practice and offer suggestions for improvement. Part IV is dedicated to extensions of the family of QCA and to framing general issues in comparative social science methodology in set-theoretic terms, such as robustness tests and case selection principles. We proceed, thus, from explaining fundamental principles (Part I), to standards of good practice (Part II), then go beyond these standards by making suggestions of our own for improved analyses (Part III), and applying the notion of sets to more general methodological issues in comparative social science (Part IV).

More specifically, our chapters deal with the following topics: in Chapter 1, we spell out in further detail what sets are and how set membership values can be attributed to single cases or, in set-theoretic methods terminology, how membership scores are calibrated. In Chapter 2, we provide a short introduction to set theory, Boolean and fuzzy algebra, and the logic of propositions, respectively. These three systems provide the notation, main terminology, and operations that are needed to perform set-theoretic analyses. In order to understand how set-theoretic methods work, it is necessary to get acquainted with these basic notions. In Chapter 3, we apply set-theoretic principles to the analysis of sufficiency and necessity relations between conditions and an outcome. This will also induce a discussion of causal complexity. In Chapter 4, we develop the analysis of sufficiency and necessity further and introduce the notion of a truth table, a concept from formal logic that is at the heart of QCA-based research.

In Chapters 5 and 6, we deal with issues that occur when applying QCA to common social science data. The problems that arise can all be captured in

terms of incomplete truth tables. In essence, there are two ways a truth table can be incomplete. In Chapter 5, we deal with contradictory, or inconsistent, truth table rows, i.e., the situation in which it is not clear whether a given truth table row is sufficient for the outcome. From this, we derive the parameter of consistency, which expresses the degree to which a given condition is a subset or superset of the outcome. We show that the consistency measure captures only one feature of subset relation and also introduce the coverage measure, which provides a numeric expression for the empirical importance of a given condition (or a combination thereof) for producing an outcome. These two parameters are also very useful in the analysis of necessary conditions. Chapter 6 deals with the second symptom of an incomplete truth table: one in which logical remainder rows occur, i.e., rows that exist only as logical combinations but which have no empirical manifestations. This situation occurs because the empirical variation in which the social world presents itself tends to be highly limited in its diversity. We discuss how logical remainders are best handled. Chapter 7 serves as wrap-up of the material learned up to this point: we put all the ingredients from Chapters 1–6 together and integrate them into the so-called Truth Table algorithm and the Standard Analysis procedure as the current predominant form of analyzing data in QCA.

In Chapter 8, we show some pitfalls in dealing with limited diversity that are not entirely resolved by the Standard Analysis procedure. We provide practical suggestions for producing what we call the Enhanced Standard Analysis procedure. In Chapter 9, we discuss various issues that arise when the analysis of necessity and sufficiency are combined. We provide solutions for avoiding the appearance of false necessary conditions and the disappearance of true necessary conditions. Along these lines, we offer a new way of identifying so-called trivial necessary conditions. We also draw attention to the more general problem of skewed set membership scores and their impact on drawing inferences in set-theoretic methods, such as, for instance, the phenomenon of one set being a simultaneous subset of another set and its complement.

Chapter 10 deals with further variants and extensions of QCA. We discuss two-step QCA as an approach for better differentiating between conditions located at different distances from the outcome; mvQCA as an attempt to work with multinomial categories; and the integration of the notion of time into QCA, with temporal QCA as the most formalized attempt at this. In chapter, we address general issues in comparative methodology from a set-theoretic perspective. We first spell out a list of standards of good QCA practice. Here, we also provide an overview of the currently available software packages that can be used for performing QCA. Then, we discuss the meaning

of "robustness" in regards to QCA results and what robustness tests should look like; spell out the logic of theory evaluation in set-theoretic methods, as opposed to hypothesis testing in statistical approaches; and present the principles of case selection for within-case studies after a QCA.

The conclusion attempts at a general evaluation of QCA as a social science method and offers an outlook on further developments in set-theoretic methods.

How to use this book

Before we enter the debate, let us give some useful hints on how to read this book. We suggest starting at the beginning. While in later chapters we at least briefly reiterate crucial points, it remains the case that issues raised in later chapters can best be understood by thoroughly reading the preceding chapters.

This book is explicitly designed to cater to both beginners and very advanced readers. In order to allow all readers to better navigate through the book and to easily identify the chapters that are most relevant to their current needs and interests, we employ several devices. First, each main chapter starts with an "Easy reading guide." This presents the content and main points made in the chapter in question. The Easy reading guides can help both more advanced readers to move directly to specific sections and beginners to identify those sections that are fundamental for understanding the method and which ones contain additional arguments and debates. The second device is "At-a-glance" boxes at the end of most sections. They summarize the key points of the respective section and are directly connected to the "Glossary," our third didactic device. It contains definitions of all key terms in set-theoretic methods that are used and introduced in the book. Terms printed in bold in the At-a-glance boxes are those that are contained in the Glossary. Finally, we provide online learning material for each chapter. The "How to" sections contain practical guidance on how to use the currently available software packages (fsQCA, 2.5, Tosmana 1.3.2, Stata, and R) in order to perform the analytic operations described in the respective chapter. The exercises and solutions are subdivided into conceptual questions, exercises that require calculations by hand, and exercises practicing the use of the software by reanalyzing published QCA.

Throughout the book, we make use of published examples of set-theoretic analyses. In the early chapters, however, when we need to separate specific

methodological issues from all the others that usually occur in applied set-theoretic methods, we often revert to hypothetical data in order to clarify our point. While there might be a slight bias towards political science in choosing our examples (due to the background of both authors), this does not suggest that set-theoretic methods are limited to this social science discipline. Set-theoretic methods are also becoming increasingly popular in sociology, psychology, anthropology, management studies, and comparative literature, to mention just a few (see www.compasss.org for more information).

Part I

Set-theoretic methods: the basics

1   Sets, set membership, and calibration

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Easy reading guide

This book is based on the conviction that the tools of set theory allow for a distinct and fruitful perspective on social science data. In order to develop the argument and to show how the analysis of empirical data works when focusing on set relations, we first clarify how sets refer to concepts (1.1). Then we discuss how set membership scores are derived from empirical and conceptual knowledge. This process is called calibration (1.2). Through calibration of sets, qualitative – and also quantitative, with fuzzy sets – differences between cases are established and expressed by set membership scores that vary between 0 and 1. The usefulness of set-theoretic methods depends on the proper calibration of sets. Beginners should read through the whole chapter with careful attention, while more advanced users might wish to skim through the text if they feel that they are well aware of the principles and practices of good set calibration.

    In the Introduction, we have already mentioned that there are two major variants of QCA, namely crisp-set QCA (csQCA, where a case is either a member of a set or it is not) and fuzzy-set QCA (fsQCA, where differences in the degree of set membership can be captured). Both these variants share one fundamental feature: they establish *qualitative* differences between those cases that are (more) in the set and those that are (more) out of the set. Beyond this, both QCA variants have much more in common than is sometimes insinuated in some of the literature. In this book, we therefore emphasize their commonalities. They both aim at identifying subset relations, which, in turn, rest on qualitative differences between cases. Indeed, a crisp set should be seen as the most restrictive form of fuzzy set, one that allows only full membership and full non-membership. Because crisp sets are a special case of fuzzy sets, most of the set operations equally apply to both variants. For all these reasons, we introduce both variants together. Admittedly, crisp sets correspond more to everyday thinking: this is why we introduce all important notions and operations by first explaining their meaning based on crisp sets. The main emphasis of this chapter is on fuzzy sets, though, because they are less intuitive and therefore require more explanation.

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1.1 The notion of sets

1.1.1 Sets and concepts

The use of the term "set" is not very broadly diffused in social science method-
ology. However, a good part of our conceptual reasoning, as Mahoney (2010)
shows, is at least based on an implicit idea of sets. According to Mahoney,
there are two basic modes of looking at concepts: if we define concepts "as
a mental representation of an empirical property" (Mahoney 2010: 2), then
we will measure cases "according to whether or the extent to which they are
in possession of the represented property" (Mahoney 2010: 2). Measurement
theory provides us with many useful techniques for doing this. This ultimately
results in the use of variables when defining a concept (Mahoney 2010: 13).
If, however, we refer to concepts as sets, defined in terms of "boundaries that
define zones of inclusion and exclusion" (Mahoney 2010: 7), then "[c]ases
are measured according to their fit within the boundaries of a set" (Mahoney
2010: 2). Sets work as "data containers" (Sartori 1970: 1039). Although this
seems to be a subtle and often overlooked differentiation, these two views
of concepts are fundamentally different. When we measure a concept by
means of traditional measurement theory, it represents a property or a group
of properties. The set-theoretic view, instead, uses set membership in order
to define whether a case can be described by a concept or not. Therefore, in
the framework of set-theoretic methods, issues of concept formation have a
somewhat different connotation than in traditional measurement theory, by
focussing on whether a case belongs to a concept (i.e., a set) or not. This pro-
cess of assigning set membership is also called "calibration" (see section 1.2).

1.1.2 The pros and cons of crisp sets

When QCA was first discussed in the 1980s and 1990s, it was limited to crisp
sets. This required a decision whether a case is a member of a set or not.
As such, this also corresponds to how sets are generally perceived, namely
as boxes into which cases can be sorted or not. However, as argued in the
Introduction, it is not always easy to make such clear-cut decisions, above
all when dealing with more fine-grained social science concepts for which
detailed and nuanced information is available. Not surprisingly, the need
for "dichotomization" has triggered some serious criticism of crisp-set QCA
(Bollen, Entwisle, and Alderson 1993; Goldthorpe 1997; for an overview and

a response, see De Meur, Rihoux, and Yamasaki 2009). This requirement certainly affected the usability and the acceptance of QCA in its early stages. The two major reservations with dichotomies seemed (and still seem) to be that (a) they represent a loss of empirical information and (b) they reduce the robustness of results due to the sensitivity of QCA findings to decisions on where to put the threshold for dichotomization, as the latter is often subject to a relatively large degree of discretion.

At the core of the argument against dichotomization is the belief that the world and large parts of social science phenomena simply do not come in a binary form. Let us take, for example, the notion of democracy again: if we think of cases such as the UK or the USA on the one hand, and North Korea or Zimbabwe on the other, then this might at first glance suggest that a clear-cut dichotomy is appropriate. The former countries are members of the set of democracies, whereas the latter two are clearly not. However, cases often fall in-between these two qualitatively different endpoints. Just think of all the so-called "electoral democracies" or any of the numerous "democracies with adjectives" (Collier and Levitsky 1997) identified in the literature. A closer look at the unquestionably democratic cases in North America and Western Europe also reveals the existence of interesting and analytically relevant differences – both across time and across countries – that defy a straightforward classification as democracies versus non-democracies (for instance, declining trust in the political class or the rise of far-right movements might be said to undermine democracy). We would probably not want to claim that any of these countries has become *un*democratic. Despite sometimes even strong deviations from perfect democracy, they are still *qualitatively* different from non-democracy. As we shall see, fuzzy sets provide the possibility to take both qualitative and quantitative differences into account.

The fact that we emphasize qualitative differences and not only quantitative variations is quite important here. In statistics, interval-scale variables are usually considered superior to dichotomous (and ordinal) variables, since their high level of measurement captures more precise quantitative differences. However, the previously mentioned limitations of dichotomous variables should not lead to the conclusion that interval-scale measurements automatically imply a greater level of validity. This is above all doubtful when the underlying concept establishes explicit *qualitative* distinctions between cases, such as, for instance, the concept of "democracy." This implies that, despite the general concerns about the use of dichotomies, not using them at all would go too far. In fact, even in applied quantitative research, where most critiques of the use of dichotomies originate, techniques like logistic

regression, which requires a dichotomous dependent variable, remain widely popular. What is more, the recent shift in the statistical literature towards the experimental design as the gold standard for causal inference has led to a renewed appreciation of dichotomies even among proponents of advanced quantitative methodology.[1]

The second type of critique aimed at using dichotomous data may seem to be a rather technical issue, but it refers back to the critique just mentioned. It is often argued that the decision on where to put the threshold is not only to a considerable extent arbitrary, but also crucially influences the results obtained. What seems to be true is that in research practice, scholars have all too often been using unconvincing criteria as to where to put the threshold for turning their raw data into crisp-set membership scores. As we will explain in section 1.2.2, a very common mistake is to use characteristics of the data at hand, such as the mean or median, as a guide to where to put the threshold.

A central critique says that arbitrariness, or simply a definition that is not perfectly accurate, could cause a case to be on the "wrong" side of the threshold, and that research results could be significantly altered through different case assignments. While true, claims about the manipulability of set-theoretic results through purposeful threshold setting (aka cheating) are largely exaggerated. First, for each concept there is only a certain, often small range where the threshold can plausibly be put. Usually, no huge differences in the results occur due to minor adjustments to the threshold.[2] If the criteria for setting the thresholds are both transparent and plausible, then hardly any chance exists for potential cheating. Finally, the effects of different thresholds on the results obtained are often so intricate that setting thresholds in order to create desired results would be a time-consuming and futile exercise for the researcher.

In sum, working with crisp sets does create some issues. At the same time, when trying to investigate relations between sets, we must establish qualitative differences between cases that are more in a set and those that are more out of the set. So what can we do in order to effectively work with concepts where there is some interesting variation between the qualitative endpoints of implicitly dichotomous social phenomena? In these situations, neither interval scale variables nor dichotomous crisp sets are ideal. The former lack the capacity to establish qualitative differences, and the latter to make differences in degree between cases of the same kind. Thus, an instrument is needed that overcomes the starkly limiting characteristics of dichotomies but which at the

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[1] We thank John Gerring for making this point (personal communication, Spring 2010).
[2] See section 11.2 on robustness tests in QCA.

same time continues to possess the potential to show qualitative differences. To this end, Ragin proposed the use of fuzzy sets (Ragin 2000).

1.1.3 Properties of fuzzy sets

The term "fuzzy set" goes back to the writings of Lotfi Zadeh (1965, 1968). The notion of fuzzy sets has triggered volumes of books in disciplines as diverse as mathematics, engineering, and philosophy. Only recently has the tool of fuzzy sets been introduced in the social sciences (Smithson 1987, 2005; Ragin 2000, 2008a, 2008b; Smithson and Verkuilen 2006). Thus, fuzzy-set theory was not invented by social scientists, and the level of complexity of this theoretical and mathematical framework goes well beyond that currently applied in fuzzy-set social sciences.

Because fuzzy-set theory refers to an established body of literature, we stick to the use of the term "fuzzy set" despite its potentially misleading interpretation and negative connotation in everyday language. One could perhaps come up with a less stigmatized adjective for sets that are not crisp, but the use of any other term would contribute to disconnecting the use of fuzzy sets in social sciences from their mathematical and epistemological background. As the extant literature makes clear, "fuzzy" does not mean "unclear" or "wishy-washy." The statement that a given case has a fuzzy-set membership score of, say, 0.8 reflects precise empirical information about that case. The fuzziness stems from imprecise conceptual boundaries. For instance, when we invoke the concept of a "bald person," we all agree that somebody with no hair at all is definitely bald. If, however, we took a person with a lot of hair and started pulling it out one strand after another, it would be difficult to point to a precise and quantifiable amount of remaining hair at which this person would have to suddenly be considered a member of the set of bald people. At the same time, we do see a qualitative difference in terms of baldness between somebody with a lot of hair and somebody with only few hairs. The problem of identifying where exactly the difference is between a bald and a non-bald person is not resolved by knowing the precise number of hairs remaining. Fuzziness, in other words, is due to *conceptual boundaries* that are not sharply defined rather than imprecise *empirical measurement*.

Fuzzy sets preserve the capability of establishing difference-in-kind between cases (qualitative difference) and add to this the ability to establish difference-in-degree (quantitative difference) between qualitatively identical cases. The term fuzzy set implies a different usage of the term "set" than we are used to from traditional set theory, which defines sets through strict membership criteria (Klir, Clair, and Yuan 1997: 48). Individual members either clearly belong to sets, or else they do not. Fuzzy sets, by contrast, allow for cases to have

partial membership in the set (Klir *et al.* 1997: 73ff.). Cases can be more in than out of a set without being full members of the set, and they can be more out than in the set without being full non-members of the set. For instance, two countries might have a fuzzy-set membership score of 0.7 and 0.8 in the fuzzy set of democracies, respectively. This indicates that both are rather more democratic than non-democratic (a qualitative property), but also that one of the two countries is slightly more democratic than the other one (a quantitative difference). Fuzzy sets are thus characterized by the fact that the boundaries between membership and non-membership are blurred. This also implies that a case – unless it has full (non-)membership in the set – is actually a partial member of both the set *and* its negation. In our example, each state is not only a member to some degree in the fuzzy set of democracies, but also of the opposite fuzzy set, that of non-democracies. The principle of the "excluded middle" whereby an element can be only a member of a set *or* of its complementary set (a fundamental rule of crisp sets) does not hold for fuzzy sets.

Fuzzy sets allow for degrees of membership, thus differentiating between different levels of belonging anchored by two extreme membership scores at 1 and 0 (Ragin 2000: 154; Ragin 2008b). In addition, a membership score at 0.5 locates the so-called point of indifference where we do not know whether a case should be considered more a member or a non-member of the set (Ragin 2000: 157). It constitutes the threshold between membership and non-membership in a set – the qualitative distinction that is maintained in fuzzy sets – and represents the point of maximum ambiguity with regard to a case's membership in the concept. Fuzzy sets explicitly require that the definition of set-membership values is based on three qualitative anchors: full set membership (1), full non-membership (0), and indifference (0.5). In crisp sets, these three anchors are all collapsed into one – the distinction between full membership and full non-membership.

Defining the precise location of the 0.5 qualitative anchor is crucial. Assigning cases a 0.5 fuzzy set membership score, however, should be avoided. It means that we are unable to say for an individual case whether it is more a member of the set or more a non-member. Because we avoid a decision on the qualitative status of the case in question, assigning the 0.5 score has important consequences for the analysis of fuzzy data that we explain in detail in Chapters 4 and 7. For all other degrees of membership and non-membership so-called fuzzy values are used to quantify the levels of membership of a case in a set. As Table 1.1 exemplifies, for each fuzzy value, linguistic qualifiers can be assigned (Ragin 2000: 156).

It is not necessary for there to be actual empirical elements corresponding to every fuzzy value, i.e., even if a fuzzy set allows for a membership of, say, 0.8 it might well be that it is not assigned to any empirical case. In particular,

Table 1.1 Verbal description of fuzzy-set membership scores

Fuzzy value	The element is …
1	Fully in
0.9	Almost fully in
0.8	Mostly in
0.6	More in than out
0.5	Crossover: neither in nor out
0.4	More out than in
0.2	Mostly out
0.1	Almost fully out
0	Fully out

Adapted from Ragin (2000: 156)

this also applies to the membership values of 1 and 0. Also, different intervals between the fuzzy-set membership scores are possible: it is perfectly fine if a fuzzy set shows membership scores of, say, 0.1, 0.4, 0.6, and 1, if theoretical considerations warrant it.

We can also imagine fuzzy scales that are differentiated even further than this. However, with increasing levels of differentiation it becomes ever more difficult to come up with theory-based and empirically observed distinctions between the values, not to mention the need to assign verbal descriptions to each value. Any such representation suggests a level of precision that is unlikely to be grounded in empirical information or theory. One should therefore not over-interpret the substantive meaning of marginal differences in set-membership scores, such as the difference between 0.62 and 0.63. Such small differences also have only a negligible impact on the analytic results.

Note that frequently some of the variation in the raw data is conceptually irrelevant. When translating raw data into corresponding fuzzy-set membership scores, this must be taken into account. Imagine that we want to assign membership scores of all countries in the fuzzy set "rich countries." If we take GDP per capita as an indicator for richness, then we find a large variation among the four countries with the highest GDP per capita (IMF data for 2010): Qatar ($88,500), Luxembourg ($81,400), Singapore ($56,500), and Norway ($52,000). Under many (if not most) definitions of "rich country," all four would be considered rich and would thus receive a membership score of 1 in the set of rich countries. The fact that Qatar is quantitatively about 1.7 times richer than Norway is deemed qualitatively irrelevant for research purposes (Ragin 2008a: 77ff.).

Fuzzy scales, with their well-defined starting- and end-points, the cross-over point, and the combination of both qualitative and quantitative differentiations, seem to defy standard classifications of measurement levels (Ragin 2008b). Both

the idea of seeing them as continuous scales (since every possible grading between 0 and 1 can be obtained) and seeing them as ordinal scales (since they display an ordered list of empirical representations of a given concept) could seem reasonable. However, the argument against interpreting fuzzy sets as continuous scales is that it downplays the establishment of *qualitative* differences between cases above and below the 0.5 anchor, which remains the essential principle of fuzzy sets. The step from a fuzzy value of 0.4 to 0.6 is something different from the step from 0.1 and 0.3. Although the quantitative difference in the degree of membership is 0.2 in both situations, there is a qualitatively different situation: in moving from 0.4 to 0.6, the qualitative anchor of 0.5 is crossed. While 0.6 indicates that the case is more like a member of the set, 0.4 tells us that it is more of a non-member of the set. The fuzzy values 0.1 and 0.3 indicate, instead, that both cases are on the same side of the point of indifference and thus both indicate non-membership, although to different degrees. This distinction does not, however, also mean that a fuzzy set will be reinterpreted as a dichotomy in the analysis: although the qualitative difference is maintained, the quantitative gradings also count. A fuzzy value of 0.3 describes something different from the fuzzy value of 0.1, although both values indicate the absence of the concept rather than its presence. Hence, fuzzy scales are neither continuous nor ordinal, since their "continuity" and their "rank order," respectively, are interrupted at the point of indifference, and since the inherent qualitative difference is dominant in the definition of the values.

Ragin (2008b) points out that this combination of qualitative anchors and quantitative gradings, which sits uneasily with mainstream social science classifications of measurement levels, is standard in disciplines that are usually regarded as more "scientific" than the social sciences, such as physics, chemistry, and astronomy. Ragin gives the example of "temperature" and the measurement "degrees Celsius." There are senses in which a temperature can be qualitatively interpreted. When falling below 0° or rising above 100°, the state of water qualitatively changes: it turns into ice and vapor, respectively. Hence, a 10-degree change from 95° to 105° implies a qualitative difference, whereas a change from 30° to 40° does not. Just using temperature at face value, without anchors that establish qualitative differences, one would miss this important information about the state of water. So far, in the social sciences it is rare to use knowledge ("the temperature at which water freezes or boils") that is external to the raw data ("mercury expanding and contracting with heat") to decide how to calibrate a scale.

1.1.4 What fuzzy sets are not

Fuzzy sets express a specific kind of uncertainty and take on values between 0 and 1. It is perhaps because of these two characteristics that fuzzy set membership

scores are sometimes interpreted as probabilities (e.g., Altman and Perez-Linan 2002: 91; Eliason and Stryker 2009). We side with those scholars who reject that view, among them Zadeh (1995) himself, whose article's title captures the essence of the argument: "Probability Theory and Fuzzy Logic are Complementary Rather than Competitive." A similar point is made by McNeill and Freiberger (1993: 185ff.), who argue that uncertainty has various aspects and that probability and fuzziness capture different forms of uncertainty. The following example helps to illustrate the difference between probability and fuzzy values.

Imagine two water glasses, each containing a different liquid, and about which the following is known. Glass A contains a liquid that has a 1 percent probability (0.01) of being poisonous. Glass B, on the other hand, contains a liquid that has a fuzzy-set membership score of 0.01 in the set of poisonous liquids. When forced to choose between the two (and assuming that we do not have suicidal tendencies), which glass is safer to drink? The answer is glass B. We know exactly what is in this glass – a liquid that is all but fully out of the set of poisonous drinks. This applies, for example, to energy drinks of the kind that are popular among college students; they are certainly not poisonous, but also not completely free of toxins as is, say, a glass of pure spring water. In contrast, we do not know what is in glass A. It is either extremely poisonous or completely non-toxic. All we know is that it comes from a population of other glasses, of which 1 out of 100 is deadly poisonous. There is a 99 percent chance that drinking from glass A is completely safe, but a 1 percent chance it will turn out to be lethal. In contrast, glass B will cause us to feel, at best, slightly bloated and a little twitchy but does not present any risk of dying.

At-a-glance: the notion of sets

The use of set theory in the social sciences requires a different perspective on concepts: cases are assessed with regard to their membership in previously defined sets.

Crisp sets are restricted to the membership values 1 (full membership of a case in a set) and 0 (full non-membership). This ultimately requires the definition of all concepts as dichotomies.

Fuzzy scales possess three qualitative anchors – the complete presence of a concept (1), its complete absence (0), and the point of indifference (0.5) – with quantitative gradings representing the degree of presence of the concept. Verbal descriptions ("linguistic qualifiers") help to connect the quantitative assessment to natural language.

Crisp sets can be seen as special cases of fuzzy sets. Thus, the rules for fuzzy sets are more general and subsume those for crisp sets.

A fuzzy-set membership score does *not* express the probability of a case's membership in a set. Fuzzy scores and probabilities express different aspects of uncertainty. The uncertainty expressed in fuzzy sets stems from conceptual rather than empirical imprecision, which, in turn, is inherent to most verbally defined concepts – especially those in the social sciences.

1.2 The calibration of set membership

Assigning set membership scores to cases is crucial for any set-theoretic method. The process of using empirical information on cases for assigning set membership to them is called "calibration." In order to be analytically fruitful, calibration requires the following: (a) a careful definition of the relevant population of cases; (b) a precise definition of the meaning of all concepts (both the conditions and the outcome) used in the analysis; (c) a decision on where the point of maximum indifference about membership versus non-membership is located (signified by the 0.5 anchor in fuzzy sets and the threshold in crisp sets); (d) a decision on the definition of full membership (1) and full non-membership (0); (e) a decision about the graded membership in between the qualitative anchors.

1.2.1 Principles of calibration

The first (and very simple) answer to the question of how to assign set-membership values is to base the calibration on the combination of theoretical knowledge and empirical evidence (Ragin 2000: 150). It is the responsibility of the researcher to find valid rules for assigning set-membership values to cases. The top priorities of this process are to make the calibration process transparent and to make it lead to a set that has high content validity for the concept of interest. When turning raw data into set-membership scores, researchers make use of knowledge that is external to the data at hand (Ragin 2008a, 2008b). Such knowledge comes in different forms and from different sources. There are, for instance, *obvious facts*. For example, it is generally true that completing the twelfth grade in the United States leads to receiving a high school diploma. If we are trying to calibrate the set "high school-educated citizens," there is a qualitative difference between completing the eleventh grade and completing the twelfth grade. There are also some *generally accepted notions in the social sciences*. In addition, there is *the knowledge of the researcher accumulated in a specific field of study or specific cases*. This requires extensive fieldwork and a very careful analysis of primary and secondary sources before proceeding to the actual calibration. As such, interviews, questionnaires, data obtained with participant observation or focus groups, and organizational analysis, quantitative and qualitative content analysis, etc., can all provide useful information sources in the process of set calibration.

1.2.2 The use of quantitative scales for calibration

Multiple non-quantitative data sources are often used for calibration. Sometimes, however, we do have one data source and it is an interval-scale measure. For instance, if we want to calibrate the set "rich countries," then a GDP per capita indicator might provide a reasonably good source of information.[3] When interval-scale data are at hand, researchers have several calibration options. In this section, we first describe what one should *not* do when calibrating sets based on interval scales. We then provide a good example of how to combine case knowledge and empirical distribution for meaningful set calibration. Then, in a separate section, we describe the direct and indirect methods of calibration (Ragin 2008a, 2008b).

When calibrating fuzzy sets, it might be tempting to simply transform the GDP per capita scale into the 0–1 interval while preserving each case's relative distances to each other.[4] When calibrating a crisp set, we might even simply want to use the arithmetic mean or the median and to define all cases above the mean or median as "in the set" and the others as "out of the set." Such purely data-driven calibration strategies are fundamentally flawed, though. Measures like the mean or median are properties of the data at hand and, as such, void of any substantive meaning vis-à-vis the concept that one aims to capture with a set. Just dropping or adding a case with an extreme value on the GDP per capita scale will change the mean. Using parameters such as the mean therefore implies that the classification of a case does not only depend on its own absolute value, but on its relative value with regard to other cases. Why, however, should the presence or absence of specific cases in the data influence the set-membership score of other cases in the set of rich countries? It should not.

This is why calibration must also make use of criteria for set membership that are *external* to the data. Certainly this does not mean that the distribution of cases on our raw data should be disregarded. It is simply another piece of evidence, but certainly not the sole guidance when calibrating. Along these lines, also consider that depending on the research context, one and the same raw data translate into different set-membership scores. This is so because the meaning of concepts, and therefore their respective sets, is highly dependent on the research context (Ragin 2008a: 72ff.). For example, in research on EU member states, a GDP per capita of, say, $19,000 (roughly the value for

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[3] Here we sidestep the substantive arguments against using GDP as a proxy for "richness" (see, e.g., Dogan 1994).

[4] The easiest method here would be to simply divide the GDP of each state by the highest value of GDP in the sample.

Table 1.2 Calibration of condition "many institutional veto points"

Country	Federalism, 1945–96	Bicameralism, 1945–96	Combined indicator	Fuzzy-set membership in "many institutional veto points"
Australia	5	4	10.00	1.00
Austria	4.5	2	7.00	0.67
Belgium	3.1	3	6.85	0.67
Canada	5	3	8.75	1.00
Denmark	2	1.3	3.63	0.00
Finland	2	1	3.25	0.00
France	1.2	3	4.95	0.33
Germany	5	4	10.00	1.00
Ireland	1	2	3.50	0.00
Italy	1.3	3	5.05	0.33
Netherlands	3	3	6.75	0.67
New Zealand	1	1.1	2.38	0.00
Norway	2	1.5	3.88	0.00
Portugal	1	1	2.25	0.00
Spain	3	3	6.75	0.67
Sweden	2	2	4.50	0.33
Switzerland	5	4	10.00	1.00
UK	1	2.5	4.13	0.00
USA	5	4	10.00	1.00

Source: Emmenegger (2011)

Hungary) would not translate into full membership in the set of rich countries. In the context of a global study, in contrast, Hungary would be a member of the set of rich countries. Set-membership values are intrinsic to the research in which they are used. They are not universal indicators of a concept (Collier 1998: 5), but directly depend on the definition of a concept, which in turn is closely linked to the research context.

A good example to illustrate the calibration of fuzzy sets based on quantitative data is Emmenegger's (2011) work on job security regulations in selected OECD countries. One of his conditions is the fuzzy set "many institutional veto points." The raw data consists of an additive index based on Lijphart's (1999) data on federalism and bicameralism (Table 1.2). Emmenegger opts for a four-value fuzzy scale (0, 0.33, 0.67, and 1). The location of the qualitative

anchors – the most important decisions to be made when calibrating sets – is derived in the following manner. All countries achieving a score lower than or equal to that of the UK (4.13 in Emmenegger's combined indicator) receive a fuzzy membership score of 0 in the set of "many institutional veto points."

Case knowledge is used in an exemplary manner in order to identify and justify meaningful qualitative anchors on the composite index that separates cases with full non-membership and partial non-membership. A prominent gap in the combined indicator between the raw values of 5.05 and 6.75 is then used to establish the point of indifference. All countries below that gap, but above the UK, are assigned a fuzzy value of 0.33. Finally, another gap in the combined indicator between 7.00 and 8.75 is used to define full set member-ship: countries higher than 8.75 are deemed full members of the set of "many institutional veto points."

While there might be room for debate about specific decisions in Emmenegger's strategy (e.g., the choice of the indicators or the way of aggre-gating them), the level of transparency and the combined use of conceptual and case knowledge for imposing qualitative anchors represent a good stand-ard of calibration practice. It allows readers to follow the reasoning behind calibration decisions and to either agree or to disagree and, if the latter, to make specific suggestions for change in the calibration.

1.2.3 The "direct" and "indirect" methods of calibration

Ragin (2008a: 85–105) proposes the so-called "direct" and "indirect" methods of calibration. Both apply only to fuzzy and not crisp sets. Unlike in the pre-vious calibration example, these two techniques are more formalized and rely partially on statistical models. The direct method uses a logistic function to fit the raw data in-between the three qualitative anchors at 1 (full membership), 0.5 (point of indifference), and 0 (full non-membership).[5] The location of these qualitative anchors is established by the researcher using criteria exter-nal to the data at hand. The "indirect method," by contrast, requires an initial grouping of cases into set-membership scores. The researcher has to indicate which cases could be roughly classified with, say, a 0.8 membership in the set; with 0.6; 0.4; and 0.2 and so on. Using a fractional logit model, these prelimin-ary set-membership scores are then regressed on the raw data. The predicted

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[5] Because a logistic function is used, the actual anchors are at 0.95, 0.5, and 0.05.
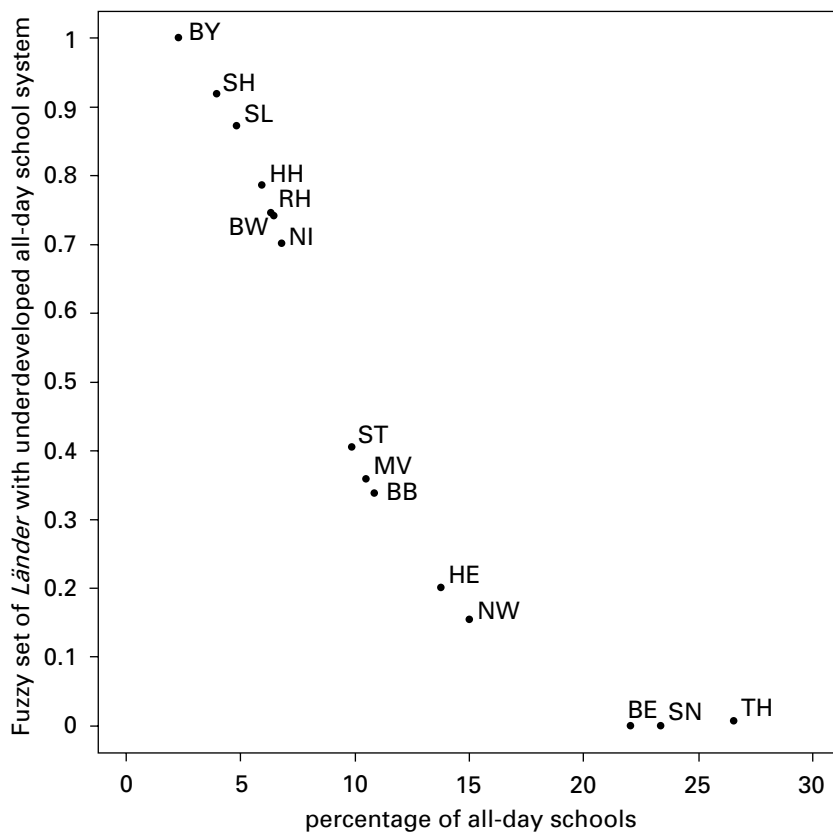
Figure 1.1   Membership in fuzzy set of *Länder* with underdeveloped all-day schools plotted against percentage of pupils enrolled in all-day schools

values of this model are then used as the fuzzy-set membership scores. Thus, if interval-scale data are at hand, the direct and indirect method of calibration can be fruitfully applied and represent progress in one of the core issues of set-theoretic methods: the creating and calibration of sets. The technical details are explained in detail by Ragin (2008a, 2008b). Conceptually, the important message is, however, that despite the complexity of the underlying statistical model, the calibration and thus set-membership scores of cases is predominantly driven by the location of the qualitative anchors. These locations, in turn, are determined by the researcher, who uses external knowledge rather than properties of the data at hand.

Freitag and Schlicht (2009) provide an example of the direct method of calibration. In their comparative work on the differences in schooling

systems in the 16 German *Länder*, they calibrate the set "*Länder* with underdeveloped all-day school system." The raw data for calibration consist of the percentage of pupils enrolled in all-day schools in a *Land*. These values vary between 2.4% (Bavaria) and 26.6% (Thuringia). Because the fuzzy set is labeled *under*development, high values in the raw data convert into low fuzzy-set membership scores and vice versa. The 0.5 qualitative anchors is located at 8.3%, which is exactly the middle of a notable gap in the raw data between 6.8% (Lower Saxony) and 9.8% (Saxony-Anhalt); the 1 anchor is located at 3% (leaving only Bavaria with full membership); and the 0 anchor at 20% (assigning 0 to Berlin, Saxony, and Thuringia).

If we plot the fuzzy-set membership scores that result from applying the direct method of calibration (for details, see Ragin 2008a: 84–94) with the qualitative anchors just described against the raw data, we clearly see the logistic nature of the transformation (Figure 1.1). We also see that despite the use of a (complex) mathematical procedure in the background, the qualitative differences between cases' set membership is clearly driven by decisions that the researcher makes based on theoretical considerations and knowledge that exist outside the raw data.

Some critiques of the direct and indirect methods of calibration have been formulated. First, partly because these calibration techniques can be performed by using the relevant software packages (fsQCA 2.5, Stata, or R), the temptation might be high to apply them in a mechanistic manner and to thus under-appreciate the importance of standards for imposing thresholds external to the data. Second, both procedures lead to very fine-grained fuzzy scales, thus suggesting a level of precision that usually goes well beyond the available empirical information and the conceptual level of differentiation that is possible. Put differently, these calibration techniques might create an impression of false precision. Another issue is the use of the logistic function for assigning set-membership scores, a choice that is not sufficiently justified. Calibration procedures using different functional forms are equally plausible and, as Thiem (2010) shows, do have a measurable impact on the set-membership scores. In other words, to some degree, the set membership of cases depends on the arbitrary choice of the functional form employed in the calibration procedure. We agree that the logistic function is arbitrary and that other functions are equally (im)plausible. Yet, as long as the 0.5 anchor remains unchanged – and its location should be determined by theoretical arguments and never by the functional form – then the effect of different functional forms on the set-membership scores remains only marginal in virtually all scenarios. The only empirical situation in which differences in

the functional form of calibration can produce differences in set membership even if the qualitative anchor remains the same is when set membership is highly skewed, i.e., when most cases are located either above or below the 0.5 qualitative anchor.

1.2.4 Does the choice of calibration strategy matter much?

Both Emmenegger and Freitag and Schlicht have (quasi-)interval-level data at hand. Yet, the first opts for a qualitative calibration while the latter apply the direct method of calibration. Does the choice of calibration strategy lead to substantively different membership scores? The general answer to this question is this: as long as the locations of the qualitative anchors are carefully chosen and thus not subject to changes in the calibration strategy (theory-guided, direct, indirect, etc.) or the functional form used in the semi-automated procedures (logistic, quadratic, linear, etc.), then the differences in set-membership scores will not be of major substantive importance.

In order to illustrate this, let us compare Emmenegger's qualitative calibration of the set of many institutional veto points with the fuzzy scores that result from applying the direct calibration method to the same data. In both procedures, we use the same qualitative anchors for full non-membership (values below 4.13) and full membership (values above 8.75). For the qualitative anchor at 0.5, it is impossible to choose the same value, though. In the qualitative calibration, Emmenegger locates it anywhere between the values of 5.05 and 6.75. The direct method of calibration, however, requires a precise location for the 0.5 cut-off. Here we encounter a major difference in calibration strategies: while in qualitative calibration no precise location for the 0.5 anchor is required, in the direct method a precise value is required. What is perhaps even more problematic is that different choices about that precise location influence the set membership scores of *all* cases, even those far above and below the point of indifference. Graphically speaking, the exact shape of the S-curve as shown in Figure 1.1 crucially depends on the location of the 0.5 anchor. Because some discretion is often exercised on the exact location of this anchor, this introduces at least some level of arbitrariness that is not found in the qualitative calibration strategy.

Table 1.3 compares Emmenegger's original fuzzy set scores with the ones obtained by such a use of the direct method of calibration. As the values in the last column indicate, the majority of cases display identical membership

Table 1.3 QUALITATIVE versus direct method of calibration for set "many institutional veto points"

	Raw data	Membership in set "many institutional veto points"		Difference
		Qualitative calibration	Direct method of calibration	
Australia	10	1	1	0
Austria	7	0.67	0.76	−0.09
Belgium	6.85	0.67	0.73	−0.06
Canada	8.75	1	1	0
Denmark	3.63	0	0	0
Finland	3.25	0	0	0
France	4.95	0.33	0.17	0.16
Germany	10	1	1	0
Ireland	3.5	0	0	0
Italy	5.05	0.33	0.19	0.14
Netherlands	6.75	0.67		−0.04
New Zealand	2.38	0	0	0
Norway	3.88	0	0	0
Portugal	2.25	0	0	0
Spain	6.75	0.67	0.71	−0.04
Sweden	4.5	0.33	0.09	0.24
Switzerland	10	1	1	0
UK	4.13	0	0	0
USA	10	1	1	0

Adapted from Emmenegger (2011)

scores. This is true for those located at the two extreme ends of the fuzzy scale. In addition, no case crosses the crucial qualitative anchor at 0.5 from one calibration strategy to the other. Only the cases with fuzzy set membership scores of 0.33 or 0.67 in Emmenegger's original calibration see a change in membership score when using the direct calibration approach. However, the difference in membership is usually too small to warrant a meaningful substantive distinction. The biggest difference occurs for Sweden, which according to the direct method of calibration is almost fully out of the set of "many institutional veto points," whereas the qualitative calibration assigns it a fuzzy value of 0.33. The reason for this is simple: Sweden's value in the raw data is just slightly higher than the UK's. This results in a marginal difference using the

direct method. However, if we just use four categories, such as Emmenegger does, then Sweden is part of the next higher category, which is described by the fuzzy value of 0.33.

When discussing the usefulness of a purely qualitative approach and of semi-automatic procedures such as the direct method, we should not forget that Emmenegger's original data (i.e., Lijphart's raw data) are not perfectly quantitative, whereas Freitag and Schlicht, for example, work with empirical quantities. Emmenegger's values are close to qualitative assessments themselves so that a complicated mathematical transformation, such as a logit function, might be a less appropriate way of reflecting the (partial) presence of a concept in given cases.

1.2.5 Assessing calibration

We have presented different ways of data calibration: starting off from theory-based, or qualitative, calibration strategies, we discussed the use of quantitative underlying scales, arriving finally at the semi-automatic direct and indirect methods. Of course, we might feel tempted to automatically resort to the latter strategies as soon as underlying quantitative measures exist. The hope of higher reliability and validity might motivate such a choice. By contrast, qualitative forms of calibration are often disregarded as being less transparent and less "scientific." However, this criticism is put in a different light if we consider that comparative research often relies on indicators generated from quantitative data of questionable quality due to issues such as low intercoder reliability; opaque aggregation strategies; or unclear content validity. For illustration, just think of the Freedom House Index as one of the most frequently used indicators of democracy used in research (see Munck and Verkuilen 2002 for a detailed critique).

Yet another reason why the critique against more theory-guided methods of calibration is somewhat misleading lies in the fact that, in practice, analytical results derived from QCA are generally robust to slight changes in the calibration method. That is to say, most results rarely vary in important ways if a case's membership value is altered slightly. We will come back to this in Chapter 11 (section 2).

In sum, it is not the principles underlying the assignment of fuzzy values which are problematic, but rather it is the temptation to disregard the central principles of calibration that causes trouble.

At-a-glance: the calibration of set membership

The **calibration** of **fuzzy-set membership scores** has to be based on theoretical knowledge and empirical evidence. Obvious facts, accepted social scientific knowledge, and the researchers' own data collection process all inform the calibration process.

Statistical distributions and parameters of underlying quantitative data can provide useful information for calibration. However, an automatic transformation of quantitative scales or the default use of statistical parameters in the calibration process is strongly discouraged, as this does not fulfill the requirement of using calibration criteria that are *external* to the data and is thus unlikely to lead to set-membership scores that reflect the meaning of the concept that is meant to be captured. A number of mathematical problems further discourage such procedures.

The **direct and indirect methods of calibration** can be applied when interval-scale data are at hand and when **fuzzy sets** (as opposed to **crisp sets**) are calibrated. These semi-automatic ways of transposing quantitative data into set-membership values are a valuable addition to the set-theoretic method toolset. Set-membership scores hinge upon the definition of the precise location of the qualitative anchors, which, in turn, are determined based on knowledge outside of the data. Thus, conceptual and theoretical knowledge remains the most important feature in these semi-automated calibration techniques.

2 Notions and operations in set theory

2.1 Conjunctions, Boolean and fuzzy multiplication, intersection, logical AND

For the purpose of demonstration, we use a hypothetical example in which a researcher is interested in two features of a country: federalism and democracy.

Table 2.1 Important operations in set-theoretic methods

Row	Sets		Operations			
	D	F	Logical AND/Boolean multiplication/ intersection/conjunction	Logical OR/Boolean addition/union/ disjunction	Negation/ complement	
			D * F	D + F	~D	~F
1	1	1	1	1	0	0
2	1	0	0	1	0	1
3	0	1	0	1	1	0
4	0	0	0	0	1	1
5	0.1	0.9	0.1	0.9	0.9	0.1
6	0.8	0.7	0.7	0.8	0.2	0.3
7	0.8	0.3	0.3	0.8	0.2	0.7
8	0.4	0.3	0.3	0.4	0.6	0.7
9	0.2	0.1	0.1	0.2	0.8	0.9
10	0.1	0	0	0.1	0.9	1

With crisp sets – and, as we shall see, with fuzzy sets also – four types of countries are logically possible: federal democracies, non-federal democracies, federal non-democracies, and non-federal non-democracies. These types are represented in rows 1 to 4 in Table 2.1. The 1-values indicate that a given case is a member of the set, the 0-values that it is not a member thereof.

When dealing with fuzzy sets, many more combinations are possible. Rows 5 to 10 provide some examples for some of them. Note, however, that rows 8, 9, and 10 all describe similar countries. All of them display fuzzy-set membership scores below the 0.5 anchor and can thus be described as rather non-democratic and rather non-federal countries, although to varying degrees. They are, therefore, fuzzified versions of the cases described in row 4. Equally, rows 1 and 6 describe countries that are similar in kind, but different in degree. The same holds for cases in row 7 (similar to row 2), and row 5 (similar to row 3). In short, even with fuzzy sets, with their potentially infinite number of combinations of membership scores in D and F, only the four *qualitatively* different types of cases are possible.

If we want to create the set of "federal democracies," we require both elements (D and F) to be present. If a country is not federal, then it cannot qualify as a federal democracy. Likewise, if a country is not a democracy, it cannot be a federal democracy. Only countries which are both federal and democratic

can be federal democracies. The general insight is this: for a specific combination of sets to be present, the presence of all its constituent components is required. In the other three potential types of countries, at least one component is missing and thus the combined set "democratic and federal" is not present.

The logic of propositions uses the operator "AND" in order to denote such a combination. If we use a D for "democracy" and an F for "federalism," then the combination of D and F would be called "D AND F," formally written as "D ∧ F." This is also called a *conjunction*, or a *logical AND conjunction*.

Combination through logical AND is also known in Boolean and fuzzy algebra, where it is called a Boolean, or fuzzy, *multiplication*. In the literature, a star (*) is often used as the symbol denoting this operation. This combination thus reads: D*F. Alternatively, we can find a dot (·), reading D·F, or, even more commonly, the letters denoting the conditions are just reported without any operator (DF).

Finally, in set theory, the combination of elements is called an *intersection*. The two components of the combination are understood as sets in which countries can be members or not. Set D contains all democracies and excludes all non-democracies, while set F contains all federal countries and excludes all non-federal countries. The area where sets D and F overlap – the intersection of D and F – is where all those countries are located that are both democratic and federal and fulfill the joint requirement. This intersection is denoted as D ∩ F.

The way to calculate a case's membership score in a conjunction is to take the minimum value of the case's membership across the sets that are combined. This is rather intuitive when using crisp sets (rows 1 to 4 in Table 2.1): when both elements are present, the case scores a value of 1 for both elements and thus receives a score of 1 in the AND combination, as 1 is the minimum across these scores. If only one of the two elements is present, a case receives a 0 score in one element (1 in the other one) and thus also a 0 score in the conjunction, since 0 is the minimum value across the elements that are combined by logical AND. Consequently, when both elements are absent (scoring 0), a case's score for the conjunction is also 0.

The same minimum rule is also used when dealing with fuzzy sets. The membership of case 5 in the intersection of D and F (i.e., its membership in the set of federal democracies) is 0.1 (the minimum of 0.1 and 0.9). Similarly, case 6 has a membership of 0.7 (the minimum of 0.7 and 0.8), etc. Just as in crisp sets, in fuzzy sets the so-called principle of the "weakest link in the chain" is used for the logical AND operator.

Notice that the minimum aggregation principle runs counter to the predominant practice of data aggregation in most social science disciplines, where the arithmetic average is often used (Goertz 2006b). For instance, a case in row 7, with membership in D of 0.8 and in F of 0.3, would receive a score of 0.65 using the average, whereas the minimum rule, as dictated by formal logic and set theory, yields a membership score of 0.3. Using the average as an aggregation strategy yields higher scores because higher values counterbalance lower ones. When using the logical AND operator, instead, both constitutive sets are seen as indispensable for the overarching concept. For cases like those in row 7, this leads to quite important differences from a set-theoretic perspective. Using the average, these cases are classified as more in than out of the set of federal democracies. The minimum aggregation rule, instead, classifies them as more out than in. Hence, different aggregation rules can lead to qualitatively different membership scores. Whether the average or the minimum (or other possible) aggregation rule makes more conceptual sense depends on the definition of the concept to be measured (Goertz and Dixon 2006).

2.2 Disjunctions, Boolean and fuzzy addition, union, logical OR

Another operator crucial for set-theoretic approaches describes logical alternatives. Such an alternative is realized if at least one of the components of a combination is present. Applied to our example, we are now interested in countries that are democratic or federal, i.e., they satisfy at least one of the requirements.

For this operation, the logic of propositions uses "OR" ("D OR F"). This logical statement is "TRUE" when one of the components can be observed. Logical OR describes a *disjunction* and is denoted as D ∨ F. The symbol "∨" is derived from the Latin word "*vel*," which is one of the two Latin terms for "or," the other being "*aut*." In English there is just one word ("or") making it more difficult to distinguish between an inclusive "or" (Latin: *vel*) and an exclusive "or" (Latin: *aut*). An inclusive "or" indicates that a logical OR connection is present if at least one of the elements connected through this operator is present. So, the disjunction D ∨ F is true if the country under study is either D or F, or if it is both, since the presence of just one of the two elements is sufficient to render the disjunction present. As column 5 in Table 2.1 shows, this is true for all but one possible type of case. If, instead, the exclusive OR (denoted as XOR or ExOR) which allows for only one element to be present (but not for

both) had been used, then the type of case in row 1 of Table 2.1 would not be an instance of D XOR F. Note that set-theoretic methods use the inclusive OR.

Boolean and fuzzy algebra denote the logical OR by using a "plus" sign ("+"), and this is called a Boolean OR in the former and a fuzzy addition in the latter.[1] This might lead to confusion for two reasons. First, most languages read the "+" sign with the word "and." As we have just learned, though, the equivalent for logical AND in the use of the logic of propositions is the multiplication (*) rather than the addition. This means that users must not read the Boolean "+" as an "and" in the conventional sense. The second potential confusion is that an addition can lead to different results in conventional linear algebra and Boolean algebra. In both algebras the following holds:

$0 + 0 = 0$; $1 + 0 = 1$; and $0 + 1 = 1$.

The difference is that in linear algebra

$1 + 1 = 2$,

whereas, in Boolean algebra, it is

$1 + 1 = 1$.[2]

In Boolean and fuzzy algebra, a case's score for a logical OR expression is calculated through the maximum value across the single components. This is straightforward with crisp sets: if both conditions democracy and federalism are absent in a case ($D = 0$; $F = 0$), then a case's score in the disjunction democracy OR federal ($D + F$) is 0, i.e., the maximum score across the single values 0 and 0. If either D or F or both are present, the score for $D + F$ is 1, again the maximum score across all conditions. The maximum scoring rule for the logical OR also holds in fuzzy sets. For example, the case in row 5 of Table 2.1 has a membership of 0.9 in $D + F$, i.e., the maximum of 0.1 and 0.9.

Just like the minimum value, the use of the maximum value runs counter to predominant practices of data aggregation, yet often seems to be the more adequate mathematical translation of concepts whose meanings are verbally defined (Goertz and Dixon 2006). This application of the maximum rule to fuzzy sets also shows well why the "+" sign means two different things in the various algebras and that the application of linear algebra is often inappropriate

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[1] The exclusive OR is denoted by a ⊕ sign.

[2] Both elements are present. Consequently, (at least) one of the alternatives is present. More than presence (= values above 1) cannot occur in Boolean algebra, nor can they in the logic of propositions or in set-theory. An element cannot have greater than full membership in a set.

when dealing with sets. Consider cases in row 6 of Table 2.1: simply adding up the membership scores in D and F, we would obtain a value of 1.5. This is, of course, meaningless from a set-theoretic and logical point of view, as cases cannot be more than full members of a set. Set membership has its absolute maximum at 1.

In set theory, the logical OR is called a *union* and is denoted as D ∪ F. It describes the set of cases that are a member in at least one participating set.

2.3 Negations, complements, logical NOT

The third crucial operator for performing set-theoretic methods is the negation (or the *complement*) of a statement. In order to denote the group of all non-democracies, propositional logic uses ¬D (the logical NOT), Boolean algebra "1−D," and set theory D̃. Calculating a case's score in the negation of a set is straightforward: simply subtract a case's score for the presence of the element from 1. A case with D = 1 has a score for ¬D of 1 − 1 = 0, a case with D = 0 has in ¬D a membership of 1 − 0 = 1. The same rule also applies to fuzzy sets. A case such as the one in row 5 of Table 2.1 with D = 0.1 has a value of ¬D = 1 − 0.1 = 0.9. In the set theory-based literature, the notations "~D" or "d" (lower case) are also frequently used to denote a logical negation.

It is important to understand that the complement of a set does not automatically denote the conceptual counterpart. For instance, the set of all *not-rich* persons is not automatically identical to the set of *poor* persons. Many of our readers would not identify themselves as rich (and are thus not members of the set of all rich persons). At the same time, most of the readers would also not qualify as being poor. More generally speaking, the complement of sets often comprises many different cases. For instance, the set of non-democracies comprises very different types of political regimes, ranging from totalitarian to sultanistic, one-party, military, or theocratic regimes. It is particularly important to take stock of this diversity when trying to attribute some causal role to the negation of a set (see section 4.3.3).

2.4 Operations on complex expressions

Each of the three operators taken alone is relatively simple and straightforward to apply. Their usefulness for social science data analysis unfolds when they are combined in order to create (sometimes quite complex) logical

expressions. In the following, we introduce commutativity, associativity, and distributivity and explain the Role of Excluded Middle and DeMorgan's law. Finally, we show how the three basic logical operators are used in order to produce complex sets and how the membership of cases in complex sets is calculated.

2.4.1 Rules for combining logical operators

A number of mathematical rules govern the three crucial operators introduced above (see for example Klir *et al.* 1997: 37). *Commutativity* means that the order in which two or more elements are connected through AND and OR is irrelevant. Instead of A * B, we could write B * A; similarly, A + B is equivalent to B + A. Note that this does not hold for the complement, though: $1 - A$ is not the same as $A - 1$.

Associativity means that, with the same operators, the sequence in which single elements are combined is unimportant. If, for instance, we want to create the conjunction of the three elements A, B, and C, then it does not matter if we first create a conjunction of A and B (A * B) and then combine the result with C – the formula for this would be (A * B) * C – or if we first combine B and C and then combine this (B * C) with the remaining factor A, that is to say, A * (B * C). Put in more formal terms:

(A * B) * C = A * (B * C) = (A * C) * B.

The same rule holds for the disjunction:

(A + B) + C = A + (B + C) = (A + C) + B.

Distributivity refers to the fact that, when both AND and OR operators are used in the same logical expression, elements which are shared by the various components can be factored out:

A * B + A * C = A * (B + C) or, using simpler notation: AB + AC = A (B + C).

There are two further operations which merit special attention. If we create the union of a set with its complement – in other words, if we combine into one single set all the elements which are members in the set *and* all the elements which are not in the set – then the universal set will result. This universal set includes all possible elements and is denoted as follows: A ∪ ~A = U. If, instead, we intersect a set with its complement, then the "empty set" will result: A ∩ ~A = Ø. There is no single element that a set and its complement have in common. This is a direct consequence of the definition of the complement: it excludes all members of the original set

and includes all non-members of the original set. While this second pattern may seem rather straightforward, it is not an unimportant insight, since it tells us that an element cannot be at the same time a member of a set and of the complementary set. However, it is important to underline that this fundamental principle, also known as the "Rule of the Excluded Middle," does not apply to fuzzy sets. With fuzzy sets, one particular case can have *partial* membership in both the original set and its complement. It should be pointed out, though, that even in fuzzy sets it is impossible for the same case to be simultaneously more in than out of both a set *and* its complement. In other words, a case can only have a set membership score above the qualitative anchor of 0.5 in either a set or its complement, but not in both. This is an important feature because fuzzy sets first and foremost establish qualitative differences between cases that are above and below the 0.5 qualitative anchor.

2.4.2 Negation, intersection, and union of complex sets

When applying set theory, the three main logical operators are not only applied to single sets but also to more complex logical expressions. For instance, and as extensively discussed in section 11.3, researchers sometimes formulate the existing theoretical literature on a given topic in the form of a Boolean expression T. Let T stand for the expression $F + G^*(\sim H + \sim I)$. The empirical analysis of the same researcher might then lead to a following solution term S. Let S stand for $\sim FG + G\sim H$. Using Boolean algebra, we can now calculate the negation of T or of S, respectively, the intersection between T and S, and the union of T and S (see 11.3).

The *negation* of (complex) logical statements is governed by DeMorgan's law. This law is based on two rules: first, if a statement is negated, then all the single elements which have been present before become absent, and vice versa. If we want to negate, for example, the very simple statement A + B, then we have to write $\sim A$ instead of A and $\sim B$ instead of B. The second rule is that the logical operators also have to be inverted: the OR operator (+) becomes the AND operator (*) and vice versa. Applying these two rules, the negation of an expression such as A + B becomes $\sim A^* \sim B$. Based on DeMorgan's law, we can therefore write:

$\sim(A + B) = \sim A^* \sim B.$

Let us apply DeMorgan's law to the expression of $T = F + G^*(\sim H + \sim I)$ to find its negation. For reasons of clarity, let us add one more (superfluous)

pair of parentheses which makes the structure clearer: F + [G*(~H + ~I)].[3] Following DeMorgan's law, the components F, G, ~H, and ~I are converted into ~F, ~G, H, and I. The previously first-level addition becomes a multiplication ("*"): ~F * […]. The second factor of this multiplication is converted from a second-level multiplication into a second-level addition: ~F * [~G + (…)]. Finally, the third-level addition becomes a third-level multiplication and thus ~F * [~G + (H*I)]. If we now apply the distributivity rule, ~F*~G + ~F*H*I results. Thus, following DeMorgan we write:

~[F + G(~H + ~I)] = ~F~G + ~FHI.

The *intersection* between T and S (T * S) is calculated in the following way:

(F + G*(~H + ~I)) * (~FG + G~H).

Following the distributivity rule, G is multiplied with both ~H and ~I and the inner parentheses of T are thus eliminated. The intermediate result is:

(F + G~H + G~I) * (~FG + G~H).

Now, every summand of the expression for T is multiplied with every summand of the expression for S:

F~FG + FG~H + G~H~FG + G~HG~H + G~I~FG + G~IG~H.

Since the intersection of G and G is G and of ~H and ~H is ~H, some expressions can be shortened, and, following the commutativity rule, the letters denoting the sets can be put in alphabetical order for the sake of clarity:

F~FG + FG~H + ~FG~H + G~H + ~FG~I + G~H~I.

The first expression F~FG represents an empty set, since it includes an intersection of F and its complement ~F. F and ~F do not have any elements in common, so that the intersection and thus the expression represent an empty set. Furthermore, the expressions FG~H, ~FG~H, and G~H~I are all subsets of the expression G~H. Since these four expressions are linked with a logical OR, it is sufficient to keep G~H and to eliminate the subsets FG~H, ~FG~H, and G~H~I: if we create the union between a set and its subset, then this corresponds to the original set. Thus, the term can be reduced to:

G~H + ~FG~I.

Following the distributivity rule, this can be further simplified and the intersection T * S be written as:

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[3] The logical expression describes a (first-level) addition whose second summand G*(~H + ~I) is itself a (second-level) multiplication whose second factor (~H + ~I) is a (third-level) addition.

G(~H + ~F~I).

The union of T and S is calculated in the following manner:

T + S = (F + G*(~H + ~I)) + (~FG +G~H).

Again, with the help of the distributivity rule, G is multiplied with both ~H and ~I and the inner parentheses of T are eliminated:

(F + G~H + G~I) + (~FG + G~H).

Since this term represents a Boolean addition (see the central +) of two Boolean additions, we can omit the parentheses and write:

F + G~H + G~I + ~FG + G~H.

G~H is mentioned twice and G~H + G~H = G~H. If we then apply the distributivity rule, the union T + S is:

F + G(~H + ~I + ~F).[4]

2.4.3 Calculating membership in complex sets

No matter how complex logical expressions are, the membership of each case boils down to one number. In order to show this, take the four cases displayed in Table 2.2. For all of them, we know the set membership scores in the single conditions F, G, H, and I. Based on this, and using the logical operators introduced above, we can calculate the membership for each case in the complex expression F + G(~H + ~I).

Let us explain how we proceed for case 3, using the example of the expression F + G(~H + ~I). As in conventional linear algebra, we have to start our calculation with the inner parenthesis, which is ~H + ~I. Because of the logical OR, we need to identify the maximum membership score. H is 0.9 and thus ~H is 0.1. I is 0.4 and ~I 0.6. The maximum of 0.1 and 0.6 is 0.6. In order to get now the value for the multiplication G(~H + ~I), we have to choose the minimum (because of the logical AND) of G (0.2) and (~H+ ~I), which we have just calculated as 0.6. This minimum is 0.2. The final OR requires us to take the maximum of this 0.2 and the value for F, 0.7. This is 0.7. Hence, the membership score of case number 3 of Table 2.2 in the complex expression is 0.7.

Determining membership in complex sets is important in set-theoretic methods, and more often than not it is necessary to perform this type of

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[4] In the online How-to section for this chapter, we explain how to use the Tosmana 1.3.2. software in order to apply DeMorgan's law.

Table 2.2 Determining membership in complex sets

| Case | Single sets | | | | Complex expression |
	F	G	H	I	F + G(~H + ~I)
1	1	0	1	0	1
2	0	0	1	1	0
3	0.7	0.2	0.9	0.4	0.7
4	0.1	0.2	0.8	0.7	0.2

operation. As we will see throughout this book, empirical results in set theory, so-called solution terms, often look like the complex expression in Table 2.2 and we will want to know the membership of our cases in such a solution. Also, there are good reasons to find the intersection between our theory-driven expectations, on the one hand, and the empirical solutions derived from our data, on the other hand, a process we describe in section 11.3 under the label theory evaluation in set-theoretic methods.

2.5 Relations between sets

All operators presented so far are used to produce new sets from existing sets. In set-theoretic methods, in general, and in QCA, in particular, the aim is to go beyond this and to investigate the *relationship* between (complex) sets. Whenever such relationships are interpreted in a causal manner, the terminology of "conditions" and "outcome" sets is applied. As an example, imagine a researcher is interested in why some countries are democratic (D). Suppose also that the claim is that being democratic depends on whether a country is located in Western Europe (W). The outcome set is the set of democracies. Some countries are members of this set; others are not. The condition set is the set of Western European countries. Again, some countries are members of this set while other are not. The goal now is to find out how these two characteristics of countries relate to each other.

From a set-theoretic perspective, conditions and outcomes are either subsets, supersets, or equivalent sets of each other. In this example, the set of all Western European countries is a subset of the set of all democracies. Every country that qualifies as Western European is also a democracy. At the same

time, however, not every democracy is located in Western Europe – just think of the USA, Canada, Australia, etc. Formally, this subset relation is denoted as W ⊂ D. This is equivalent to saying that the set of all democracies is a superset of the set of all Western European countries, denoted as D ⊃ W. Every element of W is also an element of set D. Or, whenever we see an element with characteristic W, we also see that it has characteristic D.

The commutativity rule does not apply to set relations. If W is a subset of D, then D is not a subset of W; thus

W ⊂ D ≠ D ⊂ W.

To use more straightforward language: while all elements in W also show property D (being democratic), not all elements in D also display W (being Western European), i.e., there are cases that display D without displaying W (the aforementioned non-Western European democracies). When set relations are interpreted in a causal manner, as is often done in set-theoretic methods, we are then confronted with an asymmetric causal relationship: these relations only work in a unidirectional mode, and cannot be inverted. In section 3.3.3, we elaborate further on asymmetry as a central feature of set-theoretic thinking.

The logic of propositions formulates this relation as an "if … then …" statement: *if* a country is Western European, *then* it is a democracy. Formally, we write W ⇒ D and call this an "implication." In the literature, we often find a single arrow denoting this relationship: W → D. In line with most other QCA publications, we also opt for this single arrow. In other words, one property (W) implies the other (D). Equivalent to the set-theoretic perspective, the following holds: W → D ≠ D → W: an "if … then …" statement cannot be inverted.

One important property of these inclusion relations (Smithson and Verkuilen 2006) is that they are directly related to the important notions of "sufficiency" and "necessity." These concepts, in turn, are at the core of set-theoretic methods. It is therefore fair to say that whenever set-theoretic methods are employed in order to investigate potentially causal relations between a set of conditions and an outcome set – as is done in QCA – then the aim essentially consists in unraveling necessary and sufficient conditions and combinations of these two types of causes, such as INUS (Mackie 1965) and SUIN (Mahoney *et al.* 2009) conditions. Understanding that these types of causes denote subset relations, and that, in turn, these set relations are asymmetric in nature, is crucial for grasping various methodological intricacies that one encounters when applying set-theoretic methods to social science problems.

Table 2.3 Basic operations and notations in set-theoretic approaches

Operator	Logic of propositions	Boolean algebra	Set theory
AND	Conjunction	Multiplication	Intersection
	∧	*, (·)	∩
OR	Disjunction	Addition	Union
	∨	+	∪
NOT	Complement	Negation	Negative Set
	¬, ~	1−D	
Inclusion	If–then relation		Subset
	→, ⇒		⊂

Since asymmetry and other issues, such as equifinality and conjunctural causation, are also directly related and equally constitutive of set-theoretic methods, we dedicate the whole of section 3.3 to this topic.

2.6 Notational systems in set-theoretic methods

We have explained most logical operators from three "perspectives": propositional logic, Boolean algebra, and set theory. However, note that the differences between them are not clear-cut. The fact that they often get blurred in applied QCA does not do any harm other than confusing readers about notational systems. This chapter has aimed at mitigating this confusion about terminology and symbols and at enabling users to apply a consistent use of symbols and terminology in their QCA-based research. Table 2.3 summarizes the symbols and their meaning.

It is not important which system of reference is used. As such, symbols are neither right nor wrong. What matters is that in any given context their meaning is clear and their usage consistent. In this book, we use the following symbols: We denote the logical AND (conjunction, intersection) with the symbol "*" or leave it out where appropriate; the logical OR (disjunction, union) with "+"; and the negation with "~".[5] One major reason that we choose these symbols here is simply that they are all easily available on everybody's

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[5] The use of lower-case characters in order to indicate a negation is not recommended (Goertz and Mahoney 2010, 2012) as, for many letters, it is often difficult to distinguish capital from lower-case letters (just think of P/p, L/l, I/i, M/m, etc.)

keyboards and thus do not require the use of formula editors or specific text-processing packages.

Regarding terminology, we shall use "condition" and "outcome" for what in other research contexts are often referred to as "independent variable" and "dependent variable." Where appropriate, a condition is called a necessary or a sufficient condition. If a condition consists of several conditions connected by the logical AND operator, we may refer to this as a path, a conjunction, or a term. Furthermore, if several of those paths are combined by logical OR, we refer to this as a solution term, a solution formula, or simply a solution. The term "equation" should be avoided because set relations are asymmetric in nature, i.e. set-theoretic results simply do not denote relations of equality but of inequality.[6] Accordingly, we use the symbols → (for sufficiency) or ← (for necessity) rather than the "=" sign.

We now embark on a detailed discussion of the concepts of necessity and sufficiency (Chapter 3), and then we turn to the concept of truth tables and their logical minimization as a powerful way of identifying necessary and sufficient conditions (Chapter 4).

At-a-glance: notions and operations in set theory

Set theory is closely related to the logic of propositions and Boolean algebra.

In all three mathematical sub-disciplines, it is possible to formulate set relations and basic operations. In set theory, these are called "**intersection**," if they are based on the **logical AND**; "**union**" in case of the **logical OR**; a "complementary set" (or, more simply, a "**complement**") if the opposite of a set is expressed; and a "subset" if one set fully includes another one.

The membership value of a case in an intersection of sets is determined by the minimum of all its memberships values in the components (Boolean or fuzzy multiplication). The membership value of a case in a union of sets is determined by the maximum of all its membership values in the components (Boolean or fuzzy addition). The membership value of a case in the complement of a set can be calculated by subtracting the original membership value from 1. The latter implies that the **Rule of the Excluded Middle** does not hold for fuzzy sets.

All the operations can also be combined with one another. As in linear algebra, multiplication prevails over addition.

For the logical AND and the logical OR, the **commutativity**, **associativity**, and **distributivity** rules hold.

Subset and superset relations help us to analyze necessary and sufficient conditions.

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[6] The only exception is when two sets perfectly overlap. As the subsequent chapters clearly show, this situation is very rarely encountered in applied set-theoretic methods.

3   Set relations

Easy reading guide

In the previous chapters, we have presented basic information on sets and set theory. These are important prerequisites for grasping what is the fundamental interest when applying set-theoretic methods: unraveling patterns of *necessary* and *sufficient conditions*.

We start this chapter with a presentation of *sufficient* conditions (3.1), before dealing with *necessary* conditions (3.2). For both sufficient and necessary conditions, we start by introducing general principles based on crisp sets and then move on to fuzzy sets. As before, this is done for didactic reasons, since crisp sets conform more to our everyday thinking, and it is thus easier to grasp the concepts of sufficiency and necessity based on crisp sets before extending them to fuzzy sets.

Since the notions of sufficiency and necessity are such a central building block of set-theoretic methods, it is worthwhile to discuss their logic from different angles. We do so by using a stylized data matrix, a two-by-two table, a Venn diagram, and an XY plot. In Chapter 4, we add a truth table perspective to this.

After discussing the principles and notions of sufficiency and necessity, we spell out what type of results are produced by QCA. The key concept in section 3.3 is that of causal complexity, a term that we define in detail and thus differentiate from the sort of results generated by other common research methods in comparative social sciences.

Both the technical and epistemological parts of this chapter are absolutely central for the understanding of the book. Our experience in teaching set-theoretic methods tells us that some problems of understanding at more advanced levels have their roots in a flawed notion of the basics of sufficiency and necessity and the general epistemology of QCA. Therefore, we strongly suggest an intensive reading of this chapter, even to those readers who think themselves familiar with the argument – a refresher never hurts, especially when more complicated accounts of these topics are ahead.

3.1 Sufficient conditions

3.1.1 Crisp sets

3.1.1.1 Basic logic of sufficiency

Given some plausible theoretical arguments, a condition can be considered sufficient if, whenever it is present across cases, the outcome is also present in these cases. In other words, there should not be a single case that shows the condition but not the outcome. Say, for example, we claim that being a Western European country (X) is a sufficient condition for being a democracy (the outcome Y). If this claim is true, all countries in Western Europe would also have to be democracies; no Western European country can be a non-democracy. As shown in section 2.5, this can be expressed as follows:

$X \rightarrow Y.$

This statement should be read: "if X, then Y," or "X implies Y," or "X is a subset of Y." Based on this statement, what do we know about the value of Y in cases that do *not* show a positive value for X? Asked another way, what expectations about the outcome value do we have for countries that are *not* located in Western Europe (~X)? Does our claim that X is sufficient for Y automatically mean that ~X implies ~Y? The answer is no! But why?

The statement that X is sufficient for Y generates expectations on the value of Y only for cases that display X. All cases that are *not* members of X are *not* relevant for the statement of sufficiency. That is to say, they neither help to verify nor falsify our claim, independently of whatever value of Y these cases might display. While counterintuitive at first sight – especially for anybody with thorough training in correlational methods – the statement "if X, then Y" creates expectations for values on Y only when X is present. It does not generate any such expectation, or any expectation at all, in cases where ~X is present. It follows that countries in places other than Western Europe (~X) can be stable democracies (Y) or not stable democracies (~Y) – and indeed there are plenty of both types – neither of which confirms or contradicts the statement that X is sufficient for Y.

Cases with ~X are logically irrelevant for statements of sufficiency of X because set relations are *asymmetric*. X and ~X denote two qualitatively different phenomena with potentially very different roles in bringing about the outcome. If we confirm sufficiency of X for Y (X → Y), then we cannot automatically deduce that ~X would imply ~Y. This would only work if sufficiency denoted a symmetric relation between X and Y, which it does not.

As a consequence, this also means that, if we have confirmed the sufficiency of X for Y, we have still learned close to nothing about the causes of outcome ~Y – apart from the fact that there will certainly be no case with a simultaneous occurrence of X and ~Y. This means that, with a hypothesis claiming sufficiency of X for Y, we cannot say anything about the sufficiency of ~X for Y or for ~Y, nor can we account well for ~Y. We elaborate further on this *asymmetrical causality* in section 3.3.3.

In short, when we hypothesize that X is sufficient for Y, then the following patterns in the data will confirm our hunch: first, we expect to see cases with both X and Y. Second, we expect no case with X and ~Y. Third, we do not have any expectations about the value of Y for cases with ~X. Hence, our claim X → Y is falsified if and only if we find cases that are simultaneously members of both X and ~Y.

Table 3.1 shows a stylized data matrix with four cases. "0" indicates that the condition or outcome is not present, and "1" indicates that the outcome or condition is present. With one crisp-set condition and one crisp-set outcome, four cases are logically possible. For the test of sufficiency, only those with X = 1 are of interest. If there are cases with X = 1 and Y = 1 and no cases with X = 1 and Y = 0, then we have empirical evidence supporting the claim that X is sufficient for Y. Of course, whether this empirical evidence is enough to warrant an interpretation of sufficiency ultimately depends on whether there are also convincing theoretical arguments supporting this claim.

Figure 3.1 illustrates the same claim of sufficiency in a simple two-by-two table. Again, only cases with X are relevant (column X = 1). If cell b contains cases while cell d is devoid of cases, then we have empirical support for our claim that X is sufficient for Y, regardless of how many cases might be found in cells a and c.

Another way of presenting sufficiency is through a so-called Venn diagram.[1] The diagram visualizes the relationship between sets by using overlapping circles or other shapes located within a rectangular frame. Each circle contains those cases that are members of the set that the circle represents. If, for example, we are interested in Western European countries, all cases that adhere to this criterion fall within the same circle. All other cases that are not members of the set fall outside the circle. The rectangle around the circles denotes the universal set. In social science research, this delimits the set of all cases that are relevant to the research question, i.e., that fall within the scope conditions (Walker and Cohen 1985) of the study. Venn diagrams are a powerful tool for displaying

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[1] This type of chart is named after John Venn, who – like George Boole, the father of Boolean algebra – was a nineteenth-century English mathematician.

Table 3.1 Sufficiency: stylized data matrix

Case	Condition (X)	Outcome (Y)	With respect to the sufficiency of X for Y …
A	1	1	allowed
B	1	0	not allowed
C	0	1	allowed (but irrelevant)
D	0	0	allowed (but irrelevant)

Y			
1	allowed (but not relevant) **a**	allowed **b**
0	**c** allowed (but not relevant)	**d** not allowed
	0	1

X

Figure 3.1   Two-by-two table – sufficiency

relations between sets. If, in addition to Western countries (X), we are also interested in stable democracies (Y), we draw a second circle that contains all cases with stable democracies, while all non-stable democracies are located outside this circle. Depending on if and how these two circles overlap, different set relations between X and Y can be affirmed.

If X is sufficient for Y, then the circle for set X is fully contained in the circle for set Y. X is called a subset of Y. Figure 3.2 represents such a relation of sufficiency. As we can see, set Y is larger than set X. Y contains more elements than X. The central area where both X and Y are present (X, Y) corresponds to the first row in Table 3.1 and cell b in Figure 3.1; the area outside X but inside Y (~X, Y) corresponds to the third row and cell a; and the area outside X and outside Y (~X, ~Y) corresponds to row 4 and cell c. One might wonder where row 2 from Table 3.1 or cell d from Figure 3.1 can be found in this Venn diagram. Remember that if X is sufficient for Y, then the combination (X, ~Y) must not occur, i.e., it must be empty of cases. Consequently, there is no such area in this Venn diagram.[2]

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[2] Of course, in empirical social research, it is common that the sets X and Y only partially overlap, i.e., that there are also some cases that are members of X but not of Y (X, ~Y). From Chapter 5 onwards, we discuss this issue and spell out what needs to be done when neat set theory and formal logic meet noisy social science data.
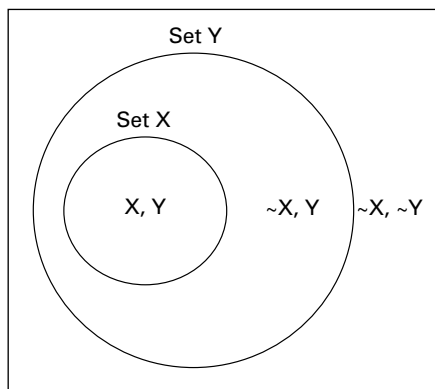
Figure 3.2   Venn diagram – sufficiency

As the figure suggests, the circle for X does not fill the circle for Y completely. Therefore, X does not account for all cases that are members of Y. If it did, X would be both sufficient and necessary for Y. This implies that in addition to X, there must be other subsets of Y which all represent other sufficient conditions of Y. In section 3.3, we address this issue under the label equifinality, which itself is one constitutive element of set-theoretic causal complexity. The smaller the area covered by X is, the smaller is its empirical importance as a sufficient condition, an issue we discuss in detail in section 5.3 under the label coverage.

3.1.1.2 A formal analysis of sufficiency in csQCA

Let us use a hypothetical example (Table 3.2) to apply our knowledge about sufficient (and later necessary) conditions. Each column denotes a different condition or the outcome, and each row a different case. The case labels are indicated in the "Cases" column.

We are interested in the conditions for a stable democracy (Y) in selected Latin American countries. We suspect membership in three sets to play a role: violent upheavals in the past (A); an ethnically homogeneous population (B); and a pluralistic party system (C). All conditions and the outcome are defined as crisp sets: a country is either a full member of it or a full non-member.

We start by asking which individual conditions (A, B, and/or C, respectively) are sufficient for outcome Y. Beginning with condition A, we ask: "Is a violent upheaval (A) sufficient for the stabilization of a democracy (Y)?" If A is sufficient for Y, then, wherever A occurs, Y should also occur. Furthermore, no case with condition A may be linked to ~Y. Since we are trying to explore a claim of sufficiency, only those cases that contain the condition of interest, i.e., those where A takes on the value of 1, are relevant. In Table 3.2, A has a

Table 3.2 Hypothetical data matrix with ten cases and set-membership scores in three conditions and the outcome

Row	Cases	Conditions			Outcome
		A	B	C	Y
1	ARG	1	1	1	0
2	PER	1	0	0	0
3	BOL	1	1	0	0
4	CHI	0	1	0	1
5	ECU	1	0	0	0
6	BRZ	0	1	1	1
7	URU	1	0	1	1
8	PAR	0	0	1	1
9	COL	0	0	0	1
10	VEN	1	1	1	0

Y = set of countries with stable democracies
A = set of countries with violent upheavals in the past
B = set of countries with ethnically homogeneous population
C = set of countries with pluralistic party system

value of 1 for Argentina, Peru, Bolivia, Ecuador, Uruguay, and Venezuela. We can see that A is linked to ~Y in the case that we begin with, Argentina. This is enough by itself to show that A is not a sufficient condition for Y, regardless of what happens in the other cases.[3]

Condition B (ethnic homogeneity) is present in Argentina, Bolivia, Chile, Brazil, and Venezuela. Were B to be a sufficient condition for Y, the Y value would have to be 1 for all these countries. We can see that this is also an incorrect statement. Argentina, Bolivia, and Venezuela are not members of the set of stable democracies (~Y), and it thus follows that ethnic homogeneity cannot be considered a sufficient condition for a stable democracy.

Condition C (a pluralistic party system) is present in Argentina, Brazil, Uruguay, Paraguay, and Venezuela. If C were sufficient for Y, there would also have to be a stable democracy in these same rows. This holds true for Brazil, Uruguay, and Paraguay, but not for Argentina or Venezuela. We therefore also conclude that C is not sufficient for Y.

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[3] From Chapter 5 onwards, we present strategies for handling less than perfect set relations.

Thus, neither condition A nor B nor C is sufficient for Y on its own. Does this mean that the empirical data displayed in Table 3.2 do not demonstrate any conditions sufficient for Y? Not really, as we have not completed all possible analyses.

So far, we have tested only the sufficiency of conditions in cases where they were *present*. Remember, though, that the data denote set-membership scores. Conditions can therefore take on two qualitatively different states: they can be either present (1) or absent (0). Because conditions and their complements denote two qualitatively distinct properties, they also need to be analyzed independently. A next step in the analysis of sufficiency is therefore a sufficiency test for the complements of A, B, and C, i.e., ~A, ~B, and ~C.

Table 3.3 facilitates this task by including the three complements to the data matrix displayed in Table 3.2, ~A (no violent upheavals in the past), ~B (an ethnically non-homogeneous society), and ~C (no pluralistic party system). Note that both data matrices contain exactly the same information; Table 3.3 just expands the presentation of information.

Is ~A a sufficient condition for Y? As we see, ~A is present in Chile, Brazil, Paraguay, and Colombia, so only those cases are relevant for the analysis of sufficiency. Just as before, if ~A is a sufficient condition, Y needs to be present in all of the same cases. This is indeed the case, so we can interpret the absence of violent upheavals as a sufficient condition for a stable democracy. Applying the same logic of analysis to conditions ~B and ~C reveals that neither qualifies as a sufficient condition for Y. Bolivia (for condition ~C) and Peru and Ecuador (both for ~B and ~C) provide evidence against these conditions having sufficiency. Our analysis of the complements thus reveals that the absence of a violent upheaval (~A) is a sufficient condition for a stable democracy (Y).

Does this fully answer the question of which conditions – according to the information contained in Table 3.2 (and Table 3.3) – are sufficient for Y? Actually, it does not. Take a look at Uruguay. It does not demonstrate condition ~A (it does not have a lack of violent upheavals), yet it displays the occurrence of Y. It achieves a stable democracy (Y) without the sufficient condition ~A. This means that there are alternative routes to achieving Y, and that Y can even occur in the presence of violent upheavals (A). This is, as explained, not evidence against the claim that ~A is sufficient for Y. It raises the question, though, of which sufficient conditions account for those instances of Y that are not explained, or *covered*, by ~A.

Table 3.3 Hypothetical data matrix with complements of three conditions

| Row | Cases | Conditions | | | | | | Outcome |
		A	B	C	~A	~B	~C	Y
1	ARG	1	1	1	0	0	0	0
2	PER	1	0	0	0	1	1	0
3	BOL	1	1	0	0	0	1	0
4	CHI	0	1	0	1	0	1	1
5	ECU	1	0	0	0	1	1	0
6	BRZ	0	1	1	1	0	0	1
7	URU	1	0	1	0	1	0	1
8	PAR	0	0	1	1	1	0	1
9	COL	0	0	0	1	1	1	1
10	VEN	1	1	1	0	0	0	0

See Table 3.2

~A = set of countries with no violent upheavals in the past

~B = set of countries with no ethnically homogeneous population

~C = set of countries with no pluralistic party system

Since the explanation (or *coverage*, as we will call it from section 5.3 onwards) of Y is not yet complete, our search for sufficient conditions continues. After examining all conditions A, B, and C and also their complements (~A, ~B, and ~C) we now turn to the investigation of the *combinations* of conditions. If A and B on their own fail the test of sufficiency, it is still possible that their simultaneous occurrence (i.e., the conjunction A*B) is sufficient for Y. Perhaps countries that are simultaneously characterized by a violent past upheaval and a homogeneous society are also stable democracies.

In order to find out whether logical AND conjunctions of single conditions qualify as sufficient conditions for the outcome, we add some further columns to our original data matrix (Table 3.4). Again, these additional columns do not alter the original empirical information displayed in Table 3.2 but simply serve as a better illustration. For reasons of presentation, we include only three conjunctions: A*B, A*C, and B*C. The logical values in their respective columns indicate whether the specific combination is present in a given case (1) or is not (0). For example, combination A*B (ethnically homogenous countries with a violent upheaval in the past) is only present in those cases where both A *and* B are present. Cases that are neither members of A nor of B, in addition to those that are non-members in both, must also not be members of conjunction A*B.

Table 3.4 Hypothetical data matrix with some conjunctions

| Row | Cases | Conditions | | | | | | | Outcome |
		A	B	C	A*B	A*C	B*C	~B*C	Y
1	ARG	1	1	1	1	1	1	0	0
2	PER	1	0	0	0	0	0	0	0
3	BOL	1	1	0	1	0	0	0	0
4	CHI	0	1	0	0	0	0	0	1
5	ECU	1	0	0	0	0	0	0	0
6	BRZ	0	1	1	0	0	1	0	1
7	URU	1	0	1	0	1	0	1	1
8	PAR	0	0	1	0	0	0	1	1
9	COL	0	0	0	0	0	0	0	1
10	VEN	1	1	1	1	1	1	0	0

See Table 3.3

A*B = set of countries with violent upheaval in the past AND ethnically homogeneous population

A*C = set of countries with violent upheaval in the past AND a pluralistic party system

B*C = set of countries with ethnically homogeneous population AND a pluralistic party system

~B*C = set of countries without ethnically homogeneous population AND with a pluralistic party system

As can be seen in Table 3.4, only a few countries are members of conjunctions. This is a direct consequence of the minimum value rule dictated by the logical AND. Notice that the predominance of zeros for conjunctions is, in fact, conducive to finding a sufficient condition: fewer rows are relevant for the test of sufficiency. Put in graphical terms, the set of, say, A*B is much smaller than the set of Y, which, in turn, makes it more likely that A*B is contained in, or is a subset of, Y. Put in common-sense terms, the more we refine country characteristics (by combining more sets through logical AND), the fewer cases display these features and the more likely it is that those that do fulfill the criteria are also members of the outcome set "stable democracy." For instance, hypothesizing that in order to be a stable democracy (Y), it is sufficient to be a small country AND rich AND located in the heart of Europe AND with a long tradition of democratic practices AND be called Switzerland is a safe bet. However, if we do this, many fewer cases will exhibit our composite set. This might make it easier to confirm the sufficiency of a combined set. But precisely because it singles out fewer

observable cases, the condition becomes empirically, and probably also the-oretically, less important.

Despite all this, we see that none of the AND conjunctions A*B, A*C, and B*C fulfills the criterion of sufficiency. Of course, many more combinations between the three conditions and their complements can be formed. These three conjunctions combine just two conditions in their presence. Other pos-sible combinations would include the conditions in their complement and would also allow AND combinations involving three conditions. Displaying all possible combinations would expand the data matrix beyond what can fit on this page, and screening it for the presence of sufficiency would get quite cumbersome. We therefore skip this step and instead, as an illustration, insert only one more column displaying the conjunction ~B*C, which happens to be sufficient for Y.

Conjunction ~B*C is present in Uruguay and Paraguay, where Y is also present. It follows that ~B*C is a sufficient condition for Y. Above we saw that sufficient condition ~A explained, or covered, the occurrence of outcome Y in Chile, Brazil, Paraguay, and Colombia. Sufficient condition ~B*C covers Uruguay and Paraguay. All cases that are members of Y are therefore covered by at least one of the two sufficient conditions. Paraguay is covered both by condition ~A and by combination ~B*C, a phenomenon we discuss in further detail in Chapter 5.

One powerful, succinct way of summing up our findings from the analysis of the information contained in the data matrix is to express it in the form of a solution formula. For our example, it looks as follows:

~A + ~B*C → Y.

The formula should be read in the following way: the absence of a violent revolution *or* the combination of an absence of an ethnically homogeneous population *and* the presence of a plural party system is sufficient for a stable democracy. We remind the reader that the logical operator OR is an inclusive (and not an exclusive) "or" and allows for both alternatives to be present at the same time. This, as shown, indeed applies to Paraguay.

3.1.2 Fuzzy sets

3.1.2.1 Basic logic of sufficiency

With crisp sets, the statement that X is sufficient for Y requires the non-exist-ence of cases where X = 1 and Y = 0. In a two-by-two table, the respective cell needs to be devoid of cases. What, however, happens once we move from
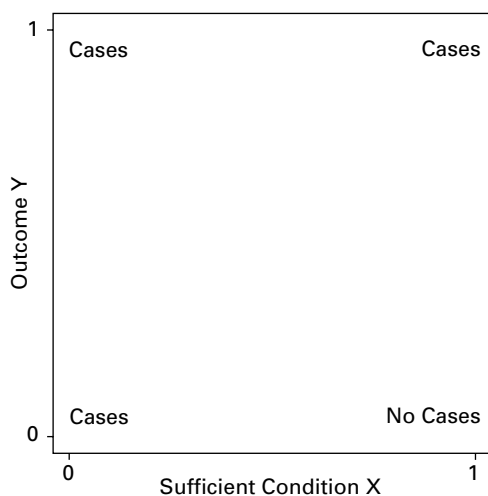
Figure 3.3   XY plots in crisp-set analysis – distribution of cases for sufficient conditions

crisp to fuzzy sets? Since the latter allow for graded membership, none of the three presentational forms discussed so far can be used.

The conceptual equivalent to a two-by-two table in fuzzy sets is a so-called XY plot (Ragin 2000). Its axes display the fuzzy-set membership scores of cases in the set of condition X and outcome Y. With crisp sets, cases can only be located exactly in the four corners of an XY plot. With fuzzy sets, however, cases can be anywhere in the area of this plot, including the corners.

In case of crisp sets, if the condition is sufficient, then cases may exist only in three out of the four corners of this plot: the top left (X fully absent, Y fully present), the bottom left (X and Y fully absent), and the top right (X and Y fully present). Only the bottom right corner cannot contain any cases; if it does, then the condition is not considered sufficient. We can represent this as shown in Figure 3.3.[4]

With fuzzy sets, if X is sufficient for Y, which areas of the XY plot must be empty of cases? In other words, which geometric figure divides the XY plot in such a way that if one of the areas does not contain cases, then X is a subset of Y?

Recall that with crisp sets the subset relation of sufficiency requires that each case's membership in X is equal to or smaller than its membership in Y. In a two-by-two table, this implies that the cell with X = 1 and Y = 0 is void of

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[4] X can stand for single condition or complex expressions. See section 2.4 for how to calculate the set-membership score of cases in complex expressions.
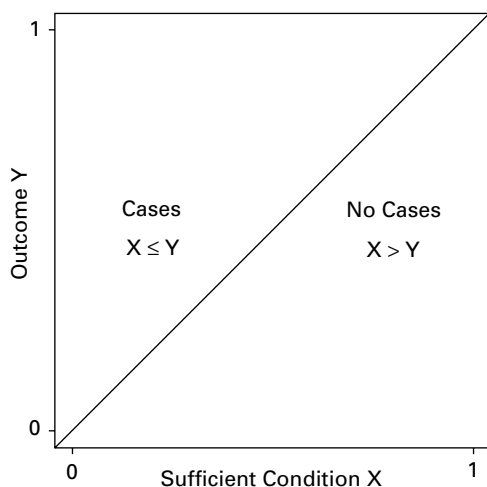
Figure 3.4   XY plot – distribution of cases for sufficient condition X

cases. The very same requirement applies to fuzzy sets. For sufficiency, each case's fuzzy-set membership score in X must be equal to or smaller than its fuzzy-set membership in Y (Ragin 2000: 237).

This subset relation can be clearly visualized in an XY plot. The main diagonal – the line that runs from the bottom left (0,0) corner to the top right (1,1) corner – divides the area into two regions. The main diagonal itself describes cases where membership in X and Y are identical (X = Y). The area above the main diagonal contains cases with a membership in X smaller than in Y (X < Y), while the area below the main diagonal contains cases with a membership in X greater than in Y (X > Y). It follows that in fuzzy sets, X is a subset of Y if all cases fall above the main diagonal (Figure 3.4).

3.1.2.2 A formal analysis of sufficiency in fsQCA

As with crisp sets, an analysis of sufficiency can be performed for every single condition, its complement, or every combination of conditions and complements. We continue with the example from above and assign fuzzy-set membership scores for each case in each condition (these fuzzy values are chosen as examples and are not based on substantive evidence). In order to maintain better comparability, crisp-set membership scores of 0 are translated into fuzzy-set membership scores smaller than 0.5 and crisp-set membership scores of 1 into fuzzy-set membership scores higher than 0.5.

Since the basic requirements for sufficiency are the same in crisp and fuzzy sets, the search for sufficiency looks identical. But because the underlying

Table 3.5 Hypothetical data matrix with fuzzy-set membership scores

Row	Cases	Conditions						Outcome	
		A	B	C	~A	AB	~BC	Y	~Y
1	ARG	0.8	0.9	1	0.2	0.8	0.1	0.1	0.9
2	PER	0.7	0	0	0.3	0	0	0.2	0.8
3	BOL	0.6	1	0.1	0.4	0.6	0	0.3	0.7
4	CHI	0.3	0.9	0.2	0.7	0.3	0.1	0.6	0.4
5	ECU	0.9	0.1	0.3	0.1	0.1	0.3	0.4	0.6
6	BRZ	0.2	0.8	0.9	0.8	0.2	0.2	0.7	0.3
7	URU	0.9	0.2	0.8	0.1	0.2	0.8	0.8	0.2
8	PAR	0.2	0.3	0.7	0.8	0.2	0.7	0.9	0.1
9	COL	0.2	0.4	0.4	0.8	0.2	0.4	1	0
10	VEN	0.9	0.7	0.6	0.1	0.7	0.3	0.3	0.7

Y = set of countries with stable democracies
~Y = set of countries with non-stable democracies
A = set of countries with violent upheavals in the past
B = set of countries with ethnically homogeneous population
C = set of countries with pluralistic party system

empirical information is not identical, the analysis does not necessarily have to yield the same solution formula. For a condition to be sufficient for Y, each case's membership in the condition must be equal to or smaller than its membership in Y. This means that all cases with non-zero membership in the condition are relevant for the test of sufficiency, regardless of whether they are above or below the qualitative anchor at 0.5.

We perform the analysis of sufficiency only for the two conditions that in the crisp-set analysis turned out to be sufficient (~A and ~BC) and for one more conjunction, AB. Argentina, Bolivia, and Venezuela contradict the statement that condition AB is sufficient for Y. Conjunction ~BC, instead, fulfills this criterion: the fuzzy value of ~BC is always less than or equal to the fuzzy value of the outcome. ~A, however, which we identified in the crisp-set example as a sufficient condition, does not pass the test based on fuzzy sets: Argentina, Peru, Bolivia, Chile, and Brazil all have higher membership scores in ~A than in Y. Thus, the result of the analysis of sufficient conditions is:

~BC → Y.

The XY plot in Figure 3.5 graphically displays our finding. All cases are above or on the main diagonal.
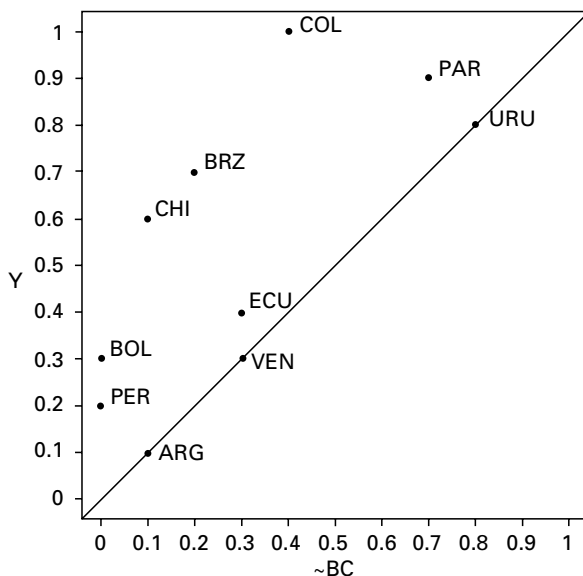
Figure 3.5   XY plot – fully consistent sufficiency solution

In sum, it is usually more difficult to find perfect subset relations for fuzzy sets than for crisp sets (Ragin 2009: 114f.). This means that some more flexibility is needed and perfect sufficiency cannot be the only goal of such an analysis. We discuss this issue of deviance from perfect set relations in Chapter 5.

3.2 Necessary conditions

3.2.1 Crisp sets

3.2.1.1   Basic logic of necessity

As we will see, the logic behind a necessary condition follows a pattern that can be viewed as the mirror image of that for a sufficient condition. Since much of what we have learned when introducing sufficiency is directly relevant for the analysis of necessity, our discussion of necessity can be shorter. We start with crisp sets and then continue on to fuzzy sets.

Generally speaking, a condition X is necessary if, whenever the outcome Y is present, the condition is also present. In other words, Y cannot be achieved without X; no case with Y displays ~X; on the presence of ~X, Y is impossible. As an example, we hypothesize that a peaceful regime transition (X) is a necessary condition for a stable democracy (Y). Based on this claim, we

expect to find only cases of peaceful transitions among stable democracies. At the same time, we have no expectations about cases that are not democracies (~Y) and whether they experienced a peaceful transition (X) or not (~X). A relation of necessity can be written as follows. We do have, however, expectations about cases with ~X, namely, they should also display ~Y.

X ← Y (read: "if Y, then X," or "Y implies X," or "Y is a subset of X").

The fact that the arrow now points from the outcome to the conditions does not, of course, mean that Y causes X. The statement "if Y, then X" entailed in the arrow only refers to the logical, or set-theoretical, relationship between two sets, not to a causal relationship. It might be better formulated as "Y logically implies X" or, in set-theoretic terms, "Y is a subset of X."

Just as with sufficiency, with necessity only some types of cases are relevant for corroborating this claim. While with sufficiency only those cases that are members of condition X matter, with necessity only those cases that are members of outcome Y matter. We have an expectation about the value of X only for cases with Y: we expect all cases with Y to also display X. Cases with ~Y are not covered by the claim that X is necessary for Y, and we are therefore not interested in the value of X in those cases.

In essence, case selection for claims of necessity comes down to this: when investigating a statement of necessity, one has to focus on cases for which the outcome is known to be present. In other words, when analyzing a necessary condition, one should select on the dependent variable and make sure that values in the outcome are constant. Influential writings on social science methodology have identified both practices as two of the biggest mistakes in comparative social science research designs and have claimed that nothing can be learned from such designs (King *et al.* 1994: 129ff.; Geddes 2003). Clearly, this view must be qualified (see also Munck 1998; Dion 2003; Brady, Collier, and Seawright 2004; Ragin 2004: 129). The asymmetric nature of set theory clearly tells us that the analysis of necessity makes it a more than plausible strategy to focus on cases where the outcome is present. Cases that do not display membership in the outcome are of much less interest. Of course, most set theory-based applications are not exclusively interested in necessity but also in sufficiency, and most of these applications need to handle deviations from perfect set relations (see Chapter 5). In both scenarios, taking into account cases of ~Y is wise advice.

Table 3.6 displays the four logically possible cases that can empirically occur when focusing on one condition and one outcome. The last column indicates which of them are allowed and which are not if X is, indeed, necessary for Y. Identical to the assessment of sufficiency, cases with (X, Y) are allowed and

Table 3.6 Data matrix – necessity

Case	Condition (X)	Outcome (Y)	With respect to the necessity of X for Y …
1	1	1	allowed
2	1	0	allowed (but not relevant)
3	0	1	not allowed
4	0	0	allowed (but not relevant)

Figure 3.6   Two-by-two table – necessity

cases with (~X, ~Y) are irrelevant. Differences exist with respect to rows 2 and 3. If X is necessary for Y, then cases with (X, ~Y) are irrelevant, while cases with (~X, Y) should not be observed.

The logic of necessary conditions can also be expressed in a two-by-two table. Only cells a and b are relevant for the test of necessity, for only they contain cases that exhibit the outcome of interest (Y = 1). Cell b, which corresponds to row 1 in Table 3.6, is allowed to contain cases, whereas cell a, which corresponds to row 3, must be empty.

Finally, Figure 3.7 shows a Venn diagram where X is necessary for Y. The area representing the set of all cases with Y is fully included within the set of all cases exhibiting X. Y is a subset of X. There are more elements in X than in Y, and – visually speaking – the set of X is larger than the set of Y. At the same time, as the area outside Y but within X shows, there are cases with X but without Y. This does not contradict the statement of necessity. It simply shows that X is only necessary, but not also sufficient for Y. Finally, Figure 3.7 does not display any area with cases that display Y but not X, as required by the logic of necessity.

3.2.1.2 A formal analysis of necessity in csQCA

The procedure for identifying necessary conditions is similar to that for testing sufficient conditions, with a different group of cases now being relevant.
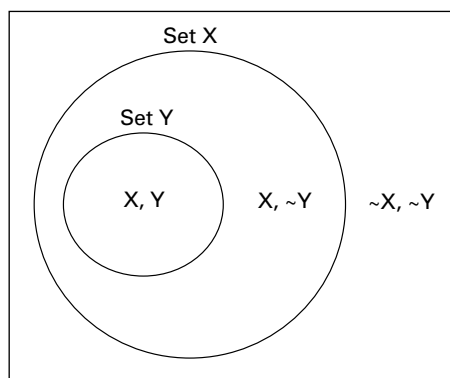
Figure 3.7   Venn diagram – necessity

The logic of a necessary condition dictates that whenever the outcome is present, the necessary condition is also present. This implies that, for tests of necessity, only those cases where the outcome is present have to be checked.

We continue with our example of the conditions for stable democracies, now also including the complements of the three conditions A, B, and C, and the logical OR combination ~A+C (Table 3.7). Outcome Y is present in Chile, Brazil, Uruguay, Paraguay, and Colombia. Testing for necessity consists of finding out, which, if any, of the conditions is present in all these cases. Let us start with condition A. It fails this test when we come to Chile, since Y is present, while A is not. Other cases cause us to arrive at the same conclusion for conditions B and C. Hence, neither A, B, or C is necessary for Y.

We now turn to the complements ~A, ~B, and ~C. Condition ~A (absence of a violent upheaval) is present in Chile, Brazil, Paraguay, and Colombia, as is outcome Y. However, outcome Y is also present in Uruguay, while ~A is absent. This contradicts the statement that ~A is necessary for Y, and ~A is therefore not a necessary condition for Y. Likewise, the necessity tests for conditions ~B and ~C reveal that these also fail to meet the criteria for a necessary condition. From this we can conclude that none of the three conditions or their complements is necessary for Y on their own.

Remember that in the analysis of sufficiency, our strategy included testing whether any logical AND combinations of single conditions passed the sufficiency criterion. In looking at necessity, however, this strategy does not make any sense. The reason for this is as follows. The logical AND operator requires taking the minimum value across the conditions. This reduces the chances that the conjunction passes the necessity test, as necessity requires

Table 3.7 Hypothetical data matrix with all complements of single conditions and conjunction ~A + C

Row	Cases	Conditions							Outcome
		A	B	C	~A	~B	~C	~A + C	Y
1	ARG	1	1	1	0	0	0	1	0
2	PER	1	0	0	0	1	1	0	0
3	BOL	1	1	0	0	0	1	0	0
4	CHI	0	1	0	1	0	1	1	1
5	ECU	1	0	0	0	1	1	0	0
6	BRZ	0	1	1	1	0	0	1	1
7	URU	1	0	1	0	1	0	1	1
8	PAR	0	0	1	1	1	0	1	1
9	COL	0	0	0	1	1	1	1	1
10	VEN	1	1	1	0	0	0	1	0

See Table 3.5

~A + C = set of countries with no violent upheavals in the past or with pluralistic party system

that membership in the condition is equal to or higher than in the outcome. Pure formal logic dictates that no AND combination can pass the test of necessity unless it is exclusively formed by single conditions that pass the test of necessity on their own. Non-necessary conditions cannot form a conjunction that is necessary. One important practical research insight derives from this: the analysis of necessity should start by investigating *single* conditions. Only if two or more single conditions pass the necessity test does it make sense to investigate whether a logical AND combination between these individually necessary conditions also qualifies as a necessary condition. In some circumstances there will be such a combination and in others there will not.

An example might be helpful here to clarify this point. Let Z stand for the set of *good exam grades*, S for *studying hard*, and G for *being in a good mood*. Furthermore, let the conjunction S*G be necessary for Z. That is, the only students who receive good grades (Z) are those who studied hard AND are also in a good mood on the exam day. In order for this statement to be true, every student with a good exam grade must have studied hard. Likewise, all students with good grades must also be in a good mood. In set-theoretic terminology: if the intersection between sets S and G fully contains the set of Y as a subset (i.e., if the area S*G is a superset of Z), then the areas for set S alone and for set G alone must also be supersets of Z.

Now that it is clear that the use of AND conjunctions is of no help in creating necessary conditions, we could simply conclude that our hypothetical

data matrix in Table 3.7 does not contain any necessary conditions. This is a perfectly legitimate result and is a frequent conclusion in applications of set-theoretic methods to observational social science data. A given social phenomenon does not need to have any necessary conditions.

Assume for a moment that we are not happy with the finding that there is no necessary condition for our outcome (*stable democracy*). How could we still produce such a condition solely on the basis of the information available in our sample data? The key to this can be found in the observation that it did not make sense to use AND combinations because their value is determined by the *minimum* of the components. Therefore, making use of the logical OR operator could be more fruitful for an analysis of necessity, since this operator follows the maximum value scoring rule: the highest value across the conditions which are part of a logical OR expression determines the membership of cases in that union. Thus, using the logical OR for combining single conditions creates a new set in which more cases tend to be members, i.e., the size of the set increases. This, in turn, makes it more likely that such an OR combination is a superset of – and thus potentially necessary for – the outcome.

For example, based on conditions ~A and C we could form the union ~A+C. It describes the set of cases that have no violent upheaval *or* do have a pluralistic party system. Column ~A+C in Table 3.7 indicates the cases in which this specific union is present. As the values in this column show, the logical OR operator leads to many 1 values. And, indeed, condition ~A+C is present wherever Y is present and thus passes the criterion for being a necessary condition for Y.

The more general insight is this: by combining conditions via logical OR, unions of conditions can be created that can pass the test of necessity even if none of the single conditions are necessary for the outcome on their own. This strategy ought to be used with care, though, because creating necessary conditions by forming unions of sets is very easy. Therefore, the important caveat is that this strategy only makes sense if there are strong and plausible theoretical or substantive arguments to support the claim that the conditions combined by logical OR operate as *functional equivalents* (Adcock and Collier 2001) of some higher-order concept. Applied to our example, claiming that ~A+C is a necessary condition only makes sense if we can plausibly argue that there is a common concept that manifests itself empirically either through the absence of a violent upheaval (~A) or a pluralistic party system (C), or through both. In our hypothetical example, no such plausible higher-order concept comes to mind. We would therefore advise against interpreting ~A+C as a necessary condition for Y.

In the literature, the practice of using functional equivalents in order to develop statements of necessity is more common than seems to be acknowledged. For instance, Emmenegger (2011) argues and empirically demonstrates that both *stable state–society relations* and *non-market coordination* are individually neither necessary nor sufficient for *strong job-security regulations*. Likewise, Mahoney *et al.* (2009: 126) refer to the democratic peace literature as a prominent example for functionally equivalent necessary conditions. For instance, if a non-democratic dyad (X) is necessary for war (Y) and X is indicated by different characteristics of a political system (absence of elections, absence of rule of law, etc.), then none of these characteristics is necessary for Y, but each of them is sufficient for X.[5]

In order to demonstrate a more straightforward example of identifying a necessary condition, we use the empirical information in Table 3.7 and choose non-stable democracies (~Y) as the outcome of interest. Are there any conditions that are shared by all members of the outcome set ~Y (cases in rows 1–3, 5, and 10)? It turns out that condition A is, in fact, present in all cases of ~Y. We therefore have empirical support for the claim that a violent upheaval (A) could be interpreted as a necessary condition for non-stable democracies (~Y). This finding can be expressed in the following formula:

$$A \leftarrow \sim Y.$$

3.2.2 Fuzzy sets

3.2.2.1 Basic logic of necessity

In the case of necessary conditions, the logic of transferring the insights from a two-by-two table to an XY plot is similar to those for sufficiency. The empirical distribution of cases that is allowed if X is necessary for Y is displayed in Figure 3.8. Parallel to sufficiency relations, for necessity, each case's fuzzy-set membership score in X must be equal to or greater than its fuzzy-set membership in outcome Y. X is a superset of Y, and, graphically, all cases fall on or below the main diagonal.

3.2.2.2 A formal analysis of necessity in fsQCA

We perform the analysis of necessity using ~Y as our outcome of interest (Table 3.5). All cases with non-zero membership in outcome ~Y are relevant for the analysis of necessity. This means only Colombia is not considered at all. While with crisp sets, condition A was a perfect superset of ~Y, the same

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[5] Such conditions are SUIN conditions (see note 2 above and section 3.3.2).
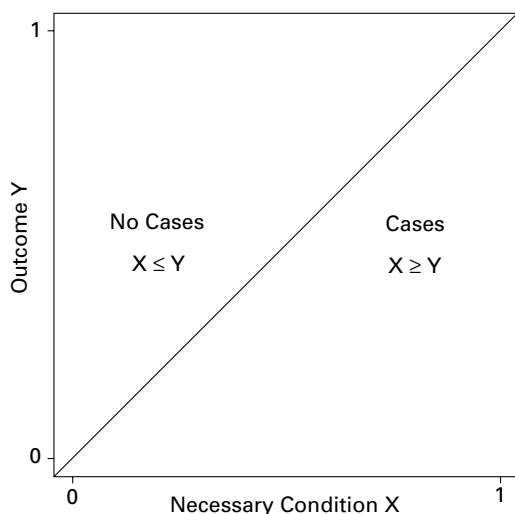
Figure 3.8   XY plot – distribution of cases for necessary condition X

does not hold true with our fuzzy-set data. Argentina, Peru, Bolivia, Chile, and Brazil all display a membership in set Y that exceeds their membership in set A, thus contradicting the statement of necessity.

The fact that A is not a superset of ~Y is neatly captured in the XY plot displayed in Figure 3.9. If A were necessary for ~Y, the area above the main diagonal would need to be empty of cases. In Chapter 5, under the label of parameters of fit, we extensively discuss strategies for handling less-than-perfect set relations.

At-a-glance: sufficient and necessary conditions

In **crisp-set** logic and in everyday thinking, a condition is **sufficient** if, whenever the condition is present, the outcome is also present. Formally, a condition is sufficient, if $X \leq Y$ for all cases. This denotes an **asymmetrical relationship**. In the representational form of **Venn diagrams**, X is a subset of Y. In an **XY plot**, all cases fall above or onto the main diagonal.

A condition is **necessary**, if, whenever the outcome is present, the condition is also present. Formally, a condition is necessary, if $X \geq Y$ for all cases. This also denotes an **asymmetrical relationship**. In a **Venn diagram**, Y is a subset of X. In an **XY plot**, all cases fall below or onto the main diagonal.

3.3 Causal complexity in set-theoretic methods

When justifying the choice of QCA, researchers often refer to the mid-sized N argument (Ragin 2000: 25; see also the section on QCA as a set-theoretic
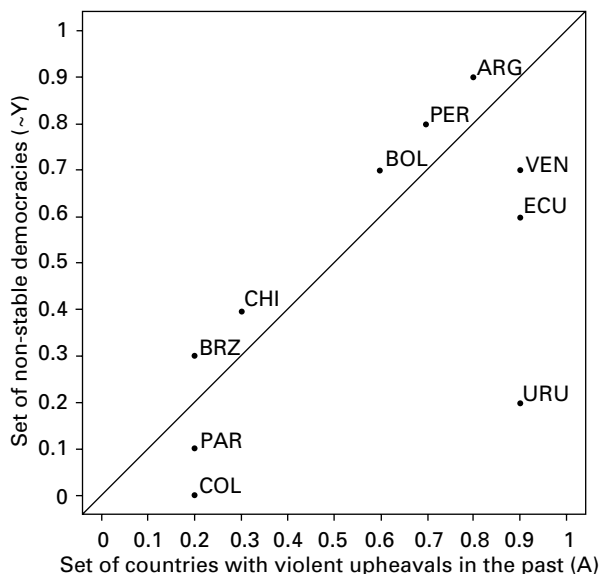
Figure 3.9   XY plot – non-consistent necessary condition

approach in the Introduction). Where the number of cases is too big for conventional small-N comparative approaches and too small to allow for the full power of (advanced) statistical techniques, so the argument goes, QCA has a distinct advantage. We believe that QCA is indeed able to fill a methodological gap in those rather common scenarios where between 10 and 50 cases (or so) are compared. However, we think that the central argument in favor of using QCA should *not* be the number of cases to be investigated. First, QCA can also be fruitfully applied to hundreds or thousands of cases (e.g., Ragin and Fiss 2008; Cooper and Glaesser 2011b; Fiss 2011). More importantly, the choice of method should be driven by the theories and expectations about the underlying causal processes at hand. If a researcher is interested in linear additive effects of single variables independent of any other causal factor, there are sophisticated and powerful statistical data analysis techniques available. QCA would be an inadequate methodological choice, regardless of the number of cases at hand. If, however, there are good reasons to believe that the phenomenon to be explained is the result of a specific kind of causal complexity, QCA is an adequate methodological choice, again, regardless of the number of cases at hand.

In the following, we define the type of causal complexity produced by set-theoretic methods, explain the logic of INUS and SUIN conditions, then spell out the meaning and implication of asymmetric causal relations, and finish with a discussion on the differences between set relations and correlation

and, by virtue of this, between set-theoretic methods and regression-based approaches.

3.3.1 Defining causal complexity

If the key to a fruitful application of set-theoretic methods is the match between the type of causal complexity derived from theory-guided hunches on the one hand, and assumptions built in by default into set-theoretic methods on the other, then it is crucial to understand just what kind of causal complexity set-theoretic methods are good at unraveling. This type of causality is defined by three characteristics: equifinality, conjunctural causation, and causal asymmetry (Lieberson 1985; ch. 9, Ragin 1987, 2000, 2008a; Mahoney 2008).

The assumption of *equifinality* allows for different, mutually non-exclusive explanations of the same phenomenon. The assumption of *conjunctural causation* foresees the effect of a single condition unfolding only in combination with other, precisely specified conditions. The assumption of *causal asymmetry* has several components. It implies that (a) a causal role attributed to a condition always refers to only one of the two qualitative states – presence or absence – in which this condition can potentially be found and (b) any solution term always refers to only one of the two qualitative states – presence or absence – in which an outcome can be found (for more on asymmetry, see section 3.3.3).

Clearly, all three features of causal complexity are intrinsically interlinked and directly derived from the notions of necessity and sufficiency. The existence of a sufficient but non-necessary condition automatically implies equifinality, as this means that there are cases in the data that achieve the outcome without the sufficient condition. Thus, at least one other sufficient condition must exist. This is a direct consequence of asymmetric causality. The existence of a necessary but non-sufficient condition automatically implies conjunctural causation, since this necessary condition must be combined with another condition (or the union thereof) in order to imply the outcome.

QCA produces results that reveal these aspects of causal complexity. In order to illustrate this, let us have a look at typical QCA solution formulas for sufficiency, one for the outcome Y and one for its non-occurrence ~Y:

A*B + ~B*C + D*~F → Y

~A*F + B*C*~D → ~Y.

Such a result displays all three aspects of causal complexity. It is equifinal, as indicated by logical OR operators. More than one path can lead to the same outcome. It is conjunctural, as indicated by the presence of logical AND operators. Single conditions play a causal role only in the context of other factors. And it is asymmetric because the solution formulas for Y and ~Y are neither identical nor logical mirror images of one another.

3.3.2 INUS and SUIN conditions

The focus on equifinality and conjunctural causality, inspired through asymmetry, makes it possible to handle types of causes that are prominent in qualitative research and which are notoriously difficult to handle in quantitative research. These are conditions that alone are neither necessary nor sufficient yet play a crucial role in bringing about the outcome. These are so-called INUS and SUIN conditions and set-theoretic methods are particularly well equipped to capture their role (Western 2001: 357).

INUS stands for a condition that is an "insufficient but necessary part of a condition which is itself unnecessary but sufficient for the result" (Mackie 1974: 62; Goertz 2003: 68; Mahoney 2008). QCA solution formulas are full of INUS conditions. Consider condition A in the following solution term:

A*B + ~B*C + D*~F → Y.

Condition A exerts its effect on Y only in combination with condition B. It is therefore insufficient on its own but needed (i.e., necessary) to form a sufficient conjunction together with B. The sufficient conjunction A*B, in turn, is not the only path to the outcome, i.e., it is unnecessary. Thus, condition A alone is, as the definition goes, insufficient, but it is a necessary part of a conjunctural condition which is itself unnecessary but sufficient for the result. It can be seen that INUS conditions are closely tied to the equifinal and conjunctural character of causal complexity. Equifinality shows that there are alternatives to sufficient conditions. Therefore, equifinal causal relations are mirrored in the "condition which is itself unnecessary but sufficient for the result" from the INUS definition. The notion of conjunctural causation refers to that part of the definition which described INUS conditions as "insufficient but necessary parts of a condition." In other words, single conditions are often not sufficient on their own, but need to be combined with others.

SUIN stands for a "sufficient, but unnecessary part of a factor that is insufficient, but necessary for the result" (Mahoney *et al.* 2009: 126). The

concept of SUIN conditions is more related to the analysis of necessity. In section 3.2.1.2, we discussed the issue of necessary conditions that are created by using the logical OR operator and interpreted these OR-combined conditions as functional equivalents of a higher-order necessary condition. SUIN conditions are the constitutive parts of such higher-order constructs. An example of SUIN conditions is shown by the following solution formula:

$$(A+B) * (C+{\sim}D) \leftarrow Y.$$

There are two necessary conditions: the unions A+B and C+~D. Each union taken alone is insufficient for producing Y. Both single components of the union – that is, A and B for the first parenthesis and C and ~D for the second – are themselves not necessary parts, but mutually substitutable (and therefore sufficient) elements of necessary conditions for Y. Condition A is an alternative necessary condition to B, B to A, C to ~D and ~D to C. Neither condition is indispensable, but they fulfill alternatively the requirements for necessity. Just like the sufficiency solution term above, this also is a causally complex statement: due to the inherent asymmetric nature of the necessity relation described by SUIN conditions, both an equifinal element ("sufficient, but unnecessary part of a factor") and a conjunctural element ("a factor that is – taken alone – insufficient, but necessary for the result") can be seen.

It is important to point out, though, that the way we define causal complexity here is only one possible definition. Other views exist and not all of them coincide with the type of results produced by QCA.[6] Most importantly, there are definitions of causal complexity that put emphasis on the unfolding of factors *over time*. Aspects such as time, timing, sequencing, or feedback loops (Abbott 2001; Pierson 2004; Grzymala-Busse 2010) are commonly dealt with in intensive analyses of few cases, often under the label process tracing (George and Bennett 2005; Hall 2006; Collier 2011). Unless special provisions are made, such as conceptualizing and measuring conditions in a way that they reflect temporal aspects or by measuring cases at different points in time, QCA produces results that are static in nature. In section 10.3, we discuss in further detail different strategies of making set-theoretic approaches more sensitive to time.

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[6] For discussions on causal complexity, see Cioffi-Revilla (1981); Ragin (1987, 2000, 2008a); Braumoeller (1999, 2003); Braumoeller and Goertz (2000, 2002); Dion (2003); George and Bennett (2005); *Political Analysis* (2006).

3.3.3 The notion of asymmetry

Among the three defining characteristics of causal complexity in set-theoretic methods, perhaps the most counterintuitive is asymmetry. We first distinguish between a static and a dynamic notion of asymmetry, and we then contrast it with symmetric notions of association and argue that DeMorgan's law is not an instance of causal symmetry.

Asymmetry as understood in QCA is a static notion. Asymmetric set relations can be detected based on cross-sectional data without any information of changes over time. Lieberson (1985), in contrast, defines asymmetry through the complete or partial irreversibility of causal processes. In his view, a causal relation is asymmetric if the outcome, when deprived of a cause at time t_1, does not take on its original value at t_0. For example, a policy measure (X), such as a job-market program, results in high job-market possibilities (Y) at t_0. In a second step at t_1, this job-market program is suspended, but the job-market possibilities remain high. Despite a dynamic character that requires at least two points in time in order to define asymmetry, Lieberson's notion has much in common with asymmetry as seen in QCA. Both share the idea that knowledge of the causal role of X for Y does not contain information on the causal role of ~X for Y. Both in Lieberson's dynamic and QCA's static notion of asymmetry, depriving an effect (Y) of its cause (X) does not necessarily mean that the effect will disappear.

Asymmetry, thus, describes the fact that insights on the causal role of a condition are of only limited use for the causal role of its absence, and the explanation of the occurrence of an outcome does not necessarily help us much in explaining its non-occurrence. This is different from symmetric notions, which are predominant in quantitative approaches. If we are able to explain positive or high values of a dependent variable, then we are also able to explain negative or low values of the dependent variable. There is no need for a separate analysis of high and low values in the outcome Y, since our equation is valid for – or better, is derived from – the whole range of values of Y. The explanation of Y automatically implies the explanation of ~Y and vice versa. The occurrence and the non-occurrence of phenomena, such as democracy/non-democracy, war/peace, wealth/poverty, etc., are each explained by the same equation. Likewise, if we know the way in which variable X contributes to the explanation, then we also know that ~X takes on the precise inverse role. In symmetric approaches it therefore makes little sense to differentiate the causal role of high versus low values of a condition, nor does it make sense to analyze Y and ~Y in separate analyses. For illustration, just consider the fact that the results of a multivariate

regression remain substantively unaffected if one inverts the scale of a dependent variable by multiplying it by −1. The coefficients of the variables remain the same; only their sign switches.

The only occasion when results produced by set-theoretic methods are symmetric is the very rare instance in which the outcome and the solution set perfectly coincide.[7] If, furthermore, the truth table is fully specified – that is, there are no logical contradictions in the data (section 5.1) – and every theoretically possible combination of conditions is represented with empirical cases (section 6.1), then DeMorgan's law (section 2.4.2) can be used to negate not only sets, but also, in a subsequent step, their relations. Remember that, following DeMorgan, the negation of a logical statement requires us to (a) invert all conditions and complements and (b) exchange the logical operators. For example, the negation of the statement A + B is ~A*~B. Now, if the expression A + B is *identical* to Y (i.e., both sets completely overlap rather than A + B being a subset of Y), then A and B are not only individually sufficient for Y, but their OR union is also necessary for Y. This means that we can put an equals sign instead of an arrow:

A + B = Y.

Dealing with an equation, it is possible to negate both sides without altering the truth value of the statement and write:

~A~B = ~Y.

This new equation can now be interpreted to mean that ~A and ~B are two necessary conditions for Y which, as an intersection, are also sufficient for Y. This indicates that, in case of a fully specified truth table, if the union of sufficient conditions is necessary for the outcome (or, parallel, if the intersection of necessary conditions is sufficient), we can interpret DeMorgan's law in terms of sufficiency and necessity: if we find a sufficient condition for the outcome, this also means that the complement of that condition is a necessary condition for the complement of the outcome, and vice versa. Formally:

X → Y => ~X ← ~Y, and

X ← Y => ~X → ~Y.

Let us illustrate this with two simple examples. If we identify having a passport as a necessary condition (X) for being allowed onto an airplane (Y), then

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[7] Yet, as we show in section 3.3.4.1, even then it is important to realize that there are crucial differences between a (perfect) set coincidence and a (perfect) correlation.

not having a passport (~X) is sufficient for not being admitted onto a flight (~Y). Likewise, if being a West European country (X) is sufficient for being a democracy (Y), then not being a West European country (~X) is a necessary condition for not being a democracy (~Y).

Two important points need to be raised. First, the formal logical symmetry revealed by DeMorgan between the solution terms for outcome Y and its negation ~Y is a far cry from the symmetry contained in equations obtained based on correlational statistical techniques (see 3.3.4). Despite being logical mirror images, the solution terms for Y and ~Y do look quite different while in multivariate regression models they do not. Second, we emphasize again that DeMorgan's law can only be applied under very peculiar situations that are rarely, if ever, encountered in empirical social research. The data must be free from logical contradictions (section 5.1) and from logical remainders (section 6.1).

3.3.4 Set-theoretic methods and standard quantitative approaches

When applying set-theoretic methods, researchers almost unavoidably create causally complex results. When applying standard regression-based methods, such types of results cannot be achieved. This may not be particularly important as many researchers might not be interested in equifinality, INUS conditions, and the like. It is important, however, that regression-based results not be interpreted as if they revealed set relations. In the following, we briefly aim at demonstrating that set relations are not correlations and that therefore any correlation-based technique is less well equipped for unraveling set relations and the form of causal complexity that comes with it. This comparison should contribute to understanding exactly what set-theoretic methods are, as well as what they are not.

3.3.4.1 Set relations are not correlations

In statistical techniques based on symmetric measures of association, the strength of a (positive) covariation between variables X and Y is determined by how many cases fall into the *two* cells in the off-diagonal in a two-by-two table. Cramer's v and the ϕ coefficients (both standardizations of the more general χ^2 measure), but also various parameters which make different use of the numbers of cells and possibly existing ties (such as τ, γ, or Y) are typical for assessing associations.[8] Importantly, when assessing a correlation all cells

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[8] A perfect correlation between the *variables* X and Y requires all cases to be in cells b and c (the main diagonal) and the two off-diagonal cells a and d to be empty. A similarly perfect (but negative) correlation would be produced by all cases being in cells a and d, with b and c being empty. In either case, the strength of a correlation is determined by the ratio of cases on the diagonal vis-à-vis those in cells off the diagonal.

Y		X	
1	not allowed **a**	allowed **b**	
0	**c** allowed (but not relevant)	**d** not allowed	
	0	1	

X

Figure 3.10   Two-by-two table – necessity and sufficiency

are taken into account, and, therefore, the correlation is a symmetric measure. In contrast, set-theoretic relations, such as sufficiency and necessity, instead require only that one off-diagonal cell is empty. In addition, the 0,0 cell (cell c in Figure 3.10) does not play any role. Likewise, when using continuous data, symmetric approaches like a regression require cases to be distributed symmetrically around the regression line in a scatter plot.[9] Instead, with fuzzy sets, asymmetric set relations are indicated in an XY plot by triangular data patterns in which either the area above or the area below the main diagonal is devoid of cases.

In short, the data pattern of a correlation looks quite different from that of a set relation. This means that it is absolutely possible for a researcher to find a perfect set relation (either necessity or sufficiency) but fail to detect any strong correlation between two variables.

In order to further highlight the difference between set relations and correlations, let us look at the extreme scenario of a condition X being simultaneously necessary and sufficient. In formal terms, such a condition can be written as follows:

$X \leftrightarrow Y$ (read: if X, then and only then Y, *or* sets X and Y fully overlap).

A Venn diagram of a necessary and sufficient condition would display only one circle, which represents both condition X and outcome Y, indicating the perfect overlap of X and Y. Figure 3.10 presents the same argument in a two-by-two table. No cases are allowed in the off-diagonal.

A perfect correlation and a perfect set coincidence (i.e., simultaneous necessity and sufficiency) seem to be the same, since they require the same two cells (a and d) to be empty. There remains an important difference,

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[9] The principles of ordinary least squares, in which the sum of the square roots of the distances between the dots and the regression line are minimized, is a direct expression of this symmetric thinking in statistical analysis.

though. While in a correlational analysis both cells on the main diagonal (b and c) have equal analytic relevance, in set-theoretic analyses only cell b is relevant, not cell c. The question of whether cases that show neither the condition nor the outcome (~X, ~Y) contribute any information about the necessity and sufficiency of X for Y is hotly debated. In philosophy this debate is often conducted under the label the "black raven paradox." The German philosopher Carl Gustav Hempel brought up the question of whether a green apple could be useful to prove the proposition "All ravens are black," that is, whether an object which shows neither the condition (it is an apple, not a raven, therefore ~X) nor the outcome (it is green, not black, therefore ~Y) can prove the positive relation between the condition and the outcome. In the social sciences, there is disagreement about whether cases without the condition and the outcome are informative. There are scholars who argue that all cases should count (Seawright 2002a, 2002b; Yamamoto 2012) and others who see only a subset of cases as relevant for particular set-theoretic claims and say that, in principle, no set-theoretic claim about the relation between ~X and ~Y implies anything about cases of X and Y (Braumoeller and Goertz 2002; Clarke 2002; Mahoney and Goertz 2006; Goertz and Mahoney 2010).

Despite these ontological disputes, with dichotomous or crisp-set data it empirically holds that a perfect correlation is automatically also a perfect set coincidence and vice versa. The same, however, is not true when moving from crisp to fuzzy sets. Here, a perfect correlation does *not* automatically indicate the presence of a condition that is simultaneously necessary and sufficient. Figure 3.11 provides a straightforward visualization of this argument. The three different lines all represent a perfect correlation between X and Y. If we interpret them as regression lines, they differ only in their slope and intercept. However, only one of these lines indicates a perfect set coincidence. It is the line on the main diagonal, i.e., the line on which all cases have identical set-membership scores in X and Y.

In sum, as Ragin (2008a: 59, n. 3) points out, not all (close-to-)perfect correlations are a sign of a (close-to-)perfect set coincidence. Among the many regression lines that indicate a perfect correlation between X and Y, only *one* is identical to perfect set coincidence: it is the regression line with a slope of 1 and an intercept of 0. From this it follows that standard measures of associations that are applied in quantitative social research are not adequate for unraveling subset relations. Standard tools provided by statistical approaches to the analysis of social science data that use symmetric correlational measures, such as regression analysis, are not well suited to analyzing necessary and sufficient conditions *separately*.
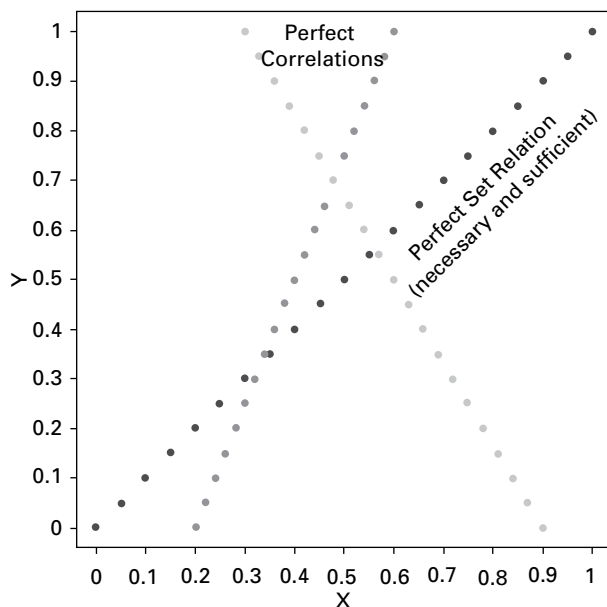
Figure 3.11 XY plot – contrasting perfect set relation with perfect correlations

3.3.4.2 Set theory and regression models

A slightly different way of showing that set-theoretic methods are an adequate choice if (and only if) researchers are interested in the type of causal complexity induced by set relations is by discussing whether regression equations can be interpreted in the light of equifinality.

When performing a statistical analysis, the aim is usually to identify the most powerful predictor for explaining variance in the dependent variable net of the explanatory power of other variables and to make probability statements for the generalization ("significance tests") from (a hopefully representative) sample to a (hopefully well-defined) underlying population. All of this makes standard statistical techniques a powerful set of tools for summarizing complex data into parsimonious equations and in extracting the "net-effect" (Ragin 2008b) of independent variables. But these techniques rest on the starting assumption that social phenomena are driven by unifinality, additivity, and symmetry. A typical example of a linear regression model, until recently the workhorse in applied social science,[10] looks as follows:

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[10] There is growing consensus between both qualitative and advanced quantitative scholars that standard regression models on observational data are in violation of too many assumptions to be able to yield valid results. For a comprehensive debate on this issue see, for instance, Goertz (2003).

$$y = a + b_1 x_1 + b_2 x_2 + b_3 x_3 + \varepsilon.$$

Th is equation shows misleading similarities to our QCA solution formula. Several components are added together with the help of a summation function that produces the dependent variable Y. However, the addition signs in the regression equation tell us that different partial effects are added together to a total effect. Th is leads to a unifinal and not an equifinal result, as in QCA. One path alone ($a + b_1 x_1 + b_2 x_2 + b_3 x_3 + \varepsilon$) explains Y. The independent variables in a regression formula are not alternatives to each other. If they were, then they would cancel each other out.

One might want to counter-argue that regression equations can be interpreted in equifinal terms as there are, in fact, innumerable combinations of values in the independent variables that produce the same value in the dependent variable. However, such an interpretation of regression equations is rather rare in the literature, perhaps simply because not much is learned by pointing out that there is a potentially infinite number of paths toward the outcome without providing any information on which of these paths do occur empirically, let alone which of them are empirically more important than others.

It is also difficult to reveal asymmetric relations of sufficiency or necessity using a standard regression approach. Clark, Gilligan, and Golder (2006) suggest "a simple multivariate test for asymmetric hypotheses" such as those of necessity and suffi ciency by specifying interaction terms in a regression analysis. Core to their argument is the formal logical rule that if X is suffi cient for Y, then ~X is also necessary for ~Y. As already mentioned and as further elaborated in Chapters 5 and 6, this logical rule makes sense only in empirical situations in which the data are void of logical contradictions and empirical information is at hand for all logically possible combinations of independent variables or in which, at least, the highly implausible argument can be made that all the missing information is substantively irrelevant. Such a "simple multivariate test for asymmetric hypotheses" might therefore work in theory, but not in research practice.

Two further arguments against the use of interaction terms for mimicking set-theoretic causal complexity exit. First, an *interaction* of interval or metric scale variables is not the same as an *intersection* of fuzzy sets (Grofman and Schneider 2009). Th e former is an algebraic multiplication whereas the latter is governed by the minimum scoring rule. Second, conjunctions in set-theoretic results often involve three or more single conditions. Yet, regression models with third-order interaction terms are rare, and those with fourth- or

even higher-order terms are virtually absent from the literature – with good reason, as the specification of such models puts high demands on both the data and the substantive interpretation skills of the researcher (Brambor, Clark, and Golder 2006; Kam and Franzese 2007).

The fact that causal complexity is difficult to reach through standard statistical techniques does not mean that it is entirely impossible. Attempts at mimicking set-theoretic approaches within the framework of statistical tools exist (Braumoeller 1999, 2003, 2004; Braumoeller and Goertz 2003; Yamamoto 2012). All of these attempts either deal with a large number of cases or handle just some aspects of causal complexity, or both.

QCA starts out by assuming a maximum level of causal complexity and then tries to simplify this complexity as much as the empirical evidence allows. In standard statistics, the most common approach is to start out with a simple model and then incorporate aspects of causal complexity in the form of interaction terms, lagged variables, multiple equation models, etc. However, methods that rely on symmetric contingency coefficient are not well suited for adequately capturing set relations in terms of necessary and sufficient conditions and all aspects of causal complexity that derive from it (Fiss 2007: 1190). Or, as Braumoeller puts it: "[a]dditive linear models are an inherently inadequate way of modeling multiple causal path processes" (Braumoeller 1999: 7), and using non-additive specifications (i.e., interaction terms) simply offers no practical solution to the problem, especially when the N is medium to low, as is often the case in macro-comparative social research (Braumoeller 1999: 9f.).

As such, neither the assumption of causal simplicity nor that of complexity can claim general superiority. Each has its strengths and weaknesses. Assuming simplicity allows for deriving parsimonious models from rather complex data, while the assumption of complexity usually enables the researcher to pay more tribute to different classes of cases within their population – both of which are valuable aims of social inquiry (Brady and Collier 2004). On the downside, the methodologically induced assumption of simplicity runs the risk of generating oversimplified representations that are not only very much detached from the cases and data patterns that underlie the analysis, but which also often present mere caricatures of the much more refined theories they claim to test (Munck 1998). The results of these procedures are often easier to interpret and are considered more "aesthetic" (Somers 1998: 761) but sometimes no longer even try to speak to the complexity of the social world. This discrepancy between method-induced simplifying assumptions on the one hand, and our hunches about the world's complexity on the other, requires us to bring our epistemological way of

thinking in closer union with our prevailing ontological view of the world (Hall 2003). In turn, the starting assumption of complexity, as implemented in set-theoretic methods, runs the risk of individualizing each and every single case without much progress towards generalization and with significant difficulties in theorizing (even *ex post facto*) this empirical complexity.

Neither the preference for causal complexity nor for simplicity should be seen as superior per se. It depends on what one expects to be an adequate account of the phenomenon being studied (Przeworski and Teune 1970: 211). Unfortunately, it is for methodological rather than for substantive reasons that scholars often succumb to a "general linear reality" (Andrew Abbott, cited in McKeown 1999) in situations where it is not appropriate (Shalev 2007). It is the increasingly strongly argued opinion in the methods-sensitive literature that the burden of proof should be on the shoulders of those researchers who believe that the world can be explained through parsimonious and elegant models, and not, as has been long asserted, on those who view a large number of social phenomena as the result of equifinal and combinatorial processes (Braumoeller 2003). On the other hand, it is of course equally wrong to assume, by default, causal complexity and to always apply set-theoretic methods without proper justification within a given research context.

In sum, set relations are not correlations. For an adequate choice of the appropriate method it is therefore essential to be clear about whether one is looking for necessary and sufficient causes or some other form of relationship between social phenomena not rooted in set theory. The application of set-theoretic methods makes sense only if there are good reasons to believe that the phenomenon under investigation is best understood in terms of set relations. QCA and other set-theoretic methods are ill equipped for detecting correlations. At the same time, the majority of standard statistical techniques are not well suited for detecting subset relations of necessity and sufficiency.

At-a-glance: causal complexity in set-theoretic methods

Three elements render the specific form of causality in QCA particularly relevant: **equifinality** refers to the characteristic that various (combinations of) conditions imply the same outcome; **conjunctural causation** draws our attention to the fact that conditions do not necessarily exert their impact on the outcome in isolation from one another, but sometimes have to be combined in order to reveal causal patterns; **asymmetrical causation** implies that both the occurrence and the non-occurrence of social phenomena require separate analysis and that the presence and absence of conditions might play crucially different roles in bringing about the outcome.

These aspects also enable us to analyze **INUS** and **SUIN conditions** with the help of QCA. INUS conditions are defined as insufficient but necessary parts of a condition which is itself unnecessary but sufficient for the result; SUIN conditions refer to sufficient, but unnecessary, parts of a factor that by itself is insufficient, but necessary, for the result.

Causally complex results produced by set-theoretic methods differ from those produced by standard statistical (regression-based) approaches. While more advanced quantitative techniques can mimic some aspects of causal complexity, achieving all of them simultaneously is a challenging and still unresolved task. However, no form of causality induced by the choice of method can be considered superior per se. Instead, researchers should choose the method that rests on assumptions which are most in line with their research question. If hunches about necessary and sufficient conditions exist (and there are many research fields for which this is the case, as Goertz and Starr 2003 and Seawright 2002b: 180f., show), then set-theoretic methods are a plausible choice.

Note that, in this chapter, we have only labeled those conditions as sufficient or necessary for which all empirical evidence was in line with these respective set relations. However, some of the examples have already alluded to the fact that there can be different degrees of deviation from perfect subset relations. In fact, when applying set-theoretic methods to social science data, such observations are the norm. We will discuss this in more detail in Chapter 5.

4 Truth tables

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Easy reading guide

In this chapter, we introduce a concept that is at the core of QCA, both in the understanding of it as an approach and as a technique: truth tables. QCA understood as an approach can be perceived as a research phase that aims to construct a truth table. Truth tables contain the empirical information gathered by the researcher, often through years of painstaking work. QCA as a technique, then, consists of the formal analysis of truth tables – the so-called logical minimization – with the aim of identifying sufficient (and necessary) conditions. As such, truth tables become *the* indispensable tool for any QCA, no matter whether we are working with crisp or fuzzy sets. This is one of the primary bases for the argument that crisp-set QCA and fuzzy-set QCA are not fundamentally that different. It also means that most of what we say in this book about truth tables, and their analysis, applies both to csQCA and to fsQCA.[1]

We deem it important to reiterate that, in this and other chapters, we mostly focus on issues related to QCA as a technique for pedagogical reasons, and we therefore take for granted the existence of empirical information upon which the truth table is constructed. However, one integral part of set-theoretic approaches – and the key to their success – consists precisely in the process of collecting this information and constructing truth tables in an iterative process, a process sometimes described as the "back and forth between ideas and evidence" (Ragin 1987). The analysis of the truth table only represents a short "analytic moment" (Ragin 2000) in the process of performing set-theoretic analysis.

In Chapter 3, we engaged in the analysis of necessity and sufficiency without making use of truth tables. One might therefore wonder why we would need truth tables if necessity and sufficiency can also be analyzed simply by screening a standard data matrix. As this chapter will show, truth tables are a much more adequate device for detecting set relations, mainly because they shift the focus from empirical cases to configurations of conditions. This leads to a radically different – and more efficient – approach to the analysis of sufficiency. The analysis of sufficiency based on a data matrix proceeds in a

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[1] There are only a few analytically relevant differences in the analysis of a truth table that follow from the difference between crisp and fuzzy sets, such as, for instance, the possibility in fuzzy sets that a given truth table row is a subset of outcome Y but also of its complement ~Y. We will discuss this in section 9.2.2.

bottom-up manner by first focusing on simple sets and then proceeding to more complex sets. In contrast, the analysis of sufficiency based on a truth table proceeds top-down, by first screening all logically possible combinations of conditions and then logically minimizing those conjunctions that have passed the test of sufficiency. Notice, however, that while for sufficiency the truth table approach is (and should be) the dominant strategy, for the analysis of necessity, the bottom-up approach is instead clearly preferable and the top-down approach is meaningless. The reason is simple: a logical AND conjunction of two or more conditions can only be necessary for Y if, and only if, all single conditions involved in the conjunction are necessary on their own.

The organization of this chapter is straightforward: after clarifying what a truth table is (4.1), we show how truth tables are constructed based on empirical information about cases (4.2). In section 4.3 we explain, step by step, how truth tables are analyzed with the help of Boolean algebra. Clearly, this chapter is central to the whole book, simply because truth tables are the *sine qua non* technique for QCA. The chapter should be read in detail and with care. This chapter also provides important information which will supply the main ingredients for the Truth Table Algorithm, the currently accepted minimum standard for a QCA, as it will be introduced in Chapter 7.

4.1 What is a truth table?

The concept of a truth table originates in formal logic. At first glance it might look a lot like a standard data matrix. Just like conventional data matrices, each truth table column denotes a different variable or, better, set. The difference consists in the meaning of rows. In a standard data matrix, each row denotes a different case (or unit of observation). In a truth table, each row instead represents one of the logically possible AND combinations between the conditions. Since each single condition can occur either in its presence or its absence, the total number of truth table rows is calculated by the expression 2^k. The letter k represents the number of conditions used and the number 2 the two different states (presence or absence) in which these conditions can occur. Each row denotes a *qualitatively different* combination of conditions, i.e., the difference between cases in different rows is a difference in kind rather than a difference in degree.

The formula 2^k yields the number of *logically possible* combinations or truth table rows or, slightly misleadingly, logically possible cases. The number of truth table rows increases exponentially with the number of conditions. With three conditions, we end up with eight configurations. With 4 conditions, we already have 16 configurations, with 5 we have 32, and with

10 we have no fewer than 1,024 logically possible cases. In social reality and therefore also in social science research practice, not all of these potential cases materialize empirically. The whole of Chapter 6 is dedicated to the phenomenon of *limited diversity* and provides strategies on how to handle *logical remainders* (Ragin 1987: 104ff.). For the time being, and in order to properly introduce the meaning and analysis of truth tables, in the current chapter we only deal with truth table rows that do not show any such logical remainders.

Venn diagrams are another way to intuitively visualize that k number of conditions produce 2^k logically possible combinations. Figure 4.1 displays three conditions (A, B, C). They all overlap in various ways, creating eight different areas. Each area in the Venn diagram corresponds to one row in a truth table, and each area can be described in the form of a Boolean expression. For example, the area in the middle of the diagram where A, B, and C overlap is the one that contains all the cases where A, B, and C are present. This can be written as A*B*C, or simply ABC (Chapter 2). The upper area of set A is where condition A can be observed (A = 1) and B and C cannot (B = 0, C = 0). This area thus denotes the set A*~B*~C or simply A~B~C. The area outside all three of the circles, but within the rectangle, denotes cases where none of the three conditions is present and can be written as ~A*~B*~C or ~A~B~C, and so on.

While Venn diagrams are generally a very useful tool for the graphical representation of set-theoretic statements, two caveats need to be made. First, as the number of conditions grows beyond four or five, it becomes difficult to draw and interpret Venn diagrams. Second, note that Venn diagrams such as the one displayed in Figure 4.1 display only sets and their intersections. In Chapter 3, however, we used Venn diagrams to visualize subset relations of sufficiency and necessity between conditions and an outcome. Of course, Venn diagrams can do both simultaneously, i.e., show the subset relation of an intersection of conditions and an outcome.

4.2 How to get from a data matrix to a truth table

4.2.1 Crisp sets

In order to show how to construct a truth table based on information on cases stored in a data matrix, let us go back to our data matrix from section 3.1.1.2. How do we get from here to a truth table? While most of the relevant software
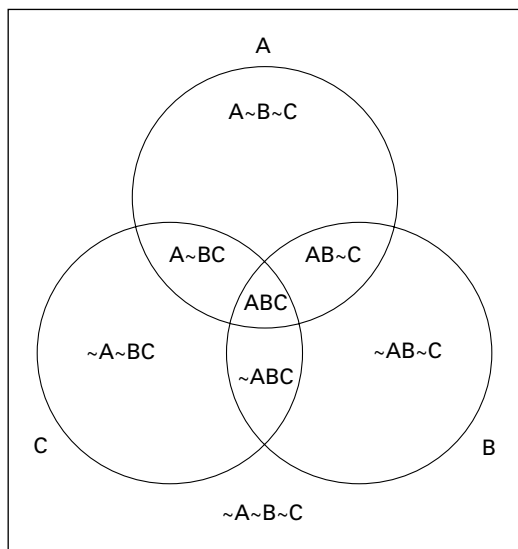
Figure 4.1   Venn diagram with three conditions

packages are able to produce a truth table based on a data matrix representing set-membership scores, it is worth spelling out the three simple steps that are needed.

First, we write down all 2^k logically possible combinations of the k conditions, leaving the column for the outcome value empty. Second, we assign each case from our data matrix to the truth table row that corresponds with its values in the k conditions. Each case can belong to only one truth table row, but individual truth table rows might contain more than one case. In our example, we observe that Argentina and Venezuela display identical values on all three conditions – they had a violent upheaval, have an ethnically homogeneous population and a pluralistic party system. They therefore belong to the same truth table row labeled A*B*C. The same holds true for Peru and Ecuador, which are both assigned to the truth table row A*~B*~C (violent upheaval, no ethnically homogeneous population, no pluralistic party system). In this way, we assign each case to one of the eight logically possible truth table rows.

Third, an outcome value has to be attributed to every truth table row. It is determined by the outcome values of the empirical cases that fall into the respective row. For instance, Colombia falls into row ~A*~B*~C and shows outcome Y. No other case falls into this row. Hence, the outcome value of row ~A*~B*~C is Y = 1. Likewise, neither Argentina nor Venezuela shows a stable

Table 4.1 Data matrix with ten cases, three conditions, and outcome

Row	Cases	Conditions			Outcome
		A	B	C	Y
1	ARG	1	1	1	0
2	PER	1	0	0	0
3	BOL	1	1	0	0
4	CHI	0	1	0	1
5	ECU	1	0	0	0
6	BRZ	0	1	1	1
7	URU	1	0	1	1
8	PAR	0	0	1	1
9	COL	0	0	0	1
10	VEN	1	1	1	0

Y = set of countries with stable democracies
A = set of countries with violent upheavals in the past
B = set of countries with ethnically homogeneous population
C = set of countries with pluralistic party system

democracy, and the outcome value of truth table row A*B*C is Y = 0.[2] Based on this procedure, the data matrix in Table 3.2 yields the truth table displayed in Table 4.2.

The truth table consists of $2^3 = 8$ rows. Strictly speaking, the columns "Row," "~Y," and "Cases" do not belong to the truth table but are included for illustrative purposes. It is important to understand the information contained in the "Outcome" column. From a case perspective, the value of 1 indicates that cases with the given characteristics also show the outcome of interest. For instance, from row 1 in Table 4.2 we learn that cases that did not have a violent upheaval and have no ethnically homogeneous population and have no pluralistic party system are stable democracies. If we shift perspective from cases to configurations, we can say that conjunction ~A~B~C (row 1) is sufficient for Y. A truth table row with outcome Y = 1 is explicitly linked (Ragin and Rihoux 2004) to this outcome. In essence, each truth table row is a statement of sufficiency (Ragin 2008a).

[2] Of course, when applied to real data, it is common that cases attributed to the same truth table row display different membership scores in the outcome. Such rows are called *contradictory rows* (Ragin 2000). Chapter 5 is dedicated to discussing this crucially important issue. For the time being, in order to present the logic of truth tables and their analysis, we present examples of truth tables that are contradiction-free.

Table 4.2 Hypothetical truth table with three conditions

Row	Conditions			Outcome		Cases
	A	B	C	Y	~Y	
1	0	0	0	1	0	COL
2	0	0	1	1	0	PAR
3	0	1	0	1	0	CHI
4	0	1	1	1	0	BRZ
5	1	0	0	0	1	PER, EC
6	1	0	1	1	0	URU
7	1	1	0	0	1	BOL
8	1	1	1	0	1	AR, VEN

See Table 3.2

~Y = set of countries with non-stable democracies

4.2.2 Fuzzy sets

The three steps for converting a data matrix into a truth table also apply when the underlying data are not crisp but fuzzy sets. We first create the truth table, then assign each case to one of these rows, and then determine the outcome value for each row. Since fuzzy sets allow for any set-membership score between 0 and 1, whereas truth tables consists of only 0s and 1s, this might seem puzzling.

The creation of the truth table is the least problematic step. Just as with crisp sets, the number of truth table rows based on fuzzy sets is given by the formula 2^k. This is because, just like crisp sets, fuzzy sets establish a *qualitative* difference between cases above the 0.5 qualitative anchor (more in than out of the set) vis-à-vis cases below that anchor (more out than in). This is why k fuzzy-set conditions yield 2^k truth table rows.[3]

The attribution of cases to specific truth table rows, a rather straightforward exercise based on crisp sets, requires more explanation when dealing with fuzzy sets. With crisp sets, in order to identify the truth table row to which a case belongs, we simply need to find the *exact* match between the case's crisp-set membership scores and the truth table rows. With fuzzy sets, however, cases with fuzzy-set membership scores in the k conditions do not

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[3] The situation is different in multi-value QCA (mvQCA). In section 10.2, we discuss the consequences of the fact that with k-number of multi-value "sets," the number of truth table rows is (much) higher than 2^k.

Table 4.3 Hypothetical data matrix with fuzzy-set membership scores

Row	Cases	A	B	C
1	ARG	0.8	0.9	1
2	PER	0.7	0	0
3	BOL	0.6	1	0.1
4	CHI	0.3	0.9	0.2
5	ECU	0.9	0.1	0.3
6	BRZ	0.2	0.8	0.9
7	URU	0.9	0.2	0.8
8	PAR	0.2	0.3	0.7
9	COL	0.2	0.4	0.4
10	VEN	0.9	0.7	0.6

A = set of countries with violent upheavals in the past
B = set of countries with ethnically homogeneous population
C = set of countries with pluralistic party system

exactly match any of the truth table rows. For instance, to which truth table row does Chile in row 4 of Table 4.3 belong, with its set membership scores of A = 0.3, B = 0.9, and C = 0.2?

In order to shed light on this, Ragin (2008a: ch. 7) refers to the concept of a *property space*, going back to Paul Lazarsfeld's (1937) initial ideas. Each set constitutes one dimension of the property space (Barton 1955). The three fuzzy-set conditions in our example thus yield a three-dimensional space as displayed in Figure 4.2. There are several important features of this property space.

First, regardless of a case's membership in conditions A, B, and C, it falls, by definition, inside the property space. This is because both set membership and the dimensions of the property space have their minimum at 0 and their maximum at 1. Second, based on the set membership in A, B, and C, each case has one precise location inside the cube. Third, each corner of the property space directly corresponds to a specific combination of values in A, B, and C. More precisely, each corner represents one specific combination of the two extreme values that are possible in fuzzy sets – full membership (1) and full non-membership (0). For example, the corner in the bottom left front of Figure 4.2 denotes the situation in which all three fuzzy sets take on the value of 0. This corner can therefore be labeled the "0,0,0" or the ~A~B~C corner. Following this logic, we can describe the lower right corner in the front as "1,0,0," the top right rear corner as "1,1,1," and so on. Fourth, because each
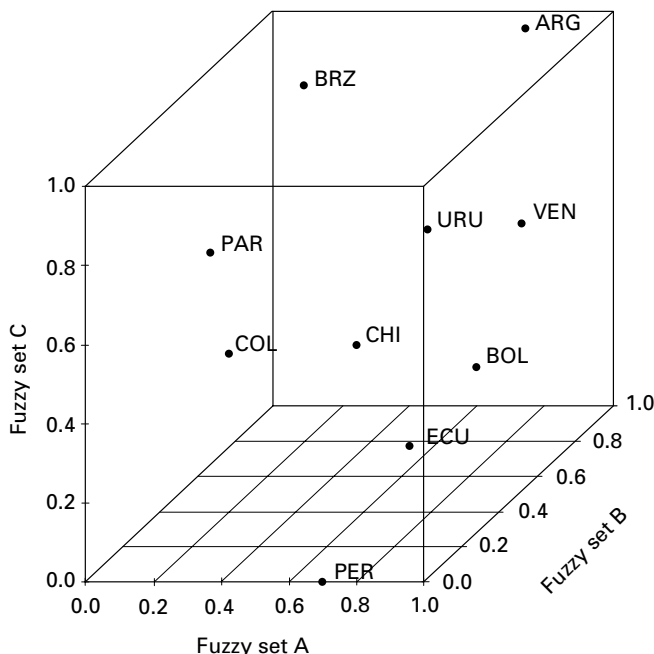
Figure 4.2   Three-dimensional property space

corner denotes a specific combination of extreme membership scores in the conditions, we can perceive of these corners as ideal-typical situations (Weber 1906). Cases that fall exactly in one of the corners are empirical instances of the ideal type denoted by that corner. Unless a case has full (non-)membership in all conditions that constitute the property space, in other words, unless a case exclusively displays crisp-set membership scores, it will not be located directly in one of the corners. Thus, most of the time in fuzzy-set analyses, many, if not all, cases get close to these ideal types only to some (varying) degree. Below, we will explain how the distance to the ideal types can be calculated.

Fifth, a property space with three dimensions has eight corners. This number should ring a bell. A truth table based on three conditions has eight rows. This is no coincidence, but directly follows from the fact that the corners of a property space, spanned by fuzzy sets, are equivalent to the rows of a truth table.[4] This equivalence exists because the corners of a property space defined by fuzzy sets denote the situation where the values of these fuzzy conditions take on the extreme values 0 or 1. In other words, the corners are where the fuzzy sets show crisp-set membership scores.

[4] The metaphor of the cube only works for three conditions. Other geometrical objects would be needed for the representation of other numbers of conditions. However, the basic principle remains the same.

We can summarize the insights gained so far in the following manner. With k number of conditions, we create a property space with 2^k corners and these corners correspond to one of the 2^k (a) ideal types; (b) truth table rows; (c) logical AND conjunctions between the k conditions.

As mentioned, with fuzzy sets cases usually have membership values between 0 and 1. Consequently, they can be located anywhere in the property space, as Figure 4.2 indicates. Some might be closer to one of the corners than to the others. We therefore have to find a way to establish two things: first, to which corner a given case most belongs, and second, how far this case is a member of this ideal type (aka truth table row).

In order to explain the principle by which the membership of cases in each corner is calculated, let us focus on two cases from Table 4.3, Venezuela and Ecuador, reproduced in Table 4.4.[5]

Looking at Venezuela, we see that its membership in all three conditions is above the qualitative anchor 0.5. If asked which of the 2^k ideal types this country resembles most, it is plausible to say that this country is more of an *ethnically relatively homogenous state with a pluralistic party system that experienced a violent upheaval* than any other logically possible type. In other words, intuitively, we would locate Venezuela closest to the ABC row of a truth table or the "1,1,1" corner of a property space, an intuition visually supported by Venezuela's location in Figure 4.2. The same logic of locating a case applies to Ecuador. It is more in than out of set A and more out than in sets B and C, respectively. This makes Ecuador closer to the "1,0,0" corner than any other and an instance of an *ethnically heterogeneous population without a pluralistic party system and without a violent upheaval* (row A~B~C).

Beyond this intuitive attribution of cases to property space corners, aka ideal types, is there a standardized way to precisely define the membership of cases in truth table rows? Yes, there is. Remember that each of the 2^k corners corresponds to one of the 2^k logically possible AND combination of conditions. Remember also (see section 2.1) that the membership of cases in an intersection is determined by their minimum set membership across the single conditions. It is therefore easy to calculate a case's membership in *all* logically possible combinations of conditions, aka corners of the property space. Table 4.5 contains this information for our two cases displayed in Table 4.4.

Venezuela has a fuzzy-set membership of 0.6 in ideal type ABC. This is the minimum across conditions A (0.9), B (0.7), and C (0.6). Ecuador has

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[5] We do not report each case's membership in outcome Y, because it is irrelevant for identifying the truth table row a case belongs to. When performing the three steps of converting a data matrix into a truth table, the outcome is added only in the third step.

Table 4.4 Fuzzy-set data matrix with two cases

| Case | Conditions | | |
	A	B	C
VEN	0.9	0.7	0.6
ECU	0.9	0.1	0.3

See Table 4.3

a membership value of 0.7 in the ideal type A~B~C, which is the minimum across A (0.9), ~B (0.9), and ~C (0.7). Both cases are not full instances of their respective ideal types, as indicated by their membership score of less than 1.

As Table 4.5 also shows, each case has a partial membership not only in its own ideal type, but also in all of the other corners of the property space. These membership scores are, however, quite low, a direct consequence of the minimum scoring rule that governs the calculation of set membership scores for conjunctions (section 2.1). The crucial point is that, while each case has a partial membership in all rows, there is only one row in which its membership exceeds the qualitative anchor of 0.5. This is a golden rule for fuzzy sets: no matter how many fuzzy sets are combined, any given case has a membership of higher than 0.5 in one and only one of the 2^k logically possible combinations.

This important mathematical property of fuzzy sets is crucial for our task at hand – identifying the truth table row to which a case best belongs, which turns out to be that truth table row in which its partial set membership is higher than 0.5.

There is one exception to the rule that each case is more in than out of one and only one logically possible combination. Whenever a case holds a membership of exactly 0.5 in one or more of the constitutive conditions, then its membership will not exceed 0.5 in any of the truth table rows. To demonstrate this, we add a third hypothetical case to our data matrix, which has a set membership of 0.5 in condition C. Since both C and ~C take on the value of 0.5, no single ideal type out of the eight possibilities can arrive at a value of greater than 0.5. No minimum from the three single conditions and their complements can be greater than 0.5. Furthermore, there are two ideal types for which the minimum is exactly 0.5. The 0.5 anchor is sometimes referred to as the point of maximum ambiguity (Ragin 2000). It expresses the fact that a case's empirical attributes are such that it cannot be decided whether the case

Table 4.5 Fuzzy-set membership in ideal types for hypothetical data matrix

Case	Conditions			Property space corners/Ideal types/Truth table rows (logically possible combinations of conditions)							
	A	B	C	ABC	AB~C	A~BC	A~B~C	~ABC	~AB~C	~A~BC	~A~B~C
VEN	0.9	0.7	0.6	**0.6**	0.4	0.3	0.3	0.1	0.1	0.1	0.1
ECU	0.9	0.1	0.3	0.1	0.1	0.3	**0.7**	0.1	0.1	0.1	0.1

Table 4.6 Fuzzy-set ideal types for hypothetical data matrix

Case	Conditions			Property space corners/Ideal types/Truth table rows (logically possible combinations of conditions)							
	A	B	C	ABC	AB~C	A~BC	A~B~C	~ABC	~AB~C	~A~BC	~A~B~C
VEN	0.9	0.7	0.6	**0.6**	0.4	0.3	0.3	0.1	0.1	0.1	0.1
ECU	0.9	0.1	0.3	0.1	0.1	0.3	**0.7**	0.1	0.1	0.1	0.1
HYP	0.8	0.1	0.5	0.1	0.1	**0.5**	**0.5**	0.1	0.1	0.2	0.2

is more a member of the set being studied or more a member of the complement of that set. It is because of this ambiguous status that such a case cannot be attributed to any of the 2^k logically possible ideal types that involve this set or its complement.

One practical lesson from this is to be careful about assigning the fuzzy-set membership score of 0.5 to cases. Doing so not only prevents the attribution of such a case to any of the truth table rows, but also represents the weakest possible conceptual statement about that case.

Getting back to our task of representing fuzzy-set data in a truth table, we now know that such a truth table has 2^k rows and that each case is more in than out of one, and only one, of these rows while holding partial membership scores in most, if not all, other rows as well. What remains to be resolved is to determine the outcome value with which each of the 2^k rows is connected. In order to answer this question, remember that each truth table row is a statement of sufficiency. This means that each truth table row should be considered a sufficient conjunction for the outcome if each case's membership in this row is smaller than or equal to its membership in the outcome (see section 3.1.2.1).

Table 4.7 Fuzzy-set membership in rows and outcome

	Conditions			Truth table rows								Outcome
Cases	A	B	C	ABC	AB~C	A~BC	A~B~C	~ABC	~AB~C	~A~BC	~A~B~C	Y
ARG	0.8	0.9	1	0.8	0	0.1	0	0.2	0	0.1	0	0.1
PER	0.7	0	0	0	0	0	0.7	0	0	0	0.3	0.4
BOL	0.6	1	0.1	0.1	0.6	0	0	0.1	0.4	0	0	0.3
CHI	0.3	0.9	0.2	0.2	0.3	0.1	0.1	0.2	0.7	0.1	0.1	0.6
ECU	0.9	0.1	0.3	0.1	0.1	0.3	0.7	0.1	0.1	0.1	0.1	0.4
BRZ	0.2	0.8	0.9	0.2	0.1	0.2	0.1	0.8	0.1	0.2	0.1	0.7
URU	0.9	0.2	0.8	0.2	0.2	0.8	0.2	0.1	0.1	0.1	0.1	0.8
PAR	0.2	0.3	0.7	0.2	0.2	0.2	0.2	0.3	0.3	0.7	0.3	0.9
COL	0.2	0.4	0.4	0.2	0.2	0.2	0.2	0.4	0.4	0.4	0.6	1
VEN	0.9	0.7	0.6	0.6	0.4	0.3	0.3	0.1	0.1	0.1	0.1	0.3
Membership in row ≤ Membership in Y				0	0	1	0	0	0	1	1	

Table 4.7 displays the fuzzy-set membership scores of our ten hypothetical cases in the three conditions, the eight truth table rows, and the outcome stable democracy (Y). For each truth table row, we asses whether each case's membership in it is smaller than or equal to its membership in Y. If so, the respective row is a subset of the outcome, thus fulfills the criterion of a sufficient condition and therefore receives a score of 1. If, however, one or more case's membership in the row exceeds that in the outcome, then the respective row is not a perfect subset of Y and receives a score of 0. As the last row of Table 4.7 shows, three conjunctions – A~BC, ~A~BC, and ~A~B~C – are perfect subsets of Y. For all other truth table rows, one or more cases deviate from the subset pattern of sufficiency and these rows are therefore not considered as sufficient for Y.[6]

We now have all the relevant information at hand to represent a fuzzy-set data matrix in a standard crisp truth table format. For each row, we know which cases belong to it and whether it is a subset of the outcome. The truth table that results from our hypothetical fuzzy-set data is shown in Table 4.8.

Before we continue and explain how a truth table is analyzed using the tools of formal logic, several important points should be underlined. First, regardless of whether crisp or fuzzy sets are used, a truth table is at the core

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[6] As mentioned, in Chapter 5 we will deal with the question of how much deviation one can or should allow for before dismissing a subset relation.

of QCA. Second, when representing fuzzy sets in a crisp truth table, the more fine-grained information contained in fuzzy sets is crucial and remains available at all times. In other words, the procedure that leads to a truth table like that in Table 4.8 does not involve any conversion of fuzzy sets into crisp sets. The information conveyed by fuzzy-set membership scores is used both when assigning cases to rows and when assessing whether a row is a subset of the outcome. Third, when producing a truth table based on fuzzy sets, the value (1 or 0) in the outcome column does not mean that all cases in that row have a membership of 1 or 0, respectively, in the outcome. Instead, the outcome column values express that the row can be considered a sufficient condition for the outcome. This is why in Table 4.8 we label the outcome column "Sufficient for Y." Fourth, when assessing the subset relation between a row and the outcome set, all cases are taken into account, not just those that are good instances of the particular row (i.e., those with a membership score above 0.5). The 0.5 qualitative anchor is thus crucial for attributing a case to a row but inconsequential when assessing the subset relation between two fuzzy sets.[7]

At-a-glance: what is a truth table? How to get from a data matrix to a truth table

Truth tables are an important tool in QCA. Although they look similar to **crisp-set** data matrices, they express a different type of information. While the single rows in data matrices correspond to actual cases (or units of observation), in truth tables, single rows denote logically possible **configurations** of **conditions**.

Three steps are needed in order to construct a truth table: First, all 2^k logically possible AND combinations of conditions are written down, with k being the number of conditions. Second, each case is assigned to the **truth table row** in which it has the highest membership. This is straightforward in **crisp-set QCA** because each case is a full member of one row and a full non-member of all the other rows. In **fuzzy-set QCA,** cases usually have partial membership in all rows but they can have a membership of higher than 0.5 in only one row. Cases are therefore attributed to this one row to which they fit best. (Exception: if one or more conditions are given a fuzzy value of 0.5, then the case will not have a membership value of greater than 0.5 in any ideal type.) Third, for each row the **outcome** value has to be defined. It is 1 for all rows that are a subset of, and thus **sufficient** for, the outcome and 0 otherwise.

These three steps yield a truth table that can be subjected to analysis, regardless of whether the underlying data consist of crisp or fuzzy sets.

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[7] In section 5.2, we qualify this statement and argue that researchers should pay attention to whether the cases that contradict the statement of sufficiency (or necessity) are located on different sides of the 0.5 qualitative anchor in the condition and the outcome, respectively. We will label these cases "logically contradictory cases."

Table 4.8 Truth table derived from hypothetical fuzzy-set data

	Conditions			Sufficient for	Cases with
Row	A	B	C	Y	membership ≤ 0.5 in row*
1	0	0	0	1	COL (0.6)
2	0	0	1	1	PAR (0.7)
3	0	1	0	0	CHI (0.7)
4	0	1	1	0	BRZ (0.8)
5	1	0	0	0	PER (0.7), ECU (0.7)
6	1	0	1	1	URU (0.8)
7	1	1	0	0	BOL (0.6)
8	1	1	1	0	AR (0.8), VEN (0.6)

* Numbers in parentheses = fuzzy-set membership of case in row

4.3 Analyzing truth tables

Truth tables can be created from both crisp-set data and from fuzzy-set data. The outcome column indicates whether the specific truth table row, or conjunction of conditions, is sufficient for the outcome of interest. If so, this is indicated by the value of 1 in the outcome column.[8] Hence, if we started our research asking which conditions are sufficient for our outcome of interest, the truth table provides a first answer: all rows that are linked to the outcome value of 1 are the sufficient conditions. This answer, however, is often not very informative and difficult to handle, simply because there might be many such rows in a truth table. Almost always, we would like to obtain a more succinct and parsimonious answer. For this, in QCA we apply the rules of Boolean algebra. The so-called Quine–McCluskey algorithm is used for logically minimizing the various sufficiency statements contained in a truth table (Klir *et al.* 1997: 61). It is important to point out that this form of truth table analysis is applicable only to the analysis of sufficiency. For the analysis of necessity, the bottom-up procedure presented in sections 3.2.1.2 and 3.2.2.2 has to be used. In fact, in section 9.1 we show that any inference about the presence or absence of necessary

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[8] Only in csQCA and only if there are no contradictory truth table rows (see Chapter 5), does the value of 1 in the outcome column indicate that *all* cases in that row are, in fact, members of the outcome. In all other scenarios – i.e., in fsQCA and/or when there are contradictory rows – a value of 1 in the outcome column of a truth table does not necessarily mean that all cases in that row are members of the outcome of interest.

conditions based on the top-down logical minimization of truth tables is prone to produce flawed results. A truth table, thus, does not play an important role in the analysis of necessity. In the following, we present the steps involved in the Quine–McCluskey algorithm (see also Ragin 1987: ch. 6).

4.3.1 Matching similar conjunctions

We return to the truth table already used in the section on crisp sets (4.2.1). Note that such a truth table could also be the result of converting a fuzzy-set data matrix into a truth table. Therefore, although we are now working with the example derived from the demonstration for crisp sets, truth table analysis is identical regardless of whether the underlying data consists of crisp or fuzzy sets.

The first step is to create a Boolean expression of all those truth table rows that are connected to the outcome to be explained. In our case, these are the rows with Y = 1 (rows 1, 2, 3, 4, and 6). Row 1 can be written as ~A~B~C, row 2 as ~A~BC, and so on. Conjunctions representing a truth table row are also called *primitive expressions*. The information contained in Table 4.9 can be expressed as follows:

> row 1 + row 2 + row 3 + row 4 + row 6
>
> ~A~B~C + ~A~BC + ~AB~C + ~ABC + A~BC → Y.

Each of these five primitive expressions has been defined as a sufficient condition for Y in the process of creating the truth table. This formula is the most complex way in which we can express the information about sufficiency contained in the truth table. The task now consists in reformulating the same logical truth in a less complex manner.

This process is called *logical minimization*. It is guided by the following first principle of logical minimization: if two truth table rows, which are both linked to the outcome, differ in only one condition – with that condition being present in one row and absent in the other – then this condition can be considered logically redundant and irrelevant for producing the outcome in the presence of the remaining conditions involved in these rows. The logically redundant condition can be omitted, and the two rows can be merged into a simpler sufficient conjunction of conditions.

Let us apply this principle to our example. Row 1 (~A~B~C) and row 2 (~A~BC) are identical except for the value condition C takes on: it is absent in row 1 and present in row 2. Thus, this information can be summarized in the

Table 4.9 Example of hypothetical truth table

Row	Conditions			Outcome
	A	B	C	Y
1	0	0	0	1
2	0	0	1	1
3	0	1	0	1
4	0	1	1	1
5	1	0	0	0
6	1	0	1	1
7	1	1	0	0
8	1	1	1	0

See Table 4.2

logically identical expression ~A~B. In other words, we can write the information about sufficiency in Table 4.9 like this:

rows 1 and 2   + row 3   + row 4   + row 6
~A~B   + ~AB~C   + ~ABC   + A~BC   → Y.

With reference to our example, this means that the absence of a violent upheaval in the past combined with an ethnically non-homogenous society (~A~B) is a sufficient condition for a stable democracy (Y), regardless of whether a pluralistic party system is in place (C) or not (~C).

   Let us apply the same logical minimization principle to the primitive expressions ~AB~C (row 3) and ~ABC (row 4). They differ only with regard to the value of condition C, which therefore can be dropped with the two rows rewritten as ~AB. Together with the previous minimization of rows 1 and 2, we can now write:

rows 1 and 2   + rows 3 and 4   + row 6
~A~B   + ~AB   +A~BC   → Y.

The same principle of logical minimization, matching a pair of primitive expressions that differ in the value of only one condition, can be equally applied to any two conjunctions that lead to the same outcome. In our example, conjunctions ~A~B and ~AB differ only in the value of condition B, which can be dropped, and the two expressions can be simplified to ~A. This means that condition ~A is sufficient for Y regardless of the values conditions B or C take. Our simplified solution formula now looks like this:

rows 1 to 4   + row 6

~A   + A~BC   → Y.

This formula is logically equivalent to the most complex formula and to all intermediate formulas.

Since it is based on the same data as our example in section 3.1.1.2, we note a difference in the solution term. In section 3.1.1.2, the very same data resulted in the solution:

~A + ~BC → Y.

The difference consists of the role attributed to condition A when it is combined with ~BC. The question is whether the inclusion of condition A is required when our aim is to find the most parsimonious solution term for the information contained in Table 4.9. The answer is that it is not required. Why? Conjunction ~BC includes both primitive expressions A~BC (row 6) and ~A~BC (row 2), i.e., by saying that ~BC is sufficient for Y, we also say that both A~BC and ~A~BC are sufficient for Y. Since these two primitive expressions differ only in the value of A, condition A can be dropped. Notice that the process of logical minimization allows for using the same primitive expression for more than one logical minimization. In our example, the primitive expression ~A~BC in row 2 can be matched with both the primitive expression in row 1 (~A~B~C, leading to ~A~B) and the one in row 6 (A~BC, leading to ~BC). This simply means that this primitive expression of row 2 is covered by more than one *prime implicant*, an issue that we address in more detail in section 4.3.2. For the moment, we can confirm the solution term:

~A + ~BC → Y.

We reiterate that this formula is one of several ways of summarizing the information on sufficiency contained in Table 4.9. All of the different solution formulas that we have reported here, as well as the intermediate steps of the minimization process, (a) are logically equivalent; (b) express the same information contained in the truth table; (c) do not contradict each other, nor do they contradict the information contained in the truth table; and (d) are acceptable summaries of the empirical information at hand.

The principle that more than one solution term is an acceptable and logically correct representation of the data in the truth table is a general feature of QCA. The decision on which solution formula to choose as the basis for the substantive interpretation of the available information depends on many research-specific issues that have nothing to do with formal logic. There are several potential reasons that we might prefer the formula

~A + A~BC → Y

over

~A + ~BC → Y.

Imagine, for instance, that the literature on the emergence of stable democracies (Y) makes a strong point that a democracy cannot stabilize in the presence of violent upheavals (A). However, as solution term A~BC (i.e., row 6 in Table 4.9) demonstrates, there is empirical evidence that warrants a qualification of this claim: if combined with ~BC, A can indeed be a causally relevant INUS condition for Y. Following our hypothetical example, and contrary to the hypothetical claim in the literature, stable democracies occur in the presence of violent upheavals – but only when these countries additionally have an ethnically non-homogeneous population (~B) and a pluralistic party system (C).[9] While it is true that the formula

~A + ~BC → Y

also contains this information, the role of condition A remains less visible. The formula that includes the term A~BC is simply more helpful in connecting the empirical findings with pre-existing theoretical knowledge and expectations on this particular topic.

A related argument to this is that more complex solution formulas help to direct attention to hitherto unexplained cases. Imagine that the literature on the stability of democracy has thus far failed to find an explanation for why a certain country, which we will call X, is a stable democracy. Further assume that country X can best be described by conjunction A~BC. By preferring the solution term that explicitly includes this conjunction as a sufficient path towards Y, we are able to demonstrate why country X displays a stable democracy in a more straightforward manner than with the more parsimonious solution term.

4.3.2 Logically redundant prime implicants

The Quine–McCluskey algorithm consists of more than the elimination of single conditions from a pairwise matching of similar conjunctions. There are situations in which this procedure yields a solution formula that can be

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[9] Caution: this should be read to mean that an upheaval *can* be violent in such circumstances but does not necessarily *have* to be. Remember that the component ~A~BC is also implicitly contained in the solution. In the scenario of a heterogeneous society with a pluralistic party system, this allows for the absence of a violent upheaval.

further minimized, but not by using the rule we have used so far. Another minimization principle is therefore needed (Ragin 1987: 95–98).

Logical equivalence can often be detected quite easily, such as in one example presented in section 4.3.1, where we show that ~A + ~BC → Y ≜ ~A + A~BC → Y. However, logical equivalence is not always so easy to detect. For this reason, we introduce a more general procedure for arriving at solution formulas that cannot be further minimized.

In order to understand this further step in the minimization procedure, we introduce the notion of prime implicants. Prime implicants can be defined as the end products of the logical minimization process through pairwise comparisons of conjunctions introduced in section 4.3.1. In other words, the solution term that we achieve through the pairwise comparison of conjunctions consists of prime implicants that are combined through logical OR. Under certain circumstances, though, it happens that one or more of those prime implicants are logically redundant. They can be dropped from the solution term in order to obtain the most parsimonious formula.

How can we identify logically redundant prime implicants? In order to answer this, we introduce some new hypothetical data. Suppose that the outcome to be explained is the presence of a consolidated democracy (C). As potential conditions, we choose whether a country is rich (R), is ethnically homogeneous (E), and has a parliamentary government (P). Suppose the empirical information contained in a truth table can be written using the following primitive expressions, aka truth table rows:

REP + RE~P + ~REP + ~R~EP → C.

Figure 4.3 shows the prime implicants obtained by applying the minimization strategy just introduced.

The complexity of the logical statement has been reduced from four paths (each composed of three individual conditions) to three paths (each composed of two individual conditions). These three new paths (RE, EP, and ~RP) are the prime implicants. They logically contain all the primitive expressions and cannot be further minimized with the minimization procedure we have described up to now.

It is nevertheless possible that this solution term contains logically redundant prime implicants. Therefore, we introduce a second rule for the minimization of solution formula: a prime implicant is logically redundant if all of the primitive expressions are covered without it being included in the solution formula. Hence, a solution formula without such a prime implicant
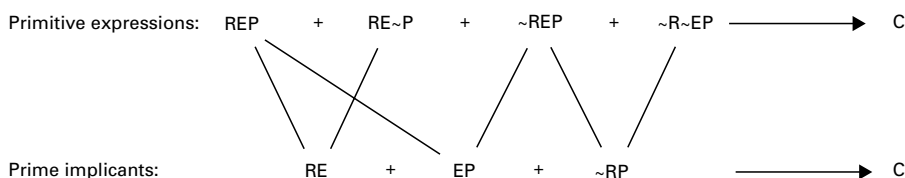
Figure 4.3   Logical minimization of primitive expressions to prime implicants

does not violate the truth value of the information contained in the truth table. Remember, the guiding principle of logical minimization is to express the same logical statement in a more parsimonious manner. The overarching requirement is that the truth value contained in the original truth table is not violated. The same logical statement can be expressed without the prime implicant in question and still adhere to this requirement.

We present the identification of logically redundant prime implicants in two different ways. First, we use the tool of a prime implicant chart, and then we use a Venn diagram. A prime implicant chart displays the primitive expressions in the columns and the prime implicants in the rows. Table 4.10 displays the prime implicant chart for our example on consolidated democracies (C). Crosses in the cells indicate which primitive expression is covered by which prime implicant(s). Each prime implicant covers at least one, but usually more, primitive expressions. In order to preserve the truth value contained in the truth table, each primitive expression must be covered by at least one prime implicant. Sometimes, there are primitive expressions that are covered by more than one prime implicant. It is here where the key to logically redundant prime implicant lies: a prime implicant is logically redundant if, and only if, all primitive expressions are covered even without it.

Take the situation displayed in Table 4.10. This table can be read as follows: the prime implicant RE covers the primitive expressions REP and RE~P, since RE is the result of the logical minimization of REP and RE~P by dropping condition P. This is indicated by the two crosses in the row for RE. The other two prime implicants, EP and ~RP, also both cover two prime implicants each, as can be seen by the two Xs in their rows.

For the truth value to be preserved, there must be at least one X per column in a prime implicant chart like that in Table 4.10. RE cannot be dropped, for it would leave primitive expression RE~P uncovered. It is therefore *not* logically redundant. ~RP cannot be dropped and is thus not logically redundant either, for it would leave primitive expression ~R~EP uncovered. Prime implicant EP, however, is logically redundant. It can be removed from the table without

Table 4.10 Prime implicant chart

Prime implicants	Primitive expressions/Truth table rows			
	REP	RE~P	~REP	~R~EP
RE	X	X		
~RP			X	X
EP	X		X	

any of the four primitive expressions remaining uncovered. REP is already covered by the prime implicant RE and ~REP by ~RP. Therefore, we can minimize our solution to:

RE + ~RP → C.

The notion of logically redundant prime implicants can also be explained by invoking the notion of intersecting sets displayed in a Venn diagram. Figure 4.4 displays the Venn diagram of our hypothetical example. In addition to the eight (2^3) different areas, which correspond to the logically possible combinations (aka truth table rows) between R, E, and P, the Venn diagram also indicates the location of the prime implicants (RE, RP, ~RP).

What Figure 4.4 demonstrates is this: the two prime implicants RE and ~RP jointly cover the entire area that is also covered by the third prime implicant EP (highlighted by the dark-gray area). Put differently, EP is logically implied, or is a subset of, the expression RE + ~RP. For this reason, EP is logically redundant and can be removed from the solution term. We say that it *can be removed*, because logically redundant prime implicants might well be of substantive interest. If so, they can and should be left in the solution formula. In these circumstances, such a formula is simply not the most parsimonious expression of the empirical information at hand.

Note that in the above example, there is only one logically redundant prime implicant (EP). This leaves no discretion to the researcher as to which prime implicant needs to be dropped in order to produce the most parsimonious solution. Very often in applied QCA, though, there are several logically redundant prime implicants and some of them are tied. Two logically redundant prime implicants are tied if either one or the other, but not both, can be dropped without violating the truth value of the solution term. This implies that in the presence of tied logically redundant prime implicants, there can be more than one most parsimonious solution term.
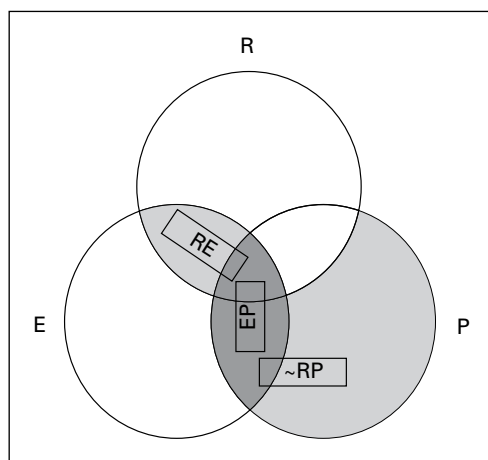
Figure 4.4   Venn diagram with logically redundant prime implicant

4.3.3 Issues related to the analysis of the non-occurrence of the outcome

Set relations are asymmetric (see section 3.3.3). One implication of this asymmetry is that the occurrence and the non-occurrence of an outcome of interest require separate analyses. All the analytic steps described so far that lead from a data matrix to a truth table and the logical minimization of the latter equally apply when the non-occurrence of the outcome is analyzed. Thus, continuing from our example displayed in Table 4.2 above, we now select ~Y (the set of countries with non-stable democracies) as the outcome of interest.

Starting with the analysis of necessity, we see that whenever ~Y is present, A is also present (see also section 3.2.1.2):

A ← ~Y.

Having experienced a violent upheaval in the past turns out to be a necessary condition for having an unstable democratic system.

For the analysis of sufficiency, we apply the Quine–McCluskey algorithm based on all rows with ~Y = 1. This yields the following result:

A~C + AB → ~Y.

This can be rewritten by factoring out condition A (section 2.4.1) as:

A (B + ~C) → ~Y.

Non-stable democracies occur in societies that have experienced a violent upheaval in the past and, at the same time, have an ethnically homogeneous

population and/or do not have a pluralistic party system. Again, this account of outcome ~Y is different from that for Y.

Three important remarks need to be made. One is of a general nature and the second and third stem from the particular simplicity of the example we have chosen. First, if, indeed, as we claim, the occurrence and the non-occurrence of a phenomenon, such as the stability of democracy and the non-stability, constitute two qualitatively different events that warrant separate explanations, then it often makes sense to resort to different theories and hypotheses to explain those outcomes. In other words, rather than just changing the outcome value from Y to ~Y in the *same* truth table, one might have to choose different conditions and thus construct an entirely *new* truth table. This directly follows from conceptual asymmetry, i.e., the fact that the negation of a concept often contains various qualitatively different notions. For instance, the set of non-democracies denotes military regimes, theocracies, and one-party regimes, to mention just a few. Likewise, for example, the set of non-married people comprises singles, widows, etc. In short, asymmetry might not only require different conditions for explaining Y and ~Y respectively. It also might require different conditions for the qualitatively different outcomes captured within ~Y.

The second and third caveats need to be made because our simple example produces two features in the solution term that usually do not hold when set-theoretic methods are applied to observational data. Discussing these features generates some general insights, though, and should help to avoid two mistakes often found in the applied QCA literature.

The second caveat is the following: in the analysis of necessity, we have identified condition A as necessary. At the same time, the analysis of sufficiency has revealed two paths, both of which involve condition A. It therefore seems that whenever a single condition is part of all sufficient paths, then this condition must be necessary for the outcome. Likewise, it might seem that that if no single condition appears in all sufficient paths, then there is no necessary condition. Both conclusions are likely to be wrong in applied QCA. They hold only if the sufficiency analysis is performed on a fully specified truth table, i.e., a truth table in which the outcome value for each of the 2^k logically possible combinations of conditions is either 1 or 0.[10] As we show in detail in sections 5.1, 6.1, and 6.2, this is hardly ever the case when formal logic meets noisy social science data. In applied QCA, truth tables

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[10] In fsQCA, even then the conclusions about necessary conditions might be erroneous for reasons that we discuss in detail in section 9.1.

almost invariably contain rows that are contradictory or logical remainders. Whenever these types of rows are present, the sufficiency analysis of a truth table runs the risk of not correctly revealing the presence or absence of necessary conditions. In Chapter 9, we spell out the detailed circumstances under which false necessary conditions appear and when true necessary conditions disappear. For the time being, it suffices to state that it is always recommended that analyses of necessity and sufficiency be kept separate and that statements of necessity and sufficiency, respectively, be based only on analyses of necessity and sufficiency, respectively.

The third caveat has similar roots to the second. Due to the simplicity of our example, it provides an exceptional instance in which it would be possible to derive the sufficiency solution formula for ~Y based on the formula for Y, without performing a separate analysis. Making use of DeMorgan's law[11] we can convert the sufficiency solution term for Y

~A + ~B*C → Y

into

A * (B + ~C) → ~Y.

This is identical to the formula derived through empirical analysis of outcome ~Y based on Table 4.2.

However, as mentioned, in social science research practice this procedure is problematic. It works properly only in a fully specified truth table, i.e., when there are no contradictions (section 5.1) or logical remainders (sections 6.1 and 6.2). Otherwise, the results produced by an application of DeMorgan's law imply claims about some truth table rows that either go unnoticed or are untenable, or both (Chapter 8 and section 9.1). Since fully specified truth tables are rare in practice, the meaningful use of the procedure described here and thus of DeMorgan's law is very limited.

For all these reasons, the standard of good practice (section 11.1) should be to always perform separate analyses of the occurrence and the non-occurrence of the outcome and to always analyze necessity and sufficiency in separate steps, not the least, since causal asymmetry also refers to the fact that substantial reasons might require us to use different causal factors for the explanation of the occurrence and the different types of non-occurrence of an outcome, respectively.

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[11] In section 3.3.3, we have described how, if we have fully specified truth tables (as in the case under examination), the arrow in the statement of sufficiency can be replaced by an equals sign (=). Consequently, it is possible to negate (e.g., through the application of DeMorgan's law, see section 2.3) both sides of the equation without altering the truth value of the statement.

At-a-glance: analyzing truth tables

The **Quine–McCluskey algorithm** uses the simplification rules of Boolean expressions on the **truth table**. It starts out by listing all **configurations** of **conditions** for which **sufficiency** has been confirmed. Subsequently, the logical expression is minimized, with the help of the rules of Boolean algebra. Examining **prime implicants** often makes possible further simplifications that are not apparent at a first glance.

Factoring out those **INUS conditions** that appear in all sufficient paths does not show that a condition is **necessary** for the outcome. Because of this, necessary and sufficient conditions must be examined separately. It is recommended that analyses of necessary conditions be performed before analyses of sufficient conditions.

The **non-occurrence** of the outcome has to be analyzed separately. Only when there are neither configurations lacking any empirical cases nor **contradictory truth table rows** can DeMorgan's law be applied.

Note that the information contained in any given truth table can be expressed through different solution terms. The principles of **logical minimization** ensure that these formulas are logically equivalent and differ only in the degree of complexity. The decision about which of these solution terms to put at the center of the substantive interpretation needs to be guided by theoretical and substantive considerations.

Part II

Neat formal logic meets noisy social science data

5 Parameters of fit

In Chapter 4, we introduced the use of truth tables in the analysis of sufficient conditions. A central point of our previous chapter was to assess for every single truth table row whether it represents a sufficient condition for the outcome. If yes, then such a row has been included in the logical minimization. If not, then it has not been included.

So far, we have assumed an ideal world that presents itself in clear and neat patterns. In reality, social science research based on observational data is characterized by noisy data. The coming chapters deal with issues that derive from this fact and describe strategies for how set-theoretic methods need to react to this. One fruitful way of looking at the discrepancy between neat set theory and the underlying empirical evidence is to frame this in terms of *incomplete truth tables*. A truth table is incomplete if it shows one or both of the following features. First, it might consist of rows that contain cases whose membership scores in that row and the outcome contradict the statement of sufficiency. These are *contradictory* or *inconsistent rows*. Second, a truth table might contain rows for which no (or, at least, not enough) empirical evidence is available. These rows are called *logical remainders*, and the presence of such rows is referred to as the phenomenon of *limited diversity*. The analytic problem caused by both forms of incomplete truth tables is that it becomes impossible to decide whether certain truth table rows represent sufficient conditions for a given outcome. This means that for some truth table rows it is not a straightforward business to establish whether they are sufficient for the outcome. Put differently, it is difficult to decide whether to include a given row in the Boolean minimization process. This represents an analytic problem, since the solution formula greatly depends on the decision of which rows are included in the minimization.

This chapter discusses the phenomenon of less-than-perfect subset relations, while Chapter 6 will deal with limited diversity. We start by introducing the notion of logically contradictory truth table rows and outline strategies of dealing with them (5.1). We introduce the consistency measure as one important strategy, and one which exists both for sufficient and for necessary conditions. After this, we also introduce the parameter of coverage, which expresses the empirical importance (sufficiency) and relevance (necessity) of a condition. We first introduce consistency and coverage formulas for sufficient conditions (5.2 and 5.3, respectively) and then for necessary conditions (5.4 and 5.5, respectively). As

in previous chapters, we introduce each argument by starting from crisp sets and by then extending it to fuzzy sets.

The notion of consistency is indispensable for understanding the logic of the Truth Table Algorithm (Chapter 7), which is at the core of QCA. A solid knowledge of the meaning and measure of the parameters of fit is therefore indispensable. More advanced readers might want to consult the At-a-glance boxes in order to assess whether they are familiar enough with these issues and, if so, simply skim through this chapter.

5.1 Defining and dealing with contradictory truth table rows

The notion of a contradictory truth table row is easier to understand with crisp sets. It describes a situation in which those cases that are members in a truth table row do not share the same membership in the outcome. Put differently, the same row leads to both the occurrence and the non-occurrence of the outcome. Since truth table rows are, in essence, statements of sufficiency, such an empirical situation suggests a logical contradiction, for it would mean that the very same combination of conditions (aka truth table row) produces both Y and ~Y. The analytic problem is that, based on the empirical evidence, it is not straightforward to decide whether this row is sufficient for Y, ~Y, or neither and, consequently, whether it should be included in the logical minimization for outcome Y, outcome ~Y, or neither. It cannot, however, be included in both minimization procedures.

There are several, mutually non-exclusive strategies for dissolving logically contradictory truth table rows in either csQCA or fsQCA *prior* to the logical minimization, and there is another set of strategies for handling such contradictory rows *during* the minimization procedure (Ragin 1987: 113–18; Rihoux and De Meur 2009). Let us first turn to the strategies for dissolving the contradiction.

The first strategy consists in *adding a condition* to the truth table. If those cases in the contradictory row that display qualitatively different outcome membership scores also show qualitatively different membership scores in the new condition, then the contradiction is resolved. This is because by adding a new condition, the contradictory row is split in two rows, separating the cases with different outcome membership scores in these two new rows. Of course, the downside of this strategy is that not only the contradictory row, but also all the other rows are split in two, thus doubling the number of truth table rows. Remember, the number of truth table rows is a direct function

of the number of conditions (k), as expressed in the formula 2^k (Chapter 4). This, in turn, increases the problem of limited diversity (Chapter 6).

A second strategy is to *respecify the definition of the population of interest*. By virtue of this, some cases might be excluded and/or new ones included. Such a change of the set of cases via a redefinition of the scope conditions (Walker and Cohen 1985) must be based in theoretical arguments. Cases cannot be excluded in an ad hoc manner simply because they contradict a statement of sufficiency. Instead, theoretical and substantive arguments must be explicitly brought forward as to why such cases are of a qualitatively different kind and therefore fall outside the scope of the analysis (Ragin and Becker 1992). The difficulty of this strategy might consist in the lack of plausible theoretical arguments. Even if these do exist, such a redefinition of the scope conditions might have to be accompanied by a change in the relevant theories. This, in turn, would have to have an influence on the choice of conditions and the outcome and their respective calibration functions, which could create new contradictory rows.

Third, one can *respecify the definition, conceptualization, and/or measurement of the outcome or condition(s)*. A closer look at the similarities and differences in the contradictory cases in a given row might reveal that the specification of the outcome or a condition was too vague, imprecise, or just plain wrong. If so, a respecification may contribute to solving inconsistencies. Just as in the case of redefining the scope conditions, a change of the meaning and thus calibration of concepts must also be based on theoretical arguments, without which such a recalibration strategy would degenerate into a blunt data-fitting exercise.

Any of these approaches can help solving contradictions. These strategies belong to the standards of good QCA practice and they represent part of what is meant by the phrase "going back and forth between ideas and evidence" (Ragin 2000), i.e., the process of updating theoretical, conceptual, and research design decisions based on preliminary empirical insights. At the same time, all strategies come at a cost and none can promise to always solve every logical contradiction. Thus, in applied QCA, it usually happens that researchers enter the process of logical minimization with truth tables that contain some logically contradictory truth table rows. There are several, mutually exclusive treatments of logically contradictory rows during the process of logical minimization.

First, one can *exclude all contradictory rows* from the logical minimization process. By doing this, one allows only perfect subset relations to qualify as sufficient conditions. As a consequence, any case that is a member of

the outcome but which falls into a contradictory truth table row will not be explained, or covered, by the solution term obtained with this strategy. Second, one can *include all contradictory rows* in the logical minimization process. This strategy is based on the argument that a contradictory row at least makes the occurrence of the outcome possible. The solution formula obtained thus represents the conjunctions of conditions that make the outcome possible. All cases that are members of the outcome will be explained, or covered, by that solution term. The downside, however, is that the solution term will also cover some cases that are not members of the outcome. Third, one can *make all inconsistent rows available for computer-generated assumptions* about their outcome value. It is then up to the computer to decide which of the contradictory rows to include in the process of logical minimization and which ones not to include. The only rationale for selecting some contradictory rows is whether their inclusion makes the resulting solution term more parsimonious. Although all three strategies for handling contradictory rows come at a price, the third strategy is usually least justifiable and is hardly ever encountered in applied QCA.

In the remainder of this chapter and, in fact, throughout the book, we advocate yet another way of dealing with contradictory rows and inconsistent truth table rows and set relations. This strategy takes into account how much, or to what degree, a given row deviates from a perfect set relation. Consider, for example, the following two scenarios. In one truth table row, nine out of ten cases share the same qualitative membership score in the outcome. Hence, one case deviates from the general pattern, or 90 percent of the evidence is in line with a subset relation. In another truth table row, six out of ten cases agree on their membership score in the outcome. Hence, only 60 percent of the empirical evidence is in line with the subset relation of sufficiency. This type of percentage can be seen as an important measure of how *consistent* a particular configuration is with the assertion that it is a sufficient condition for the outcome. We introduce this parameter as the *consistency value* in the remainder of the book.

In sum, strategies that aim at dissolving contradictions directly stem from the anchoring of set-theoretic approaches in qualitative methods. They remind us of the important fact that set-theoretic methods, in general, and QCA, in particular, are not only data analysis techniques but also research approaches with specific requirements for the research process before and after the actual data analysis. They reflect, in other words, the double nature of QCA as both a research approach and a data analysis technique (see the Introduction, section on QCA as a set-theoretic approach) (Berg-Schlosser, De Meur, Rihoux,

and Ragin 2008; Wagemann and Schneider 2010). Only if inconsistent rows still exist after these time- and energy-consuming countermeasures should one resort to those strategies that handle such rows during the process of logical minimization. Here, of greatest importance for applied QCA is the use of the consistency measure as a yardstick for guiding the decision on whether or not to include a truth table row into the logical minimization procedure.

At-a-glance: defining and dealing with contradictory truth table rows

In dealing with **contradictory truth table rows**, a decision must be made before any **logical minimization** of the **truth table** is undertaken on how to approach these contradictory rows. Some of the strategies for resolving contradictions include approaches that better specify the conditions in the explanatory model or the case selection with regard to the reference population.

   Consistency measures will be of additional help in making decisions about contradictory rows. Consistency scores should not replace, but rather complement, the qualitative strategies for dissolving contradictions.

5.2 Consistency of sufficient conditions

Starting off with csQCA, perhaps the most intuitive way of graphically displaying the notion of consistency of a sufficient condition is by means of a Venn diagram (Ragin 2006). Figure 5.1 displays Venn diagrams for three different conditions (X_1, X_2, X_3) and an outcome Y. In all three scenarios, the size of sets X and Y remains identical; only their relative location changes. Condition X_1 (Venn diagram on the left) is a perfect subset of outcome set Y, whereas both conditions X_2 and X_3 are not. Conditions X_2 and X_3 differ in the degree to which they violate the subset relation with Y. The share of set X_3 that is outside Y (area d) in relation to the overall size of X_3 (areas b and d) is larger than for condition X_2. Therefore, X_2 is more consistent than X_3 as a sufficient condition for Y.

   While Venn diagrams are good for grasping the basic notion of set-theoretic consistency, two-by-two tables are more powerful when trying to explain how to calculate this parameter of fit. Table 5.1 displays the same three conditions and outcome as Figure 5.1, and the cells (a–d) correspond to the areas (a–d) in the Venn diagrams. The numbers in the cells indicate the number of cases that show the respective membership scores in the condition and outcome.

Table 5.1 Two-by-two tables – consistent and inconsistent sufficient conditions

Outcome Y	1	80 a	100 b		80 a	90 b		80 a	8 b
	0	15 c	0 d		15 c	10 d		15 c	92 d
		0	1		0	1		0	1
			X_1			X_2			X_3
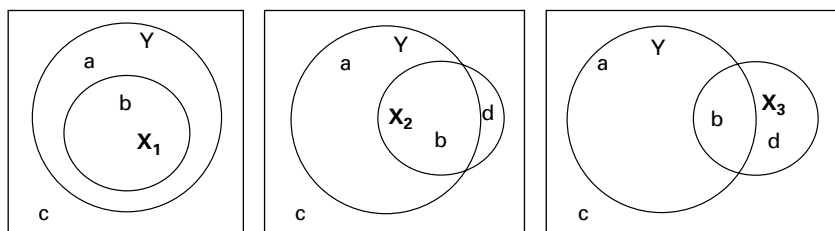
Figure 5.1   Venn diagrams – consistent and inconsistent sufficient conditions

As we can see, the difference between condition X_1, on the one hand, and X_2 and X_3, on the other, is that with the fully consistent condition X_1 all members of X_1 are located in cell b and none in cell d. Th is is why there is no area d in the first Venn diagram in Figure 5.1. Some cases in X_2 and many cases in X_3 fall into cell d rather than cell b. Recall from Chapter 3 that for statements of suffi ciency, only those cases matter that are members of the alleged suffi cient conditions (X = 1). Perfectly consistent suffi ciency requires that all cases with X = 1 are also members of outcome (Y = 1). Th erefore, no case should be in cell d. Th e more cases fall into cell d, the more consistency decreases.

Ragin (2006) suggests that the consistency of a suffi cient condition X for outcome Y be mathematically expressed by dividing the number of cases in cell b by all the cases that matter to measure suffi ciency, i.e., the number of cases in cells b and d. In csQCA, the consistency of X as a suffi cient condition for Y can therefore be calculated as follows:

$$\text{Consistency of X as a sufficient condition for } Y = \frac{\text{Number of cases where } X = 1 \text{ and } Y = 1}{\text{Number of cases where } X = 1}$$

The same can be expressed by making reference to the cells in the two-by-two table:
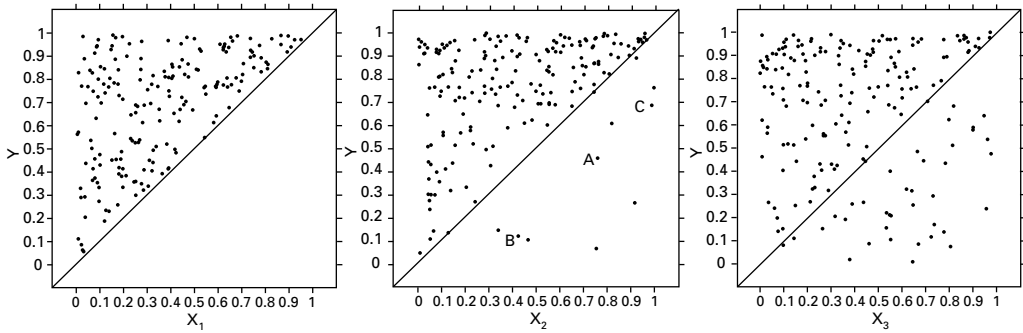
Figure 5.2   XY plot – consistent and inconsistent sufficient conditions

$$\begin{array}{l} \textit{Consistency of X as a} \\ \textit{sufficient condition for } Y \end{array} = \frac{\textit{Number of cases cell b}}{\textit{Number of cases cells } b + d}$$

Th e consistency value is 1 if a condition is fully consistent and decreases as inconsistency becomes stronger. Applied to our example in Table 5.1, the consistency values are as follows:

$X_1 = 100 / 100 = 1$
$X_2 = 90 / 100 = 0.9$
$X_3 = 8 / 100 = 0.08$.

When shift ing to fuzzy sets, the notion of subset relations is graphically best represented in the form of XY plots (section 3.1.2.1). Figure 5.2 shows three such XY plots, which, along the lines of Figure 5.1 and Table 5.1, display three different suffi cient conditions with increasing inconsistency from left (X) to right (X_3).

When trying to calculate consistency, one approach could be to proceed analogously to the calculation in crisp sets by simply counting the number of cases that are in line with the statement of suffi ciency (i.e., those above or on the main diagonal) and then dividing this number by the number of cases that are relevant for the test (i.e., those with membership in X of higher than 0). Th e plot for X shows no case below the main diagonal. Hence, consistency would be 1. For X_2 there are 10 out of 195 cases with X > 0 below the main diagonal. Hence, consistency for X_2 would be 185/195 = 0.95. For X_3, consistency would be 145/195 = 0.74.

Th is crisp approach to calculating consistency is deficient, though. Notice that it gives equal weight to all cases below the diagonal. This is not plausible. Th e distance between cases and the diagonal is clearly of interest because cases that are far below the main diagonal obviously deviate more strongly

from the alleged subset relation. For instance, case A in Figure 5.2 has a high degree of membership in the supposedly sufficient condition X, but a relatively low value of Y. It therefore contradicts the suffi ciency statement more than cases that fall only slightly below the main diagonal and/or have only weak membership in condition X and outcome Y.

The remedy for these pitfalls is to make use of the more fine-grained information conveyed by each case's fuzzy-set membership in X and Y when calculating consistency (and coverage, see 5.3). Th is is precisely what Ragin (2006; 2008a: 44–68) suggests with his formula for the consistency of a fuzzy sufficient condition. For each case, the minimum values across the membership scores in X and Y are added up and then divided by the sum of the membership values in X across all cases.

$$Consistency_{Sufficient\ Conditions\ (X_i \leq Y_i)} = \frac{\sum_{i=1}^{I} \min(X_i, Y_i)}{\sum_{i=1}^{I} X_i}.$$

If all cases have smaller or equal membership in X than in Y (as is required for fully consistent suffi ciency), then the numerator simply becomes the sum of all X_i, and this formula returns a value of 1. For cases below the main diagonal, their membership in Y provides the minimum. Th e farther they fall below the diagonal, the bigger the difference between their membership in X and Y and the smaller the sum in the numerator becomes in relation to the sum of X_i in the denominator. Th us, this consistency measure takes into account how far a case falls below the main diagonal, or how far the membership in X exceeds that in Y.

Notice, as well, that this formula is a generalization of the crisp-set consistency formulas above and yields identical consistency values to them (Ragin 2008b: 108n. 5). In case of crisp sets, the X_i in the denominator can only be 0 or 1. Th erefore, the sum of all X_i is equal to the number of cases where X = 1. In crisp sets, the numerator denotes the number of cases in cell b of our two-by-two table (Table 5.1). Th is is the only cell where the minimum across X_i and Y_i is 1, since both the X and Y values are 1. In all other cells, the minimum of X_i and Y_i is 0, and the cases contained therein are not added to the numerator.

While being a plausible way of numerically expressing the degree of a subset relation, the consistency formula has one particular shortcoming when applied to fuzzy sets. It does not take into account whether an inconsistent case is above or below the qualitative anchor of 0.5 in X and/or Y. Take, for instance, cases A, B, and C in the XY plot for X_2. Th eir distances to the main

diagonal are identical. Thus, they equally contribute to the inconsistency of X_2 as a sufficient condition for Y. There is, however, a qualitative difference between cases B and C, on the one hand, and case A, on the other, which is important for evaluating whether X_2 can be interpreted as a sufficient condition for Y. The former two cases display set membership scores in X and Y that are on the same side of the 0.5 qualitative anchor – they are either more in than out of both X and Y (case C) or more out than in of both X and Y (case B). Case A, in contrast, has qualitatively different membership scores in X and Y. Its membership in X_2 is above 0.5, making it a good empirical instance of this condition. Yet, its membership in Y is below 0.5. Hence, case A is a true logically contradictory case while cases B and C are simply inconsistent cases. To summarize this shortcoming, contradictory truth table rows can and do occur both in csQCA and fsQCA and they are, by definition, inconsistent rows. With fuzzy sets, however, not all inconsistent rows are automatically truly logically contradictory.[1] Analytically, inconsistent subset relations that also contain a true logical contradiction are less in line with a statement of sufficiency than simply inconsistent subset relations. They warrant more actions by researchers in terms of the strategies for fixing contradictory rows outlined in section 5.1 before proceeding with the logical minimization.

Which consistency level should researchers impose when identifying single truth table rows as sufficient conditions? For obvious reasons, consistency values close to, or even below, 0.5 should be ruled out, as this indicates that (almost) half of the empirical evidence contradicts the subset relational statement of sufficiency. Even values below 0.75 are often problematic as they have consequences for the subsequent analysis, which we spell out in various places in the remainder of this book (e.g., sections 5.6 and 9.1). As mentioned, with fuzzy sets, not only the consistency score, but also the presence or absence of true logically contradictory cases should be taken into account.[2] In the presence of such cases, researchers should be more reluctant to declare that row as a sufficient condition, independently of its consistency value.

Beyond these rough indications, we would like to make a strong plea for the notion that the exact location of the consistency threshold is heavily dependent on the specific research context. In other words, researchers should not justify their choice of the consistency threshold by making reference to some

[1] The notion of a true logical contradiction also extends to statements of necessity (see section 5.4). Here, a true logically contradictory case is one with X < Y and X < 0.5 and Y > 0.5.
[2] Perhaps the easiest way to do so is by producing an XY plot and checking whether the lower right area contains cases.

sort of universally accepted consistency threshold, akin to the (largely non-reflected) use of the 95 percent confidence interval in inferential statistics. Instead, researchers should guide their decision by making reference to various research-specific features.

The following guidelines should be used as some rough yardsticks. The more precise and strong the theoretical expectations that can be derived from the literature, the higher the consistency that should be used. The higher the confidence in the precision and validity of the calibration procedure for the conditions and the outcome, the higher the consistency. The lower the number of cases under investigation, the higher the consistency. The more logically contradictory cases, the higher the consistency.[3] In addition, in applied QCA a gap often exists between rows with relatively high and low consistency values that can guide the decision of where to put the consistency threshold. A less often used strategy is to employ the tools of probability theory. Ragin (2000: 109–16) suggests a binomial probability test simple for smaller N (30 or below) and a z test when the N is larger than that. Other authors also combine the assessment of set relations with tools from probability theory.[4] By now, several of the software packages (R and Stata) available for set-theoretic analyses allow for easy use of statistical tests not only of consistency, but also of coverage (see 5.3). Clearly, no precise, universal consistency value can be derived from these guidelines, often not even within a specific project. It is therefore strongly recommended that separate analyses with different thresholds of consistency be run in order to find out how sensitive the results are to the choice of the consistency level. We discuss this issue in further detail under the heading of robustness in section 11.2.

In sum, the consistency formula indicates the degree to which the statement of sufficiency is in line with the empirical evidence at hand. The more cases that deviate from the subset pattern and the stronger their deviation, the lower the consistency value. Of course, consistency can be calculated for any statement of sufficiency of arbitrary complexity. Put differently, X in the consistency formula is simply a placeholder for a set that might consist of the logical AND and OR combination of several sets. Regardless of how

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[3] As explained, with fuzzy sets cases can be inconsistent with a postulated subset relation without, however, being logical contradictory cases. A further guideline for choosing the consistency threshold for X as a sufficient condition for Y applies in fuzzy sets: with fuzzy sets, X can be a subset of both Y and ~Y. Since declaring X as sufficient for both Y and ~Y amounts to a logical contradiction, only those rows should be declared as sufficient for Y that display high consistency as sufficient conditions for Y and low values for ~Y. We return in greater detail to the more general issue of simultaneous subset relations in section 9.2.

[4] See, for instance, Braumoeller and Goertz (2003); Dion (2003); Caramani (2009); Eliason and Stryker (2009).

many sets are combined with different logical operators, each case has only *one* set membership score in that complex set. This implies that consistency values can be calculated for single truth table rows; single paths which have been identified as sufficient; or even an entire solution formula. When the consistency statement is on truth table rows, it is called "raw consistency," while the consistency value for the entire solution is called "solution consistency."

At-a-glance: consistency of sufficient conditions

Consistency provides a numerical expression for the degree to which the empirical information deviates from a perfect subset relation. This information plays a crucial role when deciding which **truth table rows** can be interpreted as **sufficient conditions** and can thus be included in the **logical minimization** process.

With **crisp sets**, inconsistency by default stems from logically contradictory cases. With **fuzzy sets**, it does not have to. Therefore, researchers are advised to check for the presence of true logically contradictory cases, in addition to the consistency value, before attributing the status of a sufficient condition to a truth table row.

Consistency can be calculated for single conditions as well as for more complex statements.

Researchers should justify their consistency threshold by making reference to research-specific features, such as the strengths of theoretical expectation and the quality of the data. The consistency value for sufficient conditions should preferably be higher than 0.75.

5.3 Coverage of sufficient conditions

Once a subset relation has been established via the use of the consistency parameter, another question can be asked: what is the relation in size between the subset (X) and the superset (Y)? The answer to this question expresses how much of outcome Y is covered by condition X, thus expressing the empirical importance of X for explaining Y.

Consider the three situations depicted in Figure 5.3. It displays three different conditions (X_1 to X_3) for the same outcome Y. All three conditions are identical with regard to their slight inconsistency as sufficient conditions (the ratio of area d over areas b and d is equal in all three Venn diagrams). What differs between the three is the size of the set of X in relation to the set of Y. Condition X_1 is larger than condition X_2 is larger than X_3. Since the set of Y is constant and the ratio between areas b and d remains the same, the varying
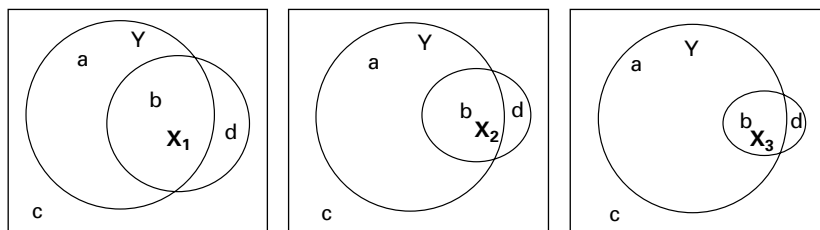
Figure 5.3   Venn diagrams – different levels of coverage sufficiency

size of the set of X means a variation in the amount of cases with Y = 1 that are *covered* by X_1, X_2, and X_3, respectively. In other words: X_1, X_2, and X_3 have different *coverages*. Th e coverage measure expresses the degree to which the consistent part of suffi cient condition X overlaps with outcome Y.

For illustration, let Y be the set of pupils with high test scores, X_1 be the set of students who study hard, X_2 be the set of students who study hard and are talented, and X_3 be the set of students who study hard, are talented, and cheat. Of course, membership in X_3 is more diffi cult to obtain because we require the joint presence of various characteristics of pupils. Th is is why less Xis smaller than X_2 and X_1. Th is also implies that fewer members of outcome Y share the features denoted by set X_3.

Table 5.2 represents the same empirical information as the Venn diagrams in Figure 5.3. In each of the three scenarios, the number of cases that are members of the outcome (Y = 1) remains the same (210) and the consistency scores are identical.[5]

What makes the three scenarios different is that as we move from condition X_1 to X_2, and then further to X_3, the number of cases that have membership in the respective condition X decreases. At the same time, the number of cases with membership in Y is constant. In terms of cells in the two-by-two tables, this means that, as more cases move from cell b into cell a, the ratio of the consistent part of X over the total number of cases with Y decreases. Th e consistent part of X accounts for an increasingly smaller portion of Y. Th e formula for calculating the coverage of X for Y can then be written as follows (Ragin 2006, 2008a: 44–68).

$$\text{\textit{Coverage of X as a sufficient condition for } Y} = \frac{\textit{Number of cases where } X = 1 \textit{ and } Y = 1}{\textit{Number of cases where } Y = 1}.$$

[5] X_1 = 200 / 208 = 0.96; X_2 = 120 / 125 = 0.96; X_3 = 24 / 25 = 0.96.

Table 5.2 Two-by-two tables – different levels of coverage sufficiency

Outcome Y

	10 a	200 b		90 a	120 b		186 a	24 b
1								
0	20 c	8 d		20 c	5 d		20 c	1 d
	0	1		0	1		0	1
		X_1			X_2			X_3

Or, in terms of our cells in the two-by-two table:

$$\text{Coverage of } X \text{ as a sufficient condition for } Y = \frac{\text{Number of cases cell b}}{\text{Number of cases cells } a+b}.$$

Based on these formulas, the coverage values for the three sufficient conditions are as follows:

$X_1 = 200 \,/\, 210 = 0.95$
$X_2 = 120 \,/\, 210 = 0.57$
$X_3 = 24 \,/\, 210 = 0.11.$

Th e graphical intuition gained by looking at the Venn diagrams in Figure 5.3 is corroborated by these coverage values: the coverage of X_1 is higher than that of X_2 is higher than that of X_3. With crisp sets, full coverage is achieved when cell a is empty of cases.

With fuzzy sets, we have to use XY plots. Figure 5.4 displays three conditions, all with identical consistency values (0.91), but different coverage. As we go from X_1 to X_2 and further to X_3, we see that cases tend to fall closer and closer to the Y-axis, i.e., where X is close to 0 – just as with the two-by-two tables above.

Just as with consistency, so also the coverage formula for sufficient conditions suggested by Ragin (2006, 2008a: ch. 3) makes use of the more fine-grained information contained in fuzzy sets looks as follows:

$$Coverage_{Sufficient\ Conditions\ (X_i \le Y_i)} = \frac{\sum_{i=1}^{I} \min(X_i, Y_i)}{\sum_{i=1}^{I} Y_i}.$$
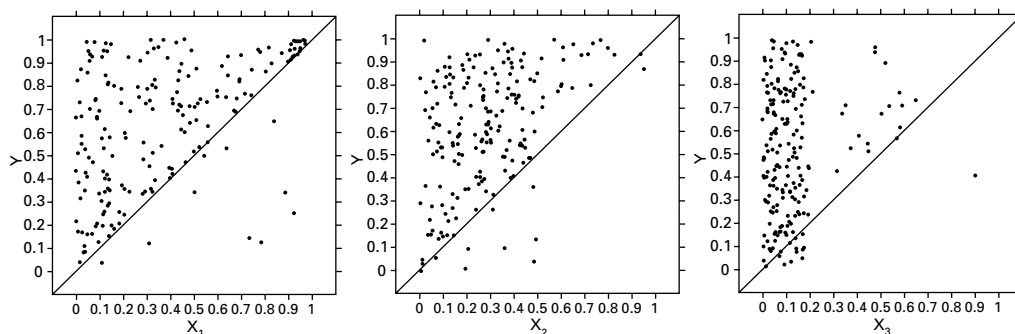
Figure 5.4   XY plot – different levels of coverage sufficiency

Applied to our three XY plots in Figure 5.3, the following coverage values are obtained:

$$X_1 = 0.81; X_2 = 0.6; X_3 = 0.19,$$

confirming our visual impression of X_1 being empirically more important than X_2, which, in turn, is empirically more important than X_3.

The more cases that are located in the upper left corner, and the farther away from the main diagonal these cases are, the lower the coverage. Those cases are good empirical instances of the outcome (high membership in Y) for which we lack, however, an adequate explanation because they are weak empirical instances of the sufficient condition (low membership in X).[6] The coverage formula takes into account how far above the main diagonal cases are located and, hence, how much of their fuzzy-set membership in Y is not covered by their membership in X. Cases in the upper left corner contribute little to the sum in the numerator (only their small X value) and much to the denominator (their high Y value).

As can be seen, with fuzzy sets, the calculation of coverage also takes into account that part of each *inconsistent* case's membership in Y that is covered by X.[7] As a consequence, coverage also increases due to cases that are inconsistent with the statement of sufficiency. This is an unfortunate property of the coverage parameter. It can be argued, though, that its effect is bound to be marginal and usually does not trigger substantive changes in the interpretation of

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[6] The upper left corner corresponds to cell a of our Venn diagram (Figure 5.3) and our two-by-two table (Table 5.2).

[7] The problem does not affect csQCA because each inconsistent case has a membership value of 0 in Y. Thus, for each inconsistent case 0 is added to both the numerator and the denominator in the coverage formula.

the results.[8] Several features reduce the effect. First, it is not the entire inconsistent case that is counted into the coverage formula but only that part of its membership in Y that is actually covered by X. Second, coverage is only calculated for conditions that have passed a threshold of consistency. This ensures that the number of cases (far) below the main diagonal is small and therefore their distorting effect on the coverage formula low. This, incidentally, provides another argument against choosing too low levels of consistency, for it would unduly boost the coverage values of (too inconsistent) sufficient conditions. From all this follows a clear rule of thumb: the consistency of a sufficient condition must always be calculated *before* its degree of coverage, and coverage should only be calculated for conditions that passed the test of consistency (Ragin 2006).[9] It makes no sense to calculate and interpret the coverage of a condition that is not sufficient.

Recall that equifinality is an important part of the epistemological foundation of set-theoretic methods, in general, and QCA, in particular (section 3.3). Different conditions (or combinations thereof) can lead to the same outcome. As a consequence of this, we can and should calculate the coverage of these different parts separately (Ragin 2006, 2008a: 54–68). It should be established how much of the outcome is covered by each of these paths. This is called *raw coverage*. We also might want to know how much of the outcome is covered only by a specific path – the *unique coverage*. The distinction between raw and unique coverage is important because different sufficient paths can overlap, i.e., the same case can follow multiple paths toward the outcome.[10] In these cases, the outcome occurs for more than one reason. Note that if no logically redundant path (section 4.3.2) is included in the solution, then all paths have a unique coverage higher than 0. We are also interested in finding out how much of the outcome is covered by the entire solution term – the so-called *solution coverage*. For instance, consider the equifinal and conjunctural solution term ~A~C + ~BC + F~D → Y. We can calculate the raw coverage and the unique coverage of sufficient paths ~A~C, ~BC, and F~D, respectively. In addition, we can calculate the solution coverage of the term ~A~C + ~BC + F~D.

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8 In section 9.2.1, we demonstrate under which circumstances this feature of the coverage formula produces misleading results and suggest alternative coverage formulas.

9 This is analogous to that in multivariate regression, where beta-coefficients should be interpreted only for significant variables.

10 Recall from section 2.2 that the logical OR operator used in QCA solution formulas is a non-exclusionary logical OR. One and the same case is allowed to be a member of more than just one sufficient condition or path.

For all three types of coverage, the coverage formula for sufficiency reported above directly applies. All that needs to be changed is what the placeholder X in this formula stands for: each case's membership in the path of interest, e.g., in the term ~A~C (raw coverage) or in the entire solution term (solution coverage). Unique coverage is calculated by subtracting from the solution coverage the amount of coverage that is obtained by all paths except the one whose unique coverage we are interested in. For instance, the unique coverage of path ~A~C would be calculated in the following way:

Unique coverage ~A~C = solution coverage − coverage (~BC + F~D).

A Venn diagram might help convey an intuitive representation of the different types of coverage. Figure 5.5 displays a Venn diagram for the solution term:

$X_1 + X_2 + X_3 \rightarrow Y$.

The rectangular box denotes all cases in the study. The largest set is outcome Y and each circle represents one of the three sufficient paths X_1–X_3. Needless to stay, X can stand for a conjunction of conditions. Furthermore, since the circles for X_1–X_3 are fully contained within the set of Y, we know that each single path and the entire solution term are fully consistent as sufficient conditions. What varies between paths is their raw and unique coverage.

The raw coverage of a single path is represented by the size of its set in relation to the size of set Y. We see that the raw coverage of X_2 is higher than that of X_1, which is higher than that of X_3, simply because area (IV) is bigger than areas (I) and (II), which are bigger than areas (II) and (III). The unique coverage is that area of a condition that does not overlap with another sufficient condition. As Figure 5.5 shows, paths X_1 and X_3 partially overlap. Therefore, the unique coverage of X_1 is equal to area (I) whereas that of X_3 is equal to area (III). Since condition X_2 does not overlap with any of the other paths, its unique coverage is the same as its raw coverage. Finally, the solution coverage of the term $X_1 + X_2 + X_3 \rightarrow Y$ is the sum of the areas (I)–(IV) in relation to the area of Y. Since the three paths jointly do not fill the entire circle for outcome Y, we can also see that the solution coverage is lower than 1.

Computationally, the calculation of coverage (and also of consistency) is not very demanding. Although all relevant software packages except Tosmana 1.3.2 automatically provide these parameters of fit, it might be helpful to demonstrate how coverage (and consistency) are calculated by hand. We do this with an example from Vis (2009),[11] who aims at explaining why some

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[11] In the remainder of the book, we repeatedly refer to data from this and other published examples, and we will introduce the study in further detail below (section 8.2).
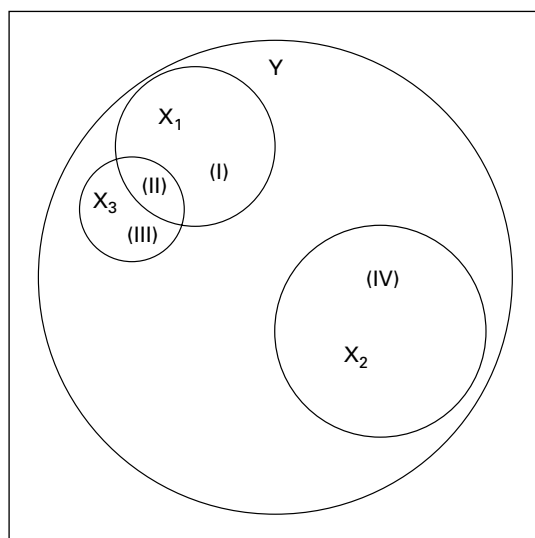
Figure 5.5   Venn diagram – equifinal solution term and types of coverage

governments in Western Europe engage in unpopular reforms (U). She finds that a weak political position of the governments (P) combined with a weak socio-economic situation (S) or a right-wing government (R) combined with a weak socio-economic situation (S) are the sufficient conditions for unpopular reform. Formally:

$P^*S + R^*S \rightarrow U$.

Table 5.3 contains the membership scores for the 25 cases, in both the solution and the outcome. In addition, the last column indicates for each case the minimum membership score across the solution and the outcome. This is a crucial quantity as it is used in the numerator for calculating both consistency and coverage. Calculating the solution coverage is straightforward. We simply add up the values in column "min(PS+RS,U)" and divide it by the sum of the membership scores in the column for outcome U. This yields a coverage for PS+RS of

Solution Coverage (PS+RS) = 10.96 / 12.74 = 0.86.

Each case with a higher membership in U than in PS+RS contributes to less-than-perfect coverage. Among them, cases like Kok I or N. Rasmussen IV are particularly striking, for they are more in than out of the outcome set U but more out than in both sufficient paths and thus remain uncovered and therefore unexplained by solution term PS+SR.

Table 5.3 Fuzzy-set membership in solution and outcome (Vis 2009)

	Solution	Outcome	
Government	PS + RS	U	min(PS+RS,U)
Lubbers I	0.83	0.83	0.83
Lubbers II	0.33	0.33	0.33
Lubbers III	0.60	0.67	0.60
Kok I	0.40	0.67	0.40
Kok II	0.33	0.17	0.17
Balkenende II	0.67	0.83	0.67
Kohl I	0.33	0.33	0.33
Kohl II	0.17	0.17	0.17
Kohl III	0.33	0.33	0.33
Kohl IV	0.67	0.67	0.67
Schröder I	0.33	0.17	0.17
Schröder II	0.83	0.83	0.83
Schlüter I	0.33	0.33	0.33
Schlüter II	0.60	0.67	0.60
Schlüter IV	0.67	0.17	0.17
Schlüter V	0.67	0.33	0.33
N. Rasmussen I	0.17	0.17	0.17
N. Rasmussen II (& III)	0.60	0.83	0.60
N. Rasmussen IV	0.33	0.67	0.33
Thatcher I	0.83	0.83	0.83
Thatcher II	0.33	0.67	0.33
Thatcher III	0.67	0.67	0.67
Major I	0.60	0.67	0.60
Blair I	0.17	0.40	0.17
Blair II	0.33	0.33	0.33
SUM fuzzy membership	(a) 12.12	(b) 12.74	(c) 10.96
Coverage sufficiency (c/b)		0.86	
Consistency sufficiency (c/a)		0.90	

As a matter of fact, the calculation of consistency for the solution PS+RS is equally straightforward. Simply add up the scores in the last column once again, but this time divide it by the sum of scores in the path PS+RS:

Solution Consistency = 10.96 / 12.12 = 0.90.

Less-than-perfect consistency is caused by cases whose membership in PS+RS exceeds their membership in U such as, for instance, Kok II, Schröder I, Schlüter IV and V.

The calculation of raw and unique coverage is equally simple. We demonstrate it for path PS. Table 5.4 displays each case's fuzzy set membership scores in path PS, outcome U, and the minimum score across these two sets.

By adding up the scores in the last column and dividing the total by the sum of column PS, we obtain a consistency value of 7.94 / 8.69 = 0.91. The raw coverage of PS is 7.94 / 12.74 = 0.62.

In order to calculate the unique coverage of path PS, we need to subtract from the solution coverage all of what can be covered by any other path in the solution except PS. Since, in our example, there is only one other path (RS),[12] we have to calculate the coverage of RS (0.71) and then subtract it from the solution coverage (0.86) in order to obtain the unique coverage of path PS:

Unique coverage PS: 0.86 − 0.71 = 0.15.

The calculation of the unique coverage of path RS (not displayed in Table 5.4) is equally simple. From the solution coverage we subtract the raw coverage of path PS:

Unique coverage RS: 0.86 − 0.62 = 0.24.

The unique coverage scores reveal that each path has some unique contributions to covering the outcome. In general, marginal differences in the coverage level should not be over-interpreted. Of equal, if not more, interest should be the cases that are uniquely covered. A case is uniquely covered if it holds a membership value higher than 0.5 in only one sufficient path (Schneider and Rohlfing in press and section 11.4, below). In our example, it turns out that out of the five cases that are more in than out of path PS, only two – Schröder II and Rasmussen II (&III) – are uniquely covered. The other three also have a membership greater than 0.5 in path RS. Path RS, in turn, has ten cases with membership greater than 0.5, and seven of them are uniquely covered by that path. Path RS is therefore empirically more important than path PS to an extent beyond what is reflected by comparing only their unique coverage formulas. The practical suggestion is that researchers should not only calculate, report, and interpret the raw and unique coverage scores, but also should go back to the cases and identify the uniquely covered cases.

Notice that for consistency, we argued that a lower threshold exists in principle, even if its precise location is subject to judgment (section 5.2). For

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[12] If the solution term consists of more than two paths, then we must calculate the joint coverage of all paths except the one we are interested in. It is not correct to simply add up the raw coverage of all these paths, because paths might partially overlap.

Table 5.4 Fuzzy-set membership in path PS and outcome (Vis 2009)

Government	Path	Outcome	
	PS	U	min(PS,U)
Lubbers I	0.33	0.83	0.33
Lubbers II	0.17	0.33	0.17
Lubbers III	0.33	0.67	0.33
Kok I	0.17	0.67	0.17
Kok II	0.33	0.17	0.17
Balkenende II	0.67	0.83	0.67
Kohl I	0.17	0.33	0.17
Kohl II	0.17	0.17	0.17
Kohl III	0.17	0.33	0.17
Kohl IV	0.67	0.67	0.67
Schröder I	0.33	0.17	0.17
Schröder II	0.83	0.83	0.83
Schlüter I	0.33	0.33	0.33
Schlüter II	0.33	0.67	0.33
Schlüter IV	0.33	0.17	0.17
Schlüter V	0.6	0.33	0.33
N. Rasmussen I	0.17	0.17	0.17
N. RasmussenII (& III)	0.6	0.83	0.6
N. Rasmussen IV	0.33	0.67	0.33
Thatcher I	0.17	0.83	0.17
Thatcher II	0.33	0.67	0.33
Thatcher III	0.33	0.67	0.33
Major I	0.33	0.67	0.33
Blair I	0.17	0.4	0.17
Blair II	0.33	0.33	0.33
SUM fuzzy membership	(a) 8.69	(b) 12.74	(c) 7.94
Coverage sufficiency (c/b)		0.62	
Consistency sufficiency (c/a)		0.91	

coverage, no lower threshold exists. The reason for this is that consistency establishes whether a subset relation exists, whereas coverage expresses how empirically important a subset relation is. Conditions with low coverage cover only a little of the outcome of interest, but that little might be of huge theoretical or substantive importance. Of course, conditions with zero unique coverage should be either disregarded or interpreted with care. Such zero coverage will always happen when logically redundant prime implicants (section 4.3.2) are included in the solution term.

At-a-glance: coverage of sufficient conditions

Coverage sufficiency expresses how much of the outcome is covered (explained) by the condition in question. The formula sums all minima of X and Y in the numerator and divides it by the sum of all Y values.

   Raw coverage indicates how much of the membership in the outcome is covered by the membership in a single path; the **unique coverage** instead indicates how much a single path *uniquely* covers. The **solution coverage** expresses how much is covered by the entire solution term.

   The empirical importance expressed by coverage is not the same as the theoretical or substantive relevance of a sufficient condition. Thus, low-coverage paths might still be of great substantive interest.

   Uniquely covered cases are those that hold a membership value higher than 0.5 in only one sufficient path. When substantively interpreting sufficient paths and assessing their importance, researchers should make reference to these uniquely covered cases.

   Unlike the case with **consistency**, there is no lower threshold for coverage.

5.4 Consistency of necessary conditions

The notions of consistency and, with some qualification, coverage can be applied to necessary conditions. If X is necessary for Y, then X is a superset of Y, whereas if X is sufficient for Y, then it is a subset of Y (Chapter 3). One consequence of this mirror-image relation between necessity and sufficiency is that the formulas for the parameter of fit are closely related. As a matter of fact, as we will see now, the formula for consistency sufficiency is mathematically identical to the standard formula for coverage necessity, and the formula for coverage sufficiency is mathematically identical to that for consistency necessity. In the following, we explain the rationale for these formulas.

   Let us start by having a look at the following three two-by-two tables. Each of them displays the same outcome Y but three different conditions (X_4, X_5, and X_6). The numbers in the cells indicate the number of cases.

   If a condition is necessary for the outcome, then no case may show the outcome without the condition. This means that cell a of our two-by-two table must be empty. When making a statement of necessity for outcome Y, then, cases that do not show the outcome are irrelevant (section 3.2.1.1). In two-by-two tables, this means that cells c and d are irrelevant for the assessment of necessity. The degree to which a condition is consistent with the statement of necessity thus depends on the ratio of cases in cells a and b. If all of these cases are located in cell b, the condition is fully consistent. The more of these cases fall into cell a, the lower consistency becomes.

Table 5.5 Two-by-two tables – consistent and inconsistent necessary conditions

		0	100		10	90		80	20
	1	a	b		a	b		a	b
Outcome Y		10	100		10	100		10	100
	0	c	d		c	d		c	d
		0	1		0	1		0	1
		X_4			X_5			X_6	

For condition X_4, cell a in Table 5.5 is empty. Thus it is a fully consistent necessary condition for Y. What about conditions X_5 and X_6? For both conditions, cell a contains cases. Certainly, X_5 and X_6 are not fully consistent necessary conditions for Y. X_6 is less consistent with the statement of necessity than X_5, because of those cases that matter (cells a and b) more cases (50) are located in the forbidden cell than for X_5 (10). Ragin (2006) suggests the following formula for calculating consistency of a necessary condition:

$$\text{Consistency of X as a necessary condition for Y} = \frac{\text{Number of cases where } X = 1 \text{ and } Y = 1}{\text{Number of cases where } Y = 1}.$$

In the numerator we add up all cases that are members of both the outcome and the necessary condition and in the denominator we add up all cases that are members of the outcome. Applied to Table 5.5, the formula can be rewritten as:

$$\text{Consistency of X as a necessary condition for Y} = \frac{\text{Number of cases cell b}}{\text{Number of cases cells } a + b}.$$

Plugging in the values for the conditions X_4, X_5, and X_6, we obtain the following consistency scores as necessary conditions:

$X_4 = 100/(0 + 100) = 1$
$X_5 = 90/(10 + 90) = 0.9$
$X_6 = 20/(20 + 80) = 0.2$.

When discussing consistency and coverage for sufficiency, we have already pointed out that with fuzzy sets, one should make use of the more fine-grained information contained in fuzzy-set membership scores. The same holds true

when dealing with necessity. With fuzzy sets, the consistency of a necessary condition is given by the degree to which each case's membership in X is equal to or greater than their membership in Y. When calculating consistency necessity, we therefore relate each case's membership in X that is consistent with the statement of necessity to the sum of each case's membership in Y. Th is logic can be expressed by the following formula (Ragin 2006):

$$Consistency_{Necessary\ Conditions\ (X_i \geq Y_i)} = \frac{\sum_{i=1}^{I} \min(X_i, Y_i)}{\sum_{i=1}^{I} Y_i}.$$

If for all cases the X values are equal to or greater than their Y values, then they are all below or on the main diagonal and the formula takes on the consistency value of 1, since the minimum of X and Y is in all cases the Y value. Th e more cases that display a membership in Y that exceeds their membership in X (and the greater the amount by which Y exceeds X in these cases), the more cases lie above the diagonal (and the farther above the diagonal they lie). In this scenario, the consistency value for the necessary condition deviates ever more from a value of 1, since smaller values go into the numerator than into the denominator.

Let us briefly demonstrate this formula with an example. Schneider, Schulze-Bentrop, and Paunescu (2010) are interested in, among other things, the necessary conditions for the high share of high-tech sector exports in proportion to all exports (EXPORT) in 19 OECD countries from 1990 to 2003 (N = 76). Th ey identify high unemployment protection (EMP); high coverage of collective bargaining (BARGAIN); high share of university-trained citizens (UNI); high share of occupation-trained citizens (OCCUP); high share of stock market capitalized indigenous firms (STOCK); and high share of cross-border mergers and acquisitions as a measure of institutional arbitrage (MA).[13] The results of applying the formula to all conditions and their complements, ordered by consistency values, are shown in Table 5.6.[14]

The condition STOCK has the highest consistency value (0.89) and researchers might see good reasons to interpret it as a necessary condition. However, an inspection of the XY plot (Figure 5.6) reveals that this is not so clear, in the end. First, a considerable number of cases fall above the diagonal. Second,

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[13] The data matrix can be found in the online appendix (www.cambridge.org/schneider-wagemann).

[14] See the online How-to section for Chapter 5 on using the software packages for calculating the parameters of fit (www.cambridge.org/schneider-wagemann).

Table 5.6 Analysis necessity, single conditions
(Schneider *et al.* 2010: 255)

Condition	Consistency
STOCK	0.89
UNI	0.81
MA	0.72
~OCCUP	0.71
BARGAIN	0.68
~EMP	0.64
OCCUP	0.58
~BARGAIN	0.50
~MA	0.50
~UNI	0.31
~STOCK	0.24
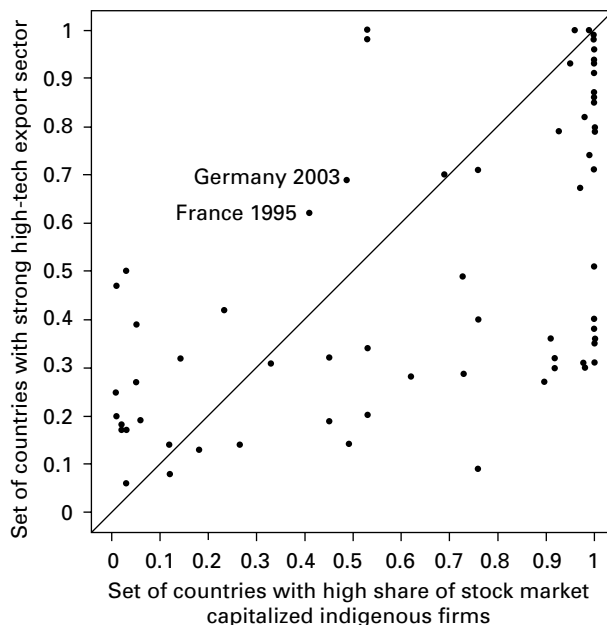
Figure 5.6   XY plot – condition STOCK, outcome EXPORT

among the inconsistent cases there are two true logical contradictory cases
(section 5.2; see footnote 1, above): France in 1995 (STOCK = 0.41; EXPORT
= 0.62), and Germany in 2003 (STOCK = 0.49; EXPORT = 0.69). Both cases
are more out of than in the alleged necessary condition while being more in
than out of the outcome. Therefore, interpreting STOCK as a necessary con-
dition for EXPORT does not seem warranted.

More generally, just as with the assessment of sufficiency, so also with necessity it is important that researchers not only use the consistency level, but also check if true logical contradictory cases exist. For necessary conditions, a consistency threshold of at least 0.9 seems advisable (Ragin 2006). One obvious rationale behind this is that higher consistency values reduce the likelihood of true logical contradictions. In section 9.1 we provide further reasons for high consistency levels for necessary conditions.[15]

The formula for the consistency of a necessary condition should look familiar to the reader. In fact, it is mathematically identical to the formula for calculating the coverage of a sufficient condition. However, the two have very different substantive interpretations. The point of a consistency test for a necessary condition is to determine the degree to which an outcome Y is a *subset* of a condition X. We expect from the very beginning that many, if not most, cases display membership values in Y that are smaller than their respective membership in X. In contrast, the purpose of a test for the coverage of a sufficient condition is to find out the portion of an outcome Y that is covered by a consistent sufficient condition X. This means that we will already know that Y is a consistent (enough) *superset* of X, such that the majority of cases will have larger Y values than X values. One practical implication for research is that the calculation of consistency must always precede that of coverage. To start with, is it meaningless to interpret the coverage of a non-consistent necessary or sufficient condition. Moreover, this procedure avoids confusion when interpreting the results obtained from the consistency and coverage formulas for necessity and sufficiency (Ragin 2008a: 63).

At-a-glance: consistency of necessary conditions

The **consistency** measure for **necessary conditions** assesses the degree to which the empirical information at hand is in line with the statement of necessity, i.e., how far the outcome can be considered a subset of the condition. As in the case of **sufficiency**, with **fuzzy sets**, the parameter takes into account both how many cases deviate from the pattern of necessity and how strongly they deviate.

The formulas for consistency of necessity, on the one hand, and **coverage** of **sufficiency**, on the other, are mathematically identical but have different substantive interpretations.

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[15] To anticipate the arguments: high consistency thresholds are also conducive to avoiding the pitfalls of (a) necessary conditions disappearing from sufficiency solution terms (*hidden necessary conditions*) and (b) false necessary conditions appearing in sufficiency solutions (*false necessary conditions*).

5.5 Coverage of necessary conditions

Th e reader might already suspect that the mutual, formal equivalence of coverage and consistency between necessary and suffi cient conditions might also be extended to the *coverage* of necessary conditions. Following this reasoning, the formula for the coverage of necessary conditions should be equal to the formula for the consistency of sufficient conditions and should therefore read for crisp sets:

$$\textit{Coverage of X as a necessary condition for Y} = \frac{\textit{Number of cases where } X = 1 \textit{ and } Y = 1}{\textit{Number of cases where } X = 1}$$

and, applied to a two-by-two table, as:

$$\textit{Coverage of X as a necessary condition for Y} = \frac{\textit{Number of cases cell b}}{\textit{Number of cases cells } b + d}$$

and, for both fuzzy and crisp sets:

$$\textit{Coverage of X as a necessary condition for Y} = \frac{\sum_{i=1}^{I} \min(X_i, Y_i)}{\sum_{i=1}^{I} X_i}.$$

And, indeed, these are the formulas for the coverage of a necessary condition as suggested by Ragin (2006a, 2008a: 61) and currently implemented in the relevant soft ware.

Th e formula for the coverage of a necessary condition expresses how much smaller the outcome set Y is in relation to set X. According to this formula, if X and Y are of roughly equal size, then the coverage of X as a necessary condition is high. Put differently, the more the size of X exceeds that of Y, the lower the coverage of X as a necessary condition.

Th e label *coverage* is misleading, though. If X has passed our consistency test as a necessary condition, then, by definition, X is a superset of Y and thus X fully covers Y. In other words, by virtue of being necessary, X always fully covers all cases of membership in Y. Ragin (2008a: 60–63) and Goertz (2006a) therefore point out that, next to consistency, the issue at stake when dealing with necessary conditions, is that of *relevance* (Ragin) or *trivialness*
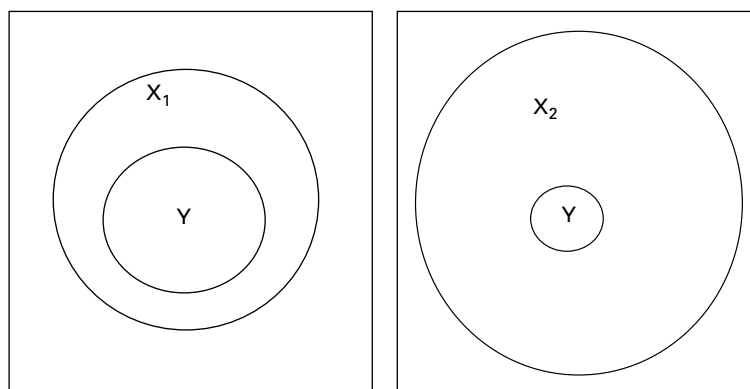
Figure 5.7   Venn diagrams – trivial and non-trivial necessary conditions

(Goertz). Thus, despite all symmetry in these parameters, the interpretation of the coverage value for necessity and that for sufficiency are fundamentally different.

In order to understand what is meant with relevance and trivialness, consider the two Venn diagrams in Figure 5.7. Let Y be the set of speeches in a country's parliament during which parliamentarians curse. X_1 is the set of male members of parliament and X_2 the set of parliamentarians born in that country. Clearly, both conditions are fully consistent supersets of the outcome and thus pass the formal requirement as necessary conditions. The relation in size of sets X_1 and Y is more in proportion than that between X_2 and Y. Hence, if we applied the coverage formula to these two empirical scenarios, X_1 (male persons) would receive a higher score and thus be deemed more relevant as a necessary condition for cursing than X_2 (being born in the country). X_2 is a trivially necessary condition for Y, simply because so many more members in parliament are born in the country (X_2) than curse during parliamentary debate (Y). The coverage formula suggested by Ragin (2006) and described here adequately captures this form of trivialness.

Let us apply Ragin's coverage formula to the example by Schneider *et al.* (2010). Calculating coverage only makes sense for those conditions that have passed the consistency threshold. Schneider *et al.* (2010: 255) convincingly argue that condition MA can be interpreted as a functional equivalent (see section 3.2.1.2) to condition STOCK. As Table 5.7 shows, the consistency value of the term MA+STOCK is above the 0.9 threshold.

Table 5.7 Analysis necessity, functional equivalents (Schneider *et al.* 2010: 255)

Condition	Consistency	Coverage
STOCK	0.89	0.72
MA+STOCK	0.92	0.68

The coverage value for the disjunction is 0.68. This is lower than the value of STOCK alone.[16] This suggests that the size of the logical OR set, compared to the outcome set, has increased, which should come as no surprise, because combining sets with the logical OR requires taking the maximum value of each case across the combined sets (see section 2.4). Since membership in outcome Y remains the same, the relation in size between sets Y and STOCK, on the one hand, and Y and MA+STOCK, on the other, increases. In an XY plot, this is graphically displayed by more cases falling further to the right-hand side of the plot. Just compare the XY plots in Figure 5.6 and Figure 5.8.

Two points are worth mentioning about the coverage formula. First, values for coverage necessity tend to be rather high. Unlike coverage sufficiency, in research practice, values far below 0.5 are rare and those close to 0 hardly ever seen. This suggests that when assessing the trivialness of necessary conditions, researchers should not be misled by seemingly high coverage values. In addition, the XY plot should always be carefully examined to ascertain whether most cases are clustering close to the vertical right axis thus suggesting trivialness.

The second issue related to the correct interpretation of the coverage formula for assessing trivialness is this: a condition X can be trivially necessary even when it is of roughly equal size to outcome Y. This happens when not only X, but also Y are very big in size and thus close to being constants (Goertz 2006a). In such a scenario, the formula for coverage necessity will yield a high value and researchers might be inclined to interpret X as a relevant necessary condition. This seems odd, though. Because of their size, both X and Y cover almost all cases and come thus very close to the universal set. Indeed, there are two sources of trivialness of a necessary condition: first, X is much bigger than Y; second, X and Y are close to being constants. Both sources of trivialness need to be taken into account and no condition interpreted as necessary in either of the two situations. The currently predominant formula

[16] However, in both analyses, the same two true logical contradictory cases occur (France in 1995 and Germany in 2003), thus providing further illustration that high consistency values alone are often not enough for a definite statement on a set relation.
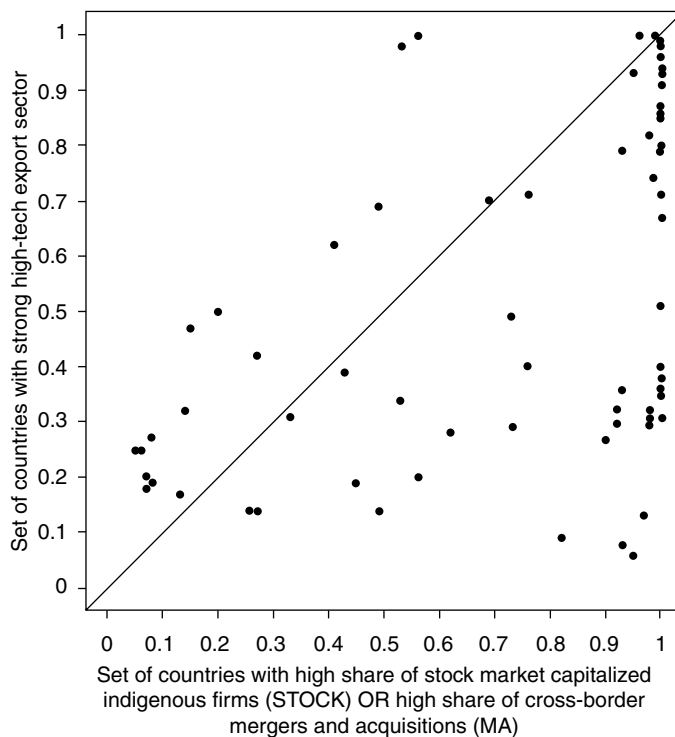
Figure 5.8   XY plot – condition MA+STOCK, outcome EXPORT

for coverage necessity, however, which we have presented here handles only the first source of trivialness. In section 9.2.1, we provide a detailed discussion of this issue and suggest an alternative formula for calculating the relevance of a necessary condition which also takes into account the second source of trivialness.

At-a-glance: coverage of necessary conditions

The standard **coverage** measure for **necessary conditions** is better interpreted as a measure of the **relevance** of a necessary condition.

High values indicate relevance, whereas low values indicate **trivialness**.

Conditions that pass the **consistency** test as a necessary condition should not be deemed to be relevant necessary conditions unless they also obtain a high value in the relevance measure.

The coverage measure for necessity captures only one source of trivialness, though. It detects whether the outcome set is much smaller than the condition set but is not capable of capturing whether both the condition and the outcome are (close to) universal sets.

5.6 Issues related to consistency and coverage

The concepts of consistency and coverage contribute in important ways to making set-theoretic methods, in general, and QCA, in particular, a more adequate and useful tool for analyzing social science questions. They allow for the use of set theory and formal logic to find patterns in noisy social science data. Despite – or perhaps precisely because of – their usefulness researchers employing QCA should resist the temptation to reduce this method to a simple hunt for high values of consistency and coverage. This would clearly be against the spirit of set-theoretic methods and would deprive them of their main strength: being grounded in the qualitative research practices of engaging in an iterative dialogue between ideas and evidence. Consistency and coverage are better thought of as numerical summaries that describe the data patterns in the underlying dataset. QCA is above all a *qualitative* data technique, and its primary purpose consists in interpreting and understanding the cases under study. Neither should specific consistency values obtain the status of universally applicable thresholds. Nor should individual cases disappear behind, or be hidden by, consistency and coverage values. Instead, researchers must carefully judge and then explicitly argue which consistency threshold is adequate for their specific research and then also perform several analyses with consistency values that vary within a reasonable range. When using fuzzy sets, we also advise paying close attention to which cases are true logical contradictions (consistency), uniquely covered, and which ones are not covered at all.

Consistency is the parameter which should always be assessed first. The reason is straightforward. It only makes sense to calculate the coverage of a sufficient (or necessary) condition if that condition has already been identified as being consistently sufficient (or necessary). If the consistency value is too low for the condition to be considered sufficient (or necessary), the calculation of coverage is meaningless. Along these lines, while there are consistency levels below which a condition cannot be considered as sufficient (or necessary), such lower-bound thresholds do not exist for coverage. With sufficiency, very low coverage values indicate that only a small portion of the outcome of interest is explained by that condition. However, that little bit might still be of great theoretical and substantive importance. With necessity, low levels of coverage indicate trivialness whereas high levels might or might not indicate relevant conditions, an issue we come back to in section 9.2.1.
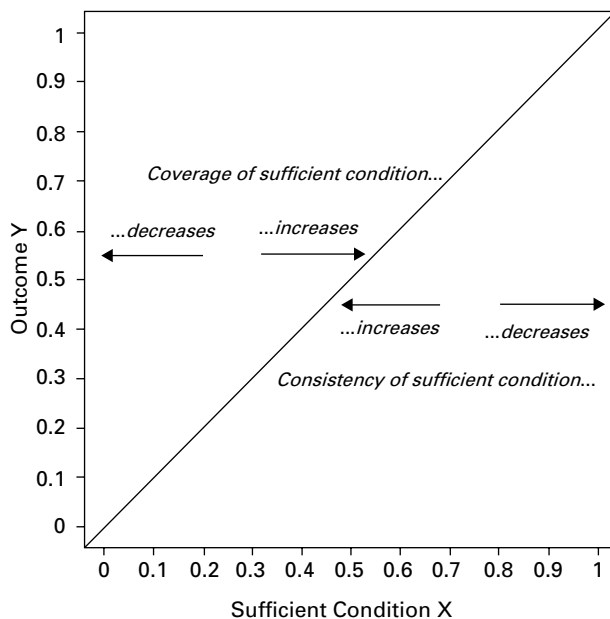
Figure 5.9   XY plot – the tension between consistency and coverage of sufficient conditions

Also note that, in research practice, higher consistency values often come at the price of lower coverage values. In the analysis of sufficiency, this works the following way. We can increase consistency by adding single conditions through logical AND. For instance, we might enlarge conjunction A*B*C to conjunction A*B*C*D*E. The more conditions that are combined, the more difficult membership in it becomes (section 2.1). This makes the set ever smaller, and thus makes it more likely to be a consistent subset of the outcome. At the same time, however, and precisely because membership becomes more and more difficult, long conjunctions cover less and less of the outcome simply because so few cases are members of this conjunction. A similar logic applies to the analysis of necessity. Here we increase consistency by adding conditions through logical OR. For instance, we extend expression A+B+C to A+B+C+D+E. The more conditions that are added, the easier membership in it becomes. This makes the set ever bigger and thus more likely to be a consistent superset of the outcome. But at the same time, and as with the process just described for conjunctions, long OR expressions cover more and more cases of the entire set of cases under study since membership becomes ever easier,

and they thus risk becoming trivial necessary conditions (section 9.2.1). The inherent tradeoff between consistency and coverage is graphically depicted in the XY plot in Figure 5.9 for an analysis of sufficiency, but works in the same way in the analysis of necessity.

> ## At-a-glance: issues related to consistency and coverage
>
> **Consistency** is the central measure for the assessment of set relations. Only if consistency is satisfactory should **coverage** be calculated.
>
> Often it is not possible to achieve high values for the consistency and coverage measures at the same time. Indeed, there is a tradeoff between the two: to increase consistency often means to decrease coverage and vice versa.
>
> Parameters of fit are not an end in themselves. The main focus should always be on the cases under study. Researchers should identify the cases that contribute to inconsistency and to low coverage.

6   Limited diversity and logical remainders

Easy reading guide

As seen in Chapter 4, the analysis of truth tables is at the core of QCA. In Chapter 5 we presented, among other things, the consistency value as a parameter for assessing whether a given truth table row could be considered a subset of, and thus sufficient for, the outcome. What if, however, there is not enough empirical evidence for a given row in order to assess whether it is sufficient? In other words, what if a row consists of a conjunction of properties that is logically possible but not empirically observed? Treating these so-called *logical remainder rows* in a conscious manner is both crucial for, and an asset of, set-theoretic methods. As this chapter shows, assumptions about remainders do have a direct impact on the results obtained and some assumptions are more plausible than others.

The presence of logical remainders is called *limited diversity*. This can be defined as the set of all logically possible combinations of conditions for which either no or not enough empirical evidence is at hand. It is a universal phenomenon in comparative social science research. The effect that these logically possible, yet empirically unobserved, "cases" have upon the possibilities for drawing evidence-based inferences is perhaps among the most understudied topics in social science research methodology. This is why we dedicate extensive space here to the different strategies that researchers facing limited diversity should be aware of. In Chapter 8, we add further strategies that go beyond the current best practice approach.

In this chapter, we first explain how to detect logical remainders (6.1). Second, we discuss why virtually all social science data is limited in its diversity. We do so by differentiating between different sources, and thus different types, of logical remainders (6.2). Since limited diversity afflicts the capacity for drawing inference, regardless of which specific method is applied, be it statistical or not, we then delimit in section 6.3 the phenomenon of logical remainders from other seemingly related notions in the social science methodology literature (such as missing values). In the final, main section of this chapter, we spell out the principles of the so-called Standard Analysis (Ragin 2008b) as the currently predominant procedure in applied QCA for making plausible assumptions about logical remainders (6.4). The aim of this chapter is to formulate set-theoretic strategies that help to keep the impact of logical remainders on inferences under the conscious control of the researcher.

The proper handling of logical remainders is of central importance for QCA. This chapter is certainly a must-read for beginners. Even experienced users will profit from studying

the procedures and pitfalls connected to this topic in further detail. The example of how to use directional expectations (6.4.3.2) is quite complex. It can be skipped without losing the understanding of what directional expectations are. In general, a good command of the material provided in this chapter is indispensable for successfully mastering the more advanced issues raised in Chapter 8. The current chapter contributes the last ingredients to the Truth Table Algorithm, which we then present in Chapter 7.

6.1 Limited diversity in set-theoretic methods: how to see it when it is there

Logical remainders are truth table rows that lack enough empirical evidence to be subjected to a test of sufficiency. This raises two questions. First, what do we mean by empirical evidence? Second, how much is enough?

The answer to the first question is straightforward when using crisp sets. As explained in section 4.2.1, with crisp sets, each case is a full member of one, and only one, of the 2^k rows of a truth table. In csQCA, empirical evidence thus refers to the number of cases in a particular truth table row. Therefore, logical remainders are simply those rows without enough cases in them.

The question of where to find logical remainders is somewhat more intricate with fuzzy sets. As shown in section 4.2.2, most cases have partial membership in most rows and hardly ever have full membership in any of the rows. From this, two opposing claims could be made. On the one hand, one could argue that limited diversity does not exist with fuzzy sets, for each row contains at least the partial membership of some cases. On the other hand, one could claim that limited diversity is everywhere, because with fuzzy sets hardly any row ever contains a single case with full membership. Both of these answers to the question of how to conceptualize limited diversity in fsQCA are problematic.

Ragin (2008a) therefore proposes an alternative. Recall two things from section 4.2.2. First, k fuzzy sets create a k-dimensional property space and the 2^k corners of this property space directly correspond to the 2^k truth table rows. Second, any given case is more in than out of one and only one of these rows. With fuzzy sets, then, a logical remainder is defined as a truth table row that does not contain enough cases with a fuzzy-set membership score higher than 0.5. This conceptualization of logical remainders crucially rests on the insight that the 0.5 qualitative anchor establishes a qualitative difference between cases, and it avoids the pitfalls of the other approaches outlined above.

For the second question – how much empirical evidence is enough? – no distinction between csQCA and fsQCA exists. Instead, the question on how many cases must be more in than out of a row before we cease to call it a logical remainder is largely a function of the overall number of cases in the study. For small- to medium-sized N studies (roughly 10–100 cases), the frequency threshold per row is usually set to at least one case. With larger Ns, it is often reasonable to require not just one case, but rather a certain percentage of the overall number of cases to be a member of a specific row in order not to declare it a logical remainder (e.g., Ragin and Fiss 2008).

At-a-glance: limited diversity in set-theoretic methods: how to see it when it is there

Logical remainders are **truth table rows** without enough cases in them that have a membership of higher than 0.5 in that row. The decision of how many cases with high membership a row must have depends on the characteristics of the research project, and mostly on the overall number of cases. Since QCA is largely applied to a medium-sized number of cases, the most often used frequency threshold is a minimum of one case per row.

6.2 Sources of limited diversity

Limited diversity occurs in social science observational data for various reasons. While logical remainders always make causal inference more difficult, the possibility of employing them for counterfactual arguments (see section 6.4) does depend on the source of the remainder. In the following, we distinguish between three, mutually non-exclusive, sources. The first possible source is that the number of truth table rows simply outnumbers the cases at hand (arithmetic remainders). Second, the social reality we come to observe tends to be pre-structured by multiple social, political, historical, and other processes, thus ruling out the occurrence of some truth table rows (clustered remainders). And, third, some of the conditions in a study could create combinations that not only do not exist in the world as we know it (clustered remainders), but also cannot exist in a world that we are able to imagine in either the past or the future (impossible remainders). In the following, we describe each source of limited diversity in further detail.

6.2.1 Arithmetic remainders

One common reason that logical remainders occur is simply that *the num-ber of logically possible combinations of conditions outnumbers the cases under study*. In the absence of a better term, we label them arithmetic remainders. Imagine, for instance, a study of the 27 EU member states, and that this study includes 5 conditions. The truth table will consist of 32 rows, but there are only 27 available cases. This means that there must be at least five logical remainders, none of which necessarily has to be an impossible remainder of the sort described above and below. We say "at least five," for the actual num-ber of logical remainders depends on the characteristics of the data. Most likely, the number of logical remainders is significantly higher because social phenomena tend to occur in clusters (see 6.2.2). Several cases will therefore probably be analytically similar, i.e., they will fall into the same truth table row. The more that cases cluster in a few truth table rows, the higher the num-ber of logical remainders.

6.2.2 Clustered remainders

Another reason for the presence of a logical remainder is that *a type of case does not exist in social reality as we know it, for this reality is structured by his-torical, social, cultural, and other processes.* As an example, consider Ragin's (2000) discussion of two conditions of a strong welfare state (Y): the exist-ence of a strong left-wing party (L) and of a strong trade union (U) (see also Grofman and Schneider 2009). In his data, there is not a single country that has a strong trade union but no strong left-wing party. Hence, any truth table row implying the combination ~L*U is a logical remainder. There are good reasons to suspect that this is no coincidence. The current literature on the historical development of trade unions tells us that the presence of a strong left parties (L) is crucial for the existence of a strong union (U). Hence, in the absence of L, U usually does not occur. Put in set-theoretic terms: the set of countries with left parties (L) is an almost completely consistent superset of the set of countries with trade unions (U). Thus, L might be interpreted as being a necessary condition for U.[1] In short, ~L*U is a clustered remainder because L

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[1] Baumgartner (2008, 2009) points out that the Quine–McCluskey algorithm – the common procedure for logical minimization of truth tables in set-theoretic methods (see section 4.3) – produces results that overlook such causal dependence between conditions and are thus misleading. He develops his own minimization algorithm that promises to take this complexity into account.

is, by and large, necessary for U. In short, clustered remainders occur because of a causal relationship between two (or more) of the conditions.

Sometimes, the set-theoretic, or causal, relationship between conditions that produces clustered remainders changes over time. For illustration consider the following example. For decades, both the set of "Caucasians" and that of males used to be supersets, and thus necessary, for the set of US presidents. Any study that attempted to discern the role played by the ethnicity and gender of US presidents on their actions (e.g., social expenditure, fiscal discipline, or war-proneness) was hampered by the severe presence of logical remainders as there simply were no non-white (male or female) or white female US presidents. As the election of Barack Obama in 2008 has shown, the causal interdependence between ethnicity and being US president has changed, though. The hitherto logical remainder of a non-white male US president has become an empirically observed case. Still, diversity continues to be limited, as there has not yet been any female US president. While in practice it might still take another while before a woman becomes US president, in principle this is no longer a far-fetched scenario.

6.2.3 Impossible remainders

A third source for limited diversity is that *a particular configuration is impossible in the light of what we know about the world*. Let us call this type of remainder an impossible remainder (Elman 2005). As an example, imagine a study that aims at explaining why some individuals are good car drivers (Y). Among the six conditions are the following two sets: the set of females and the set of pregnant persons. The truth table of this analysis will consist of 2^6 = 64 truth table rows, 16 of which will describe "cases" of non-female AND pregnant persons. For obvious reasons, any such row will be void of empirical instances, no matter how large-N the study is.

The proverbial pregnant man might seem too obvious an example, and might thus suggest that impossible remainders are not a serious issue in social science research practice. This impression is wrong, though. Especially in micro-level studies it is common to include a very long list of individual characteristics, mostly as control variables. More often than is perhaps recognized and realized, such models contain variables with values that cannot jointly occur. The same can happen at the macro-level as well. Consider, for example, the study by Ragin, Shulman, Weinberg, and Gran (2003), who aim at identifying the sufficient conjunctions for collective action in 41 villages in India. Among their crisp-set conditions, we find the set of villages located

on channel MN (M), the set of villages located on channel V (V), and the set of villages that are irrigated (I). By default, their truth table contains several rows denoting villages that would be located on the MN (or V or both) channel that are *not* irrigated. These combinations are empirically impossible, as the authors themselves correctly point out, simply because being located on a channel logically always implies being irrigated (Ragin *et al.* 2003: 331).

As a matter of fact, any study – whether on the micro- or macro-level – that includes conditions that go back to the transformation of single variables into two or more mutually exclusive conditions produces impossible remainders by default. For instance, consider the interval-scale variable GDP. For some justifiable reason, the researcher is interested both in the set of rich countries (R) and in the set of poor countries (P) but not in middle-income countries. Cases with a GDP higher than 75 percent of the GDP scale are members of R and those with GDP of lower than, say, 25 percent of that scale are members of P. Any truth table based on, among others, the conditions R and P will contain rows that imply countries that are simultaneously rich and poor (R*P). Such truth table rows must be logical remainders, for rich-poor countries simply cannot exist.[2]

In sum, impossible remainders are remainders that cannot exist in social reality as we know it. They run counter to fundamental truisms in biology (pregnant man), geography (tropical village in the Himalayas), or similar fields that are certain to remain in place in any foreseeable future. Slightly different forms of impossible remainders are those that result from transforming multinomial, ordinal, or interval variables into mutually exclusive categories. These impossible remainders will remain impossible, regardless of any future development, and their impossibility is uncontested by social scientists and laypersons alike.

The crucial difference between impossible remainders, on the one hand, and clustered remainders, on the other, is that the latter could theoretically exist, whereas the former cannot unless the entire world we live in radically changes.[3] This makes clustered remainders accessible for thought experiments, or counterfactual claims, whereas impossible remainders should not be the objects of such speculations. It makes sense, for instance, to make counterfactual speculations about, say, the policy profile of a hypothetical white

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[2] Notice that there will also be rows that contain the combination ~R*~P, i.e., countries that are neither rich nor poor. This is not an impossible remainder, but instead simply defines the set of non-rich and non-poor (aka middle-income) countries.

[3] Rather than a clear-cut dichotomy, this distinction is better perceived of as a continuum, for some clustered remainders are more likely to occur than others and some impossible remainders are more absurd than others.

female US president (or even a non-white female US president). These logical remainders can be imagined to exist in the not-so-distant future. In contrast, speculating on the driving skills of pregnant men is questionable. A world in which pregnant men can exist requires imagining a world that looks completely different (Emmenegger 2011). In section 8.1, we will therefore label counterfactuals of impossible remainders *implausible assumptions*, which are one type of *untenable assumptions*.

At-a-glance: sources of limited diversity

Impossible remainders describe hypothetical "cases" whose existence defies common-sense knowledge about the world (e.g., pregnant men, a country that is both rich and poor at the same time, or a tropical village in the Himalayas).

    Arithmetic and **clustered remainders** could potentially exist in the empirical world as we know it, but cannot be empirically observed. This is because either the number of logically possible combinations outnumbers the cases at hand (arithmetic remainders), and/or past historical, social, cultural, and other processes have so far prevented their occurrence (clustered remainders).

6.3 What limited diversity is not

Generally speaking, limited diversity is an understudied problem in social science methodology. This is surprising, for not only are logical remainders omnipresent in empirical social science research based on observational (and, to some degree, also experimental) data, but also their presence has serious implications for our capacity of drawing (causal) inferences. For example, Mill's methods, still quite popular among many qualitative scholars, produce logical remainders by their very design and force the researcher to make assumptions about them.[4] In large-N, statistical approaches, limited diversity is the almost unavoidable consequence of the "curse of dimensionality" (Ho, Imai, King, and Stuart 2007: 209), i.e., the simple fact that the number of potential "cases" grows exponentially with the number of conditions specified in an analysis – similar to what happens in QCA. This relation between the number of variables and the dimensionality of the property space is one reason why even a highly skilled quantitative researcher like Christopher Achen (2005) deems it difficult to really understand what is going on in the data once

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[4] In the online appendix (www.cambridge.org/schneider-wagemann) we provide empirical illustrations of the role of (impossible) logical remainders in Mill's methods and logistic regression.

the number of variables exceeds even the seemingly modest figure of three. Sometimes, the unavoidable assumptions that are made by applying a specific statistical model to these non-existent cases are reasonable, and sometimes they are not. Thus, the most problematic issues with regard to limited diversity are that researchers are often not aware of the assumptions made, how they affect their results, and what their theoretical or commonsensical status is. Even if researchers do know about the presence of logical remainders in their analysis, there is currently little debate as to what to do about them within the framework of statistical approaches. Some exceptions exist and the debates come under different labels, such as empty cells, structural zeros (Timpone 1998; Achen 2008), or convex hulls (King and Zeng 2007a, 2007b; Morrow 2007; Sambanis and Doyle 2007; Schrodt 2007). This latter discussion shows, however, that even among leading quantitative scholars there is little agreement on how to handle the omnipresent phenomenon of limited diversity within a multivariate statistical framework.[5]

While, as we argue, the issue of limited diversity is rarely taken head-on, several well-known topics within the statistical literature might mistakenly be seen as a treatment of limited diversity. As we argue now, neither the literature on missing values nor that on degrees of freedom provides much guidance for dealing with limited diversity.

The simple reason why the literature on *missing values* does not tackle the problem of limited diversity is straightforward. Missing values refer to situations in which, for one or more empirically observed case(s), information is missing on one or more variable. Thus, the concept of missing values makes sense only in reference to real and existing cases for which empirical information on at least some variables is at hand. Logical remainders, in contrast, are by definition non-existing cases, i.e., there is no empirical information on either any of the independent or the dependent variable. It seems nonsensical to invoke the notion of missing values when empirical information is lacking on *all* variables, i.e., to interpret a logical remainder as an extreme scenario of missing values on all conditions and the outcome. To the best of our knowledge, none of even the most sophisticated techniques for dealing with missing values imputes values for *all* variables of a "case," thus literally inventing a case from scratch. Limited diversity, in contrast, exclusively focuses on precisely these logically possible but empirically non-existent "cases." In sum, the conceptual difference between missing values and logical remainders is

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[5] This debate also nicely illustrates that logical remainders and missing values are two separate issues (see below). The concept of missing values is not invoked in this discussion.

twofold. The former focuses on empirically observed cases and provides principles for imputing values for (one or more, but not all) independent variables but not the dependent variable, while the latter focuses on empirically unobserved cases and provides principles for imputing values for the outcome.[6]

Another concept from the statistical literature that could mistakenly be interpreted as dealing with the issue of limited diversity refers to the notion of *degrees of freedom*. Strictly speaking, degrees of freedom refer to how many parameters a researcher is "free" to choose in an inferential model. This has often been simplified to "there must be enough cases for drawing inferences on the variables." In order to see that degrees of freedom address an issue different from logical remainders, simply imagine a very large-N study (N = 100,000) on the driving skills of individuals with only, say, three dichotomous conditions, two of which are the set of male persons and the set of pregnant persons, respectively.[7] Out of the eight logically possible combinations, at least four will not contain any cases because they denote impossible remainders (see section 6.2.3). With three variables and 100,000 cases, degrees of freedom are clearly not an issue. Yet, limited diversity is present. This demonstrates that these two concepts are independent of each other. It is not the number of conditions itself that matters for limited diversity, but rather the number of logically possible combinations of conditions, which increases exponentially. If most cases cluster in a few truth tables, i.e., if most cases are analytically identical (falling in the same truth table rows), then adding more cases of the same type to the dataset will increase the degrees of freedom, but will do nothing to lower the number of logical remainders. Because limited diversity can also occur in the absence of the lack of degrees of freedom, it should be clear that *significance tests* are not an appropriate means to detect or determine the extent and effect of limited diversity.

These are also the reasons why it is futile to try to assess the presence of logical remainders by focusing on single variables in isolation and checking whether they empirically vary enough. Even if all possible values on their respective scales occur empirically for all variables, we still can have logical

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[6] Researchers performing set-theoretic analysis might encounter missing values. However, for various reasons, in this book we will not discuss the treatment of missing values in any further detail. First, their occurrence is very rare. Especially when set-theoretic methods are used in a smaller-N setting, the requirement, akin to purely qualitative approaches, is that if missing values exist one needs to go back to the field (or the library) and dig up this missing information. When set-theoretic methods are applied to a large number of cases, missing values do occur more often. In such a scenario, there is no set-theoretic method-specific proposal for handling missing values. Instead, the general literature on missing values applies.

[7] For more detail on this illustrative example, see the online appendix (www.cambridge.org/schneider-wagemann).

remainders. As an illustration, consider, again, the study on driving skills and the two conditions "being female" (yes/no) and "being pregnant" (yes/no). For both conditions, both possible values empirically materialize in the data, i.e., there will be females and males, as well as pregnant people and non-pregnant people. Yet, diversity will be limited as there will be the logical remainder of a pregnant man about whom we have no empirical information as to whether he would be a good car driver.[8]

In sum, limited diversity is the rule rather than the exception in comparative social science research. The examples show that limited diversity can hardly ever be remedied by a "better" case selection, where "better" in the eyes of many would be equivalent to "more (and randomly selected) cases." Rather, limited diversity is the inevitable result of logical constraints, features of common research designs, and the clustered way in which social reality presents itself. All this applies regardless of the specific technique that is employed to analyze the data. The general aim should be to be aware of their existence and to make careful decisions on them – something which cannot happen if limited diversity is overlooked, disregarded, or not even debated.

> ### At-a-glance: what limited diversity is not
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> **Logical remainders** also influence analytic results in non-set-theoretic data analysis techniques. **Limited diversity** does not correspond to the concepts of missing values or degrees of freedom.

6.4 The Standard Analysis procedure: identifying logical remainders for crafting plausible solution terms

Limited diversity is essentially omnipresent, and assumptions about logical remainders influence the solution formulas from a truth table. This is why it is very important to handle logical remainders in a conscious manner, i.e., to develop strategies that allow for transparent and informed decisions on which remainders can serve as the basis for counterfactual claims while others cannot. This issue becomes even more important once we understand that,

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[8] Limited diversity is also not the same as the fallacy of predicting, based on regression equations, values that fall outside the observable range of the dependent variable or which rely on values of the independent variable outside of the range for which empirical information exists. None of these issues takes stock of the fact that what is missing are specific *combinations* of characteristics.

from a pure formal logical and set-theoretic perspective, assumptions about any logical remainders are permissible. Regardless of which remainders are included, logical minimization will yield a solution formula that never contradicts the empirical information at hand. That is to say, any solution term will be a superset of the truth table rows that contain empirical information and that are sufficient for the outcome of interest.

If formal logic provides no guide for selecting logical remainders for creating plausible solution terms, other criteria are needed. In this section we explain the logic of the Standard Analysis procedure as a major advancement in the treatment of logical remainders (Ragin and Sonnett 2004; Ragin 2008a). In order to explain its logic, we introduce three different dimensions based on which the different solution terms and logical remainders that go into them can be classified. The first dimension is that of a *set relation*. Different solution formulas are in a subset relation if the truth table rows that are used for the logical minimization are in a subset relation as well. Second, the dimension of *complexity* captures the degree of complexity, or parsimony, of a solution term. And third, the dimension of *type of counterfactual* classifies the assumptions about logical remainders according to their substantive, theoretical, and formal logical qualities (see section 6.2). In the following, we explain each dimension in further detail.

6.4.1 The dimension of set relations

As we now show, no matter which assumptions a researcher imposes on any of the logical remainders, the solution term resulting from these assumptions will never contradict the empirical evidence contained in the data. To illustrate, Table 6.1 displays a truth table with three logical remainders, indicated by a question mark in the column for outcome Y (rows 6–8). If we are interested in finding the conditions for outcome Y = 1, then rows 1, 4, and 5 of Table 6.1 are relevant and have to be included into the minimization process; if we are interested in explaining Y = 0, then rows 2 and 3 are relevant. The question arises what to do with the logical remainders in rows 6, 7, and 8. Should they be included in the logical minimization of outcome Y, of outcome ~Y, of both, or of neither?

Different assumptions on different logical remainders produce different solution formulas. To illustrate this point, Table 6.2 displays the eight different truth tables that result by making all logically possible combinations of assumptions on the three logical remainder rows. All eight truth tables displayed in Table 6.2 are identical with regard to their values in column Y for

Table 6.1 Truth table with three conditions and limited diversity

| Row | Conditions | | | Outcome |
	A	B	C	Y
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	1	1
5	1	1	0	1
6	1	1	1	?
7	0	0	0	?
8	1	0	0	?

the empirically observed rows 1–5, but differ in the values assigned to Y for the logical remainders in rows 6–8 (highlighted in bold font).

Each of these truth tables can be logically minimized by using the rules described in section 4.3. Choosing Y as the outcome of interest and applying the principles of logical minimization to each of the eight truth tables, we obtain eight slightly different solution terms. Although they appear different, as we show, they share quite a lot.

(a) AB~C + ~BC   → Y
(b) AB + ~BC   → Y
(c) A~C + ~BC   → Y
(d) A + ~BC   → Y
(e) ~A~B + ~BC + AB~C   → Y
(f) ~A~B + ~BC + AB   → Y[9]
(g) ~B + A~C   → Y
(h) ~B + A   → Y.

Solution (a) is the result when no assumptions about any logical remainder are made. This solution is often referred to as the complex solution term. For reasons that become clear when we discuss the dimension of complexity below (section 6.4.2), however, we suggest referring to it as the *conservative solution term*. Conservative because in producing it, the researcher refrains from making assumptions about any logical remainder and is exclusively guided by the empirical information at hand. Notice that when we are interested in the sufficient paths towards Y = 1, then assigning the value of Y = 0

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[9] Due to logically redundant prime implicants (see section 4.3.2), a logically equivalent solution is ~A~B + AB + AC → Y.

Table 6.2 Truth tables with all logically possible combinations of simulated values for logical remainders

(a)

Row	A	B	C	Y
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	1	1
5	1	1	0	1
6	1	1	1	**0**
7	0	0	0	**0**
8	1	0	0	**0**

(b)

Row	A	B	C	Y
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	1	1
5	1	1	0	1
6	1	1	1	**1**
7	0	0	0	**0**
8	1	0	0	**0**

(c)

Row	A	B	C	Y
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	1	1
5	1	1	0	1
6	1	1	1	**0**
7	0	0	0	**0**
8	1	0	0	**1**

(d)

Row	A	B	C	Y
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	1	1
5	1	1	0	1
6	1	1	1	**1**
7	0	0	0	**0**
8	1	0	0	**1**

(e)

Row	A	B	C	Y
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	1	1
5	1	1	0	1
6	1	1	1	**0**
7	0	0	0	**1**
8	1	0	0	**0**

(f)

Row	A	B	C	Y
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	1	1
5	1	1	0	1
6	1	1	1	**1**
7	0	0	0	**1**
8	1	0	0	**0**

(g)

Row	A	B	C	Y
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	1	1
5	1	1	0	1
6	1	1	1	**0**
7	0	0	0	**1**
8	1	0	0	**1**

(h)

Row	A	B	C	Y
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	1	1
5	1	1	0	1
6	1	1	1	**1**
7	0	0	0	**1**
8	1	0	0	**1**

to a logical remainder indicates only that this remainder is not sufficient for Y = 1. It does *not* imply that this remainder is a sufficient condition for the outcome ~Y. This is because the very same remainder could also be excluded from the logical minimization procedure for ~Y. In other words, a remainder row might well be considered insufficient for both Y and ~Y.

The crucial commonality of all eight seemingly different solution terms is that they all imply the rows with empirical evidence for sufficiency for Y (rows 1, 4, and 5). In addition to these three rows, solutions (b) through (h) imply one, two, or even all three logical remainders. It follows that all the solutions from (b) to (h) are supersets of the conservative solution (a). By virtue of this subset relation, none of the solution terms is in contradiction to the empirical evidence at hand and expressed in solution (a).

Note that this subset/superset logic also implies that one of the eight solution terms must be the superset of all the others. Intuitively, we might suspect that if the solution based on no assumptions on logical remainders is the subset of all other solutions, then the solution based on assuming that all remainders are sufficient for the outcome might be the superset of all other solutions, and this intuition would be correct. In our example, solution (h) is the one based on the assumption that all logical remainders are linked to the outcome Y. The more general insight is this: a solution term (X_1) is a superset of another solution term (X_2), if all rows implied by X_2 are also implied by X_1, and X_1 implies at least one more row. Hence, solution (f) is a superset of solution (e) but not of solution (c). Likewise, solution (d) is a superset of solution (c) but not of solution (f), and so on.

In sum, we see that different assumptions about logical remainders produce different solution formulas that all imply, and thus do not contradict, the empirical information at hand. The conservative solution rests on no assumptions and is the subset of all the other terms, whereas the solution term based on the assumption that all remainders are sufficient for the outcome is the superset of all others. All other solutions in-between (i.e., those that make assumptions on only some remainders) are only in a subset relation to each other if all assumptions used for one solution are also all used for another solution.

At-a-glance: the dimension of set relations

In the presence of **limited diversity**, the same **truth table** can yield different **solution terms**, depending on the **assumptions** made about the **logical remainders**. None of these solutions ever contradicts the empirical evidence at hand.

> The **conservative solution** does not rest on any assumption about the logical remainders and is the subset of all the other solution terms. The solution for which all logical remainders have been included in the **logical minimization** is a superset of all other solutions.
>
> Solution terms that rest on assumptions on some, but not all, remainders form subset relations with some, but not all, of the other solution terms.

6.4.2 The dimension of complexity

In addition to set relations, there is another dimension on which we can differentiate between our eight solution terms. We call it the dimension of complexity. The complexity of a solution term is defined by the number of conditions and the logical operators AND and OR that it involves. For instance, the solution term

A*B*C + ~F*D → Y

is more complex than the solution term

A*B + ~F → Y,

which, in turn, is more complex than the term

A + ~F → Y.

Among the different solution terms that are produced by altering the assumptions on the logical remainders, one (or, in the presence of tied [Mendel and Ragin 2011: 36] prime implicants, more than one) does consist of fewer conditions and logical operators than the others. This solution formula is known as the most parsimonious solution term (Ragin 1987).

Let us go back to our solution formulas (a) through (h) from the example above (section 6.4.1). They clearly vary in their degree of complexity. Solution (h) (~B + A → Y) is the most parsimonious one. It contains just two sets representing the conditions (A and ~B), which are linked by only one operator (OR). Solution (h) therefore describes the empirical information in the most parsimonious manner.

Among the eight solution terms, we can also identify the most complex term. It is solution formula (e) (~A*~B + ~B*C + A*B*~C → Y). It consists of six logical operators and invokes five different conditions (A, ~A, B, ~B, and ~C). The level of complexity of all the other solution terms falls between that of the most complex formula (e) and the most parsimonious formula (h).

It is important to point out that the dimensions of complexity, on the one hand, and that of subset relations, on the other, do not necessarily run in parallel. As our example demonstrates, the most complex solution term (e) is *not* identical with the subset solution of all the others (a). This is why we suggest labeling the subset solution the conservative solution term rather than the complex solution term, for, quite obviously, solution terms can get more complex by making assumptions about logical remainders, and the formula without any assumptions is not necessarily the most complex one.

Likewise, the most parsimonious solution term does not necessarily have to be identical to the superset of all the others. The fact that, in our example, the solution term (h) is both the most parsimonious and the superset solution is a coincidental property of our data and cannot be generalized. In applied QCA, the extreme ends of the dimensions of set relations and complexity more often than not refer to different solution formulas – the subset formula is usually not the most complex, and the superset formula not the most parsimonious one.

It is for two reasons that we emphasize the distinction between the two dimensions of set relations and complexity. First, part of the established terminology misleads users into thinking that these two dimensions are the same. More specifically, the terms *most parsimonious solution term* and *complex solution term* seem to imply that their respective properties refer to the same underlying dimension of complexity. As shown, however, the most parsimonious solution term has no distinct set-theoretic property other than being one of many other supersets of the conservative solution term. In other words, the most parsimonious solution can be anywhere on the dimension of set relations. At the same time, the so-called complex solution (which we have renamed *conservative*) has no distinct property on the complexity dimension other than being more complex than the most parsimonious solution term.

The second, related, reason for stressing this distinction is that it is crucial for understanding both the fundamental logic of the Standard Analysis procedure and its potential pitfalls (Chapter 8). The Standard Analysis narrows the range of solutions by imposing the requirement that each of them must be a subset of the most parsimonious solution term.[10] Since in our example in Table 6.2 the most parsimonious solution is (by coincidence) also the superset of all other solutions, this requirement does not exclude any of the eight solution terms. If, however, the most parsimonious solution is not the superset of

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[10] In addition, any solution term must be a superset of the conservative solution. Since all solution terms, by default, comply with this, it does not serve as a selection criterion.

all others, then several logically possible solution terms are ruled out simply because they are not a subset of the most parsimonious solution. For the sake of argument, imagine that solution (f) in Table 6.2 was the most parsimonious solution term. Apart from the conservative solution (a), only solutions (b) and (e) are subsets of (f).[11] Hence, if solution (f) was the most parsimonious one, the Standard Analysis would consider only four out of eight solution terms (solutions (a), (b), (e), and (f)).[12]

At-a-glance: the dimension of complexity

Solution terms differ in their degree of complexity. The least complex solution is called the **most parsimonious solution**.
 The dimension of complexity does not run parallel to the dimension of subset relations.

6.4.3 The dimension of types of counterfactuals

The Standard Analysis uses the most parsimonious solution as the anchor and allows all of its subset solutions to enter the next round of acceptable solution terms. Notice that neither the subset dimension nor the complexity dimension requires any substantive knowledge of the meaning of the conditions and, by virtue of this, of the plausibility of the assumptions made on the logical remainders. In the Standard Analysis, such considerations come into play only *after* the criterion of parsimony has been used to select some logical remainders for counterfactual claims. In the following, we introduce only those criteria for classifying assumptions that are used in the Standard Analysis. In Chapter 8, we add further quality criteria and show that unless these are also taken care of, there is no guarantee that the Standard Analysis procedure will not yield solution formulas that are based on what we label untenable counterfactual claims.

The first classification of assumptions has, in fact, already been introduced in discussing the dimension of complexity. Some assumptions contribute to making a solution term more parsimonious, while others do not. The former are called *simplifying assumptions*, whereas the others are called *assumptions* or *counterfactuals*.

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[11] Solution (f) implies the remainders in rows 6 and 7, solution (b) row 6, solution (e) row 7, and solution (a) no remainder row.

[12] In Chapter 8 we formulate two critiques. First, interesting solution terms might be rejected; second, some solution terms might be accepted although they rest on what we will call untenable assumptions.

By definition, the Standard Analysis allows only for simplifying assumptions. Within that group, a distinction is made between *difficult counterfactuals*, on the one hand, and *easy counterfactuals*, on the other (Ragin and Sonnett 2004). Both easy and difficult counterfactuals are simplifying assumptions, or – put differently – easy counterfactuals and difficult counterfactuals are each subsets of, and jointly constitute, the set of simplifying assumptions.

Easy counterfactuals are defined as those simplifying assumptions that are in line with both the empirical evidence at hand and existing theoretical knowledge on the effect of the single conditions that compose the logical remainder. These theory-guided hunches about conditions are often called *directional expectations* (Ragin 2008b). Difficult counterfactuals, in contrast, are in line only with the empirical evidence at hand, but not with directional expectations. Some examples should further clarify the notion of easy and difficult counterfactuals and the role of directional expectations.[13]

6.4.3.1 The principle of directional expectations

In the following, we illustrate the practice of crafting intermediate solution formulas with a hypothetical example from research on welfare states (Ragin 2000; Grofman and Schneider 2009). Imagine a researcher interested in the outcome strong welfare state (W). She has empirical evidence that in countries with well-developed neo-corporatist systems of interest intermediation (C), strong trade unions (U), and the absence of a left-wing government($\sim$L) a strong welfare state can be observed. In Boolean notation the conservative solution reads as:

$CU\sim L \rightarrow W.$

Furthermore, her study does not include any country with a well-developed neo-corporatist system of interest intermediation, strong trade unions, and a left-wing government. In other words, CUL is a logical remainder. Thus no empirical information exists as to whether this conjunction of conditions produces W or $\sim$W or neither. In Boolean notation:

$CUL \rightarrow ?$

The conservative solution, by definition, does not make assumptions for any logical remainder, including the remainder CUL.

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[13] In principle, the dividing line between what is an easy and what a difficult counterfactual is not a dichotomy but more a continuum, because the theoretical hunches on which they rest vary in their level of certainty (Ragin 2008a: 162). In research practice, however, the distinction is a crisp one, because no weight is attached to the directional expectations that distinguish between easy and difficult counterfactuals.

Imagine further that the most parsimonious solution based on the truth table reads as follows:

CU → W.

The most parsimonious solution term rests on the assumption that remainder CUL would produce a strong welfare if it existed. This yields two rows linked to W, the empirically observed row CU~L and the counterfactual CUL, which can then be logically minimized to CU (section 4.3.1).

From a substantive point of view, the question is whether the simplifying assumption

CUL → W

is an easy or a difficult counterfactual. There are good reasons to deem this an easy counterfactual. Indeed, most scholars would agree that, ceteris paribus, the presence of a left-wing party (L) rather than its absence (~L) is conducive to the presence of a strong welfare state (W). So, if W already occurs when ~L is combined with C*U, it is not too far-fetched to assume that W would also occur when L is combined with C*U. Hence, the claim that CUL is sufficient for W is an easy counterfactual for two reasons. First, we empirically observe that CU~L is sufficient for W. Second, we know from theory that L – rather than ~L – is expected to contribute to outcome W (our directional expectation).

Imagine, however, a slightly different empirical situation. This time, we have empirical evidence that CUL produces W, but the conjunction CU~L is a logical remainder. In order to produce the more parsimonious statement CU → W, we would need to assume that the remainder CU~L is sufficient for W. This is a difficult counterfactual, as it runs counter to pre-existing theory-based hunches, which state that the presence of L, not its absence (~L), is expected to be conducive for bringing about W.[14]

6.4.3.2 Using directional expectations for crafting intermediate solution terms

In the following, we provide a detailed discussion of how to arrive at the intermediate solution term. In our example, five conditions (A–E) form a truth table with 32 rows (Table 6.3), of which only 12 contain empirical information. Hence, there are 20 logical remainders – no less than 62.5 percent of all truth table rows, a rather common scenario in applied QCA. The conservative solution term is:

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[14] Notice that here directional expectations are formulated for single conditions. In section 8.3.2, we elaborate on the idea of formulating conjunctural directional expectations.

Table 6.3 Hypothetical truth table with five conditions and limited diversity

	Conditions					Outcome	Row included in		
Rows	A	B	C	D	E	Y	conservative solution	most parsimonious solution	intermediate solution
1	0	0	0	0	1	0			
2	0	0	0	1	0	0			
3	0	0	1	0	1	0			
4	0	1	0	1	0	1	X	X	X
5	0	1	1	0	0	1	X	X	X
6	0	1	1	0	1	1	X	X	X
7	0	1	1	1	0	0			
8	1	0	0	0	0	1	X	X	X
9	1	0	0	1	1	1	X	X	X
10	1	0	1	1	1	1	X	X	X
11	1	1	0	1	1	1	X	X	X
12	1	1	1	1	0	1	X	X	X
13	0	0	0	0	0	?			
14	0	0	0	1	1	?			
15	0	0	1	0	0	?			
16	0	0	1	1	0	?			
17	0	0	1	1	1	?			
18	0	1	0	0	0	?		X	
19	0	1	0	0	1	?		X	
20	0	1	0	1	1	?		X	X
21	0	1	1	1	1	?			
22	1	0	0	0	1	?		X	X
23	1	0	0	1	0	?		X	X
24	1	0	1	0	0	?		X	X
25	1	0	1	0	1	?		X	X
26	1	0	1	1	0	?		X	X
27	1	1	0	0	0	?		X	X
28	1	1	0	0	1	?		X	X
29	1	1	0	1	0	?		X	X
30	1	1	1	0	0	?		X	X
31	1	1	1	0	1	?		X	X
32	1	1	1	1	1	?		X	X

ABCD~E + A~BDE + A~CDE + A~B~C~D~E + ~ABC~D + ~AB~CD~E
→ Y.

This is a very complex statement; interpreting it in a theoretically mean-ingful manner is likely to cause some headaches. There is much heterogeneity

among the cases that are members of the outcome. This is why they fall into very different truth table rows, and why very little logical minimization is possible.

In order to produce the most parsimonious solution term, we need to identify those remainders that, if assumed to produce outcome Y, would make the solution term more parsimonious. With truth tables such as the one in Table 6.3, this is a complex task that needs to be performed by the relevant software packages.[15] For our example, the most parsimonious formula is the following:

$$A + B{\sim}C + B{\sim}D \rightarrow Y.$$

This term represents the shortest way of expressing the empirical evidence on the sufficient conditions for Y. It rests on numerous counterfactual claims about logical remainders. Most likely, not all of them are in line with existing theoretical knowledge and are thus difficult counterfactuals. This is why any substantive interpretation of the most parsimonious solution term should be treated with care.

Nevertheless, in the Standard Analysis procedure, the most parsimonious solution plays a crucial role. It defines the set of remainders that are eligible for the intermediate solution term. We therefore need to know which of the 20 logical remainders in Table 6.3 have been assumed to produce the outcome Y. As the column "most parsimonious solution" in Table 6.3 shows, simplifying assumptions have been made on the remainders in rows 18–20 and 22–32.

In principle, the creation of the intermediate solution term is based on barring all difficult counterfactuals from the simplifying assumptions and allowing only the easy counterfactuals to be included. This suggests that researchers would need to pass judgment on each and every individual simplifying assumption. This would be a complex task, and not only because the number of simplifying assumptions can often be quite high. More importantly, for each single assumption this judgment involves complex considerations that simultaneously need to take into account several pieces of information: (a) which conjunctions are empirically observed to be sufficient for the outcome (as expressed in the conservative solution term); (b) which single conditions are available for any intermediate solution (as expressed in the most parsimonious solution); and (c) which theory-guided expectations exist for single conditions (as expressed in the directional expectations).

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[15] See the online How-to section for Chapter 6 for practical guidance (www.cambridge.org/schneider-wagemann).

```
ABCD~E +

A~BDE +

A~CDE +                     A +                       A +

A~B~C~D~E +               B~CD +                    B~C +

~ABC~D +                  BC~D                       B~D

~AB~CD~E
```

conservative;   *intermediate*   *most parsimonious;*
subset   *superset*
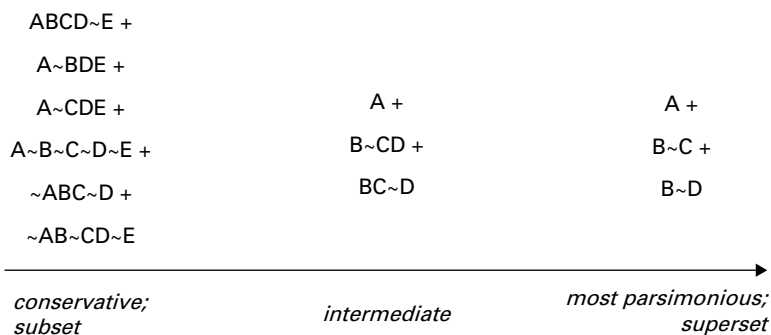
Figure 6.1   Conservative, intermediate, and most parsimonious solution terms

In practice, therefore, the strategy for crafting an intermediate solution term focuses not on truth table rows but on the conservative and most parsimonious solution terms and decides which of the single conditions that appear in the conservative but not the most parsimonious solution can be dropped, using directional expectations as a guide. In the following, we demonstrate the logic of this procedure. For the sake of simplicity, the directional expectations in our example are: each single condition (A–E) is expected to contribute to the outcome Y when it is present rather than absent.

The strategy for crafting an intermediate solution term follows two principles (Ragin and Sonnett 2004). First, no single condition can be dropped from any sufficient path of the most parsimonious solution term, because the most parsimonious solution term would otherwise not be a superset of the intermediate solution. This principle can be restated as follows: only those single conditions can be dropped from sufficient paths in the conservative solution that do not appear in the most parsimonious solution term. Second, only those conditions that are in line with the directional expectations can be dropped from the conservative solution term. If, according to the first principle, condition ~X is eligible for being dropped, and our directional expectation is that X contributes to the outcome, then ~X can be dropped. In the following, we show how these principles yield the intermediate solution term reported in Figure 6.1.

Remember, each single condition in this hypothetical example is expected to contribute to the outcome in its presence rather than absence. Let us start with path B~C from the most parsimonious solution. This is the superset of path ~AB~CD~E from the conservative solution. Any intermediate

solution must contain conjunction B~C. Let us check which conditions can be dropped from the conservative path. Following our directional expectations, these conditions could be ~A, ~C, and ~E, because our expectation is that A, C, and E contribute to the outcome in their presence rather than their absence. However, ~C cannot be dropped, since – as just mentioned – any intermediate solution must contain B~C. This leads to the intermediate solution term B~CD. This term rests on assumptions about the remainder rows 20 (~AB~CDE) and 29 (AB~CD~E) of Table 6.3. They denote easy counterfactuals because if ~AB~CD~E (row 4) – which is different from rows 20 and 29 in having only ~A instead of A and ~E instead of E, respectively – already implies the outcome, then we have every reason to believe that rows 20 and 29 would also imply the outcome if they were empirically observed.

Path B~D from the most parsimonious solution term is a superset of path ~ABC~D of the conservative solution. The intermediate solution path must contain the conjunction B~D but can drop all those conditions that appear in their absence in the conservative path (again due to our directional expectations that conditions will be sufficient in their presence, not their absence). In this example, we can only drop condition ~A, thus yielding the intermediate path BC~D. Path BC~D rests on assumptions on remainders in rows 30 (ABC~D~E) and 31 (ABC~DE). They are easy counterfactuals: row 30 is different from row 5 (~ABC~D~E) – a row which is empirically connected to the outcome – only in condition A, which, however, following our directional expectations, contributes to the outcome in its presence rather than in its absence. Also, row 31 (ABC~DE) is different from row 6 (~ABC~DE; connected to the outcome) only in condition A and is thus also an easy counterfactual.

It becomes a bit more complicated for path A from the most parsimonious solution. It is the superset of all remaining paths in the conservative term, namely those that contain condition A (ABCD~E, A~BDE, A~CDE, and A~B~C~D~E). The most promising path is A~B~C~D~E. Following the logic explained above, we can drop all those conditions that appear in their negation, because our directional expectations are that each condition should contribute to the outcome when present. There is no negated condition in the path of the most parsimonious solution (A) which would have to be maintained. Since in path A~B~C~D~E all conditions but condition A appear in their negation, this procedure yields simply A. This means that we have used the remainder rows 22 to 32 as easy counterfactuals. Rows 22 (A~B~C~DE), 23 (A~B~CD~E), 24 (A~BC~D~E), and 27 (AB~C~D~E) are easy counterfactuals since row 8 (A~B~C~D~E) differs in just one condition from each of

them and since the directional expectations suggest that, if row 8 is sufficient for the outcome, rows 22, 23, 24, and 27 can be considered easy counterfactuals. If we consider row 24 (A~BC~D~E) as an easy counterfactual, then we can consider row 25 (A~BC~DE) and 26 (A~BCD~E) easy counterfactuals, too, because they differ in just one condition (E in the case of row 25 and D in the case of row 26) from row 24, and are in line with our directional expectations. By a similar logic, if we consider row 27 an easy counterfactual, then rows 28 through 30 are also easy counterfactuals.[16] If assumptions about row 30 are allowed, then row 31 is also an easy counterfactual.[17] Finally, row 32 (ABCDE) is an easy counterfactual, because it differs in just one condition from row 10 (A~BCDE), row 11 (AB~CDE), and row 12 (ABCD~E), which all imply the outcome and for which the condition differing from row 32 is absent (so that row ABCDE confirms our directional expectations). Row 32 also differs from row 31 (ABC~DE) – which we have already defined an easy counterfactual – in one condition, for which we can follow the same logic and which we have also already defined as an easy counterfactual above.

This procedure yields three paths that can be summarized in the following intermediate solution:

A + BC~D + B~CD → Y.

The last three columns of Table 6.3 reveal several of the properties of the three solution terms. First, they all imply the same rows that contain empirical evidence. Second, the set of assumptions about remainders that are used for the intermediate solution is a subset of the set of assumptions used for the most parsimonious solution. Lastly, the two simplifying assumptions in rows 18 and 19 are deemed difficult counterfactuals.

In sum, all three solution terms are true in the sense that they capture the empirical facts expressed by the truth table in Table 6.3. Any of them can be used as the center of the substantive interpretation, and which one is chosen depends on the specific features of the research. We do suggest, though, that researchers present all three solution formulas in their results, together with the assumptions on which they are based. This allows readers to make their own judgments with regard to the plausibility of each of the solution terms.

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[16] Another reason why row 29 (AB~CD~E) is an easy counterfactual is that row 4 (~AB~CD~E) implies the outcome and differs only in A from row 29; this comparison follows the logic in our directional expectations. A parallel argument can be made for row 30 with the help of row 5.

[17] Row 6 being linked to the outcome also confirms that row 31 is an easy counterfactual.

At-a-glance: the dimension of types of counterfactuals

Intermediate solutions can be crafted in two ways. We can drop from the **conservative solution** any of those single conditions that do not appear in the **most parsimonious solution** and are in line with the **directional expectations**. Or we can screen all **simplifying assumptions** that go into the most parsimonious solution term and accept only those that can be classified as **easy counterfactuals**. Both strategies yield the same intermediate solution. In research practice, relevant software packages can be used for putting the notion of directional expectations into practice.

As long as no outcome value is altered for the empirically observed **truth table rows**, no **solution term** that is produced by changing **assumptions** on **logical remainders** can ever contradict the empirical evidence at hand.

The intermediate solution has various important properties. It is a subset of the most parsimonious and a superset of the conservative solution. It is less parsimonious than the most parsimonious solution and more parsimonious than the conservative solution. Unlike the most parsimonious solution, it does not rest on **difficult counterfactuals**. Unlike the conservative solution, it incorporates theoretical hunches in the form of easy counterfactuals.

6.4.4 The Standard Analysis procedure in a nutshell

The Standard Analysis is the strategy suggested by Ragin (2008a and Ragin and Sonnett 2004) when confronted with a truth table that contains logical remainders. It consists of producing the conservative solution (no assumption about logical remainders); the most parsimonious solution (all simplifying assumptions); and the intermediate solution (only easy counterfactuals). As argued above, the latter is – by definition – a subset of the most parsimonious solution and a superset of the conservative solution, a direct result of the requirement that easy counterfactuals are a subset of all simplifying assumptions. The intermediate solution term is also in between the conservative and the most parsimonious solution in terms of complexity. The rationale for creating intermediate solution terms is that, on the one hand, the conservative solution often tends to be too complex to be interpreted in a theoretically meaningful or plausible manner and that, on the other hand, the most parsimonious solution term risks resting on assumptions about logical remainders that contradict theoretical expectations, common sense, or both. Intermediate solution terms therefore aim at striking a balance between complexity and parsimony, using theory as a guide as to which logical remainders should be assumed to have a link to the outcome.

In short, the basic difference between the various solution terms consists of which logical remainder rows are used as simplifying assumptions. In applied
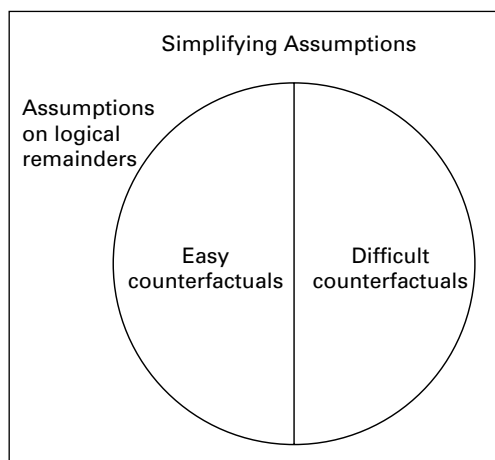
Figure 6.2   Venn diagram – types of counterfactuals in Standard Analysis procedure

QCA, researchers are often confronted with a very large number of remainders. The Standard Analysis procedure can be considered a powerful way of drastically reducing the number of remainders that need to be considered. It starts from the principle that only those remainders are eligible that contribute to parsimony (aka simplifying assumptions) and then proposes those for logical minimization that are in line with theoretical expectations (aka easy counterfactuals). Figure 6.2 displays a Venn diagram of the types of logical remainders as defined by the Standard Analysis procedure. The rectangular box contains all logical remainders, or better, assumptions about all remainders. A subset of these consists of the simplifying assumptions (the circle in the center of Figure 6.2). They yield the most parsimonious solution term and consist of the two mutually exclusive sets of easy and difficult counterfactuals. The easy counterfactuals alone produce the intermediate solution term. The drastic reduction of eligible remainders compared to all logically possible remainders is graphically displayed by the area outside the circle for simplifying assumptions. Those remainders will never be considered in the Standard Analysis procedure, simply because they do not contribute to parsimony.

This reduction has the virtue of lowering the number of possible solution terms to a more manageable number. In Chapter 8, however, we show that this reduction comes with two risks. First, by choosing parsimony as the criterion for selecting eligible remainders, it can (and often does) happen that impossible remainders are selected, which unavoidably leads to untenable assumptions. Directional expectations do not prevent this from happening, so that even intermediate solutions might be based on some untenable assumptions.

Second, some remainders that are never considered might actually provide very useful grounds for good counterfactual claims. This should not be a surprise. The only reason that they are discarded by the Standard Analysis is that they do not contribute to parsimony. Parsimony, however, is never the sole goal when it comes to good theoretical and substantive argumentation.

Before introducing extensions of the Standard Analysis in Chapters 8 and 9, we put all the insights from Chapters 1–6 together and describe in detail the so-called Truth Table Algorithm introduced by Ragin (2008a) in Chapter 7.

7   The Truth Table Algorithm

Easy reading guide

This chapter should be seen as a gateway in this book. It summarizes and systematizes the previous chapters and thus describes the current default way of performing QCA. In so doing, it also provides a fruitful opener for understanding several of the pitfalls and extensions that we are going to address in the remainder of the book.

In the preceding chapters, we outlined the various ingredients of the Truth Table Algorithm. More specifically, in Chapter 4, we introduced the truth table as the central analytical tool for QCA. Chapters 5 and 6 added to this by discussing various issues that arise when this formal logical tool meets noisy social science data. In Chapter 5, we introduced several parameters of fit. The consistency parameter enabled us to decide whether a given truth table row can count as a sufficient condition and be included in the logical minimization. In Chapter 6, we discussed logical remainder rows as a second feature of incomplete truth tables and presented strategies for dealing with them. We now have all the components of the so-called Truth Table Algorithm (Ragin 2008a), the predominant mode of sufficiency analysis in QCA. Strictly speaking, no new major insights are contained in this chapter. Nevertheless, Chapter 7 is crucial, as it illustrates in detail how the various aspects discussed previously relate to one another and how they are part of the Truth Table Algorithm. The algorithm is composed of several steps: the conversion of a data matrix into a truth table (7.1), which heavily draws on knowledge gained in Chapter 4; the attribution of an outcome value to every truth table row (7.2), which follows the guidelines of Chapter 4 and adds the knowledge about the consistency parameter from Chapter 5; and the logical minimization of a truth table (7.3), which takes into account the rules of Chapter 4, enriched with our knowledge about the Standard Analysis from Chapter 6. After the presentation of these steps, we refer to some important issues (and misunderstandings) related to the Truth Table Algorithm (7.4).

As mentioned, this chapter is also the basis for more advanced refinements which will subsequently be presented in Chapters 8 and 9. Readers both at the beginner and the more advanced level should only proceed with the book if and when they feel confident that they have mastered the material up to this point and which is presented in a condensed manner here in Chapter 7.

Note that this chapter does not amount to a full-fledged recommendation of best practices for QCA (for this, see section 11.1). Moreover, the Truth Table Algorithm is about the analysis of sufficient conditions. No reference to the analysis of necessity in made in this chapter. Issues in the analysis of necessity have been addressed in sections 3.2, 5.4, and 5.5 and we will further elaborate on these in Chapter 9.

7.1 From the data matrix to truth table

In order to illustrate the Truth Table Algorithm, we use Pennings' data (2003).[1] His is a fuzzy-set QCA, but the same principles and practices apply to crisp-set QCA. Pennings is interested in the conditions for high constitutional control of the executive in parliamentary democracies (K). He identifies the presence of a consensus democracy (C), a strong presidentialist component in the political system (P), whether the democracy is new (N), and the rigidity of the constitution (R) as potential conditions. The data matrix in Table 7.1 displays each case's fuzzy-set membership in all conditions and the outcome.

We first have to convert this data matrix into a truth table. As laid out in section 4.2, this requires three steps: the identification of all logically possible configurations; the assignment of each case to one of these truth table rows; and the definition of the outcome values for each row. Step 1 is easy and straightforward: 4 conditions produce a truth table with 16 rows.

In step 2, each case is assigned to that row in which its membership exceeds 0.5. For example, let us consider Australia. Its membership in C, P, N, and R is 0.33, 0, 0.33, and 0.83, respectively. Australia is thus closest to the ideal type of a country with a low level of consensus democracy (~C), without a strong president (~P), a democracy that is not new (~N), and with a rigid constitution (R). Its membership in row ~C~P~NR is 0.67 and lower than 0.5 in all other truth table rows, for, as shown in Chapter 4, each case can have a membership greater than 0.5 in one and only one row. Canada (0.83) and Ireland (0.55) are also members of the ideal type ~C~P~NR. For further illustration, let us take Austria. Its best-fitting ideal type, or row, is CP~NR, in which it has a membership of 0.55. Finland (0.67) shares this row with Austria.

Based on this procedure, all cases can be attributed to their respective ideal type. The result of this exercise is displayed in Table 7.2. For instance, the ideal type ~C~P~NR is denoted by row 2 with C = 0, P = 0, N = 0, R = 1. As we have seen, this truth table row contains three cases: Australia, Canada, and Ireland (shown in the last column of Table 7.2).

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[1] Ours is not meant to be a reanalysis, but merely an illustration. In the original study, Pennings uses the so-called Inclusion Algorithm (Ragin 2000) (see also section 7.4), and each case set with a membership score of 0.5 has been recoded by us to 0.55. Furthermore, we have excluded the Russian Federation from our analysis because of incomplete data.

Table 7.1 Fuzzy values data matrix, 44 cases

Country	Conditions				Outcome
	C	P	N	R	K
Australia	0.33	0	0.33	0.83	0.33
Austria	0.55	0.55	0.33	0.67	0.33
Bangladesh	0.17	0.17	1	0.67	0.33
Belgium	1	0	0.17	0.67	0.17
Botswana	0	1	0.67	0.17	0
Bulgaria	0.55	0	1	0.17	0.67
Canada	0.17	0	0.17	0.83	0.33
Czech Rep.	0.55	0.83	1	0.17	0.55
Denmark	0.83	0	0.17	0.55	0.55
Estonia	0.83	0.33	1	0.17	0.67
Finland	0.83	0.67	0.33	0.67	0.17
France	0.67	0.55	0.67	0.55	0
Germany	0.55	0.17	0.55	0.83	0.67
Greece	0.33	0.67	0.83	0.55	0.55
Guyana	0	0.33	0.67	0.67	0.17
Hungary	0.67	0	1	0.33	1
Iceland	0.67	0.83	0.33	0	0.17
India	0.33	0.55	0.55	0.67	0.33
Ireland	0.33	0	0.33	0.55	0.55
Israel	1	0	0.55	0	0.83
Italy	1	0.67	0.55	0.33	0.67
Jamaica	0	0	0.67	0.67	0.33
Japan	0.17	0	0.55	1	0.33
Latvia	0.83	0.17	1	0.67	0.83
Lithuania	0.33	0.33	1	0.33	0.17
Luxembourg	0.67	0	0.17	0.33	0.55
Macedonia	0.55	0	1	0.67	1
Malta	0	0.17	0.83	0.33	0.33
Namibia	0	0.83	1	0.33	0.17
The Netherlands	0.83	0	0.17	0.67	0.55
New Zealand	0.67	0	0.17	0	0.17
Norway	0.67	0	0.17	0.67	0.55
Pakistan	0.17	1	0.83	1	0
Poland	0.55	0.55	1	0.33	0.55
Portugal	0.17	0.67	0.83	0.67	0.55
Romania	0.83	0.33	1	1	0.55
Slovakia	0.83	0.83	1	0.17	0.67
Slovenia	0.83	0.55	1	0.17	0.33
South Africa	0	0.67	1	0.67	0.55
Spain	0.33	0	0.83	0.33	0.55

Table 7.1 (*cont.***)**

Country	Conditions				Outcome
	C	P	N	R	K
Sri Lanka	0.33	1	0.83	0.33	0.17
Sweden	0.67	0	0.17	0	0.55
Turkey	0.55	0.17	0.83	1	0.17
UK	0	0	0	0	0.17

Adapted from Pennings (2003)
C = Consensus democracy
P = Strong president
N = New democracy
R = Rigid constitution
K = Constitutional control of the executive

Table 7.2 Distribution of cases to ideal types

Row	Description of ideal type = truth table row = corner of property space				Number of cases with membership > 0.5 in this ideal type	Cases with membership > 0.5 in this ideal type
	C	P	N	R		
1	0	0	0	0	1	GB
2	0	0	0	1	3	AUS, CDN, IRL
3	0	0	1	0	3	E, LT, M
4	0	0	1	1	4	BD, GUY, J, JA
5	0	1	0	0	0	No cases
6	0	1	0	1	0	No cases
7	0	1	1	0	3	CL, NAM, RB
8	0	1	1	1	5	GR, IND, P, PK, ZA
9	1	0	0	0	3	L, NZ, S
10	1	0	0	1	4	BE, DK, N, NL
11	1	0	1	0	4	BG, EW, H, IL
12	1	0	1	1	5	D, LV, MK, RO, TR
13	1	1	0	0	1	IS
14	1	1	0	1	2	A, FIN
15	1	1	1	0	5	CZ, I, PL, SK, SLO
16	1	1	1	1	1	F

7.2 Attributing an outcome value to each truth table row

Step 3 of the Truth Table Algorithm consists of determining the outcome value for each truth table row. A row can either be sufficient for the outcome, not sufficient for the outcome, or a logical remainder row. The latter type of row is identified first.

In our data, none of the cases under examination has a membership higher than 0.5 in either ~CP~N~R (row 5) or ~CP~NR (row 6). Thus, rows 5 and 6 are logical remainders. Several other rows each contain only one case with membership higher than 0.5 (rows 1, 13, and 16).[2]

After identifying logical remainders, all remaining rows are subjected to a test of sufficiency. This consists of calculating each row's consistency value as a sufficient condition for the outcome. For this calculation, we need to check if each case's membership in a given row is equal to or smaller than its membership in the outcome (section 3.1.2.1). Note that the consistency test of all non-remainder rows always involves all cases under study and not just those that hold a membership higher than 0.5 in the respective row.[3]

As an example, let us demonstrate a consistency test for row 2 (~C~P~NR). First, each case's membership in this row is calculated (column ~C~P~NR in Table 7.3). We see that most cases have low membership in ~C~P~NR. Only the aforementioned cases of Australia, Canada, and Ireland hold a membership higher than 0.5 in this ideal type. For calculating the consistency of ~C~P~NR as a sufficient condition for outcome K, we apply the formula introduced in section 5.2. For this, we need to sum up the minimum values across ~C~P~NR and K over all the cases. The value is 5.17 and goes into the numerator of the formula. Then we need the sum of all cases' membership in condition ~C~P~NR. This value is 6.86 and goes into the denominator. The consistency value for ~C~P~NR then is:

Consistency sufficiency ~C~P~NR = 5.17 / 6.86 = 0.75.

While it is at the lower bound of acceptable consistency levels, researchers might still classify this row as a sufficient condition for the outcome. If we

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[2] As mentioned earlier (section 6.1), researchers might set the frequency threshold for defining logical remainders higher than one case, either because theirs is a large-N study and/or because the set calibration is thought to be imprecise and the presence of only one case in a row probably due to measurement error. The higher the frequency threshold, the lower the solution coverage is likely to be (section 5.3).

[3] This explains why most of the relevant software packages report consistency values also for logical remainder rows.

Table 7.3 Fuzzy-set membership scores of cases in ideal type ~C~P~NR

Case	Conditions				Outcome	Membership in:		Consistent with subset relation
	C	P	N	R	K	~C~P~NR	Min (~C~P~NR; K)	~C~P~NR ≤ K
AUS	0.33	0	0.33	0.83	0.33	0.67	0.33	No (TLC)
A	0.55	0.55	0.33	0.67	0.33	0.45	0.33	No
BD	0.17	0.17	1	0.67	0.33	0	0	Yes
BE	1	0	0.17	0.67	0.17	0	0	Yes
RB	0	1	0.67	0.17	0	0	0	Yes
BG	0.55	0	1	0.17	0.67	0	0	Yes
CDN	0.17	0	0.17	0.83	0.33	0.83	0.33	No (TLC)
CZ	0.55	0.83	1	0.17	0.55	0	0	Yes
DK	0.83	0	0.17	0.55	0.55	0.17	0.17	Yes
EW	0.83	0.33	1	0.17	0.67	0	0	Yes
FIN	0.83	0.67	0.33	0.67	0.17	0.17	0.17	Yes
F	0.67	0.55	0.67	0.55	0	0.33	0	No
D	0.55	0.17	0.55	0.83	0.67	0.45	0.45	Yes
GR	0.33	0.67	0.83	0.55	0.55	0.17	0.17	Yes
GUY	0	0.33	0.67	0.67	0.17	0.33	0.17	No
H	0.67	0	1	0.33	1	0	0	Yes
IS	0.67	0.83	0.33	0	0.17	0	0	Yes
IND	0.33	0.55	0.55	0.67	0.33	0.45	0.33	No
IRL	0.33	0	0.33	0.55	0.55	0.55	0.55	Yes
IL	1	0	0.55	0	0.83	0	0	Yes
I	1	0.67	0.55	0.33	0.67	0	0	Yes
JA	0	0	0.67	0.67	0.33	0.33	0.33	Yes
J	0.17	0	0.55	1	0.33	0.45	0.33	No
LV	0.83	0.17	1	0.67	0.83	0	0	Yes
LT	0.33	0.33	1	0.33	0.17	0	0	Yes
L	0.67	0	0.17	0.33	0.55	0.33	0.33	Yes

Table 7.3 (*cont.*)

Case	Conditions				Outcome	Membership in:		Consistent with subset relation
	C	P	N	R	K	~C~P~NR	Min (~C~P~NR; K)	~C~P~NR ≤ K
MK	0.55	0	1	0.67	1	0	0	Yes
M	0	0.17	0.83	0.33	0.33	0.17	0.17	Yes
NAM	0	0.83	1	0.33	0.17	0	0	Yes
NL	0.83	0	0.17	0.67	0.55	0.17	0.17	Yes
NZ	0.67	0	0.17	0	0.17	0	0	Yes
N	0.67	0	0.17	0.67	0.55	0.33	0.33	Yes
PK	0.17	1	0.83	1	0	0	0	Yes
PL	0.55	0.55	1	0.33	0.55	0	0	Yes
P	0.17	0.67	0.83	0.67	0.55	0.17	0.17	Yes
RO	0.83	0.33	1	1	0.55	0	0	Yes
SK	0.83	0.83	1	0.17	0.67	0	0	Yes
SLO	0.83	0.55	1	0.17	0.33	0	0	Yes
ZA	0	0.67	1	0.67	0.55	0	0	Yes
E	0.33	0	0.83	0.33	0.55	0.17	0.17	Yes
CL	0.33	1	0.83	0.33	0.17	0	0	Yes
S	0.67	0	0.17	0	0.55	0	0	Yes
TR	0.55	0.17	0.83	1	0.17	0.17	0.17	Yes
UK	0	0	0	0	0.17	0	0	Yes
					Sum	6.86	5.17	

TLC = true logically contradictory case

have a look at which cases violate the statement of sufficiency, some doubts about this judgment should arise, though. As the last column of Table 7.3 shows, the cases that contradict the statement of sufficiency of C~P~NR are: Australia, Austria, Canada, France, Guyana, India, and Japan. Two of these cases (Australia, Canada) represent true logical contradictions (TLCs; see section 5.2). They are more in than out of the hypothesized sufficient condition (~C~P~NR > 0.5), but more out of than in the outcome (K < 0.5). What is more, these two are also among the only three cases for which ~C~P~NR is the best-fitting ideal type. Hence, it is precisely those cases that are good empirical representations of the alleged sufficient condition that contradict the very statement of sufficiency. Therefore, despite its still acceptable consistency level of 0.75, there are good arguments against defining the truth table row ~C~P~NR as a sufficient condition for K.

The practical implication for research is that looking only at the consistency values is often not enough in evaluating sufficiency. Since the consistency formula does not reflect the presence of true logical contradictions, the judgment of sufficiency should always be based also on a more case-oriented perspective (Ragin 2009). If inconsistency stems from true logical contradictions, then the statement of sufficiency is put more in question than if it does not.

Table 7.4 displays the consistency value for each truth table row except for the two remainder rows. Based on the consistency scores – and, as we argue, based on the presence of true logical contradictions – we have to decide whether a given row can be considered sufficient for outcome K.

We see that consistency ranges from 0.91 (row 11) to 0.62 (row 14).[4] If we accept a threshold of 0.8, then five rows (rows 3 and 9–12) are deemed sufficient for outcome K and thus included in the logical minimization procedure (section 4.3). As suggested, researchers should also take into account the presence or absence of true logical contradictions before declaring a row sufficient for the outcome. For row 3, out of the five cases that are inconsistent with the statement of sufficiency, two (Lithuania and Malta) are true logical contradictions. For row 9, there are six inconsistent cases, one of which (New Zealand) is a true logical contradiction. Row 10 displays six inconsistent cases, of which one is a true logical contradiction (Belgium). For row 11, four inconsistent cases exist, none of which is a logical contradiction. Finally, five inconsistent cases exist in row 12, of which one is a true logical contradiction (Turkey).

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[4] In research practice, it is highly advisable to sort the rows by their consistency value in order to check if there are larger gaps in consistency that could be used as a threshold for consistency (Ragin 2008a: 144).

Table 7.4 Consistency values of ideal types

Row	Description of ideal type = truth table row = corner of property space				Number of cases with membership >0.5 in this ideal type	Consistency score
	C	P	N	R		
1	0	0	0	0	1	0.76
2	0	0	0	1	3	0.75
3	0	0	1	0	3	0.85
4	0	0	1	1	4	0.79
5	0	1	0	0	0	–
6	0	1	0	1	0	–
7	0	1	1	0	3	0.66
8	0	1	1	1	5	0.68
9	1	0	0	0	3	0.80
10	1	0	0	1	4	0.80
11	1	0	1	0	4	0.91
12	1	0	1	1	5	0.87
13	1	1	0	0	1	0.68
14	1	1	0	1	2	0.62
15	1	1	1	0	5	0.78
16	1	1	1	1	1	0.77

Just as they are for the choice of the precise consistency threshold – which should depend on research-specific characteristics – researchers should also be flexible and explicitly justify which rows containing true logical contradictions are declared sufficient and which ones not. For instance, one might argue that the difference in consistency of 0.8 (rows 9 and 10), on the one hand, and 0.79 (row 4) is too minor to declare the former two rows sufficient for K, but not the latter. Notice, however, that for row 4 there are eight inconsistent cases, of which no fewer than four are true logical contradictions.

We decide to include rows 3, 9, 10, 11, and 12 for logical minimization, although all rows but one (row 11) contain true logical contradictions. Any solution term based on this truth table will therefore not only be less than fully consistent, but also contain true logically contradictory cases.

7.3 Logically minimizing the truth table

Table 7.5 displays the truth table that results from our previous decisions about which rows to declare as logical remainders (those with fewer than one

Table 7.5 Truth table based on fuzzy-set data matrix

Row	Description of ideal type = truth table row = corner of property space				Number of cases with membership >0.5 in this ideal type	Consistency	Is row deemed sufficient for K?
	C	P	N	R			
11	1	0	1	0	4	0.91	1
12	1	0	1	1	5	0.87	1
3	0	0	1	0	3	0.85	1
9	1	0	0	0	3	0.80	1
10	1	0	0	1	4	0.80	1
4	0	0	1	1	4	0.79	0
15	1	1	1	0	5	0.78	0
16	1	1	1	1	1	0.77	0
1	0	0	0	0	1	0.76	0
2	0	0	0	1	3	0.75	0
8	0	1	1	1	5	0.68	0
13	1	1	0	0	1	0.68	0
7	0	1	1	0	3	0.66	0
14	1	1	0	1	2	0.62	0
5	0	1	0	0	0	Logical remainder	?
6	0	1	0	1	0	Logical remainder	?

case with a membership higher than 0.5), which ones to declare sufficient conditions (those with a consistency score of 0.8 or higher without too many logically contradictory cases), and which ones not (those below the consistency threshold of 0.8 and/or too many logically contradictory cases). The column labeled "Is row deemed sufficient for K?" reflects this classification for each truth table row.[5]

At this point, we can revert to the tools for the logical minimization of truth tables introduced in sections 4.3 and 6.4. We are interested in the sufficient conditions for the presence of constitutional control of the executive (K). This is why we calculated each row's consistency as a sufficient condition for K. In other words, this truth table is valid only for the analysis of the sufficient conditions for the presence of outcome K, and neither for the absence (~K) nor the analysis of necessary conditions (see section 9.1).

[5] Note that this column is called "outcome" or "Y" in most software packages that perform the Truth Table Algorithm. This is misleading, for the values in this column do *not* reflect a dichotomized version of the empirically observed outcome value of cases in the respective row. Instead, they reflect the researcher's assessment of whether the given row can be considered sufficient for the outcome.

As explained in section 6.4, in the presence of limited diversity, the Standard Analysis procedure allows for different solution terms to be produced depending on whether assumptions on logical remainders are made. If no assumptions are made, the conservative solution is produced, if simplifying assumptions are made, the most parsimonious solution results, and if only easy counterfactuals are permitted, an intermediate solution emerges. For presentational purposes, we produce only the conservative solution here.

The Boolean expression of all sufficient rows for outcome K is this:

C~PN~R + C~PNR + ~C~PN~R + C~P~N~R + C~P~NR → K.

This can be minimized to:

C~P + ~PN~R → K.

We identify two sufficient paths for high constitutional control of the executive: a combination of a consensus democracy with the absence of a strong president, or the combination of the absence of a strong president with the presence of a new democracy and the absence of a rigid constitution.[6] The parameters of fit for this solution are as follows:

|	C~P +		~PN~R	→ K
Raw coverage	0.69		0.57	
Unique coverage	0.23		0.11	
Uniquely covered				
Covered cases*	BE,DK,LV,NL,LX		LIT,MAL,SP	
	NZ,S,RO,G,MC,TK			
Consistency	0.77		0.84	
Solution coverage	0.80			
Uncovered cases**	I,SK,RU,PL,CZ,IR,GR,SA,PT			
Solution consistency	0.75			

*   cases uniquely covered by path, that is membership > 0.5 in only this path and not the other

** cases with membership in outcome > 0.5 and of < 0.5 in any path

In addition to reporting the parameters of fit, another, more case-oriented strategy for assessing and interpreting this solution term is to produce XY plots for each path and/or the entire solution term. Figure 7.1 displays such

[6]   We might be tempted to interpret ~P as a necessary condition, since it is part of all paths of the solution for sufficiency. However, if we do so we risk creating so-called false necessary conditions. In section 9.1.2, we provide a detailed discussion of this fallacy and potential remedies.
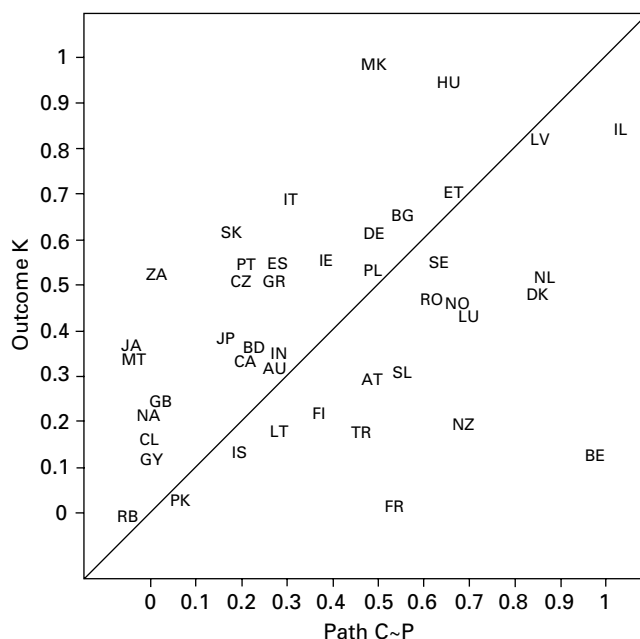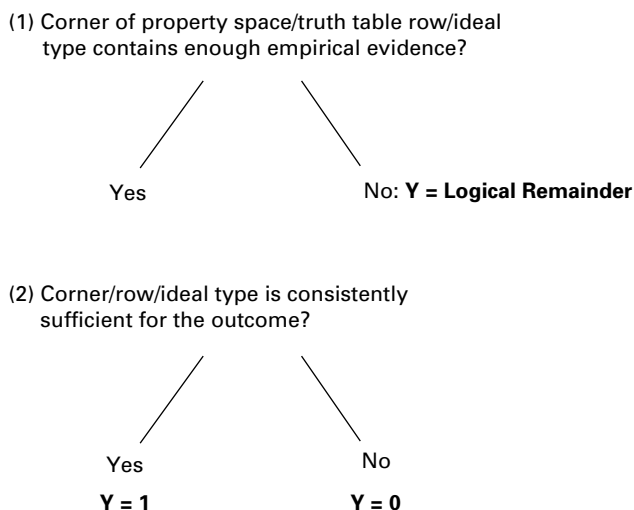
Figure 7.1   XY plot for path C~P

an XY plot for path C~P. The rather low consistency value is visualized by the number of cases below the main diagonal and their distance from it. The worst offender against the statement that a consensus democracy combined with the absence of presidentialism (C~P) is sufficient for the presence of strong constitutional control of the executive (K) is Belgium with its perfect membership in path C~P and its virtual non-membership in outcome K. In addition to Belgium, New Zealand (NZ) and Turkey (TR) are also true logically contradictory cases to the statement that C~P is sufficient for Y. Researchers should report this evidence and adequately include it in their substantive interpretation of the findings. For instance, it would be implausible to refer to Belgium, New Zealand, or Turkey as empirical instances that demonstrate that condition C~P is sufficient for K, or, even worse, to use these cases for within-case analysis trying to unravel the causal mechanisms linking C~P with K.[7] Furthermore, we see that several cases are more in than out of the outcome set but have low membership in path C~P. These are cases like Italy, Slovakia, and others in the upper left corner of the XY plot. In principle, they

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[7] In section 11.4, we discuss the issue of post-QCA case selection strategies in further detail.

(1) Corner of property space/truth table row/ideal
type contains enough empirical evidence?

Yes   No: **Y = Logical Remainder**

(2) Corner/row/ideal type is consistently
sufficient for the outcome?

Yes   No
Y = 1   **Y = 0**

Figure 7.2   Steps in the Truth Table Algorithm

could be covered by the other sufficient path – in practice, however, they are not and thus remain uncovered by the entire solution term.

In sum, the Truth Table Algorithm consists of a sequence of steps that starts with the crisp- or fuzzy-set data matrix; attributes cases to rows; defines rows either as sufficient, not sufficient, or as logical remainders; and then logically minimizes the truth table. Figure 7.2 provides a graphical summary of the steps of the Truth Table Algorithm. For each logically possible combination of conditions, it needs to be decided whether the underlying fuzzy-set evidence warrants treating it as a logical remainder. Among those configurations that are not logical remainders, a distinction needs to be made between those that are consistent enough to be considered sufficient for the outcome and those that are not.

7.4 Implications of the Truth Table Algorithm

Several important features of the Truth Table Algorithm are worth pointing out (again). First, once a truth table is constructed, different treatments of logical remainders can be chosen, either by applying the Standard Analysis (section 6.4) or the Enhanced Standard Analysis (section 8.2). Second, the truth tables produced by the Truth Table Algorithm like that in Table 7.5

are no proper base for performing analyses of necessity, because they do not display each case's membership in the outcome. As a consequence, no inferences about the presence or absence of necessary conditions should be drawn based on the Truth Table Algorithm, a point we elaborate in more detail in section 9.1.

Third, the truth table produced by the algorithm is only valid for either the presence of the outcome or its negation, never for both. Thus, if we wanted to analyze the sufficient conditions for the absence of constitutional control of the executive (~K), then we *cannot* use the truth table displayed in Table 7.5, which has been constructed with K as the outcome. That is, when interested in outcome ~K, we cannot simply logically minimize all rows that are denoted with the value 0 in the outcome column of Table 7.5. Instead, when analyzing ~K, we have to start the Truth Table Algorithm from the beginning and calculate each row's consistency as a sufficient condition for ~K. The reason for the need to create separate truth tables is that the decision on whether a row can be deemed sufficient for the outcome is partially based on the consistency value it obtains. It might well be – and it frequently happens in applied QCA – that a given row is sufficient neither for K nor for ~K, because it does not pass the consistency threshold for either of the two outcomes and researchers therefore do not include it in any of the two logical minimization procedures.[8]

Fourth, the algorithm can be applied to both crisp and fuzzy sets. As a matter of fact, since the Truth Table Algorithm rests on a truth table, which, in turn, exclusively consist of 1s and 0s, some might think that what the Truth Table Algorithm does is nothing more than turning fuzzy sets into crisp sets by dichotomizing them. However, this is false. The more fine-grained information contained in fuzzy sets is never lost and is used at various steps: (a) when calculating the membership of cases in truth table rows and thus when identifying logical remainders; (b) when calculating the consistency of a row and thus when identifying sufficient rows; and (c) when calculating the parameters of fit for the solution formula. In addition, after the analysis of fuzzy data with the Truth Table Algorithm, each case's fuzzy-set membership in the solution term and outcome can and should be displayed, such as, for instance, in the XY plot in Figure 7.1.

As a matter of fact, whenever a concept is not a natural dichotomy and when fuzzy sets are available, there is never a good reason for turning them

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[8] With fuzzy sets, it can happen that one and the same row passes the consistency threshold for both outcomes and might thus be deemed sufficient for both Y and ~Y. In section 9.2.2, we provide an extensive treatment of this issue and several other analytic challenges that all derive from skewed membership in X, or Y, or both.
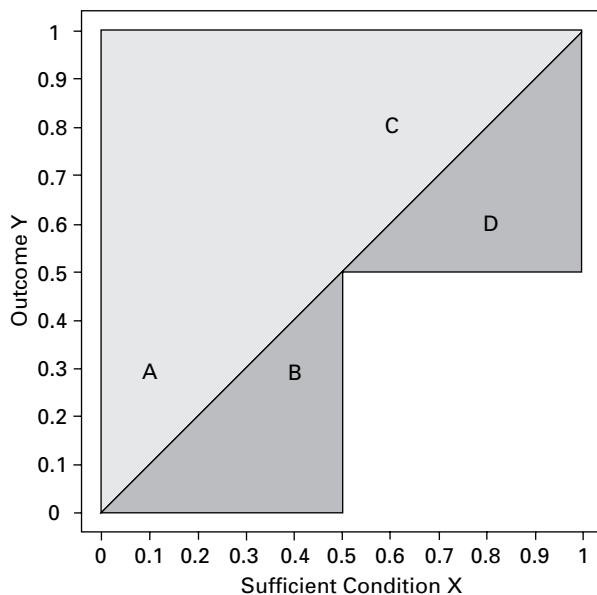
Figure 7.3   XY plot combined with two-by-two table

into crisp sets (Ragin 2008a: 138–41). First, with fuzzy sets, the test for suffi-ciency is more conservative. If we turned fuzzy sets into crisp sets, using the qualitative anchor of 0.5 as the threshold for membership, it can then happen that a condition that is not consistently sufficient when using fuzzy sets all of a sudden becomes fully consistent when using the crisp version of the same data. To illustrate this, consider Figure 7.3, which displays an XY plot with a two-by-two table superimposed.

With fuzzy sets, all cases need to be above the main diagonal in order for X to be fully consistent as a sufficient condition for Y. In other words, cases that fall into areas B and D are inconsistent with the statement that X is sufficient for Y. For instance, a case with X = 0.8 and Y = 0.6 would fall into area D and would thus contribute to the inconsistency of X. Similarly, a case with, say, X = 0.4 and Y = 0.3 would fall into area B and make condition X more incon-sistent. Now, consider what happens if we turn a fuzzy set into a crisp set. All cases in area D obtain crisp-set membership scores of 1 in both X and Y and all cases in area B obtain membership scores of 0 in both X and Y. Because of this, they no longer contradict the set-theoretic statement that X is sufficient for Y. In other words, by eliminating the finer-grained fuzzy-set membership scores and retaining only the information on their qualitative differences (as reflected by their membership scores above or below the qualitative anchor

0.5), we have turned a hitherto inconsistent condition into a more (or even fully) consistent sufficient condition. By dichotomizing the fuzzy-set membership scores, we therefore artificially increase the consistency of set relations – an argument that also applies to set relations denoting necessity.

Finally, the Truth Table Algorithm has by now replaced the so-called Inclusion Algorithm described in Ragin (2000) as the predominant protocol for analyzing fuzzy sets. The major advantage of the Truth Table Algorithm is that, unlike with the Inclusion Algorithm, researchers can make use of the powerful tool of the truth table. By virtue of this, researchers have a better grip on logical remainders and the analytic processes of crisp-set QCA and fuzzy-set QCA become virtually identical rather than being two different types of data analysis approaches.

At-a-glance: the Truth Table Algorithm

The **Truth Table Algorithm** is the central tool to analyze **sufficient conditions** and consists of the three steps. First, the data matrix is converted into a **truth table**. Second, each **truth table row** is classified either as a **logical remainder**, as **consistent** for the outcome of interest, or as not consistent. Third, the truth table is **logically minimized**.

The Truth Table Algorithm can be applied both to **crisp** and **fuzzy sets**. Dichotomizing fuzzy sets and executing a crisp-set analysis lead to different results.

The outcome and the non-occurrence of the outcome have to be analyzed separately.

Based on the Truth Table Algorithm, **necessary conditions** are commonly not correctly identified.

Part III

Potential pitfalls and suggestions for solutions

8 Potential pitfalls in the Standard Analysis procedure and suggestions for improvement

Easy reading guide

In the previous chapter, our presentations of the various elements of QCA have culminated in the Truth Table Algorithm. It describes the data analysis procedure from the creation of a truth table based on empirical data and its subsequent logical minimization. For the latter, the Standard Analysis is applied, which produces the conservative, most parsimonious, and the intermediate solution through different strategies for handling logical remainders. In Chapters 8 and 9, we develop suggestions on how to improve the Standard Analysis. We consider these chapters a valuable addition to the current debate, because the Standard Analysis can result in real analytic pitfalls that are more than just cosmetic nuisances.

There is no doubt that the Standard Analysis is a very useful strategy for dealing with limited diversity. In particular, intermediate solutions have several advantageous properties: they strike a balance between parsimony and complexity; they are the result of supplementing the empirical information at hand with a controlled dose of theory-guided assumptions; and they handle logical remainders in a conscious, yet practical manner. These properties make intermediate solution terms preferable to both the complex and the most parsimonious solution term. While intermediate solutions are indeed highly useful, in this chapter we aim to qualify the idea that they are *always* preferable. We argue that the Standard Analysis does not guard against two pitfalls. One is that it might create solution terms that are based upon what we label untenable assumptions. The other is that it leads researchers to overlook interesting and useful logical remainders, and thus insightful solutions, simply because these remainders are not contributing to parsimony.

What is thus needed is a more differentiated view on counterfactuals (8.1). We demonstrate with published QCA that the problem of making untenable assumptions does in fact occur when the Standard Analysis is applied. We offer the Enhanced Standard Analysis as a remedy (8.2). This, in essence, consists of barring untenable assumptions from being included in any solution term. The Theory-Guided Enhanced Standard Analysis, in turn, presented in (8.3), consists in replacing parsimony with theoretical soundness as the primary decision rule as to which logical remainders should be used for counterfactual claims. This includes the extension of the notion of directional expectations from single conditions to conjunctions of conditions, with entire truth table rows being the most extreme form of this form of conjunctural directional expectations.

Note that this chapter and the next expand on the current literature on QCA. Most likely, readers who stop reading the book at this point and correctly implement all their knowledge from the previous chapters in their QCA-based research will not be seen as falling short of current standards for how to perform QCA. The issues we address from now on are real, however, and they do impact the quality of inferences drawn from QCA. We therefore recommend that readers at beginner's level first make sure that the basics addressed in Chapters 1–7 have been adequately digested, and then gain at least a general overview of the topics dealt with in the coming chapters. More advanced readers might especially enjoy the following chapters because it is highly likely that most of them have been confronted with one or more of these issues without yet having found sound solutions.

8.1 Beyond the Standard Analysis: expanding the types of counterfactuals

In section 6.4.3, we introduced the distinction between different types of assumptions, or counterfactual claims: simplifying assumptions, and easy and difficult counterfactuals. In the following, we further differentiate between types of counterfactuals. We will label them *good counterfactuals* and *untenable counterfactuals*, the latter consisting of implausible and incoherent counterfactuals.

Incoherent counterfactuals are defined as assumptions that contradict claims made about the same remainder at a different moment of the analytical process. This fallacy can happen in two ways. First, the researcher performs separate analyses for both Y and ~Y and includes the same remainder into both minimization procedures. By doing so, the researcher is effectively saying that the same logical remainder is sufficient for both the occurrence *and* the non-occurrence of an outcome. This type of assumption is already discussed in the literature under the label *contradictory assumption* (Yamasaki and Rihoux 2009).[1] The second form of incoherent counterfactuals can occur when researchers make a claim of necessity but then also allow a logical remainder to be part of a sufficiency solution that contradicts that claim of necessity. To see how this happens, recall that if X is necessary for Y, then Y cannot occur in the presence of ~X. Formally: if X ←Y, then ~X → ~Y (section 3.3.3). Hence, if a researcher claims that X is necessary for Y, then any

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[1] Commonly, they are labeled contradictory simplifying assumptions. We prefer to drop the adjective "simplifying," for such assumptions do not have to contribute to parsimony in order to be contradictory.

logical remainder row that implies condition ~X cannot be used for a counterfactual claim following which it would produce outcome Y. This is, so far, a widely overlooked pitfall.

Implausible counterfactuals are defined as assumptions about impossible remainders. Recall from section 6.2.3 that impossible remainders denote those configurations that run counter either to pure formal logic (e.g., a country that is rich and poor at the same time) or to common sense (e.g., the pregnant man). Claiming that any such impossible remainder is sufficient for any outcome strikes us as an implausible counterfactual which should, thus, be avoided.

A further category is that of *good counterfactuals* (Lewis 1973; Tetlock and Belkin 1996; Lebow 2010; Goertz and Mahoney 2012: ch. 9). These are claims about logical remainders that fulfill the criterion of being theoretically sound counterfactuals – regardless of, and this is the main point here, whether they contribute to parsimony.[2] In other words, good counterfactuals are chosen because the researcher has strong theoretical and substantive arguments that this specific remainder (or set of remainders) would produce the outcome, if only it existed.

Figure 8.1 is an updated version of a similar Venn diagram in section 6.4.4 (Figure 6.2). It graphically summarizes our point and provides a visualization of where the pitfalls of the Standard Analysis (SA) can be found. The rectangle denotes the set of all logical remainders. The inner circle denotes all simplifying assumptions, which are decomposed into difficult and easy counterfactuals. We go beyond this and further differentiate between tenable and untenable simplifying assumptions (both easy and difficult ones) and among the tenable assumptions between good and not good assumptions.

The SA differentiates neither between tenable and untenable, nor between good and not good assumptions. It therefore allows for the inclusion of untenable easy and difficult counterfactuals and the exclusion of those good counterfactuals that are not simplifying. In essence, then, the difference between the SA and our suggestions for the treatment of logical remainders is the following. The SA certifies any solution as long as it is based on logical remainders that fall into the inner circle of Figure 8.1. We, instead, argue that only solution terms should be accepted that are based on those logical remainders that fall into the grey-shaded area of Figure 8.1. This area consists of only good counterfactuals that can be simplifying (easy counterfactuals) or not.

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[2] According to Emmenegger's (2012) review of the literature on this topic, these criteria are: clarity of antecedent and consequent assumptions; plausibility of antecedent; conditional plausibility of consequent; projectability; and minimum rewrite rule.
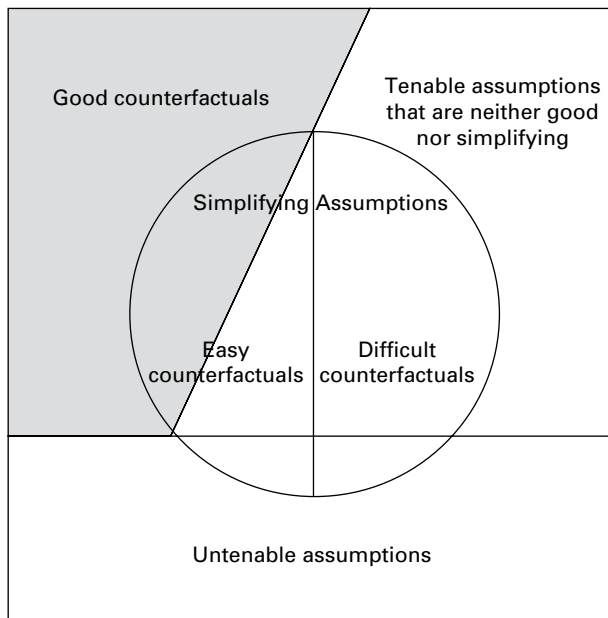
Figure 8.1   Venn diagram – types of counterfactuals, extended list

To be fair, most easy counterfactuals are good counterfactuals, and most good counterfactuals are easy counterfactuals. In this sense, the size of the areas in Figure 8.1 does not reflect the frequencies with which the different types of counterfactuals usually occur in applied QCA. However, when assessing the role of counterfactuals, it is not so much the quantity but their quality that matters. If it turns out that a specific intermediate solution term for outcome Y rests, among many other assumptions, on the counterfactual claim that the pregnant man produces Y, then the problem is not how many impossible remainders implying the pregnant man have been used but that at least one such remainder has been used.

8.2 The Enhanced Standard Analysis: forms of untenable assumptions and how to avoid them

We demonstrate how the primacy given to parsimony by the SA procedure can produce implausible and incoherent counterfactuals. We show that this pitfall can also occur when using only easy counterfactuals. Hence, there is the risk that the SA produces intermediate solution terms that are based

on untenable assumptions. In the final section, we describe the Enhanced Standard Analysis as a strategy for avoiding untenable assumptions. In the following, we use published data, without, however, either replicating the original analyses or always claiming that the original authors have committed the analytic fallacies we describe with the help of their data.

8.2.1 Incoherent counterfactuals I: contradicting the statement of necessity

The first example stems from Vis (2009), which we have already introduced in section 5.3. In her fsQCA, she investigates the conditions under which governments pursue unpopular social policies (U). She specifies three conditions: weak political position, with parties in government expecting losses at the next election (P); a deteriorating socio-economic situation (S); and a government dominated by parties from the right of the political spectrum (R). In her article, Vis analyzes the occurrence of the outcome (U). For presentational purposes, we analyze the sufficient conditions for ~U – the non-occurrence of unpopular reforms.

Table 8.1 represents the information on the 25 cases in the form of a truth table.[3] There are only eight truth table rows. Nevertheless, three of them remain without enough empirical evidence and are thus classified as logical remainders (sections 6.1 and 7.1).

The analysis of necessity reveals that parties in government *not* expecting losses is a necessary condition for the absence of unpopular reform. Formally:

~P ← ~U.[4]

For the analysis of sufficiency, we choose a frequency threshold of 1 and a consistency threshold of 0.8. For the intermediate solution, we impose the following directional expectations:

~S → ~U, ~R → ~U.

In plain words, we expect that the lack of socio-economic deterioration (~S) and the absence of rightist parties in government should both contribute to ~U. We have no directional expectation on condition P.

Based on this setup, the SA produces the following three solution terms:[5]

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[3] The fuzzy-set data matrix can be found in the online appendix (www.cambridge.org/schneider-wagemann).

[4] ~P has a consistency of 0.93 and a coverage of 0.7 as a necessary condition for ~U.

[5] For ease of reading and because they are irrelevant for the argument we make, we do not report the parameters of fit.

Table 8.1 Truth table for outcome ~U (Vis 2009)

Row	P	S	R	~U	Consistency	Cases
1	0	0	0	1	0.829	Kok I, II; Schröder I; N. Rasmussen I, IV; Blair I, II
2	0	0	1	1	0.911	Lubbers II; Kohl I, II, III; Schlüter I; Thatcher II
3	1	1	1	1	0.836	Balkenende II; Kohl IV; Schlüter V[6]
4	0	1	1	0	0.706	Lubbers I, III; Schlüter II, IV; Thatcher I, III; Major
5	1	1	0	0	0.696	N. Rasmussen II/III; Schröder II
6	0	1	0		0.887	–
7	1	0	0	α ε	0.916	–
8	1	0	1	α ε β	0.958	–

Rows 6–8: logical remainder

α simplifying assumption for most parsimonious solution term for outcome ~U

β easy counterfactual for intermediate solution term for outcome ~U

ε the negation of the necessary condition is present

α ε and α ε β assumption contradicting statement of necessity

conservative: ~P~S + PSR   → ~U

most parsimonious: ~S + PR   → ~U

intermediate: ~P~S + PR   → ~U.

As foreseen by the SA (section 6.4), the most parsimonious solution is a superset of the intermediate one, which, in turn, is a superset of the conservative solution term. For the latter, by definition, no assumptions on logical remainders are made. The most parsimonious solution rests on the simplifying assumption that the remainders in rows 7 and 8 are sufficient for the outcome (P~S~R + P~SR → ~U). This is indicated by the Greek letter α in column ~U of Table 8.1.

The problem with these simplifying assumptions is that they contradict the statement of necessity from above. If we claim that ~P is necessary for ~U, then this means that outcome ~U cannot be present without condition ~P being present. In other words, their cannot be a simultaneous presence of

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[6] This particular row not only has a high raw consistency for outcome ~U, but also for U (0.91). In her original analysis of outcome U, Vis (2009: 44) therefore correctly attributes a value of 1 to this row. Row 3 in Table 8.1 is an example of a simultaneous subset relation with both the outcome and its negation – another important analytic pitfall which we address in detail in section 9.2.2.

~U and P (the complement of ~P). Or, even more simple, in the presence of P, ~U cannot occur. From this follows that no logical remainder with P present (denoted by ε in Table 8.1) can be included into the logical minimization process, for it would mean that we assume such remainders to be sufficient for ~U. Now, as Table 8.1 shows, assumptions on remainders containing P have been made (rows 7 and 8, denoted by α ε): the most parsimonious solution rests on the assumption that when P is combined with either ~SR or ~S~R, it is sufficient for ~U. Remainders with P present that are included in minimization constitute incoherent counterfactual claims and should be avoided.

One might think that this is less of a problem because at the center of substantive interpretation should be not the most parsimonious solution, but rather the intermediate solution term. And because the latter allows only for easy counterfactuals, the problem of untenable assumptions would be solved. Unfortunately, this is not true by default. Notice that the intermediate solution term for outcome ~U also rests on an assumption that contradicts the statement of necessity. As column ~U in Table 8.1 shows, the logical remainder in row 8 is included in the intermediate solution term despite the fact that it is an incoherent counterfactual (denoted by the symbol β).

This example demonstrates that easy counterfactuals can, indeed, be logically untenable assumptions and that neither the SA, in general, nor the directional expectations, in particular, are safeguards against making untenable assumptions.[7]

8.2.2 Incoherent counterfactuals II: contradictory assumptions

In the textbook on configurational comparative methods edited by Rihoux and Ragin (2009), data on the social requisites for the survival of democracy (Lipset 1959) is used to illustrate the different variants of QCA. Ragin (2009) presents fsQCA using the survival of democracy during the inter-war period in Europe as the outcome (S) and the following five conditions: economically developed countries (D); urbanized countries (U); countries with high literacy rate (L); industrialized countries (I); and politically stable countries (G) (Ragin 2009: 93). The fuzzy-set membership scores for the 18

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[7] Note that the conservative solution also seems to contradict the statement of necessity, since ~P is only part of one sufficient path, but not of both. This is a different phenomenon from the inclusion of untenable assumptions, and we will discuss this topic of "hidden necessary conditions" in section 9.1.1.

European countries during the interwar period examined (Ragin 2009: table 5.2) with a threshold for raw consistency value of 0.8 (section 5.2) produce the truth table displayed in Table 8.2. In the following, we are using these data as an example for the second type of incoherent counterfactual: contradictory assumptions.[8] This problem may arise when the occurrence and the non-occurrence of the outcome are analyzed in separate steps based on the same truth table.[9]

The most parsimonious solution for outcome S is:

D~I + UG → S.

In order to obtain this solution, 12 of the logical remainders are used as simplifying assumptions (denoted by the Greek letter α in column S of Table 8.2). The most parsimonious solution for the non-occurrence of the outcome ~S is:

~D + ~G → ~S.

From column S in Table 8.2, we see that 18 of the 23 logical remainders have been used for producing the most parsimonious solution term for outcome ~S. They are indicated by the Greek letter β (rows 10–22, 24, 26, 27, 29, and 31). The problem is that several of these are contradictory simplifying assumptions. More specifically, the logical remainders in rows 15, 17, 19, 21, 22, 26, 27, and 31 (denoted as α β) have already been assumed to produce outcome S and are now also assumed to produce outcome ~S. But we cannot have it both ways. Either we assume that the remainders under question are sufficient for S or we assume that they are sufficient for ~S. Contradictory simplifying assumptions are clearly untenable and they occur because the selection of remainders is exclusively driven by the goal of parsimony.

Unfortunately, even if we allow only for easy counterfactuals, contradictory simplifying assumptions can occur. We call them contradictory easy counterfactuals. For illustration, consider the following directional expectations for our analyses of S and ~S, respectively:

~D → S; ~U → S: ~I → S

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[8] With this data, it is also possible to generate counterfactuals that are incoherent with the statement of necessity. An analysis of necessity reveals L (consistency: 0.99) and G (consistency: 0.92) as necessary conditions for S. Now, if we specify the directional expectations ~L → S, ~D → S, ~U → S, ~I → S, both the most parsimonious solution (D~I + UG → S) and the intermediate solution (D~IG + UG → S) are based on incoherent assumptions about necessary conditions.

[9] As mentioned, this is not meant to be a reanalysis of the original study. For didactic reasons, we use a different consistency threshold (0.8 rather than 0.7) and different directional expectations than the original study.

Table 8.2 Truth table, Lipset data (Ragin 2009)

Row	Conditions					Outcome	Cases
	D	U	L	I	G	S	
1	1	1	1	1	1	1	BE; CZ; NL; UK
2	1	0	1	0	1	1	FIN; IRL
3	1	0	1	1	1	0	F; S
4	0	0	1	0	1	0	EE
5	0	0	1	0	0	0	H; PL
6	1	1	1	1	0	0	D
7	1	0	1	1	0	0	A
8	0	0	0	0	1	0	I; RO
9	0	0	0	0	0	0	GR; P; E
10	0	0	0	1	0	β δ ε	
11	0	0	0	1	1	β δ ε	
12	0	0	1	1	0	β δ ε	
13	0	0	1	1	1	β δ	
14	0	1	0	0	0	β δ ε	
15	0	1	0	0	1	α β δ ε	
16	0	1	0	1	0	β δ ε	
17	0	1	0	1	1	α β δ ε	
18	0	1	1	0	0	β δ ε	
19	0	1	1	0	1	α β γ δ	
20	0	1	1	1	0	β δ ε	
21	0	1	1	1	1	α β γ δ	
22	1	0	0	0	0	α β ε	
23	1	0	0	0	1	α ε	
24	1	0	0	1	0	β δ ε	
25	1	0	0	1	1	ε	
26	1	0	1	0	0	α β ε	
27	1	1	0	0	0	α β ε	
28	1	1	0	0	1	α ε	
29	1	1	0	1	0	β δ ε	
30	1	1	0	1	1	α ε	
31	1	1	1	0	0	α β ε	
32	1	1	1	0	1	α γ	

Adapted from Ragin (2009: table 5.8)

Rows 10–32: logical remainders

α simplifying assumption for most parsimonious solution term for outcome S

β simplifying assumption for most parsimonious solution term for outcome ~S

γ easy counterfactual for intermediate solution term for outcome S (directional expectations: ~D → S; ~U → S; ~I → S)

δ easy counterfactual for intermediate solution term for outcome ~S (directional expectations: D → ~S; U → ~S; ~L → ~S; I → ~S; ~G → ~S)

ε the negation of the necessary condition is present

α β contradictory simplifying assumption

γ δ contradictory easy counterfactual

D → ~S; U → ~S; ~L → ~S; I → ~S; ~G → ~S.

The directional expectations for ~S take into account the finding that L and G are necessary conditions for S (consistency values: 0.99 and 0.92, respectively). There are no necessary conditions for ~S, but, following our interpretation of DeMorgan's law (section 3.3.3), if L ← S and G ← S, then ~L → ~S and ~G → ~S.[10]

These produce the following intermediate solution terms:

D~ILG + UGL   → S
~D + ~GI   → ~S.

In the analysis of outcome S, easy counterfactuals are made about the logical remainders in rows 19, 21, and 32, indicated by γ in Table 8.2. In the analysis for outcome ~S, easy counterfactuals are made for the remainders in rows 10–21, 24, and 29 (indicated by δ). Hence, there are two *contradictory easy counterfactuals* (indicated by γ δ) in rows 19 and 21. The intermediate solutions for S and ~S, respectively, rest on the claim that these logical remainders are sufficient for S and ~S, respectively.

While this example demonstrates that even intermediate solutions do not, by default, rule out contradictory counterfactuals, an element of caution is required. Much depends on how the directional expectations are formulated. The methodological point we aim at making here is not that contradictory easy counterfactuals *often* occur. Rather, we want to draw attention to the fact that they *can* occur even when following current best practices by producing the intermediate solution term.

8.2.3 Implausible counterfactuals: contradicting common sense

In section 6.2.3, we defined impossible remainders as combinations of conditions that describe cases that simply cannot exist in the world as we know it. To some readers it might seem that impossible remainders are just too obvious and rare in social science research to constitute a serious threat to the task of drawing inferences based on observational data. As the following example shows, not only are there more intricate impossible remainders than the hypothetical pregnant man we have referred to, but their number might also be quite large in a given truth table. If so, then researchers need to be

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[10] If a condition X is necessary for Y, one could argue that then the directional expectation for condition X should be X → Y. In fact, Mahoney *et al.* (2009) hint in this direction. We believe, however, that the only logical consequence for directional expectations that follows from the empirical finding X ← Y is the directional expectation ~X→ ~Y.

particularly alert in order not to make any implausible assumptions about such impossible remainders.

In section 6.2.3, we have already introduced Ragin *et al.*'s (2003) study of collective action in communal irrigation in some villages in India's Andhra Pradesh state. Three of their conditions are: village is located on channel MN (M), village is located on channel V (V), and village is irrigated (I). Their truth table (Table 8.3) contains these three conditions (plus two more, which are irrelevant to our current point). By pure formal logic, several logical remainder rows denote hypothetical villages that are located on channel MN or V or both that are *not* irrigated. These combinations are, of course, impossible, since any village on a channel is irrigated by definition – as the authors themselves correctly point out (Ragin *et al.* 2003: 331). There are even more impossible remainders. Given the geographical features, no village can be close to the V channel without being also close to the MN channel. Hence, the set of impossible remainders can be expressed in Boolean terms as follows:

Impossible remainders = M~I + V~I + ~MV.

As Table 8.3 shows, the majority of the truth table rows are logical remainders (20 of the 32 rows). Seventeen out of the 20 logical remainders are, in fact, impossible remainders, indicated by the Greek letter ζ. The most parsimonious solution and the conservative solution differ quite considerably in their degree of complexity:
conservative:

IM~VW + IMWD → CA

most parsimonious:

MW   → CA.

The most parsimonious solution is based on simplifying assumptions about five logical remainders (indicated by α in column CA of Table 8.3). We see that four of the five simplifying assumptions used for the most parsimonious solution term are, indeed, made on impossible remainders and thus constitute *implausible counterfactuals* (denoted as ζ α). Only the simplifying assumption about the remainder in row 32 does not constitute an untenable assumption.

Even intermediate solution terms might be based on implausible assumptions. As a demonstration, let us formulate the directional expectation that, if a village is not irrigated, inhabitants engage in collective action. Formally:

~I → CA.

Table 8.3 Truth table, outcome CA (Ragin *et al.* 2003)

Row	Conditions					Outcome
	I	M	V	W	D	CA
1	1	1	1	0	1	0
2	0	0	0	0	0	0
3	1	1	1	1	1	1
4	1	1	0	0	0	0
5	0	0	0	0	1	0
6	1	0	0	0	1	0
7	1	0	0	1	1	0
8	1	1	0	0	1	0
9	1	1	0	1	0	1
10	1	0	0	0	0	0
11	1	0	0	1	0	0
12	1	1	0	1	1	1
13	0	0	0	1	0	
14	0	0	0	1	1	
15	0	0	1	0	0	ζ
16	0	0	1	0	1	ζ
17	0	0	1	1	0	ζ
18	0	0	1	1	1	ζ
19	0	1	0	0	0	ζ
20	0	1	0	0	1	ζ
21	0	1	0	1	0	ζ α γ
22	0	1	0	1	1	ζ α γ
23	0	1	1	0	0	ζ
24	0	1	1	0	1	ζ
25	0	1	1	1	0	ζ α
26	0	1	1	1	1	ζ α γ
27	1	0	1	0	0	ζ
28	1	0	1	0	1	ζ
29	1	0	1	1	0	ζ
30	1	0	1	1	1	ζ
31	1	1	1	0	0	
32	1	1	1	1	0	α

α simplifying assumption used for most parsimonious solution
term for outcome CA
γ easy counterfactual for intermediate solution term for outcome
CA (directional expectation ~I → CA)
ζ impossible remainder
ζ α and ζ α γ implausible counterfactual

This yields the following intermediate solution:

M~VW + MWD → CA.

This solution is based on assumptions about three logical remainders (indicated by γ in Table 8.3). All three of them are impossible remainders and the assumptions thus *implausible easy counterfactuals* (denoted as ζ α γ). Together with the examples by Vis and Ragin from above (see sections 8.2.1 and 8.2.2), this shows that intermediate solution terms are, in fact, prone to include any type of untenable assumptions (incoherent and implausible assumptions).

8.2.4 Putting the Enhanced Standard Analysis procedure into practice

We think it is plausible to require that no solution formula obtained by set-theoretic methods in general, or by QCA in particular, should incorporate untenable assumptions. In the following, we explain how the Standard Analysis as the current best practice in handling logical remainders should be modified. We label this slightly altered procedure the Enhanced Standard Analysis procedure (ESA).

The remedy for avoiding untenable assumptions that we suggest is straightforward. All logical remainder rows that would provide the basis for untenable assumptions must be excluded *prior* to the application of the SA procedure. In the online How-to section to Chapter 8,[11] we show how barring remainders from being included in the logical minimization procedure is done in the different software packages. Here, it is important to point out the various benefits of ESA.

First and foremost, the most obvious advantage is that the ESA makes sure that no solution term rests on untenable assumptions. This implies two things. First, with ESA no contradictory simplifying assumptions are made. This is because contradictory simplifying assumptions are a subtype of untenable assumptions. Second, necessary conditions do not disappear from the solution term for sufficiency.[12]

For demonstrational purposes, we apply ESA to our examples from above. The analysis of the lack of unpopular reforms (~U: Vis 2009), barring any remainder from the minimization that would lead to untenable

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[11] www.cambridge.org/schneider-wagemann.

[12] To be more precise, ESA assures that necessary conditions do not disappear from solution terms of sufficiency due to assumptions made on logical remainders. As we discuss in detail in section 9.1.1.2, necessary conditions can disappear for reasons unrelated to the treatment of logical remainders.

assumptions (denoted as α ϵ in Table 8.1), produces the following *enhanced most parsimonious solution term*:

~P~S + PSR → ~U.

This solution is equivalent to the conservative and the intermediate solution because the only remainder that is not untenable is the one in row 6 of Table 8.1 (~PS~R). No assumption is made about this remainder because it does not contribute to parsimony.

   Let us apply ESA to our Lipset data (section 8.2.2). In the analysis of outcome S, we need to bar those remainders that would lead to assumptions that contradict the statement that conditions L and G are necessary for S. That is, we need to exclude the logical remainders in rows 10–12, 14–18, 20, and 22–31 (marked ϵ in Table 8.2) from the minimization procedure:[13]

Enhanced most parsimonious solution:	D~ILG + UGL → S
Enhanced intermediate solution:	D~ILG + UGL → S.

The enhanced most parsimonious solution differs from the one produced by SA, whereas the enhanced intermediate solution is identical to the one derived with SA. This is because the intermediate solution did not rest on assumptions that were incoherent with the statement of necessity, that is, there are no rows labeled γ ϵ in the table.[14]

   Since the enhanced most parsimonious solution and the enhanced intermediate solution coincide with the intermediate solution derived in the SA, it just happens in our example that the Greek letter γ also denotes those remainder rows that have been used for producing the ESA solution. Hence, none of the remainders in rows 19, 21, and 32 can be used for the analysis of the non-occurrence of the outcome (~S). They are thus barred from the analysis of ~S. All other remainders are available. This yields the following enhanced solution terms:

Enhanced most parsimonious solution:	~D~U + ~G → ~S
Enhanced intermediate solution:	~D~U + DI~G → ~S.

Both solutions for ~S produced with ESA differ from those obtained with SA (most parsimonious solution: ~D + ~G → ~S; intermediate solution: ~D + I~G → ~S).

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[13] The consistency threshold and the directional expectations are the same as in the analyses performed in section 8.2.2.

[14] The intermediate solution for S did rest on contradictory simplifying assumptions, though. Assumptions on such remainders can be made as long as they are made only in the analysis of outcome S or ~S but not in both. We decide to bar contradictory simplifying assumptions from the analysis of ~S (below) and keep them for the analysis of outcome S.

Finally, applying ESA to the analysis of collective action in Indian villages (Ragin *et al.* 2003), we obtain the following. For the enhanced most parsimonious solution, we need to bar all implausible counterfactuals (denoted as ζ α γ in Table 8.3).

Enhanced most parsimonious solution term: IMW → CA.

The enhanced most parsimonious solution is only slightly more complex than the most parsimonious one. But we can now be sure that it does not incorporate any untenable assumptions.

Imposing the directional expectation ~I → CA, we obtain the following:

Enhanced intermediate solution: IM~VW + IMWD → CA.[15]

In addition, this ESA solution is slightly more complex than the SA intermediate solution. However, unlike the solutions from SA, ESA does not imply anything about collective actions in villages that cannot possibly exist. In sum, solutions produced with ESA tend to be less parsimonious, but researchers can rest assured that they are not based on untenable assumptions.

At-a-glance: the Enhanced Standard Analysis: forms of untenable assumptions and how to avoid them

The **Standard Analysis** risks producing results based on **untenable assumptions**. This concerns both the **most parsimonious** and the **intermediate solution**. **Assumptions** can be untenable because they are **implausible** (i.e., they contradict common sense), are **incoherent** with findings for necessity, or because they are **contradictory assumptions**. The **Enhanced Standard Analysis (ESA)** restricts the choice of **logical remainders** for **counterfactuals** by excluding any **remainder** that would create untenable assumptions.

8.3 Theory-Guided Enhanced Standard Analysis: complementary strategies for dealing with logical remainders

Our ESA strategy restricts the range of usable remainders and, by virtue of this, the range of acceptable solutions. Now we outline a strategy for handling logical remainders that broadens the set of usable remainders without, however,

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[15] See the online appendix (www.cambridge.org/schneider-wagemann) for a detailed analysis of this example.

bringing untenable assumptions back in. The key to doing this is to drop the central premise of both SA and ESA, namely that parsimony is the overarching goal and guiding principle for choosing logical remainders for counterfactuals. Our basic critique is that both procedures categorically rule out the use of logical remainders that are theoretically sound but do not contribute to parsimony. Since parsimony should not, simply by default, be regarded the highest goal, we suggest that researchers – in addition to using ESA – should always carefully think about which of the logical remainders represent good counterfactuals, regardless of whether their inclusion produces a more parsimonious solution term. As mentioned in section 8.1, we label these assumptions *good counterfactuals*. Some of the good counterfactuals are also easy counterfactuals, which, by definition, contribute to parsimony (Figure 8.1). Other good counterfactuals do not contribute to parsimony and are thus, by definition, not easy counterfactuals. Thus, the solution based on good counterfactuals will be more complex than the conservative solution term, but we do not think that this should rule them out by default. For lack of a better term, we label the following two strategies Theory-Guided Enhanced Standard Analysis (TESA).[16]

When identifying hitherto overlooked remainders by SA and ESA, researchers can either identify entire remainder rows or specify conjunctural directional expectations. In the following, we illustrate both strategies with examples from applied QCA.

8.3.1 Choosing entire truth table rows as good counterfactuals

For an illustration of the strategy of choosing entire truth table rows for counterfactuals based on theoretical reasoning, we turn to Koenig-Archibugi's (2004) study on why some members of the European Union are in favor of a supranational foreign and security policy (SUPRA). He identifies four conditions: a Europeanized public (IDENTITIES); high policy conformity with other European member states (CONFORMITY); domestic multilevel governance structures (REGIONALISM); and high capabilities of a country (CAPABILITIES). As Table 8.4 shows,[17] the 13 countries fall into 9 different truth table rows, thus producing 7 logical remainders (rows 10–16).[18]

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[16] The SA part in the acronym TESA is slightly misleading, for TESA does not prioritize parsimony as SA and ESA do, and therefore represents a deviation rather than an extension from the latter two.

[17] The fuzzy-set data can be found in the online appendix (www.cambridge.org/schneider-wagemann).

[18] The point we make about the treatment of logical remainders is unaffected by the fact that Koenig-Archibugi uses fuzzy sets.

Table 8.4 Truth table (Koenig-Archibugi 2004)

	Conditions				Outcome	
Row	IDENTITIES	CONFORMITY	REGIONALISM	CAPABILITIES	SUPRA	# cases
1	1	1	1	0	1	2
2	0	1	1	0	1	1
3	0	1	1	1	1	1
4	1	1	1	1	1	1
5	0	1	0	0	0	3
6	1	1	0	0	0	2
7	0	0	0	0	0	1
8	1	0	0	1	0	1
9	0	0	0	1	0	1
10	1	0	1	1	α	0
11	0	0	1	0	α	0
12	0	0	1	1	α	0
13	0	1	0	1	θ	0
14	1	0	0	0		0
15	1	0	1	0	α η	0
16	1	1	0	1	θ	0

Rows 10–16: logical remainders

α simplifying assumptions for most parsimonious solution term for outcome SUPRA

η good counterfactual, entire row selected by Koenig-Archibugi for producing theory-guided solution term

θ good counterfactual, selected based on conjunctural directional expectation CONFORMITY * CAPABILITIES → SUPRA

The author formulates hypotheses for each of the four conditions. Although they are not formulated in terms of set relations, that is, in terms of necessity or sufficiency, we can, for the sake of the argument, derive directional expectations from them. In Boolean notation, they look as follows:

CAPABILITIES → SUPRA; IDENTITIES → SUPRA; REGIONALISM → SUPRA; CONFORMITY → SUPRA.

SA produces the following solution formulas.[19]

conservative: REGIONALISM*CONFORMITY   → SUPRA

most parsimonious: REGIONALISM   → SUPRA.

[19] For the sake of readability – and because they are irrelevant for the argument we make – we do not report the parameters of fit.

Four simplifying assumptions are made in order to obtain this solution (indicated by α in Table 8.4).

The intermediate solution is identical to the conservative term:

REGIONALISM*CONFORMITY   → SUPRA.

This means that there is no solution term that is simultaneously a subset of the most parsimonious and a superset of the conservative solution. This should not come as a surprise, as the conservative solution is already quite parsimonious, consisting of just one path with only two conditions.

The SA procedure, thus, produces only two solutions. Does this mean that there is no other theoretically meaningful and substantively interesting formula that can be derived based on the empirical evidence at hand? We think there is. And, in fact, Koenig-Archibugi does derive a theory-guided solution term. In order to obtain this solution, Koenig-Archibugi allows just one assumption: countries are in favor of a supranational foreign and defense policy (SUPRA) if their population has a strong European identity AND they do not expect conformity AND they have a domestic multilevel governance structure AND if they do not have high power capabilities. In short, only the logical remainder in row 15 in Table 8.4 is assumed to produce the outcome. This yields the following TESA solution:

	CONFORMITY * REGIONAL +	IDENTITIES*REGIONAL*~CAPABIL
Consistency	1	1
Raw coverage	0.494	0.29
Unique coverage	0.211	0.07
Cases*	Germany, Belgium, Austria, Italy, Spain	Belgium, Spain
Solution consistency	1	
Solution coverage	0.501	

* cases with membership > 0.5 in path

Clearly, this solution is more complex than any of the others mentioned before. In addition to the path that appeared in the conservative solution, a second is identified. This is the result of choosing a logical remainder as a plausible counterfactual – irrespective of whether it contributes to parsimony. Koenig-Archibugi can justify the counterfactual claim on the logical remainder in row 15 and he provides a *theoretically* plausible interpretation

of this solution term. In addition, the second path, which makes the solution term more complex than the conservative solution, is also *empirically* not redundant. Its unique coverage (section 5.3), while small, is not 0. More importantly, for two out of the nine cases that are members of outcome SUPRA – Belgium and Spain – this path provides a good description because their membership in it is higher than 0.5. In short, the more complex solution term both rests on solid theoretical arguments and is empirically non-redundant.

8.3.2 Formulating conjunctural directional expectations

Another, related strategy of dealing with logical remainders that can go beyond SA and ESA is to formulate directional expectations on conjunctions of conditions. The argument in favor of this practice seems plausible. In many cases, researchers already have expectations about the effect of *combinations* of conditions on the outcome (see, e.g., Schneider 2009). If this is the case, specifying directional expectations on single conditions is not in line with theoretical knowledge and can lead to making assumptions that run counter to the theoretical knowledge at hand. Put differently, while the current practice of limiting directional expectations to single conditions does not violate QCA's focus on conjunctural causation per se, it does underemphasize the role this notion of causality can and should play when formulating directional expectations. And, in fact, in the literature the notion of conjunctural directional expectations is gaining ground (see, for instance, Amenta & Poulsen 1994; Amenta, Caren, and Olasky 2005; Maggetti 2007; Schneider 2009; Blatter, Kreutzer, Rentl, and Thiele 2009). So far, it has not yet been treated in a more systematic manner and its implications for the treatment of logical remainders have not been spelled out in detail. We think that an adequate strategy consists in formulating conjunctural directional expectations.

For a demonstration, let us return to the example of Koenig-Archibugi and assume that the existing literature on supranational arrangements in foreign and defense policy pointed to the following conjunctural directional expectation. Countries with high power capabilities (CAPABILITIES) can be expected to be in favor of supranational arrangements (SUPRA) if, in addition, there is also a high policy congruence with other EU member states (CONFORMITY). In Boolean notation this conjunctural directional expectation looks as follows:

CONFORMITY * CAPABILITIES → SUPRA.

The two logical remainders in rows 13 and 16 (Table 8.4) imply this conjunction. If, based on our directional expectations, we assume that these two, and only these two, remainders produce the outcome SUPRA, then we obtain the following solution:

	CONFORMITY * REGIONALISM	CONFORMITY* CAPABILITIES → SUPRA
Consistency	1	1
Raw coverage	0.494	0.334
Unique coverage	0.185	0.025
Cases*	Germany, Belgium, Austria, Italy, Spain	Germany, Italy
Solution consistency	1	
Solution coverage	0.519	

* cases with membership > 0.5 in path

The strategy of conjunctural directional expectations yields a solution that is different from any of the others produced based on different strategies for dealing with the logical remainders. It is, as expected, more complex than the conservative solution (REGIONALISM*CONFORMITY)[20] but, at the same time, less complex than the solution obtained by including an entire truth table row (REGIONALISM*CONFORMITY + IDENTITIES*REGIONALISM*~CAPABILITIES). Both paths are empirically non-redundant, as indicated by their unique coverage scores and the fact that both paths contain cases with membership higher than 0.5.

Notice that strategy of selecting entire logical remainder rows for counterfactual claims as described in section 8.3.1 above can be seen as an extreme way of formulating conjunctural directional expectations. By formulating directional expectations that involve only two or three conditions, researchers are less rigid in their treatment of remainders than when choosing entire truth table rows. At the same time, conjunctural directional

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[20] The difference between the TESA solution and the conservative solution consists of path CONFORMITY * CAPABILITIES. This, in turn, is exactly our conjunctural directional expectation. The fact that the path added fully coincides with the conjunctural directional expectations is a coincidence in the present example and not a general rule, though. In other words, even if directional expectations are formulated on combinations of conditions, these conjunctions do not have to form a path on their own.

expectations are more restrictive than directional expectations on single conditions only.

Notice as well that some counterfactuals based on conjunctural directional expectations contribute to parsimony, while others do not. If a researcher accepts only those counterfactuals that contribute to parsimony, then all assumptions allowed for are, in fact, a subset of the easy counterfactuals. In such a scenario, the practice of conjunctural directional expectations should be considered as a further refinement of the (Enhanced) Standard Analysis. If, however, also non-simplifying counterfactuals are accepted, then the practice of conjunctural directional expectations is an alternative to the (Enhanced) Standard Analysis.

At-a-glance: Theory-Guided Enhanced Standard Analysis: complementary strategies for dealing with logical remainders

If the central emphasis in the treatment of **logical remainders** is not placed on parsimony but theoretical plausibility, then different **counterfactuals** can be used and **solution terms** produced than with the (**Enhanced) Standard Analysis** procedure. We label it the **Theory-Guided Enhanced Standard Analysis (TESA)** for which entire **truth table rows** can be chosen as **good counterfactuals,** or **directional expectations** can be based on **conjunctions** instead of on single conditions (or their **complements**) only. Generally, researchers should always feel encouraged to take a direct look at the **logical remainders** and engage in careful theoretical thinking as to which of them might be **good counterfactuals**, regardless of whether they contribute to **parsimony**.

8.4 Comparing the different strategies for the treatment of logical remainders

QCA, mainly due to its reliance on truth tables in the analytic process, has a competitive advantage in confronting the omnipresent phenomenon of limited diversity. Thanks to the introduction of the Standard Analysis procedure (Ragin and Sonnett 2004; Ragin 2008a), much progress has been made in handling logical remainders in a conscious and theory-guided manner. In this chapter, we have tried to further improve the treatment of logical remainders.

Table 8.5 Types of assumptions included in Standard Analysis vis-à-vis additional strategies

		Tenable assumptions	
		No	*Yes*
Good assumptions	*Simplifying*	SA	SA; ESA; TESA
	Non-simplifying		TESA

SA = Standard Analysis
ESA = Enhanced Standard Analysis
TESA = Theory-Guided Enhanced Standard Analysis

Our message, in a nutshell, is this: in their substantive interpretation of the empirical evidence at hand, researchers should only rely on solution terms that are exclusively based on counterfactual claims that are both good and tenable. Graphically, this refers to all those solution terms that result when only including counterfactuals that fall into the grey-shaded area in Figure 8.1. The same message is conveyed in a different form in Table 8.5. It juxtaposes criteria for counterfactuals. Good assumptions are divided into simplifying and non-simplifying assumptions, while tenable assumptions are either present or not. First, it shows that the Standard Analysis (SA) does exclusively rely on simplifying assumptions (upper row in Table 8.5). Second, among those, some counterfactuals are actually not tenable (upper left cell). These should not be allowed. This is precisely the argument in favor of the Enhanced Standard Analysis (ESA). Third, ESA accepts the predominance of parsimony in selecting counterfactuals, but tames it by strictly barring any untenable assumptions (upper right cell). Fourth, the theory-guided analysis of logical remainders (TESA), in contrast, does put theoretical plausibility at the top of the priority list when selecting logical remainders for counterfactual claims. This might or might not come at the cost of parsimony (right-hand column).

How relevant are the potential pitfalls in handling logical remainders in applied set-theoretic methods? No doubt, by only allowing easy counterfactuals, the SA has greatly reduced the risks of untenable assumptions which occur when producing the most parsimonious solution. It is also true that most of the time, easy counterfactuals used in the SA are indeed tenable – as required by ESA – and good – as required by TESA. Nevertheless, when it comes to making counterfactual claims, it is less relevant *how many* untenable assumptions are made. A single one is already enough to cast doubt on the solution term that is derived from it. In this chapter we have shown that all

the different types of untenable assumption can occur in the SA. Among these types, we think that assumptions contradicting the statement of necessity are most likely to occur, followed by contradictory assumptions, and finally implausible assumptions. In general, the higher the number of conditions, the more likely the occurrence of untenable assumptions and the higher the benefits of applying ESA or TESA instead of SA.

9 Potential pitfalls in the analysis of necessity and sufficiency and suggestions for avoiding them

Easy reading guide

Just as in Chapter 8, so in this chapter we offer solutions to common problems that go beyond the currently dominant best practices in applied QCA. Here, we concentrate on pitfalls that can occur when analyzing sufficiency and necessity at the same time. As emphasized throughout this book, set-theoretic methods are, in essence, about unraveling subset patterns – which, in turn, are interpreted in terms of necessity and sufficiency and their derivatives, such as INUS and SUIN conditions. However, as it is applied (and taught), most QCA puts much more emphasis on the analysis of sufficiency, to the detriment of the analysis of necessity. One reason for the prevalence of sufficiency is the central role that truth tables play in QCA. Each truth table row is interpreted as a statement of sufficiency and, thus, the analysis of a truth table as described in Chapters 4–7, is an analysis of sufficiency.

This "sufficiency bias" in QCA-based research increases the risks for several analytic pitfalls to occur. First, the analysis of necessity is often erroneously treated as being essentially the same as the sufficiency analysis and is assumed not to require any separate examination. Second, and perhaps as a result of the negligence with which necessity tends to be treated in applied QCA, some strands of the literature fully separate the analysis of necessity from QCA. As an illustration, consider the fact that the seminal work on necessary conditions by Goertz and Starr (2003) does not contain a separate entry on QCA. Third, much of QCA-based research remains silent about the presence or absence of necessary conditions, even if there are good theoretical and empirical reasons to investigate them. Fourth, and finally, some studies address the presence or absence of necessary conditions but do so based on the results of a sufficiency analysis, thus assuming that the former are correctly identified as an automatic byproduct of the latter.

In this chapter, we provide conceptual and technical arguments as well as empirical evidence that the common neglect with regard to necessity comes at an analytic cost. It can happen that necessary conditions remain invisible from the result of a sufficiency analysis. We call this *the problem of hidden necessary conditions* (9.1.1). In turn, it can also happen that the result of the sufficiency analysis suggests that a condition is necessary when, in fact, it is not. We call this *the problem of false necessary conditions* (9.1.2). As we show in this chapter, the remedy is easy: simply perform separate analyses for necessity and sufficiency, preferably with the analysis of necessity coming first (Ragin 2000: 106).

Even when following our advice to perform separate analyses of necessity and sufficiency, intricate analytic issues can arise. One of these is how to handle necessary conditions during the sufficiency analysis. We addressed this in section 8.2.1, when we stated that necessary conditions should be kept in the analysis of sufficiency but no assumptions about logical remainders ought to be made that contradict the statement of necessity.

Another pitfall consists in naming a condition as necessary when, in fact, it is merely trivially necessary. We have already touched upon this issue in section 5.5. Here, we expand on this and demonstrate that to correctly capture the triviality of necessary conditions, it is useful to frame the concept in terms of "skewed set-membership scores" not only in condition X, but also in outcome Y. We also show that skewed set membership triggers analytic problems that go well beyond those of trivial necessary conditions and also affects statements of sufficiency in a similar manner. This is why we provide a more general discussion on the potential pitfalls of skewed set-membership scores in set-theoretic methods (9.2).

As with Chapter 8, the reading of this chapter requires a solid knowledge of the analysis of truth tables and (with regard to section 9.2) of the parameters of consistency and coverage. In set-theoretic terminology, a necessary condition for getting the most out of this chapter is that readers feel sufficiently familiar with the topics addressed in Chapters 1–8.

9.1 Pitfalls in inferring necessity from sufficiency solution terms

There are two pitfalls related to the analysis of necessity: the disappearance of true necessary conditions and the appearance of false necessary conditions. We illustrate both fallacies using data from published QCA. As before in the book, we do not aim at reanalyzing the original studies but rather alter them in order to better demonstrate our methodological arguments.

9.1.1 Hidden necessary conditions

Hidden necessary conditions can occur due to two, mutually non-exclusive features of the data at hand. One reason consists in the kind of assumptions made on logical remainders. The other reason rests in the treatment of less-than-perfect set relations. In the following, we provide one example for each source. From these examples, we then derive the general conditions under which this phenomenon occurs.

9.1.1.1 Hidden necessary conditions due to incoherent counterfactuals

Stokke (2004) aims at identifying the conditions under which the strategy of shaming makes hitherto non-compliant countries observe international fishing rules (SUCCESS). He identifies five conditions: advice (A); commitment

Table 9.1 Truth table (Stokke 2004)

| Row | Conditions | | | | | Outcome |
	A	C	S	I	R	SUCCESS
1	0	0	0	1	0	0
2	1	0	0	0	0	1
3	1	0	0	1	0	0
4	1	0	0	1	1	0
5	1	0	1	1	1	1
6	1	1	1	0	0	1
7	1	1	1	1	0	0
8	1	1	1	1	1	1
9–32	logical remainders					?

(C); shadow of the future (S); inconvenience (I); and reverberation (R). The ten countries evenly split into five successful and five unsuccessful incidences of shaming. The cases fall into eight different truth table rows (four connected to success and four to the lack of success). Given a truth table with 2^5 conditions, there are 32 − 8 = 24 logical remainders (Table 9.1).

The most parsimonious solution formula is:

~I + SR → SUCCESS.

From this one might be tempted to conclude that no condition is necessary, for none of them appears in both sufficient paths. However, as a glance at the truth table readily reveals, condition ADVICE (A) is present in all instances of successful shaming. It therefore empirically qualifies as a necessary condition.[1] Why then does the most parsimonious solution term not contain condition A?

The answer to this question rests in the treatment of logical remainders. Recall from Chapters 6 and 8 that the most parsimonious solution rests on assumptions about some of the logical remainders. Those remainders are chosen in such a way that the logically minimized solution term is most parsimonious. Let us therefore first identify those logical remainders for which the occurrence of SUCCESS has been assumed in order to produce the most parsimonious solution term.[2] The Boolean expression of the 16 simplifying assumption looks as follows.

[1] This has, of course, not escaped the attention of the authors and has been discussed by Stokke (2007) and also by Ragin and Sonnett (2004) and Ragin (2008a: ch. 9).
[2] See the online How-to section for Chapter 6 on how to identify simplifying assumptions (www.cambridge.org/schneider-wagemann).

~A~C (~S~I~R + ~S~IR + S~I~R + S~IR + SIR) +

~AC (~S~I~R + ~S~IR + S~I~R + S~IR + SIR) +

A~C (~S~IR + S~I~R + S~IR) +

AC   (~S~I~R + ~S~IR + S~IR).

The most parsimonious solution for SUCCESS rests on the assumption that 16 out of 24 logical remainders would produce SUCCESS if they were empirically observed. Note that 10 out of the 16 counterfactual claims involve remainders that display the absence of the necessary condition ADVICE (i.e., ~A). It is because of these incoherent assumptions that the necessary condition A is deemed logically redundant and is thus minimized away from the parsimonious solution term. For instance, truth table row 2 (A~C~S~I~R), which is linked to outcome SUCCESS, is logically minimized through combination with the logical remainder ~A~C~S~I~R by dropping condition A to the new statement:

~C~S~I~R → SUCCESS.

Combined with a number of other remainders, this new term can be simplified to ~I and becomes a prime implicant of the most parsimonious solution.[3] This means that every single combination containing ~I either empirically implies the outcome or is assumed to imply it, regardless of whether it logically contradicts the statement that A is necessary for SUCCESS.

This example suggests that the disappearance of necessary conditions from statements of sufficiency is caused by wrong-headed assumptions about logical remainders. In fact, if condition A is necessary for SUCCESS (A ← SUCCESS), then this implies that there cannot be any simultaneous occurrence of ~A and SUCCESS. In other words, whenever we see a configuration containing ~A, we expect the outcome SUCCESS *not* to occur. Assuming that in the presence of ~A outcome SUCCESS occurs – as we do for the most parsimonious solution when including remainders containing ~A – contradicts our conclusion drawn from empirical observation, namely that SUCCESS occurs only when condition A is present, and that the latter should therefore be interpreted as a necessary condition. In section 8.2, we have labeled such assumptions incoherent counterfactuals.

If necessary conditions disappear due to incoherent assumptions on logical remainders, the way to avoid this problem is straightforward: do not make

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[3] A~C~S~I~R and ACS~I~R exist empirically (rows 2 and 6) and ~A~C~S~I~R, ~A~C~S~IR, ~A~CS~I~R, ~A~CS~IR, ~AC~S~I~R, ~AC~S~IR, ~ACS~I~R, ~ACS~IR, A~C~S~IR, A~CS~I~R, A~CS~IR, AC~S~I~R, AC~S~IR, and ACS~IR are part of the simplifying assumptions listed above.

any such incoherent assumptions, i.e., assumptions that contradict the claim that a specific condition is necessary. More specifically, logical remainders containing the absence of the necessary condition must be barred from the logical minimization for outcome Y.

Let us apply this strategy to the example provided by Stokke (2004). If we produce the *enhanced most parsimonious solution term* (section 8.2.4) by blocking all incoherent assumptions,[4] we obtain the following result for outcome SUCCESS from Stokke's data:

	A~I +	ASR → SUCCESS
Consistency	1	1
Raw coverage	0.6	0.4
Unique coverage	0.6	0.4
Solution consistency	1	
Solution coverage	1	

As we see, the exclusion from the process of logical minimization of all those logical remainders that would contradict the statement of necessity produces a solution in which the necessary condition is part of all sufficient paths toward SUCCESS. The danger of overlooking the presence of a necessary condition is averted with the enhanced most parsimonious solution term.

As an alternative strategy against the disappearance of necessary conditions Ragin and Sonnett (2004; see also Ragin 2008a: ch. 9 and Stokke 2004) propose to simply add the necessary condition to each sufficient path *after* the analysis of necessity. This strategy always yields the same solution as our enhanced most parsimonious solution term and, by virtue of this, also avoids the pitfall of incoherent assumptions that contradict the statement of necessity. Note, however, that our strategy of excluding incoherent counterfactuals *before* the analysis of sufficiency has the practical advantage that the software correctly calculates the consistency and coverage values for the resulting solution formula. If necessary conditions are reinserted into a sufficiency solution formula, no correct parameters of fit are calculated.

In sum, the remedy for avoiding the disappearance of necessary conditions due to incoherent assumptions boils down to what in section 8.2.1 we have already introduced as part of our Enhanced Standard Analysis (ESA): during

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[4] See the online How-to section for Chapter 8 for practical advice (www.cambridge.org/schneider-wagemann).

Table 9.2 Truth table with logical contradictions and hidden necessary condition

| Row | Conditions | | | Outcome | | | |
	A	B	C	Y	Cases with Y	Cases with ~Y	Consistency for Y
1	0	0	0	0	0	1	0
2	0	0	1	1	20	0	1
3	0	1	0	0	0	39	0
4	0	1	1	0	0	15	0
5	1	0	0	1	10	0	1
6	1	0	1	1	15	0	1
7	1	1	0	1	4	1	0.8
8	1	1	1	0	0	2	0

the analysis of sufficiency, do not allow any assumptions on remainders that would contradict the statement of necessity. This requires, of course, that the analysis of necessity is performed *prior* to the analysis of sufficiency.

9.1.1.2 Hidden necessary conditions due to inconsistent truth table rows

Are incoherent assumptions about logical remainders the only reason for the disappearance of necessary conditions? Alas, unfortunately not. Necessary conditions can disappear even from the conservative solution term, which, by definition, makes no assumptions at all on logical remainders. This can happen when inconsistent truth table rows are included in the logical mini- mization that contain the *absence* of the necessary condition. Let's have a look at the hypothetical example displayed in Table 9.2.

This truth table does not suffer from limited diversity. Therefore, no sim- plifying assumptions are made on logical remainders. Row 7, however, is a contradictory row: it contains four cases displaying Y and one case with out- come ~Y. The consistency score of AB~C as a sufficient condition for Y is 4/5, or 0.8 (section 5.2). If we accept this score as high enough for a sufficient condition, and thus include ~A~BC in the minimization procedure, we then obtain the conservative solution term for outcome Y:

	A~C +	~BC → Y
Consistency	0.93	1
Raw coverage	0.286	0.714
Unique coverage	0.286	0.714
Solution consistency	1	
Solution coverage	0.98	

Table 9.3 Test of necessity, outcome Y

Condition	Consistency
A	0.59
~A	0.41
B	0.08
~B	0.92
C	0.71
~C	0.29

There are two sufficient paths. Both are highly consistent, and jointly they cover all cases that are members of outcome Y. What is important for our point here is that no single condition appears in both paths. One is therefore tempted to conclude that there is no necessary condition for Y. Is this conclusion correct? In order to find out, let us run a proper test of necessity. This, as explained in section 3.2.1.2, starts with the test of single conditions in isolation. Testing each condition and its complement yields the following consistency scores for being a necessary condition of Y (Table 9.3).

We see that condition ~B is almost fully consistent (0.92) with the statement of being necessary for Y. Out of the 49 members of Y, only 4 are not also members of condition ~B and thus contradict the statement of necessity. Since ~B also has a high coverage score (0.98), we can conclude that it is a relevant necessary condition (section 5.5). Hence, based on the empirical evidence, we have good reasons to consider ~B to be a relevant necessary condition for Y.

Why, then, is ~B not part of all sufficient paths in the conservative solution? Necessary condition ~B is logically minimized from the sufficiency solution by matching row 5 of Table 9.2 (A~B~C) with the inconsistent row 7 (AB~C) into the sufficient path A~C. In this example, thus, the necessary condition disappears from the sufficiency solution because both the former and the latter are not fully consistent. In other words, this is an example of a hidden necessary condition due to inconsistent subset relations.

When necessary conditions disappear due to inconsistent set relations, an imperfect remedy is to increase the raw consistency threshold for truth table rows and/or for necessary conditions. By increasing the raw consistency threshold for truth table rows, it becomes less likely that the necessary condition is minimized out of the sufficiency solution. For instance, if in our example above we had chosen a consistency threshold of 1, truth table row

8 of Table 9.2 would not have been included in the logical minimization and necessary condition ~B would thus not have been logically minimized away.[5] By increasing the consistency threshold for necessity, we make it less likely that any condition is deemed necessary in the first place. Obviously, if no condition is considered as necessary, none can disappear from the sufficiency solution term. If in the same example we had required that a necessary condition be fully consistent, we would not have accepted ~B as a necessary condition, and the sufficiency solution term A~C + ~BC → Y would not be in contradiction to any statement of necessity.

In general, when researchers select consistency thresholds and thus deviate from perfect set relations, they must pay attention to the potential danger of disappearing necessary conditions. At the same time, to always impose very high – if not perfect – consistency thresholds is not a practical one-size-fits-all solution. First, social science data deviate more often than not from perfect set relations. And, second, even with a consistency threshold for necessary conditions well above the suggested 0.9 consistency benchmark, the problem of disappearing necessary conditions might occur, as our example above demonstrated. Condition ~B had a consistency score of 0.92 as a necessary condition for Y, yet it disappeared from one of the two sufficient paths and thus remained invisible as a necessary condition.

In sum, several lessons can be drawn from the fact that necessary conditions might be overlooked when only performing an analysis of sufficiency. First, always perform separate analyses of necessity and sufficiency. Second, unless these two separate analyses are run, do not infer necessity from an analysis of sufficiency and vice versa. Third, of the two reasons for the disappearance of necessary conditions, that stemming from limited diversity is analytically more disturbing: necessary conditions disappear even though this disappearance is not based on any *empirical* counterexamples. In contrast, when necessary conditions disappear from the solution for sufficient conditions due to inconsistency, the hidden necessary conditions themselves are not fully consistent. Nonetheless, since subset relations are usually at least slightly inconsistent in applied set-theoretic methods, this issue is of high practical relevance.

9.1.2 The appearance of false necessary conditions

A related important issue consists of the fallacy of postulating the presence of a necessary condition based on the sufficiency solution formula when, in fact,

[5] Imposing a raw consistency threshold of 1 would yield the solution formula ~BA + ~BC → Y.

there is no necessary condition. A good example for discussing this intricate phenomenon is provided by Vis (2009), which we have already used in sections 5.3 and 8.2.1.[6] She aims at explaining the conditions under which governments pursue unpopular social policies (U). As analytically relevant, Vis identifies three conditions: a weak political position (P); a weak socioeconomic situation (S); and a rightist government (R). The 25 cases (Table 9.4) – cabinets in various Western European countries during the 1970s to 1990s – fall into 5 different truth table rows, leaving the 3 remaining rows as logical remainders (Table 9.5).

We use a consistency threshold of 0.85, thus including rows 3 and 4 in the logical minimization procedure. The conservative solution term (i.e., with no assumptions on logical remainders) is:

	PS~R +	~PSR → U
Consistency	1	0.857
Raw coverage	0.154	0.462
Unique coverage	0.154	0.462
Solution consistency	0.889	
Solution coverage	0.615	

Let us disregard the coverage scores and focus only on the consistency values. There are two highly consistent paths toward outcome U, and the overall solution term is also highly consistent. We note that both paths contain condition S. This seems to suggest that the outcome U would not occur without condition S being present. If so, why should we not declare S a necessary condition for U, as Vis (2009: 31ff., 48), based on her fuzzy-set data, does?

The direct test of necessity reveals the following surprise: the consistency of S as a necessary condition for U is a meager 0.77 – far too low to be considered a necessary condition (Schneider and Wagemann 2010; see also section 5.4, above). Indeed, looking at the data (Table 9.4), we see that in three cases (the governments of N. Rasmussen IV, Thatcher II, and Kok I) unpopular reforms were implemented in the absence of the alleged necessary condition S, a clear contradiction to the claim of necessity. Since 3 out of a total of 13 relevant cases (i.e., cases with membership in outcome U) do not support it, the claim of necessity of S needs to be rejected.

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[6] For didactic purposes, we are using a dichotomized version of Vis' (2009) original fuzzy-set data. Note, though, that the same problem of suspecting a necessary condition when there is none also occurs with the original fuzzy-set data and is not an artifact of our recoding the data into crisp sets.

Table 9.4 Crisp-set membership scores (Vis 2009)

Government	Conditions			Outcome
	P	S	R	U
Lubbers I	0	1	1	1
Lubbers II	0	0	1	0
Lubbers III	0	1	1	1
Kok I	0	0	0	1
Kok II	0	0	0	0
Balkenende II	1	1	1	1
Kohl I	0	0	1	0
Kohl II	0	0	1	0
Kohl III	0	0	1	0
Kohl IV	1	1	1	1
Schröder I	0	0	0	0
Schröder II	1	1	0	1
Schlüter I	0	0	1	0
Schlüter II	0	1	1	1
Schlüter IV	0	1	1	0
Schlüter V	1	1	1	0
N. Rasmussen I	0	0	0	0
N. Rasmussen II (and III)	1	1	0	1
N. Rasmussen IV	0	0	0	1
Thatcher I	0	1	1	1
Thatcher II	0	0	1	1
Thatcher III	0	1	1	1
Major I	0	1	1	1
Blair I	0	0	0	0
Blair II	0	0	0	0

Why, however, is S included in all sufficient conjunctions for U such that it seems to be a necessary condition? The answer to this question lies in the fact that some truth table rows are inconsistent. In our example, the three governments that are members of U but do *not* show the false necessary condition S, and which therefore lower the consistency score of S as a necessary condition, are all part of those truth table rows (rows 1 and 2) that are not included in the logical minimization, due to their low raw consistency values. It is merely a coincidence that all those truth table rows that are included in the logical minimization contain condition S.

In sum, the problem of false necessary conditions depends on particular properties of not fully specified truth tables. From this it does not follow that,

Table 9.5 Truth table, outcome U (Vis 2009)

| Row | Conditions | | | Outcome | | | | |
	P	S	R	U	N	Consistency for U	Cases
1	0	0	0	0	7	0.29	Kok I, II;Schröder I; N. Rasmussen I, IV; Blair I, II
2	0	0	1	0	6	0.17	Lubbers II; Kohl I, II, III; Schlüter I; Thatcher II
3	0	1	1	1	7	0.86	Lubbers I, III; Schlüter II, IV; Thatcher I, III; Major I
4	1	1	0	1	2	1.00	Schröder II; N. Rasmussen II/III
5	1	1	1	0	3	0.67	Balkenende II; Kohl IV; Schlüter V
6	0	1	0	?	0	Logical remainder	–
7	1	0	0	?	0	Logical remainder	–
8	1	0	1	?	0	Logical remainder	–

in the presence of inconsistent rows, false necessary conditions always appear. Nor does it mean that false necessary conditions appear only in the presence of inconsistent truth table rows. To illustrate the latter point, imagine we had imposed a consistency threshold of 1 for truth table rows in Table 9.5. This would have yielded the following conservative solution term:

$$PS{\sim}R \rightarrow U.$$

There is just one path toward outcome U, corresponding to row 4 in Table 9.5. From this, one might want to infer that PS~R is also necessary for U, but this would obviously be wrong. The consistency score of PS~R as a necessary condition for U is a dismal 0.15.[7] The reasons for this low value is again that many cases that are members of U are located in truth table rows that are not fully consistent with sufficiency, and which therefore have not been included in the logical minimization. Thus, even if all sufficient paths and the entire solution term for sufficiency are fully consistent, false necessary conditions can appear. For this to happen, the entire solution term (and thus at least one path) must have a coverage of less than 1. This is because less-than-perfect coverage means that there are cases in the data that are not members of the

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[7] For each single condition, the consistency scores as necessary conditions for U are: P (0.31), S (0.77), ~R (0.31).

sufficiency solution but are members of the outcome. If those unexplained members of Y happen not to be members of the alleged necessary condition, then this condition ceases to be necessary.

The strategy for avoiding the analytic pitfall of declaring conditions as necessary when, in fact, they are not, is to simply perform a separate test of necessity prior to the analysis of sufficiency. Applying the strategy to the example by Vis (2009), one would first perform a test of necessity and conclude from this that no condition is necessary for the outcome. Then, when confronted with the solution term PS~R + ~PSR → U, or with only PS~R → U, one simply does *not* interpret any condition as necessary. At this point, it would perhaps also be wise for the researcher to explicitly name those cases that contradict the statement that condition S is necessary for U (the governments of N. Rasmussen IV, Thatcher II, and Kok I, see Table 9.4).

In sum, necessary conditions are the set-theoretic mirror image of sufficient conditions. In principle, i.e., in an ideal world that neatly fits into subset relations (with no inconsistencies) and into fully specified truth tables (with no logical remainders), it would therefore be possible to correctly derive the presence or absence of necessary conditions by simply performing an analysis of sufficiency. However, social scientists hardly ever encounter an ideal world. Sets are often not in perfect subset relations, and truth tables lack information for a wide number of reasons that are beyond the control of the researcher. Because of this lack of perfect information and the strategies used to handle this lack – assumptions about logical remainders and using the notion of consistency – necessary and sufficient conditions cease to be perfect mirror images of each other. In practical terms, this implies that necessary conditions cannot be simply identified by performing an analysis of sufficiency, or vice versa. Instead, separate analyses of necessity and sufficiency must be performed in order to avoid the pitfalls of hidden or false necessary conditions.

At-a-glance: pitfalls in inferring necessity from sufficiency solution terms

When analyzing **necessary** and **sufficient conditions**, researchers are often tempted to derive the necessary conditions from their analysis of sufficient conditions. This, however, may lead to two problems.

First, a condition which has been identified as necessary may not be visible in all paths of the analysis of sufficient conditions (a **hidden necessary condition**), due to the inclusion in the **logical minimization** either of remainder rows which contradict the statement of necessity or of not fully consistent **truth table rows**.

> Second, a condition can appear in all sufficient paths, even though it is not a necessary condition (a **false necessary condition**). This might happen if only those rows which include the false necessary condition are included in the logical minimization.
>
> These pitfalls can be avoided if necessary and sufficient conditions are analyzed in two separate steps, with the necessary conditions analyzed first. Furthermore, it is useful to choose rather high **consistency** levels in the assessment of necessary conditions and to make sure not to allow any **incoherent assumptions** about **logical remainders**.

9.2 The analytic consequences of skewed set-membership scores

So far, in this chapter we have presented various scenarios in which the inference about the presence or absence of necessary conditions might be flawed. In this section, we add another potential source of flawed inference about set relations. What the following pitfalls have in common is that they are based in one way or another on skewed distributions of membership in either the condition(s), or the outcome, or both. Membership in a set is skewed if the large majority of cases holds high or low membership in the set. Unlike the previous sources, this problem is not restricted to inference about necessity, but also has an equally distorting impact on inferences about sufficiency.

In the literature, two issues that are intimately linked to aspects of skewed distributions have already been discussed without, however, making reference to each other. One of these debates addresses the notion of trivial necessary conditions. Parameters have been developed that aim at distinguishing trivial from non-trivial necessary conditions (Braumoeller and Goertz 2000; Dion 2003; Goertz 2003; Mahoney 2004; Ragin 2008a). Below, we discuss the notion of trivialness and suggest an updated version for calculating this parameter of fit (9.2.1). Second, and less well discussed,[8] are the consequences of a perplexing characteristic of fuzzy sets. The same set X can be a subset of both set Y and its complement ~Y, meaning that X can pass the test of sufficiency both for Y and ~Y. Furthermore, the same set Y can be a superset of both set X and its complement ~X, meaning that both X and ~X pass the test of sufficiency for Y. This is possible because in fuzzy sets the Rule of the Excluded Middle does not hold (section 2.4.1). Combined with skewed membership scores, this property of fuzzy sets provides fertile grounds for drawing wrong inferences about sufficient and necessary conditions. In the following, we first turn to the notion of trivialness, then discuss the practical

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[8] For an exception, see Ragin (2008a: 137f.).

implications of the absence of the Rule of the Excluded Middle for the analysis of sufficiency, and finally engage in a broader reflection on the consequences of skewed distributions for set-theoretic data analysis techniques. In other words, we start with two special phenomena that are occasionally discussed in the literature, but we then integrate these debates into the broader frame of skewed set membership.

9.2.1 The coverage of necessary conditions and the problem of trivialness

9.2.1.1 The two sources of trivialness

In section 5.5, we presented trivialness as a possible interpretation of the coverage value of a necessary condition. There, we alluded to the fact that there are two sources for the trivialness of a necessary condition rather than just one – X can be much bigger than Y and X can be close to a constant – and that the current formula for calculating relevance (Ragin 2006) adequately captures only the former of these two sources.

As a demonstration, we return to our example from section 5.5. Let Y_1 be the set of speech acts in parliament during which members of parliament curse. X_1 is the set of male members of parliament and X_2 the set of parliamentarians born in the country. To these, we add another outcome: speech acts during which reference to government policies is made (Y_2). The three Venn diagrams in Figure 9.1 display three empirical patterns. In all three, the condition is a fully consistent superset of the outcome and thus easily passes the formal test for necessity (aka their consistency score is 1).

If we apply the standard formula for the relevance of a necessary condition, we obtain the following results. First, X_1 would obtain a *high* value and thus be deemed a relevant necessary condition for Y_1. Second, X_2 would obtain a *low* value and thus be deemed a trivial necessary condition for Y_1. Both of these results are plausible. The problem occurs when we turn to the assessment of necessity of X_2 for Y_2. Both sets are of roughly equal size. Hence, the standard relevance formula yields a *high* value. This, in turn, might lead researchers to infer that being born in the country (X_2) is a *relevant* necessary condition for addressing government policies in parliamentary speeches (Y_2). This inference is problematic, though. In this research context (speech acts in parliament), X_2 (the set of members of parliament born in the country) is almost a constant. It is therefore, by default, a superset of whatever the outcome set is – any other set could scarcely avoid being a subset of X_2. Hence, by being close to a constant, X_2 automatically passes the formal requirement of being classified as a necessary condition for whatever the outcome set is. It does so
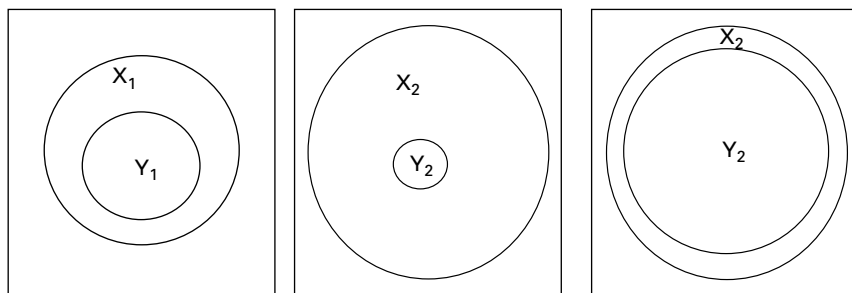
Figure 9.1   Venn diagram – different sources of trivialness necessity

not by virtue of its substantive or causal relevance but by its empirical distribution. The reason why this clear trivialness is not detected by the standard relevance formula is that in addition outcome set Y is very big.

This example reveals that in order to establish the relevance or trivialness of a necessary condition, two pieces of information have to be taken into account: the relation in size of sets X and Y, and the relation in size of X and ~X. If X is so much bigger than ~X, then it is very easy for X to be a superset of whatever the outcome set Y consists of. This should make clear that the issue of trivialness is linked to the more general topic of skewed membership distributions (section 9.2.3). Saying that X is much bigger than ~X is nothing more than saying that the membership of cases in X is skewed toward high membership in X.

Ragin's formula for the relevance of necessary conditions (section 5.5) assesses correctly the relation in size between X and Y, but is insensitive to the relation in size between X and ~X. Arithmetically, if set membership in X is always 1 (the most extreme case of skewedness), the denominator of the relevance formula is equal to the number of cases under investigation. The minima of X and Y which are used in the numerator all correspond to the Y value (since X is always 1). In other words, the formula is equal to the arithmetic mean of all Y values. Now, if Y itself almost exclusively contains cases with high membership scores (such as in the example above of speech acts with references to government policies), then this arithmetic mean, and with it the coverage value, will be high.

In sum, the coverage parameter as proposed by Ragin (2006) correctly captures trivialness understood as the relation between the size of the sets X and Y. The formula does not, however, account for trivialness that is due to the necessary condition being (close to) a constant. What, then, could a formula look like that captures both sources of trivialness?
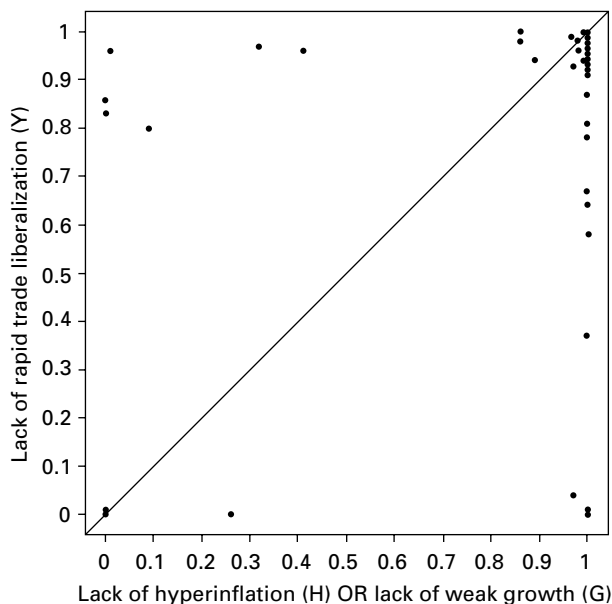
Figure 9.2   XY plot – trivial necessary condition

9.2.1.2 Suggestions for an updated formula for trivialness necessity

For demonstrational purposes, take Samford's (2010) study on trade liberal-
ization in Latin America. Let the set of countries without rapid trade liberal-
ization be the outcome to be explained (Y). Furthermore, let us argue that two
of his seven conditions form a functionally equivalent necessary condition
(section 3.2.1.2): the lack of hyperinflation (H) or the lack of weak growth
(G). Put differently, all governments in Latin America that abstain from rapid
trade liberalization are either not confronted with hyperinflation, or not con-
fronted with weak growth, or neither. Formally:

$$H + G \leftarrow Y.$$

Empirically, this claim seems to be supported by the high consistency (0.9)
and coverage (0.87) scores for condition H + G. A look at the XY plot of Y and
H + G (Figure 9.2), however, should trigger some doubts. We notice that both
the sets of Y and of H + G are skewed toward high membership. This can be
seen from the fact that most cases cluster in the upper right corner of the XY
plot. This pattern corresponds to the Venn diagram for X_2 and Y_2 in Figure
9.1. Thus, H + G is trivially necessary despite its high coverage value.

Two proposals have been made in order to capture trivialness triggered
by skewed set membership, which, as we have stated, goes unnoticed by the

standard formula for relevance necessity. First, Goertz suggests "that the trivialness of X be defined as the average distance between x_i and 1, standardized by the maximum importance that this value can attain based on y_i" (Goertz 2006a: 95). Based on this reasoning, he arrives at the following formula for trivialness:[9]

$$\text{Trivialness of Necessity} \mid (\text{Goertz}) = \frac{1}{N}\sum \frac{(1-x_i)}{(1-y_i)}.$$

If we apply Goertz's formula to our necessary condition H + G, a major shortcoming is revealed. For H + G, it returns the value of 1.58. Th is is disturbing for two reasons. First, high values are supposed to indicate non-trivialness. Since we already know from looking at the XY plot that H + G is trivially necessary, this high value is misleading, if not wrong. Second, the value is higher than 1, suggesting that, unlike the case for all other parameters of fit introduced so far, there is no upper limit for Goertz's trivialness measure. Th is increases the diffi culty of interpreting this parameter.

Th e reason for both problems is that this formula is unduly sensitive to inconsistency. With only minor deviations from perfect consistency (such as in our example with 0.9), it might return values that are greater than 1 and indicate non-trivialness when, in fact, a condition qualifies as trivial.

Goertz's formula yields more plausible results than Ragin's when applied to (close to) constant conditions. In fact, for a perfectly constant condition – which, by default, will also be perfectly consistent – Goertz's formula returns the value of 0,[10] while Ragin's would not always. Goertz's formula returns implausible values in the presence of inconsistency, though. In response to this, we suggest the following formula for the assessment of trivialness:

$$\text{Relevance of Necessity (Schneider and Wagemann)} = \frac{\sum (1-x_i)}{\sum (1-\min(x_i, y_i))}.$$

Th is parameter aims at including the positive features of both Ragin's and Goertz's formulas while avoiding the pitfalls of each. Like Ragin's parameter, it is not affected too much by inconsistent cases.[11] Like Goertz's, our parameter

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[9] Naming this formula "trivialness" is prone to contribute to confusion, because *low* values indicate trivialness and high values non-trivialness.

[10] Th is is because in this scenario the numerator is 0 for all fractions of the sum.

[11] Th is is achieved both by using the minima instead of y values and by first summing up and then dividing (like Ragin's formula) instead of first dividing and then summing up (Goertz). Incidentally,

takes into account whether X is (close to) a constant. If X is constant, our formula returns the value of 0. At the same time, our formula can never return values higher than 1.

Applied to condition H + G, our formula yields a value of 0.56. This value is lower than Ragin's and thus more correctly suggests that condition H + G is trivial and not relevant as a necessary condition for rapid trade liberalization (Y).

We think our parameter can be deemed a valid assessment of the relevance of a necessary condition. Low values indicate trivialness and high values relevance.[12] Hence, researchers should substantively interpret only those conditions that pass the consistency test and receive high values on our formula.

As we have suggested, one might assume that researchers will simply know if there are many more instances of the alleged necessary condition than of the outcome, and thus that a necessary condition is trivial. There are situations, however, when the size of the sets of X and Y are less visible. This usually is the case when applying QCA to larger data sets and/or when using fuzzy sets. In addition, the identification of trivial necessary conditions by pure eyeballing is particularly difficult when the notion of functional equivalents, in the form of SUIN conditions (Mahoney *et al.* 2009), is invoked. All this points to the need to use parameters that effectively reveal trivialness and relevance when making statements about necessary conditions.

9.2.2 The consistency of sufficient conditions and the problem of simultaneous subset relations

In the previous section, flawed inferences about necessity could occur because the set of condition X was very big. As we now show, similar problems can occur in the analysis of sufficiency. If X is very small, then it can pass the formal

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this latter feature reveals another potential shortcoming of Goertz's formula for trivialness: it might lead to the mathematically undefined situation of a division by zero. This did not go unnoticed by Goertz himself, who proposed fixing the value of the fraction at 1 (Goertz 2006a: 108). However, when making this proposal, full consistency is consumed where X = 1 and Y = 1. The problem of a value of 0 in the denominator of his formula can also occur in case of inconsistency (e.g., X = 0.9 and Y = 1), though.

[12] Goertz (2006a), instead, separates trivialness from relevance and suggests calculating both with different formulas. For him, the relevance of X as a necessary condition is indicated by how consistent X is as a sufficient condition (Goertz 2006a: 91). The formula is: Relevance of Necessity (Goertz) = $(1/N) \Sigma (y_i/x_i)$ (Goertz 2006a: 96). In addition, Goertz proposes the notion of the importance of a necessary condition and calculating it by taking the arithmetic mean of his relevance and the trivialness parameters (2006a: 98). Our use of the term "relevance" of a necessary condition makes it a synonym of "non-trivialness" rather than "also sufficient," as suggested by Goertz (2006a).

Table 9.6 Simultaneous consistent subset relation of X with both Y and ~Y

Case	Condition	Outcome		Subset relation	
	X	Y	~Y	X<Y	X<~Y
A	0.1	0.8	0.2	Yes	Yes
B	0.2	0.4	0.6	Yes	Yes
C	0.3	0.3	0.7	Yes	Yes
D	0	0.9	0.1	Yes	Yes
E	0.2	0.7	0.3	Yes	Yes

test of being sufficient both for Y and ~Y. Clearly, claiming that one and the same condition is sufficient for an outcome and its complement amounts to a logical fallacy. In the following section, we first provide a simple example that demonstrates the simultaneous subset relation of X with both Y and ~Y and explain the circumstances under which this phenomenon occurs. After this, we present various strategies for confronting simultaneous subset relations in fsQCA, using published data for illustration.

9.2.2.1 Sources of simultaneous subset relations

A fuzzy set X can be a perfectly consistent subset of both fuzzy set Y and its logical complement ~Y. For illustration, consider the situation displayed in Table 9.6. All cases have a membership in X[13] that is smaller than their respective membership in Y. X, thus, is a subset of Y and could thus be considered as a fully consistent sufficient condition for Y. At the same time, and perhaps surprisingly, each case's membership in X is also smaller than its membership in ~Y. Hence, X is also a subset of ~Y and thus passes the test as a fully consistent sufficient condition for ~Y.

The phenomenon of simultaneous subset relations to be discussed now occurs only with fuzzy sets – unlike the occurrence of misleading values for the assessment of relevance necessity due to skewed set membership discussed above (9.2.1), which affects both crisp and fuzzy sets. Simultaneous subset relations cannot occur in crisp sets, because if X is a subset of Y, then all elements in X are also elements of Y. Now, if (some) elements of X also were elements of ~Y, then the set of Y and of ~Y would partially overlap. This, however, is impossible with crisp sets. This has become known as the principle of the excluded middle. With fuzzy sets, however, the Rule of the Excluded Middle

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[13] Notice that X can be a placeholder for a complex Boolean statement, such as, for instance, a truth table row involving several single conditions combined with a logical AND.

does not hold (section 2.4.1). As a consequence, and as shown in Table 9.6, cases can have membership scores in X that are smaller than their values in Y and ~Y, respectively. In substantive terms, it is, of course, nonsense to claim that the same condition is sufficient for producing both the outcome and its non-occurrence.[14] Such inferences must be avoided.[15] In the following, we show how this can be done.

For developing our argument, it is crucial to distinguish between different forms of simultaneous subset relations. Notice that in the example in Table 9.6, all membership values in X are smaller than 0.5. According to the current principles in fsQCA, X would be deemed a logical remainder, for there is not a single case that is more in than out of the set of X (section 6.1). In other words, if membership in a fuzzy set X is so highly skewed toward non-membership that no (or not enough) cases display a membership higher than 0.5 in X, then X is treated as a logical remainder and the problem of fully consistent simultaneous subset relations seizes to exist.[16]

In applied fsQCA, it frequently happens, though, that a condition X is not a logical remainder and is still a simultaneous subset of both Y and ~Y. The only difference to the situation depicted in Table 9.6 is that then simultaneous subset relations of non-remainders can never be fully consistent. Yet, they easily can be – and in applied fsQCA, they often are – well above any reasonable thresholds for consistency of a sufficient condition. The problem of simultaneous subset relations is thus a real one.

For purposes of illustration, take the slightly expanded version of the data from Table 9.6 as displayed in Table 9.7. To cases A–E, we add case F with X = 0.7 and Y = 0.6. Condition X is not a logical remainder anymore because F's membership is above the 0.5 qualitative anchor. Case F's membership in X, however, exceeds its membership both in Y and in ~Y. Case F therefore contradicts the statement that X is sufficient for Y and also the statement that X is sufficient for ~Y. If we take the information about all cases into account and calculate the consistency scores of X as a sufficient condition for Y and ~Y, respectively, we discover that X passes reasonable thresholds in both. Its

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[14] Single conditions can play a causal role in producing Y and ~Y only as INUS conditions (section 3.3.2), a phenomenon known as multifinality.

[15] As shown in section 8.2.2, a similar problem can occur when making contradictory assumptions about logical remainders. In both scenarios, the same truth table row is included in the logical minimization for outcome Y and outcome ~Y.

[16] If X stands for a truth table row, we might still decide to include X into the logical minimization procedure, i.e., we might use X for counterfactual claims (section 6.4). If so, however, we are allowed to do so either in the analysis of Y or of ~Y, not in both. Otherwise, we make a contradictory assumption (section 8.2.2). Furthermore, consistency does not usually play any role when selecting remainders for counterfactual claims.

Table 9.7 Simultaneous inconsistent subset relation of X with both Y and ~Y

Case	X	Y	~Y	X<Y	X<~Y
A	0.1	0.8	0.2	Yes	Yes
B	0.2	0.4	0.6	Yes	Yes
C	0.3	0.3	0.7	Yes	Yes
D	0	0.9	0.1	Yes	Yes
E	0.2	0.7	0.3	Yes	Yes
F	0.7	0.6	0.4	No	No

consistency for Y is 0.93 and for ~Y it is 0.8. So even if X is not a perfectly consistent subset, it still passes conventional tests of sufficiency for both Y and ~Y.

Notice, though, that inconsistent simultaneous subset relations always imply the presence of at least one true logically contradictory case (section 5.2).[17] For sufficiency, these are those inconsistent cases (i.e., X > Y) that have membership in X > 0.5 and in Y < 0.5. In other words, if X is an inconsistent subset both for Y and ~Y, then there must be a true logically contradictory case hidden behind at least one of the two consistency scores. Above and below (sections 5.2, 7.2, 11.1.6, and 11.4), we argue that researchers should capitalize on this characteristic of fuzzy sets when deciding which of the two subset relations they interpret in terms of sufficiency.

In sum, in fuzzy sets, a condition with most cases having low membership scores (skewed distribution) can be a simultaneous subset of outcome Y and outcome ~Y. Such simultaneous subset relations are usually not fully consistent (unless X is a logical remainder), yet can pass conventional thresholds of consistency. Since the statement that a given condition in and of itself is sufficient for both Y and ~Y must be avoided, and given that the standard toolset for identifying sufficient conditions does not automatically capture simultaneous subset relations, additional strategies need to be developed.

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[17] Remember that consistency is defined as the sum of the minima of the condition and outcome values, divided by the sum of the values of the condition. If X is not a remainder (i.e., X > 0.5 in at least one case), then X is automatically greater than Y or 1 − Y (= ~Y) (or both) in those cases for which X > 0.5. This means that the consistency value for Y or ~Y (or both) cannot be 1 anymore. Depending on how distant X is from Y (or ~Y) for the inconsistent case(s), this reduction of the consistency can vary: if both values are close to 0.5, then the consistency value will still remain high, perhaps even high enough to suggest Y as a consistent sufficient condition. Furthermore, if one case with X > 0.5 exists, such case *must* have a value of < 0.5 in either Y or ~Y, and therefore at least one truly inconsistent case exists for either Y or ~Y.

Table 9.8 Consistency of truth table rows for outcome and its complement

	Conditions			Consistency with outcome	
Row	P	S	R	U	~U
1	1	1	1	0.911	0.836
2	0	1	1	0.918	0.706
3	1	1	0	0.911	0.696
4	0	0	1	0.719	0.911
5	0	0	0	0.642	0.829
6–8	Logical remainders			–	–

9.2.2.2 Strategies for handling simultaneous subset relations

For demonstrating potential strategies, and also to provide further evidence that simultaneous subset relations are a more common problem in applied fsQCA than is currently recognized, we return to the example of Vis (2009) and her study on the conditions for the implementation of unpopular reforms (sections 5.3 and 8.2.1). Table 9.8 displays the truth table of her data. Out of the eight rows, three are logical remainders and are not displayed here, for they are not central to the problem of misguided inferences about simultaneous subset relations.

The table also displays each relevant truth table row's consistency as a sufficient condition for outcome U (unpopular reform) and ~U (no unpopular reform). If we apply the reasonable threshold of raw consistency of 0.8, we find that row 1 (PSR) passes the threshold for sufficiency for both U and ~U. In essence, the conjunction of characteristics of a cabinet being in a weak political situation (P), in a weakening socio-economic situation (S), and being dominated by parties on the right (R) formally qualifies as being sufficient for such cabinets to implement unpopular reforms but also not to implement such reforms. This, of course, amounts to a contradictory and thus untenable claim. PSR can be interpreted as sufficient either for U or for ~U or for neither, but certainly not for both.

The XY plots for condition PSR and outcomes U and ~U, respectively (Figure 9.3), provide graphical evidence that PSR could be considered sufficient for U and ~U. Furthermore, the plots show what we have already alluded to above (see especially note 17), namely that with simultaneous subset relations at least one relation *must* contain at least one true logically contradictory case. In our example, the analysis of U contains one such case: the Schlüter IV government (PSR = 0.6; U = 0.33). The analysis of ~U contains two of them: Balkenende II (PSR = 0.67; ~U = 0.17) and Kohl IV (PSR = 0.67; ~U = 0.33).
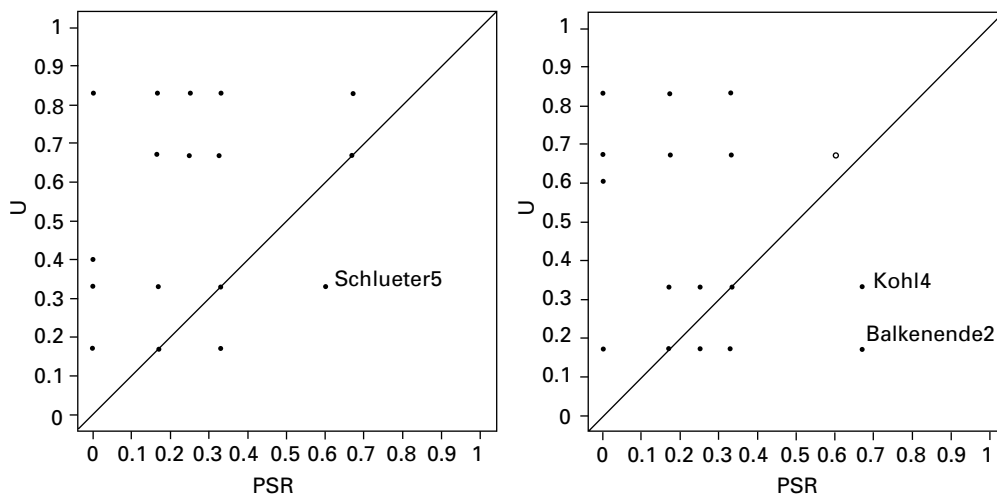
Figure 9.3   XY plots for condition PSR and outcomes U and ~U

Th e first, and so far only, scholar to have taken up the task of tackling the analytic troubles caused by simultaneous subset relations is Ragin.[18] He proposes a formula that uses the notion of a proportional reduction in error akin to many statistical measures of associations.[19] The so-called PRI measure,[20] which is also reported by default when using the Truth Table Algorithm in the fsQCA 2.5 soft ware package, provides a numerical measure of (roughly speaking) how much it helps to know that a given X is specifically a subset of Y and not a subset of ~Y. The formula for calculating PRI is as follows:

$$\text{PRI} = \frac{\sum \min(X,Y) - \sum \min(X,Y,\sim Y)}{\sum \min(X) - \sum \min(X,Y,\sim Y)}.$$

PRI is sensitive to X being a subset of both Y and ~Y. In general, if the consistency scores for Y and ~Y are close to each other, then PRI yields lower scores. If the consistency scores for Y and ~Y are very different, then PRI yields high scores. Th is holds true irrespective of the level of consistency reached by X as a sufficient condition for Y or ~Y. Thus, it can happen that PRI is high

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[18] Th e following arguments rest on personal exchanges with Charles Ragin, who generously shared his as yet unpublished thoughts on this intricate issue with us (see also Mendel and Ragin 2011).

[19] Specifically, it corresponds to the Proportional Reduction in Error (PRE) calculation, which is the principle of most statistical measures of association. The well-known R^2 of regression analysis is nothing more than the PRE, measuring how the error is reduced by the introduction of the independent variable(s), as compared to a situation in which the values of the dependent variable are estimated by their arithmetic mean.

[20] PRI stands for proportional reduction in inconsistency (Mendel and Ragin 2011: 38).

Table 9.9 Consistency, PRI, and PRODUCT for simultaneous subset relation

Row	Outcome U			Outcome ~U		
	Consistency	PRI	PRODUCT	Consistency	PRI	PRODUCT
PSR	0.911	0.647	0.589	0.836	0.353	0.295

even if X is not consistent enough as a sufficient condition for either Y nor ~Y. This should make clear that PRI *cannot* substitute the conventional consistency measure. Instead, both measures need to be combined. Ragin proposes multiplying the consistency measure and PRI. In the fsQCA 2.5 software, this measure is labeled PRODUCT in the Truth Table Algorithm. If both the consistency and the PRI measure are high, then PRODUCT will also be high. Conjunctions with a high PRODUCT value are those for which there is a clear non-simultaneous subset relation, and there is no problem in interpreting them as sufficient conditions for the outcome of interest.

Applying the PRI measure to row 1 (PSR) in Table 9.8, we obtain the values displayed in Table 9.9. While the standard consistency measure suggests that PSR can be interpreted as sufficient for both U and ~U, the PRI measure now says that by taking into account the simultaneous subset relation, PSR should be considered as a sufficient condition only for U, and not for ~U. The value of PRI (and thus also PRODUCT) is high for the analysis of U and low for the analysis of ~U.[21]

We deem the PRI and PRODUCT parameters to be important improvements of applied fsQCA. They are calculated by some of the relevant packages, and making use of them comes at no cost, producing potentially huge gains. Researchers should make use of them even if they are not interested in analyzing the non-occurrence of the outcome. PRI and PRODUCT help to identify, and potentially reject as sufficient conditions, those sets that pass the consistency threshold only because they are so small, i.e., because their membership scores are highly skewed.

PRI, just like consistency, is a continuous measure. This, of course, raises the question of where to put the threshold above which PRI (and PRODUCT) are high enough to consider the condition under consideration to be a sufficient condition. In the following, we provide some clues. First of all, interpreting PRI and PRODUCT only makes sense, of course, for those conditions that have passed the consistency threshold in the first place. In addition, we

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[21] This inference is also supported by the fact that for outcome ~U there are two true logically contradictory cases while for outcome ~U there is only one (Figure 9.3).

encourage readers to always inquire which of the two consistency values (for Y and for ~Y) are driven by true logically contradictory cases. Researchers could use the presence of logically contradictory cases as an argument for ruling out as evidence of sufficiency one of the two subset relations of X with Y and ~Y.

In sum, researchers should combine considerations of PRI and PRODUCT with a more case-oriented focus and take true logically contradictory cases head-on. Researchers should also be aware that skewed membership scores in conditions are a frequent phenomenon when using the Truth Table Algorithm (Chapter 7). The reason is simple. In the Truth Table Algorithm, each truth table row is tested for its consistency as a sufficient condition for the outcome. As shown above (section 4.2 and Chapter 7), most cases have low membership in most truth table rows. In other words, most truth table rows denote small sets. This might occur even when each individual set in the analysis is not skewed at all. Thus, the problem of skewedness might be hidden from the eyes of the researcher who believes him- or herself to be on the safe side because no single condition is skewed.

9.2.3 A general treatment of skewed set membership in fuzzy-set analyses

The deeper, underlying issue in all the potential pitfalls addressed so far in this section is the impact of skewed distributions of set membership scores. An extensive treatment of all the potential pitfalls, and their potential cures, is a still-pending task in the set-theoretic literature – and one which goes well beyond the scope and aims of this book. Our following overview is intended simply to create a sensitivity to this topic and to outline potential avenues for further work on this issue.

In order to visualize our argument, we use a modified version of an XY plot (Figure 9.4). In addition to the usual diagonal running from the bottom left to the top right, we add another diagonal running from the top left to the lower right. Just as the commonly plotted diagonal denotes where X = Y, the new diagonal denotes where X = ~Y. By using two diagonals, we can therefore visualize subset relations for Y (standard main diagonal) and ~Y (new diagonal) (see also Cooper and Glaesser 2011a). For instance, if X is a subset of ~Y, all cases fall below the new diagonal. If X is necessary for ~Y, then all cases fall above the new diagonal.

With two diagonals, four different areas are formed. In the following, we spell out the subset relations that one would infer if all cases fell into only one or two of the four areas. We specify these areas for all the eight logically
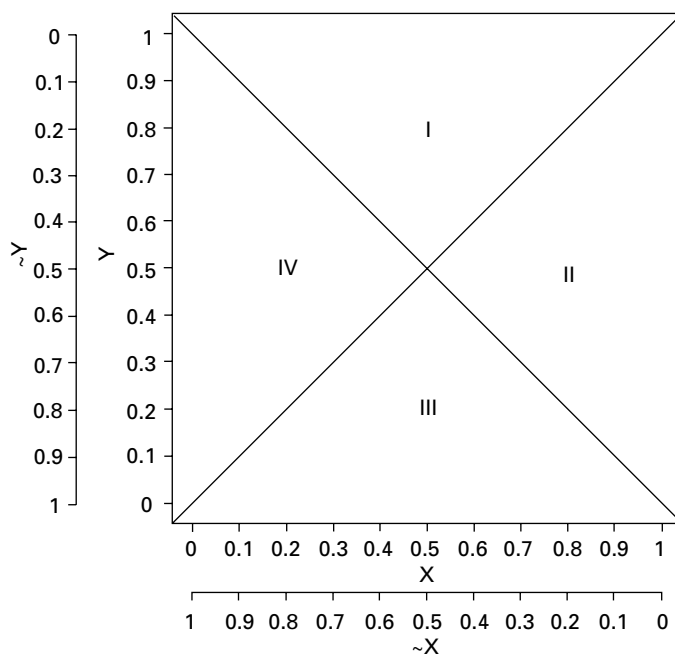
Figure 9.4   XY plot – four areas and eight potential subset relations

possible set relations between conditions X and ~X, on the one hand, and outcomes Y and ~Y, on the other:

1. All cases in areas I and/or IV: $X \rightarrow Y$.
2. All cases in areas II and/or III: $X \leftarrow Y$.

   If X is sufficient for Y, then this means that $X \leq Y$ for all cases. We can negate this statement and obtain $\sim X \geq \sim Y$. Hence, if all cases fall in the areas I and IV, this not only means that X is sufficient for Y ($X \leq Y$), but also that ~X is necessary for ~Y ($\sim X \geq \sim Y$). Similarly, an area which denotes necessity of X for Y ($X \geq Y$) also describes the subset relation $\sim X \leq \sim Y$. We can therefore say:

3. All cases in areas I and/or IV: $\sim X \leftarrow \sim Y$.
4. All cases in areas II and/or III: $\sim X \rightarrow \sim Y$.

   As mentioned above, if X is a subset of ~Y, all cases fall below the new diagonal. Similarly, if ~Y is a subset of X, then all cases fall above the new diagonal. Hence:

5. All cases in areas III and/or IV: $X \rightarrow \sim Y$.
6. All cases in areas I and/or II: $X \leftarrow \sim Y$.
    We can again negate statements 5 and 6 and get the following:
7. All cases in areas III and/or IV: $\sim X \leftarrow Y$.
8. All cases in areas I and/or II: $\sim X \rightarrow Y$.

As we see, in order to denote sufficiency and necessity, *two* areas of the XY plot are always examined. Note that, through the expression "and/or," we have indicated that the cases can fall into both areas or also into just one of them – what matters is that they cannot fall into any other area in order to be fully consistent set relations.

Now, consider the consequences if all cases fall into just *one* of the four areas. We are then confronted with an interesting phenomenon. In order to explain, let us start with area I. If all cases fall into area I, then – based on our list from above – the following statements are true:

All cases in area I: $X \rightarrow Y$ & $\sim X \leftarrow \sim Y$ & $X \leftarrow \sim Y$ & $\sim X \rightarrow Y$.

We see that, with all cases in area I, both X and ~X are sufficient for Y and necessary for ~Y. This clearly is a logical contradiction. It occurs when membership in Y is skewed towards high membership. The stronger the skewedness in Y towards high membership, the more membership in X can vary for this phenomenon to still occur.

If all cases only fall into area II, then the following holds:

All cases in area II: $\sim X \rightarrow Y$ & $\sim X \rightarrow \sim Y$ & $X \leftarrow Y$ & $X \leftarrow \sim Y$.

In words: ~X is sufficient for both Y and ~Y and X is necessary for both Y and ~Y. This logical pitfall occurs when membership in X is skewed towards high membership. The stronger the skewedness in X towards high membership, the more membership in Y can vary for this phenomenon to still occur.

If all cases fall in area III, then the following holds:

All cases in area III: $X \leftarrow Y$ & $\sim X \leftarrow Y$ & $X \rightarrow \sim Y$ & $\sim X \rightarrow \sim Y$.

In words: both X and ~X are necessary for Y and both X and ~X are sufficient for ~Y. This logical pitfall occurs when membership in Y is skewed towards low membership. The stronger the skewedness in Y towards low membership, the more membership in X can vary and still allow for this phenomenon to occur.

If all cases fall into area IV, then the following holds:

All cases in area IV: $X \rightarrow Y$ & $X \rightarrow \sim Y$ & $\sim X \leftarrow Y$ & $\sim X \leftarrow \sim Y$.

In words: X is sufficient for both Y and ~Y, and ~X is necessary for both Y and ~Y. This logical pitfalls occurs if membership in X is skewed towards low membership. The stronger the skewedness in X towards low membership, the more membership in Y can vary and still allow for this phenomenon to occur.

Apart from formal logical exercises, what are the insights of this for applied fsQCA? First, and importantly, skewed set-membership scores do have serious impacts on drawing inferences with set-theoretic methods as they can lead to illogical statements. Second, all of these analytic problems can occur even if neither X nor Y is a constant. Instead, as the above XY plot shows, all that is required is that cases tend to fall into specific triangular regions of the XY plot. This, in turn, already happens when set membership scores are skewed rather than being constant. Third, and related to the previous point, it is enough that either X or Y is skewed for these pitfalls to occur. The stronger the skewedness in one set, the more membership in the other set can be distributed normally and the logical pitfall still arise. Both points make it harder for researchers to detect the presence of skewedness and its impact on their findings. Fourth, by allowing for less than perfect consistency scores, not all cases have to be located in one specific area, yet the analytic pitfalls could still occur.

Fifth, the literature has so far only addressed some of the four scenarios caused by skewed set membership. More precisely, the scenario with most cases in area IV is dealt with by our discussion of the non-existence of the Rule of the Excluded Middle in fuzzy sets in section 9.2.2. As explained there, the PRI and the PRODUCT parameters are possible instruments to remedy the fallacy of declaring X sufficient for both Y and ~Y.[22] Furthermore, the scenario with most cases in area II is addressed in the discussion on relevant and trivial necessary conditions in section 9.2.1. The different formulas for the relevance of necessary conditions aim at not pointing to those sets of X that are highly skewed towards high membership as relevant necessary conditions even if membership in Y is also highly skewed towards high membership.[23]

Sixth, as point five shows, only the analytic consequences of skewed membership in X have been addressed in the literature so far. The consequences

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[22] We now see that PRI and PRODUCT could also be used to avoid the fallacy of declaring ~X necessary for both Y and ~Y.

[23] Following our logic as discussed here, these formulas could also be used to avoid the fallacy of declaring ~X sufficient for both Y and ~Y.

of cases only falling into area I or area III, respectively, are not addressed and stem from skewedness in outcome Y. Luckily, they are of somewhat less practical research concern because skewedness in Y seems less likely or, at least, less likely to go unnoticed. First, unlike condition X, the outcome Y usually consists of a single set rather than of several sets combined by logical AND (produces small sets) or logical OR (produces large sets). Second, being rooted in the qualitative mode of reasoning, set-theoretic methods tend to be more Y-oriented and thus researchers tend to pay closer attention to how their cases score in the outcome set. This, in turn, makes awareness of skewed set membership more likely – though not necessarily awareness of the analytic consequences it triggers.

Clearly, skewed set membership can lead to flawed conclusions. Here are some proposals for how to avoid such pitfalls. The most straightforward suggestion is to avoid skewed membership distributions. This, however, is often easier said than done. For one, sometimes all cases under investigation simply have low membership in a given set. For instance, most cases in research on public debt among EU member states might simply have low membership in the set of, say, countries with a very light public debt burden. If so, researchers might want to mitigate skewedness by reconceptualizing and therefore also recalibrating the set into, say, countries with a modest public debt burden. Second, as mentioned above, skewedness is often present even if single conditions are not skewed at all. In the analysis of sufficiency, this occurs because we are usually looking at logical AND combinations and often many cases have low membership in such conjunctions, i.e., they denote sets that are skewed toward low membership scores. In the analysis of necessity, we are sometimes interested in logical OR combinations (functionally equivalent necessary conditions, see section 3.2.1.2) and often many cases have high membership in such expressions, i.e., they denote sets that are skewed toward high scores.

All of this illustrates why it is of great importance that researchers keep an eye on the distribution of set-membership scores and whether it could influence their results. With our formula for the relevance of necessity, and with Ragin's PRI and PRODUCT formulas, some initial parameters are available for this task. In addition, researchers should always visualize their findings through an XY plot and see if their cases cluster in one of the four areas of Figure 9.4.

At-a-glance: the analytic consequences of skewed set-membership scores

Sets with skewed membership scores can lead to flawed inferences in the analysis of **sufficiency** and **necessity**.

Researchers should always check whether any of the single sets or their **solution formulas** are characterized by skewed membership scores.

In the analysis of necessity, the Schneider–Wagemann formula for **relevance** can help researchers avoid flawed statements of necessity. In the analysis of sufficiency, the parameters of **PRI** and **PRODUCT** are helpful and should be used more frequently in applied QCA.

Skewedness has further intricate implications for which no ready-made fixes currently exist. This makes it all the more important that researchers do not lose touch with their cases and their membership scores in the sets under consideration.

Part IV

Variants of QCA as a technique meet QCA as an approach

10 Variants of QCA

Easy reading guide

In the previous parts of this book, we have first presented the basics of QCA (Chapters 1–6), built them together into the Truth Table Algorithm (Chapter 7), and then elaborated on strategies for avoiding potential pitfalls (Chapters 8 and 9). We have done so by referring to static crisp-set or fuzzy-set QCA. In this chapter, we present further variants of QCA. More precisely, we discuss two-step approaches in QCA (10.1); multi-value QCA (mvQCA, 10.2) and how causally relevant notions of time can be dealt with in set-theoretic methods (10.3).

Readers at beginner's level should turn to these topics only having gone through Chapters 1–9. Advanced readers are likely to have already come across these three additional forms of QCA but might be interested in further reflections and critical assessments of them.

10.1 The two-step approach

The two-step approach (Schneider and Wagemann 2006; Schneider 2008) is based on the idea that often in comparative social science research endeavors, "conditions … can be divided into two groups, which can be labeled 'remote' and 'proximate' factors" (Schneider and Wagemann 2006: 759; see also Kitschelt 2003). Remoteness and proximity can be defined along various dimensions. For instance, remote factors are relatively stable over time. Also, their origin is often remote from the outcome to be explained on the space and time dimensions. As a consequence, remote factors cannot usually be easily altered by actors and are treated as exogenously given. In many research settings, remote factors are adequately labeled structural factors, contexts, historical legacies, etc. Proximate factors, by contrast, vary over time and originate not so far back in the past. They can be relatively easily modified by actors; often, they even describe human action itself. Remoteness and proximity can thus be interpreted not only in terms of space and time, but also with regard to the causal closeness which they are assumed to have.

Two-step thinking is of course not exclusive to QCA. Many sub-disciplines of the social sciences refer to causal processes (proximate factors) which unfold within certain contexts (remote factors). Kitschelt (1999), for instance, argues that explanations that rely exclusively on remote (structural) factors provide for causal depth, but fall short of demonstrating the causal mechanisms that link deep, distant causes with an outcome. By contrast, explanations based on proximate factors display causal mechanisms (often, but not necessarily, at the micro-level). Consequently, a good causal statement consists of finding the right balance between the two core features: causal depth and causal mechanisms (Schneider and Wagemann 2006: 761f.).

The general principle of the two-step QCA approach is as follows. In step one, a truth table is constructed exclusively based on remote conditions and the outcome. This truth table is then logically minimized, yielding a solution term that unravels what we label "outcome-enabling conditions" (Schneider and Wagemann 2006: 761). Because of the deliberate exclusion of proximate factors that are expected to matter for the outcome, the analysis in step one is under-specified. It should therefore be performed using lower consistency thresholds, in order to leave room for improvement once the proximate conditions are brought into the picture in step two.

The second step consists of constructing truth tables for each outcome enabling context from step one and the proximate conditions. Hence, if step one produced, say, three outcome-enabling contexts, then one truth table is constructed for each context and the proximate conditions. The logical minimization of these tables yields the sufficient paths towards the outcome. In step two, the consistency thresholds should be high and no assumptions about logical remainders should be made.[1] The purpose of step two is to unravel the configuration of proximate conditions that link a well-specified remote context to the outcome.

One positive side-effect of the two-step approach is that, by splitting the conditions into two groups, the number of logical remainders is drastically reduced (Schneider and Wagemann 2006: 762). The reduction in the number of remainders stems from barring any of the combinations between remote conditions that have not shown themselves as being outcome-enabling, on

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[1] Alternative treatments of logical remainders in step two are possible and plausible. For instance, Schneider (2009: ch. 6), who is interested in the configuration of political institutions (proximate conditions) that are sufficient for the consolidation of democracy within specific societal (remote) contexts, suggests refraining from any logical minimization in step two. Otherwise, the crucial information on the precise institutional configuration gets lost. Another option could be to prevent any logical remainder that implies the remote context being used as a counterfactual and to produce the intermediate solution term.

the one hand, and, proximate factors, on the other. Of course, the plausibility of excluding these combinations of conditions crucially depends on the plausibility of the division into remote and proximate conditions imposed by the researcher.

We nevertheless think that, despite being contingent on such theory-based decisions by the researcher, the two-step approach represents a viable research strategy that holds potential when researchers are confronted with a large number of conditions and/or when a clear division into remote and proximate factors suggests itself (e.g., Roehner 2011). The two-step QCA approach can be used both with crisp and fuzzy sets, and with multi-value "sets" (section 10.2).[2]

10.2 Multi-value QCA

So far, in this book, we have almost exclusively referred to crisp-set and fuzzy-set QCA. Next to these two QCA variants, multi-value QCA (mvQCA) is often presented as a further type of QCA (Cronqvist 2005; Cronqvist and Berg-Schlosser 2008). It uses some of the key principles of QCA – first and foremost, the logical minimization of truth tables – but operates not on crisp or fuzzy sets but on multi-value "sets."

The argument most often encountered for introducing yet another form of QCA is that many social phenomena do not manifest themselves in explicit (crisp sets) or implicit (fuzzy sets) dichotomies.[3] Rather, many of them straightforwardly consist of multinomial categories. Think, for instance, of the geographical location of a country (Europe, America, Asia, etc.), family status (married, single, widowed), professional affiliations (lawyer, academic, unemployed, etc.), to mention just a few. With any multinomial variable, there is no clear way to assign membership scores of cases into *one* crisp or fuzzy set.

In standard csQCA or fsQCA, a multinomial variable with c categories can be captured by creating c−1 different sets.[4] For instance, the multinomial variable marital status with three categories (married, single, widowed) can be

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[2] For a further discussion of the two-step approach, see also Mannewitz (2011).
[3] Recall (section 1.1.3) that fuzzy sets, despite their "continuous" membership scores, also maintain the notion of dichotomy through the 0.5 qualitative anchor that separates cases into two qualitatively different groups with regard to their membership in the fuzzy set.
[4] This procedure is akin to creating multiple dummy variables in multivariate regression.

captured by the two sets "married" and "single." The third category (widowed) is implied by the combination "not married" and "not single."

As Cronqvist and Berg-Schlosser (2008: 70–72) point out, this strategy of capturing multi-value concepts through multiple sets creates two problems. First, it increases the number of truth table rows and thus the problem of limited diversity. Second, some of these remainders are, by default, what we have called impossible remainders (sections 6.2.3 and 8.2.3). For instance, by capturing all three categories of the variable marital status through the two sets "married" and "single," we create at least one implausible truth table row that denotes individuals who are both married and single. Proponents of mvQCA see as its strength the fact that it mitigates limited diversity and the related analytic challenges.

In the following, we first present the principles and then a critical assessment of mvQCA. Here, we focus both on whether mvQCA really handles the issue of limited diversity in a superior manner compared to existing variants of QCA as well as on limitations that are inherent to mvQCA.

10.2.1 Principles of mvQCA: notation and logical minimization

Since mvQCA operates with conditions that do not indicate the presence or absence of one specific trait of a case, but allows different statuses, the notational system of just using letters with or without tilde (e.g., A and ~A) in order to indicate the presence or absence of a condition cannot be used anymore. Instead, the status of the condition is either indexed or indicated in brackets. For example, if a case takes on the value 2 in condition A, the indexed notation would be A_2 and the version using brackets A{2}. A combination of the conditions A, B, and C, where A takes on the value 2, B the value 0, and C the value 3, would therefore be $A_2B_0C_3$ in the indexed form and A{2}B{0}C{3} in the form using brackets. In this chapter, we use brackets.[5]

Truth tables and their logical minimization are also at the core of mvQCA. The principles of logical minimization are very similar to those in csQCA and fsQCA (section 4.3.1). Two or more expressions can be simplified if they fulfill the following two criteria. First, all but one conditions have to have the same value in all the expressions (e.g., condition A has to have the value 2, or condition B has to have the value 1 in all expressions, i.e., A{2}B{1}). Second, the remaining condition (C in our example) has to be present in

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[5] This presentational form is also used in the Tosmana 1.3.2 software.

every primitive expression resulting from the first step (A{2}B{1}), taking on every possible value. If, e.g., every condition can take on the values 0, 1, 2, and 3, then condition C has to be combined with A{2}B{1} in all these values. Only then can this condition be excluded through logical minimization. For instance, in the following logical expression, all the prime implicants consist of A{2}B{1} and C takes on all of its possible values across these prime implicants:

A{2}B{1}C{0} + A{2}B{1}C{1} + A{2}B{1}C{2} + A{2}B{1}C{3} → Y.

Condition C therefore is logically redundant and can be eliminated. This yields the new, more parsimonious expression:

A{2}B{1} → Y.

If any of the prime implicants were missing, then this logical minimization would not be possible. For example:

A{2}B{1}C{0} + A{2}B{1}C{1} + A{2}B{1}C{3} → Y

cannot be minimized into A{2}B{1}, since C does not appear in the status C{2}, i.e., the prime implicant A{2}B{1}C{2} is missing.

Continuing with the example, A{2}B{1} could be further simplified into A{2}, if A{2}B{0}, A{2}B{2}, and A{2}B{3} were also available for logical minimization. However, in order for this to be the case, the following conjunctions would need to be connected to the outcome:

A{2}B{0}C{0}, A{2}B{0}C{1}, A{2}B{0}C{2}, and A{2}B{0}C{3}

in order to make A{2}B{0} available;

A{2}B{2}C{0}, A{2}B{2}C{1}, A{2}B{2}C{2}, and A{2}B{2}C{3}

in order to make A{2}B{2} available; and

A{2}B{3}C{0}, A{2}B{3}C{1}, A{2}B{3}C{2}, and A{2}B{3}C{3}

in order to make A{2}B{3} available.

As can be seen, the principles of logical minimization in mvQCA are similar to those in the truth table analysis of csQCA and fsQCA.[6] The difference is that in csQCA and fsQCA, only one partner conjunction is needed in order to render minimization possible. In mvQCA, by contrast, more (sometimes

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[6] According to Cronqvist and Berg-Schlosser (2008: 74), the logical minimization used on mvQCA is as a generalized version of that in csQCA.

many more) such partner conjunctions are needed, simply because single conditions can appear in various statuses. Because the logical minimization in mvQCA requires more from the data, conservative solution terms tend to be more complex and most parsimonious solution terms based on more simplifying assumptions than the respective solutions produced by other variants of QCA.

In sum, mvQCA is in principle very similar to the main variants of QCA. First, a data matrix is converted into a truth table. Then, the truth table is logically minimized, applying rules very similar to those in QCA, and producing solution formulas that can be interpreted in terms of sufficiency. In practice, however, some qualifications are apt.

10.2.2 An assessment of mvQCA

In the following, we raise some issues related to mvQCA, some of which have not been discussed in detail so far and that should help researchers in deciding which variant of QCA to choose. We address the question of the set-theoretic status of mvQCA and shed some light on whether mvQCA is superior to csQCA and fsQCA in handling both multinomial concepts and limited diversity.

10.2.2.1 Is mvQCA a set-theoretic method?

In the following, we raise three issues that put the status of mvQCA as a set-theoretic method in doubt. First, it is argued (Vink and van Vliet 2009) that the data – multi-value variables – squarely fit into the notion of sets. Second, within the category of multi-value variables, two quite different types exist (one ordinal and one categorical). Third, mvQCA tends to de-emphasize one of the core epistemologies of set-theoretic methods: the focus on subset relations of necessity and sufficiency, respectively.

The distinctive feature of mvQCA is that it can handle variables with multiple categories. Can such multi-value variables be perceived as sets, though? One distinct feature of sets is that some cases are members while other are not and that the set label already expresses this qualitative distinction between members and non-members. For instance, the (crisp or fuzzy) set of rich people establishes a qualitative difference between those that are (more) in that set (rich people) and those that are not (not-rich people). Now, let us try to do the same with a typical multi-value variable, say, "professional status" with the four categories "white collar (1)"; "blue collar (2)"; "farmers (3)"; "managers (4)." It is impossible to assign non-membership

to any person in this multi-value variable. If there is no non-membership, it suggests that multi-value variables are indeed not sets. Along these lines, notice that the acronym mvQCA stands for multi-*value* QCA and not multi-*set* QCA, suggesting that the set-theoretic status of mvQCA is at least not entirely clear.

One counterargument in favor of mvQCA being a straightforward set-theoretic method could be that multinomial variables simply stack together multiple crisp sets. Rather than focusing on one professional category (e.g., the crisp set "blue collar workers") and thus relegating all others into the residual categories of non-blue collar workers, as is done with crisp sets, it is argued that mvQCA unpacks the set of non-members by allowing for multiple categories. From this perspective, each category in a multi-value variable is a set in itself (Vink and van Vliet 2009: 273). This interpretation, though appealing, has a major shortcoming. It leaves open the crucial question of how non-membership in a set is handled in mvQCA. If we take, for instance, the professional category "farmer," it is not clear of which set of other professional categories "farmers" are non-member.[7] One fundamental feature of sets is that they establish the qualitative difference between members and non-members of a given set. Multi-value "sets" are ambiguous about this. The only solution to establishing clear membership and non-membership is the aforementioned strategy of creating (crisp or fuzzy) sets for each of the categories in a multinomial variable. Only then it is possible to correctly identify all the sets in which a farmer, for example, is a non-member (see Vink and van Vliet 2009: 273).

A second reason why the set-theoretic status of mvQCA is put in doubt is the fact that many mvQCA applications do not operate on multinomial variables. Instead, they use ordinal variables, often derived from underlying interval-scale level data (Vink and van Vliet 2009: 271).[8] For instance, in a textbook example introducing mvQCA (Cronqvist and Berg-Schlosser 2008), the only multi-value variable used is called "GNPCAP" and contains three different categories, with countries in category 1 displaying lower GDP per

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[7] The mvQCA minimization algorithm, where all combinations of possible values have to be observed in order to logically minimize an expression (see section 10.2.1), suggests the interpretation that a case with given value in a multi-value "set" has a membership of 0 in all the other possible values of the same "set." If this interpretation is correct – the literature on mvQCA is not explicit on that, though – then mvQCA could be seen as a set-theoretic method. Then, however, one cannot simultaneously sustain the claim that mvQCA reduces the number of logical remainders because multi-value "sets" de facto are a conglomerate of multiple crisp sets (see section 10.2.2.2).

[8] Also consider that only a small fraction of all published QCA are performed with mvQCA (Rihoux *et al.* in press).

capita than those in category 2, which, in turn, display lower GDP per capita than those in 3. Hence, while the primary motivation for mvQCA is to handle multinomial categories, in research practice, and methodological introductions, this is extended to (or conflated with) ordinal notions of multi-value variables.[9] For multi-value variables with an ordinal notion, however, it is not clear what the advantage over fuzzy sets is.

Finally, the impression that mvQCA is different from the main QCA variants is fostered if we recall what the main aims – and strengths – of set-theoretic methods are: the unraveling of set relations that are interpreted in terms of necessity and sufficiency and all other forms of conditions, such as INUS and SUIN, that can be derived from it (see section 3.3, in particular 3.3.2). Precisely the notions of set relations, necessity, and sufficiency are strikingly missing in publications on mvQCA, though.[10]

In sum, when confronted with multi-value variables, we have two choices. Either we apply mvQCA but create uncertainty about the set-theoretic status of such data (Vink and van Vliet 2009: 286) and, in consequence, of mvQCA, or else we create multiple (crisp or fuzzy) sets and apply csQCA or fsQCA, thus creating certainty about the set-theoretic status of the method we use. If mvQCA operates on ordinal multi-value variables, then its distinct methodological contribution is less clear.

10.2.2.2 Does mvQCA reduce limited diversity?

One of the arguments in favor of mvQCA and against the strategy of using multiple (crisp or fuzzy) sets for representing multinomial concepts is that, with mvQCA, the number of conditions is kept lower and thus the problem of limited diversity is more under control (Cronqvist and Berg-Schlosser 2008: 70ff.). It is easy to see, however, that this claim is difficult to sustain.

With three crisp- or fuzzy-set conditions A, B, and C, there are eight logically possible combinations, i.e., truth table rows. But if we allow for, say, three statuses of conditions A and B and for four statuses of condition C, then no less than $3 \times 3 \times 4 = 36$ logically possible combinations result. Five conditions, each with four possible statuses, will already lead to $4^5 = 256$ truth table rows.

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[9] In the same publication, another illustration of multi-value variables also uses a metric scale – the age of children – to group them into categories 0, 1, 2, and 3 of a multi-value variable.

[10] In their seminal introduction to mvQCA, Cronqvist and Berg-Schlosser (2008) do not mention any of these terms once. Furthermore, Tosmana, the software developed by Cronqvist for mvQCA, does not provide a possibility for analyzing necessary conditions.

Clearly, many of these rows will be logical remainders, and several of them might even be impossible remainders.

One strategy for mitigating this problem is to limit the number of categories of conditions. Cronqvist and Berg-Schlosser (2008: 84) propose a maximum of four categories. However, even in such a scenario, many truth table rows result. Ironically, then, the best way of keeping limited diversity at bay is to make mvQCA as similar to csQCA or fsQCA as possible.

A second strategy for reducing limited diversity consists of representing a given multinomial concept with c categories with neither a multi-value condition with c values nor with c−1 crisp sets. Instead, a researcher could establish a new crisp set that captures each case's membership and non-membership in only *one* of the categories (e.g., professional categories: farmers and non-farmers), thus focusing on just one of the possible values the concept can take (Vink and van Vliet 2009: 271f.). Clearly, with this strategy, there is a tradeoff between reducing limited diversity, on the one hand, and measurement validity of a concept, on the other. First, because many different cases are captured within the same category (e.g., "non-farmers"), this strategy is prone to increase the number of contradictory truth table rows. Second, and related, whenever the negation of such a set is involved in a solution, either on its own or as an INUS or SUIN condition, it is difficult to substantively interpret this finding, for all that is known about the non-members of the set of farmers is, well, that they are non-farmers but not whether they are doctors, academics, or street cleaners.

Recall that one argument in favor of mvQCA is that it reduces the problem of limited diversity vis-à-vis the strategy of converting multi-value concepts with c categories into c−1 crisp or fuzzy sets (Cronqvist and Berg-Schlosser 2008: 72; Herrmann and Cronqvist 2009: 39). It is argued that by converting multinomial variables into crisp or fuzzy sets, one unavoidably creates truth table rows that are impossible.[11] This argument would only be in favor of mvQCA and against csQCA and fsQCA if it were the case that in mvQCA *fewer* of these impossible remainders were produced than with the aforementioned strategy of converting multinomial variable into various crisp or fuzzy sets. This, however, is not the case. As can be shown, both strategies for handling multinomial concepts – mvQCA and the creating of c−1 crisp or fuzzy

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[11] If a condition A with three values is split into three mutually exclusive crisp sets A1, A2, and A3, then these three conditions form eight truth table rows. Of these eight rows, only three are possible, namely all those rows that describe a situation in which cases have membership in just one of the sets. The remaining five rows refer to hypothetical cases which have membership in more than one set, or in no set, and are, thus, impossible (see also sections 6.2.3 and 8.2.3).

sets – produce exactly the same number of possible truth table rows.[12] Hence, nothing is gained by using mvQCA in terms of reducing the number of relevant logical remainders.[13] This might come as a somewhat surprising conclusion, for one of the arguments in favor of mvQCA is that it avoids the pitfalls of capturing multi-value variables through multiple crisp sets, which, as outlined above, consisted in an increase of limited diversity and the creation of impossible remainders.

Th e initial problem raised by mvQCA the fact that many social phenomena do not neatly fit into one crisp set – is a valid concern. Researchers applying set-theoretic methods would be well advised to be aware of this issue and ready to deal with it. We do think that the use of mvQCA during an initial upstream (Rihoux and Lobe 2009) or pre-QCA (Schneider and Rohlfing in press) research phase can potentially be of help in learning more about cases and potential patterns in the data. As has become clear, however, we are skeptical whether the most important analytic goals that can be achieved with either crisp- or fuzzy-set QCA can equally be achieved with mvQCA. Most importantly, we argue that it is impossible to simultaneously claim that mvQCA operates on sets and reduces limited diversity. Instead, to us it seems that mvQCA is a set-theoretic technique (but unnecessarily conceals this fact) and therefore is not superior to other variants of QCA when it comes to limited diversity.

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[12] Let c be the number of categories in a multi-value variable. Then every multi-value concept can be expressed in $c - 1$ crisp sets. From m such multi-value concepts, $\Sigma_m (c - 1) = \Sigma_m (c) - m$ crisp sets can be derived. Consequently, the final truth table will contain $2^{\Sigma(c) - m}$ truth table rows. Each multi-value concept which is added with $c - 1$ crisp sets, multiplies the number of truth table rows by $2^{c - 1} = \frac{1}{2} * 2^c$. However, for every of the m ex-multi-value concepts, only those truth table rows will be plausible which show only exactly *one* of the categories to be present, and the others absent. Th is is equal to the number of categories of the respective concept, c. Thus, the actual multiplication of truth tables by multi-value concepts transformed into crisp sets is not by $\frac{1}{2} * 2^c$, but by $c/(\frac{1}{2} * 2^c)$, which expresses the share of the plausible truth table rows (on all added truth table rows). If performing these multiplications for each of the former m multi-value concepts (now crisp sets), then the formula $2^{\Sigma(c) - m} * \prod_m (c/(\frac{1}{2} * 2^c))$ results. This is equal to the number of truth table rows in the original mvQCA analysis which could be expressed as the product of all numbers of categories for all multi-value conditions, that is $\prod_m (c)$. Th e mathematical proof is this: $2^{\Sigma(c) - m} = \prod_m (2^c)/2^m$ and each single one of the m other factors, $c/(\frac{1}{2} * 2^c) = 2c/2^c$. Th us: $\prod_m (2^c)/2^m * \prod_m (2c/2^c) = \prod_m (2^c) / 2^m * \prod_m (2c) / \prod_m (2^c) = \prod_m (2^c) / \prod_m (2^c) * \prod_m (2c) / 2^m = \prod_m (2c) / 2^m$. Furthermore: $\prod_m (2c) = 2^m * \prod (c)$. Th us, the formula becomes $2^m * \prod_m (c) / 2^m = \prod_m (c)$. And this is precisely the number of truth table rows in an mvQCA.

For our example, with A and B having three categories each, and C four, mvQCA produces $3 \cdot 3 \cdot 4 = 36$ truth table rows. For csQCA, we calculate, following our formula, $2^{3+3+4 - 3} * (3/(\frac{1}{2} * 2^3)) * (3/(\frac{1}{2} * 2^3)) * (4/(\frac{1}{2} * 2^4)) = 2^7 * (\frac{3}{4}) * (\frac{3}{4}) * (\frac{4}{8}) = 128 * 0.75 * 0.75 * 0.5 = 36$ plausible truth table rows.

[13] Note that the conversion of multi-value concepts into $c-1$ crisp sets is only *one* source for impossible remainders. All other types of impossible remainders, as discussed in sections 6.2.3 and 8.2.3, can of course also occur in mvQCA, as in csQCA and fsQCA.

In closing, we deem it important to address two further misunderstandings related to mvQCA. First, mvQCA cannot make the use of crisp sets obsolete. For one, the outcome in mvQCA has to be a crisp set (Cronqvist and Berg-Schlosser 2008: 84).[14] Only the conditions can be multi-value, but as shown, even there the advice is to have as many crisp sets as feasible, even in an mvQCA. Second, the choice of mvQCA vis-à-vis other variants of QCA is not (and should not be) driven by the number of cases studied. Thus, the claim according to which csQCA is appropriate for small-N, mvQCA for mid-sized-N, and fsQCA for large-N (Herrmann and Cronqvist 2009) is not convincing, for it seems to assume that small-N studies – by default – exclusively invoke dichotomous concepts and that large-N studies deal with concepts that can always be best captured by fuzzy sets. We, instead, think that the decision of which QCA variant to apply should be based exclusively on the characteristics of the underlying concepts. If in a small-N analysis all concepts lend themselves to being represented by fuzzy sets, so be it.

At-a-glance: multi-value QCA

mvQCA is designed for handling multinomial concepts that are not dichotomous in nature. The outcome has to be a **crisp set**, though.

mvQCA uses an indexed form of notation or brackets, such as A{2}. The principles of **logical minimization** are similar to those in **csQCA** and **fsQCA**, but solutions obtained with mvQCA tend to be more complex or based on a greater number of **simplifying assumptions**, thus raising the risk of making **untenable assumptions**.

The set-theoretic status of mvQCA is unclear, for multi-value variables squarely fit into the notion of sets; two quite different types of multi-value variables exist (ordinal and categorical); and mvQCA de-emphasizes the focus on subset relations of necessity and sufficiency.

An alternative to using mvQCA is to transform multinomial concepts with k categories into k−1 crisp or fuzzy sets and to use csQCA or fsQCA. This strategy produces exactly the same number of **logical remainders** once **impossible remainders** are discarded.

10.3 Set-theoretic methods and time

So far, we have introduced set-theoretic methods as cross-sectional data analysis tools. When reporting solution formulas, the order by which conditions

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[14] There is, however, no reason why, in principle, multi-value outcomes could not also be handled (Cronqvist and Berg-Schlosser 2006). In that case, each analysis would still focus on just one of the various values of the outcome, though. If four values in the outcome were allowed, then different solution formulas could be derived for Y{0}, Y{1}, Y{2}, and Y{3}.

are linked through logical AND or OR does not matter. A*B is equivalent to B*A and so are A+B and B+A. Just like their false friends in normal algebra, these logical operators follow the commutative law (section 2.4.1). Thus, unless specific strategies are adopted, solutions produced by QCA are largely insensitive to the potential causal role different aspects of time might play.

It is obvious that any cross-sectional method that turns a blind eye to the relevance of time in its analysis will be subject to criticism. Since one of the trademarks of qualitative research is its careful treatment of different aspects of time (Mahoney 2000; Grzymala-Busse 2010), this omission is particularly awkward for a technique called "*qualitative* comparative analysis." The plausibility of causal claims made in qualitative research very often depends crucially on arguments that elaborate sequences (Mahoney *et al.* 2009), use causal process observations (Brady and Collier 2004, 2010), detect slow-moving processes (Pierson 2003), use process tracing (George and Bennett 2005; Hall 2006), invoke the notion of path dependency (Bennett and Elman 2006), and the like. In the following, we outline strategies for making set-theoretic approaches more time sensitive. For this, we first differentiate between different forms of causally relevant notions of time (section 10.3.1), discuss informal ways of integrating some of these notions into the analysis (section 10.3.2), and then present a formalized set-theoretic treatment of sequence elaboration (section 10.3.3). In the major part of this section, we explain the principle and practice of the so-called temporal QCA (tQCA) as a further variant of QCA (section 10.3.4).

10.3.1 Forms of causally relevant notions of time

There are different ways in which time, broadly understood, might be causally relevant. We only briefly mention some of them here (for extensive treatments, see, e.g., Mahoney 2000; Pierson 2000, 2004; Abbott 2001; Cappocia and Kelemen 2007; Grzymala-Busse 2010).

The *temporal order of events* might matter. Former communist countries that first establish a functioning legal framework (A) and then liberalize their economy (B) might be more likely to achieve a proper market economy (Y) than countries that do it the other way around. If we introduce the notation "/" for a logical THEN, capturing thus the aspect of temporal order, we can say while A/B is sufficient for Y, B/A is not. Similarly, the *causal sequence of events* might matter. This is the classical form of a causal process argument and could be denoted as A → B → C → Y. Unlike with temporal order, here conditions A, B, and C are causally connected in a sequence at the end of

which C implies the outcome of interest Y. A third possibility is that the *speed of the processes* might matter. Some events unfold their causal impact because they happen suddenly, last only briefly, and are very visible. Military coups, the onset of a war, or even natural disasters are typical instances of this category of time-related causes.[15] Often they are interpreted as *critical junctures* (Collier-Berins and Collier 1991; Mahoney 2000). Other processes, in contrast, produce an outcome precisely because they are evolving very slowly, often over several decades, if not centuries. Because the changes are incremental, they often remain invisible and derive their causal role precisely from these characteristics (Pierson 2003, 2004). Even more intricate time-related arguments abound in the qualitative literature. Scholars detect *feedback loops* (A produces B, which, in a subsequent phase, has an effect on A) or *anticipated effects* (B occurs because the occurrence of A is anticipated, even if A then actually does not occur).

In short, without any attempt to include at least some of these aspects in set-theory based research, its credentials as an essentially qualitative research tool are hampered, an issue hinted at early on by Ragin (1987: 162f.). Neglecting time often might also mean that wrong conclusions are drawn and conditions deemed irrelevant when they are not (or vice versa), a danger common to any purely cross-sectional approach. Finally, the integration of time into set-theoretic techniques would be beneficial because it greatly facilitates the combination of case studies with cross-sectional approaches in a multi-method research design (section 11.4).

In the following, we describe several strategies for integrating some of these notions of time into set-theoretic-methods-based research. Depending on the research design and interest, some are more adequate than others.

10.3.2 Informal ways of integrating notions of time into set-theoretic methods

A first and very basic way to consider time in QCA is to run separate QCA for different points in time. For instance, one might be interested in the change over time of the conditions that explain fiscal discipline of EU member states. One could produce truth tables on identical sets of countries and conditions for, say, 1980, 1990, 2000, and 2010. The logical minimization of these truth tables will yield different solution terms. The time-related analytically relevant information would be stored in the difference between these solution terms.

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[15] See Grzymala-Busse (2010) for an intriguing discussion of the differences between "tempo" and "duration."

Another strategy is to collect data for each case at different points in time and then sort them as different cases into one pooled truth table. Here, the relevant information is provided by how cases move over time in the property space, i.e., how they move from one truth table row to another. In such a setup, a researcher might not necessarily want to engage in the logical minimization of a pooled truth table, for the meaning of such a solution term is dubious. Instead, the strategy of pooling data over time seems more promising when analyzing multidimensional concepts over time. When using only two or three fuzzy sets, two- or three-dimensional plots provide a useful graphical representation of each case's movement over time (for an example from welfare state research, see Kvist 2007).

Four further, straightforward strategies exist. The first three incorporate the notion of time through the calibration of conditions. First, raw data can be averaged over a given time period and this information then used for set calibration. Second, the over-time difference in raw scores (or set-membership scores) is calculated and a new set is created that captures these differences. The third strategy is already mentioned by Ragin (1987: 162f.). It entails producing a condition that expresses the sequence of two or more events. For instance, a researcher might have information not only on each case's membership in the set of high inflation (A) and slow growth (B), but also on whether in a given case A occurred before B or vice versa. Based on this, it is possible to assign a membership score for each case in the set called "High inflation occurred before slow growth" (A_before_B). This sequence-capturing condition is then used along the other conditions to form a truth table. This, in essence, is the strategy that is at the heart of temporal QCA (tQCA), which we describe in further detail in section 10.3.4.

A fourth way is to adapt the two-step approach (section 10.1) such that the difference between remote and proximate conditions is framed in an exclusively time-related way. Remote conditions would be those occurring prior to the proximate conditions. The first step of the analysis, taking into account only the remote conditions, would give us some indications of historical legacies which are favorable for the occurrence of the outcome and the second step reveals which conjunctions of subsequently occurring factors are sufficient for producing the outcome within a specific historical legacy.

10.3.3 Sequence elaboration

Mahoney *et al.* (2009) develop an approach called sequence elaboration. In essence, this can be interpreted as an attempt at translating well-known

research principles of macro-historical approaches into set-theoretic language and practices. The aim of sequence elaboration is to specify the relative importance of individual conditions that are part of a sequence of causal factors. According to the authors, such analyses usually start with a bivariate relationship between X and Y, which is then extended, or elaborated, through the introduction of a further condition Z. Z can either be an antecedent or an intervening factor. Furthermore, X and Z can be any of the five possible types of cause in set-theoretic methods: necessary, sufficient, necessary and sufficient, INUS, and SUIN.

The authors suggest the following notations for a research design aiming at elaborating a sequence by introducing an antecedent and an intervening condition, respectively:

Antecedent condition: Z --?→ X −n(s)→ Y
‖ --------?--------↑.
Intervening condition: X --?→ Z – ?→ Y
‖ -------n(s)-----↑.

Y always denotes the outcome of interest, X the initial cause, and Z the condition that is subsequently added to the analysis. The symbol ‖ denotes the beginning and ↑ the end of the sequence; "n" stands for necessity, "s" for sufficiency, and a "?" for an unknown set relation. The arrow → simply indicates the direction of causality. Contrary to standard notation in set theory, it does not indicate sufficiency but, if preceded by the letter "n," it denotes necessity.

Mahoney *et al.* (2009) show that depending on its position in the sequence, Z either contextualizes or diminishes the causal role of X. Consider, for instance, the "antecedent condition" scenario. A researcher interested in explaining the sequence of events that leads to the outbreak of large-scale protests in authoritarian regimes (Y) might find that egregious vote-rigging in national elections (X) is a sufficient condition. She then finds that youth poverty (Z) is a necessary condition for X and a sufficient condition for Y. Formally:

Z −n→ X −s→ Y
‖ -------s-------↑.

According to Mahoney *et al.*'s (2009: 135) inventory of sequence elaboration results, this is an instance in which Z *diminishes* the initial relationship between X and Y. Anti-regime street protests (Y) do occur when youth poverty is high (Z), even if no vote-rigging (X) took place.

Imagine the very same example with the only difference being that, this time, Z is found to also be necessary for Y. Formally:

Z –n→ X –s→ Y
‖-------n-------↑.

Now, the introduction of the antecedent condition Z *contextualizes* the initial role of X in bringing about Y. Vote-rigging (X) produces anti-regime protest (Y) only in countries that display high levels of youth poverty (Z).

In sum, by translating common arguments found in the path-dependence literature into the language of set theory and formal logic, sequence elaboration tremendously enlightens the underlying structure of many, if not most, arguments that are commonly made in historical approaches to social phenomena. Put the other way round, sequence elaboration makes set-theoretic approaches sensitive to time by showing that historical institutionalists essentially make set-theoretic statements. For researchers interested in unraveling the sequences by which a given phenomenon is produced, set-theory-based sequence elaboration offers an appealing framework for analysis. Sequence elaboration helps clarify the arguments that are made and prevents researchers from making logically impossible claims.[16]

On a more cautious note, sequence elaboration also reveals the perplexing complexity that researchers are buying into when attributing causal relevance to the sequence of events. Notice that although Mahoney et al.'s (2009) discussion is limited to only two conditions (X and Z) and only two set relations (necessity and sufficiency), the number and complexity of sequences is already remarkable. The mapping of sequences – and with this the capacity for avoiding logically flawed claims – quickly reaches its limits when more conditions and forms of set relations (INUS and SUIN) are involved. This, as we will see again below when discussing temporal QCA (section 10.3.4), is a common feature of paying attention to time in social research: it massively increases the complexity of the analysis and therefore faces remarkable substantive and methodological hurdles.

Sequence elaboration does not involve truth tables or standard principles of logical minimization. Nor is it meant to be comparative. Sequence elaboration, therefore, does not represent a strategy that helps to make QCA more time sensitive. In the following, we describe the most elaborate strategy as of now: temporal QCA.

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[16]   The authors show that several formal logically possible sequences do describe, in fact, logical impossibilities.

10.3.4 Temporal QCA

So far the most formalized approach to making QCA more amenable to capturing the causally relevant role of time stems from Caren and Panofsky (2005) and Ragin and Strand (2008). We first present the general difficulties that emerge when trying to formalize time as causally relevant information in a QCA. Then we explain the logic and principles of temporal QCA (tQCA), followed by some notes of caution.

10.3.4.1 Time and property space: keeping logical remainders at bay

Among the many different notions of time mentioned above (section 10.3.1), tQCA tries to incorporate information on the sequence of events. In addition to the logical OR (A+B) and AND (A*B), in tQCA the temporal operator "/" (A/B, read: "A then B") is also allowed for. This has tremendous consequences for the number of logically possible combinations. With two conditions A and B, eight logically possible sequences are created.[17] Three conditions yield 48 sequences and four conditions already the staggering number of 384 sequences. The formula for calculating the number of logically possible sequences is:

$$k! * 2^k,$$

with k being the number of conditions. Thus, in a QCA based on a moderate number of, say, five to eight conditions, the complexity of a truth table spins out of control if all possible sequences of events are taken into account. Fortunately, several research design strategies are available for reducing the number of logical remainders in tQCA. In the following, we present some of them.

The first strategy, suggested both by Caren and Panofsky (2005: 158) and Ragin and Strand (2008), is to only focus on sequences that involve the *presence* of condition events. The substantive argument for this restriction sounds plausible. If event A did not happen (~A), then it is futile, if not impossible, to determine whether ~A "happened" before event B (~A/B) or after B (B/~A). This argument obviously extends to the sequencing of more than two conditions and when more than one event condition did not occur.

The second strategy is to focus only on event pairings and their sequences rather than the sequence of three or more events. For instance, with three conditions A, B, C, there are three event pairings (A*B, A*C, B*C), yielding $2^3 =$ 8 logically possible event sequences. As Ragin and Strand (2008: 439f.) show,

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[17] These are: A/B, A/~B, ~A/B, ~A/~B, B/A, B/~A, ~B/A, ~B/~A.

some sequences of two-event pairings are logically possible but intransitive. This means they cannot occur and are thus impossible remainders (sections 6.2.3 and 8.2.3).[18] Such meaningless sequences must be excluded from any logical minimization procedure, thus reducing the problem of limited diversity. When considering only sequences that involve the *presence* of two conditions, the formula for calculating the number of transitive truth table rows that are created by transitive sequences is k!,[19] with k being the number of initial conditions to be sequenced (Ragin and Strand 2008: 439). With three initial conditions, then, there are only six event sequences.[20] While this number of truth table rows is much more manageable, it still implies that with more than a handful of events to be sequenced, things are prone to get out of control. This is why the authors suggest not involving more than four sequenced events (Ragin and Strand 2008: 439f.).

Yet another powerful way of drastically reducing the property space is to fix the temporal order for one or more of the initial conditions (Caren and Panofsky 2005; Ragin and Strand 2008). To demonstrate how far this strategy reduces complexity, consider Caren and Panofsky (2005), who try to find the sequences of events that lead to the recognition of graduate student unions at research universities (Y). Their four conditions are whether the potential union is at a public university (P), has elite allies (E), has a national union affiliation (A), and organizes a (threat of a) strike (S). Based on substantive arguments grounded in the meaning of these sets, it is a credible claim to say that condition P (or its negation ~P) always comes first and condition S (or ~S) always comes last in any empirically observable sequence of events. This means that the only events that can occur in different sequences are E and A. In principle, these would be eight sequences (see note 20), but because the authors allow only the presence of events to be sequenced (and not their negation), this number is down to two. Cases are either members of the set E_before_A or the set A_before_E.

10.3.4.2 The logic and principles of temporal QCA

The introduction of sequences of events into formal logical statements requires additional rules of logical minimization. According to Caren and Panofsky (2005: 160–62), these are as follows.

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[18] For instance, if A occurs before B and C before A, then B cannot occur before C.

[19] The complete formula would be k! * 1^k. The expression 1^k indicates that for each single condition only its presence and not its negation is taken into account. 1^k can be omitted regardless of the value of k, however, since it always yields the value of 1.

[20] These are A/B, A/C, B/C, B/A, C/A and C/B.

First, researchers need to check the causal relevance of temporal borders. A statement like

A/B + B/A → Y

can be reduced to

A*B → Y,

because if both sequences of these two events lead to Y, their sequence is irrelevant.

Second, conditions can be logically minimized only if they belong to the same temporal block. For instance,

A/B/C + A/B/~C → Y

can be logically minimized to

A/B → Y.

In other words, within the sequence A/B, the presence or absence of condition C does not make any difference in bringing about Y.[21]

In the example discussed by Caren and Panofsky (2005) and Ragin and Strand (2008), five sequences lead to the recognition of student unions. Following the logical minimization rules, these five sequences can be reduced to only three sequences, as shown in Figure 10.1.

There are three sequences that are sufficient for the recognition of student unions at universities. In public universities students first obtain elite support and then threaten to strike or become affiliated with national associations (P/E/(S+A)) or students become first affiliated, then receive elite support, and then threaten to strike (A/E/S).

Rather than doing the logical minimization by hand, Ragin and Strand (2008) suggest using the software package fsQCA 2.5. This is done in the following manner: the condition capturing the sequence "E occurs before A" is added to the truth table, which then consists of five conditions (P, E, A, S, and E_before_A). The membership of each case in E_before_A is determined in the following manner: cases that are members of both conditions E and A need to be separated into two groups, those in which E occurred before A and those in which A occurred before E. The former group of cases has

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[21] According to this rule, the expression A/B/D + A/B/C/~D → Y cannot be logically minimized, for D and ~D are parts of different sequences (A/B/D vis-à-vis A/B/C). This expression can be rewritten only by using the rules for factoring out conditions: A/B/(C/~D + D) → Y.

P/E/A/S + P/A/E/S + P/E/A/~S + P/E*~A/S + ~P/A/E/S ⟶ Y

P/E*A/S + P/E/A + A/E/S + P/E*~A/S ⟶ Y

P/E/S + P/E/A + A/E/S ⟶ Y
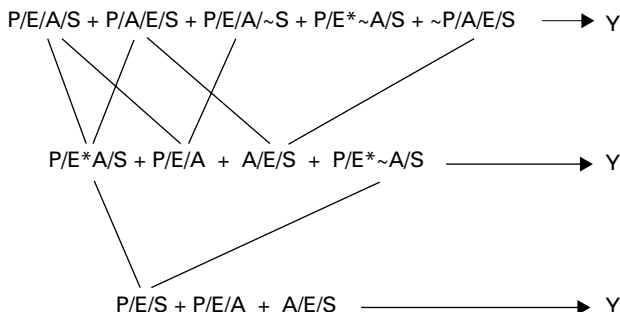
Figure 10.1   Logical minimization of sequence of events

full membership in condition E_before_A, whereas the latter has full non-membership in E_before_A. This is straightforward. There is, however, a third group of cases, namely all those that are not members in E, in A, or in either. Because in tQCA we assume that sequences involving the negation of event conditions cannot be formed, the membership of these cases in set E_before_A should be denoted with a dash (–) (Ragin and Strand 2008). When encountering a dash in a combination of conditions, the software treats this condition as irrelevant in the process of logical minimization. For instance, in the example on student unions, there is a truth table row P~EAS– (where the "–" represents the logical status of condition E_before_A).[22] This row is equivalent to the statement P~EAS(E_before_A) + P~EAS~(E_before_A).[23] Using this coding for condition E_before_A, the software produces the following logically minimized result:

PES + EAS(~E_before_A) + PEA(E_before_A) → Y.[24]

It might be surprising that this solution term is void of any operator indicating the sequence of events. Yet, we can nevertheless infer the temporal order of events. First, we know by assumption that P always comes first and S always last. Second, condition E_before_A provides the information on the sequence of conditions E and A. We therefore can rewrite the solution term using the temporal THEN as follows:

P/E/S + A/E/S + P/E/A → Y.

[22] See the online appendix for the truth table (www.cambridge.org/schneider-wagemann).
[23] Notice that using a dash (–) in the coding the outcome, the software can be asked to replace the dash with either a 1 or a 0 according to which decision yields the most parsimonious solution term. Instead, when a dash is assigned for a condition, the software does not assign any value to that condition. We thank Charles Ragin for pointing this out to us (personal communication, March 2011).
[24] Note that the software accepts a dash as an entry in a condition only when choosing the option [Analyze] [Crisp Sets] [Quine (QCA 3.0)] but not when choosing [Truth Table Algorithm].

This, in turn, is exactly the same result as that obtained through logical minimization by hand above. Because logical minimization by hand is prone to lead to wrong conclusions, Ragin and Strand (2008) strongly recommend making use of the computer software when analyzing sequences of events in QCA.

10.3.4.3 Evaluating temporal QCA

Whenever researchers have theoretical hunches that a limited number of sequences involving a limited number of events are causally relevant, tQCA provides a formal, logically sound way of integrating these hunches into the analysis. Such analysis of sequences can even be performed with the help of the computer, using either fsQCA 2.5 or R. tQCA therefore is a valuable extension of QCA that should be used more often.

It should be recognized, though, that the level of complexity that is introduced when taking time into consideration is kept at a manageable level in tQCA only through a set of limiting assumptions which can – as can any assumption – be questioned. First of all, it is assumed that sequences can only be formed with the occurrences of events. Notice, though, that sometimes the absence of an event is also an event itself. For instance, if X denotes the set "riots in response to IMF measures," then ~X ("no riots in response to IMF measures") is clearly also an event that can be located in time and space. Hence, a sequence of events that includes ~X is possible and plausible (Caren and Panofsky 2005: 158). Second, only sequences involving two conditions are investigated. Qualitative researchers often have hunches about longer and thus more complex sequences, though. Third, only a subset of the initial conditions is allowed to be part of a sequence. This is achieved by restricting some conditions to always occur at a specific place in a sequence (condition P always first and S always last in the example above). In research practice, such clear-cut classifications might often not be feasible.

At-a-glance: set-theoretic methods and time

The inclusion of time is of great importance in understanding social phenomena. Many macro-historical approaches make set-theoretic arguments, as revealed by the sequence elaboration approach. By including time, however, the level of **complexity** of the analysis increases dramatically. One useful way of grasping this is by calculating the number of logically possible sequences.

Informal ways include the idea of running separate QCA for different points in time, using time-related information when calibrating sets, or adjusting the two-step approach accordingly. So far, tQCA is the most more formalized strategy for including time. It either

uses the logical THEN operator and **logically minimizes** "by hand," or else specifies conditions that express the sequence of events and uses the computer software for logical minimization. Both strategies produce the same result.

The integration of time aspects into a formal logical approach is practically feasible only when looking at a very reduced notion of time and only a small number of sequences. It is important to emphasize that this is an intrinsic problem and unrelated to whatever data analysis technique is employed. If we concentrate only on plausible sequences, excluding non-events, then k! of transitive sequences can be formed. The **property space** is further reduced by fixing the temporal order for one or more conditions.

11 Data analysis technique meets set-theoretic approach

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Easy reading guide

The double nature of QCA as both a research approach and a data analysis technique is paramount (see Introduction). Throughout this book, we have mostly concentrated on issues surrounding QCA as a technique. In this chapter, we strengthen our perspective on QCA as a research approach and discuss various salient comparative methodological issues from a set-theoretic perspective. This chapter should help researchers – beginners and advanced – to increase the quality of their set-theoretic-method-based research endeavors as well as further insert set-theoretic principles into the broader methodological social science literature.

 We start off with a recipe for a good QCA (11.1). Even if no new technical insights are provided, a careful reading of this recipe is nevertheless recommended to both beginning and advanced users of QCA, as it contains guidelines on how to perform QCA not only in a technically correct manner, but also in a plausible and complete way. All subsequent sections can be read separately and in the order of the reader's choice. We discuss fundamental methodological issues from a set-theoretic perspective: robustness (11.2); theory evaluation (11.3); and case selection principles in set-theory-based multi-method research designs (11.4).

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11.1 Recipe for a good QCA

The following recipe is a summary of what an ideal QCA should look like.[1] We believe that providing a profile of an idea-typical QCA is fruitful, even if in research practice most published QCA comes short of fulfilling all the criteria, mostly due to restrictions in space and time. Our list can help authors, readers, and reviewers to become aware of how a given study deviates from an ideal application and to provide explicit arguments for why certain items from the list have been omitted.

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[1] For an extended exposition, see Schneider and Wagemann (2010).

11.1.1 The appropriateness of set-theoretic methods

The first and fundamental question to be asked is whether the use of set-theoretic methods makes sense for the research project at hand. Throughout this book, we have repeatedly argued that the use of set-theoretic methods makes sense only if a researcher has good reasons to believe (or straightforward hypotheses to be tested) that the phenomenon of interest is best understood in terms of set relations. This unavoidably implies that the researcher buys into the idea of producing causally complex accounts (section 3.3). In other words, one must find it plausible to claim that the outcome of interest is based on equifinal, conjunctural, and asymmetric relations in terms of necessary, sufficient, INUS, and/or SUIN conditions.

It seems obvious that not all – according to some, perhaps even only a minority of – research questions are adequately dealt with through set-theoretic methods. For our methodological purposes, there is no need to enter into this debate. It suffices to state that set-theoretic methods such as QCA are an adequate tool if, and only if, a researcher is interested in set relations and not correlations. Standards of good practice require that an explicit statement is made that a researcher is interested in relations between sets rather than correlations between variables.

Beyond this, there are different, more specific aims one can pursue when using QCA. Berg-Schlosser, De Meur, Rihoux, and Ragin (2008; see also Ragin and Rihoux 2004: 6) mention several possible aims of using QCA: to summarize data, best done by representing it in the form of a truth table; to check whether the empirical evidence at hand is in line existing with claims of subset relations, i.e., the evaluation of existing hypotheses and theories;[2] and to develop new theoretical arguments. To this, we add the use of QCA as a means of creating empirical typologies (for more details, see Kvist 2006, 2007). Of course, it is possible to pursue all or just some of these purposes of QCA in the framework of a research project.

11.1.2 The choice of the conditions and the outcome

The number of conditions included in a QCA should be kept at a moderate level. Too many conditions in QCA are dysfunctional. Most importantly, the number of logical remainders grows considerably, leading to severe problems

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[2] Recall that if hypothesis testing is the aim, these hypotheses need to be formulated in terms of subset relations (see also section 11.3).

of limited diversity (Chapters 6 and 8). As a consequence, solution terms tend to be either too complex or based on too many assumptions on remainders. Complex solutions often apply only to single cases and are difficult to interpret in a theoretically meaningful manner, whereas a high number of assumptions increases the risk that some of those are untenable (section 8.2).

Several mutually non-exclusive strategies exist for the reduction of the number of conditions (Amenta and Poulsen 1994). For instance, higher-order constructs can be created (Ragin 2000: 321–28) through so called master or macro-variables (Berg-Schlosser and De Meur 1997; Rokkan 1999). Furthermore, the selection of conditions and the conceptualization of the outcome should occur via an iterative dialogue between prior theoretical knowledge and empirical insights gained during the research process (section 11.4 and Schneider and Rohlfing in press). Since social science theories, by and large, provide only limited guidance as to exactly which conditions to choose, the ongoing refinement and reduction of the number of conditions forms an integral part of a good QCA.

11.1.3 The choice of the QCA variant

The choice between different types of QCA depends on whether the types of concepts involved and the empirical data at hand lend themselves to being captured in crisp or fuzzy sets. Whenever feasible, fuzzy sets should be used. They contain more information than crisp sets (section 1.1.2) and set higher standards for subset relations (Chapter 5). Needless to say, if a concept by its very nature presents itself in a dichotomous form, then it must be represented by a crisp set. Crisp-set conditions, though not crisp-set outcomes, can easily be integrated into an fsQCA, whereas the inverse is not true. The choice of multi-value QCA should be handled with care for reasons specified in section 10.2. The choice of the QCA variant is *not* driven by the number of cases.

11.1.4 Calibration of set-membership scores

The calibration of set-membership scores should be discussed and documented in detail (Ragin 2008a, 2008b: chs. 4 and 5). The most important issue here is which criteria are used in order to qualify cases as members of a set. Theoretical knowledge external to the data is needed in order to determine and justify where the qualitative anchors 0, 0.5, and 1 are located (or the cut-off between 0 and 1 in csQCA, respectively). Beyond this, explicit arguments should also be provided which empirical evidence qualifies for the differences

in degree in set-membership scores that are established between cases on the same side of the 0.5 qualitative anchor. Membership values can be attributed to cases by using conventional forms of index-building or through the direct or indirect method of calibration (section 1.2.3 and Ragin 2008a: ch. 5). Using multiple empirical sources for calibrating one single set is encouraged.

11.1.5 Analysis of necessary conditions

The analysis of necessary conditions should be separate from and should precede the analysis of sufficient conditions. Statements about necessity should be made only if specific tests for necessity have been performed; necessity must not be automatically inferred from the results of a sufficiency analysis (section 9.1). The consistency values for necessary conditions have to be higher than those for sufficiency. A threshold of 0.9 or even higher is recommended. If researchers make use of the possibility to combine single conditions through a logical OR in order to create a new set that then passes the consistency test as a necessary condition (functional equivalents), such new OR conditions need to be carefully justified on theoretical grounds. Only those conditions should be declared necessary that pass the test of relevance and are thus not trivially necessary (sections 5.5 and 9.2.1). When statements of necessity are made, researchers, in the analysis of sufficiency, must avoid assumptions that are incoherent with those statements of necessity, something that is achieved by applying the Enhanced Standard Analysis (section 8.2.1).

11.1.6 Analysis of sufficient conditions

For the analysis of sufficiency, the researcher should always directly consult the truth table and decide for each row whether it is a logical remainder and, if not, whether it can be interpreted as a sufficient condition for the outcome of interest. According to the Truth Table Algorithm (Chapter 7), these decisions are based on the number of cases in each row and its level of consistency. We add to this that researchers should keep an eye on simultaneous subset relations (section 9.2.2) and the presence of true logically contradictory cases (section 5.2) before declaring a non-remainder row sufficient for the outcome.

11.1.6.1 Threshold level for raw consistency values

Logical contradictions in the truth table should be resolved prior to the minimization process. The raw consistency score of each truth table row has to be

stated explicitly and the minimum threshold for the raw consistency has to be reported. As mentioned, true logically contradictory cases (section 5.2) might be hidden behind the consistency score and should be identified. The presence of true logical contradictions should make researchers more hesitant in assigning the status of sufficient condition to this row.

The choice of an appropriate level of consistency for a sufficient condition is specific to every individual research project and needs to be explicitly justified. It varies with the number of cases in the research; the knowledge the researcher has about the cases; the quality of data; the specificity of theories and hypotheses at hand; the research aims; whether there is a large gap in consistency between two groups of rows; and whether a row contains logically contradictory cases. In general, consistency levels (well) above 0.75 are advisable. Including rows with a consistency below 0.5 does not make any sense at all, since there is more evidence against the claim of sufficiency than in favor of it.

11.1.6.2 Logical remainders and choice of solution terms

The treatment of logical remainders should be transparent. This requires, as a first step, to specify whether logical remainders exist and, if so, what type(s) of logically possible configurations are not observed empirically (preferably expressed in a Boolean expression). Researchers should bar from any logical minimization all logical remainders that would lead to untenable assumptions (section 8.2). The directional expectations, both about single conditions and (theory permitting) conjunctions of conditions (section 8.3), should be explicitly formulated. After that, the conservative, the enhanced most parsimonious, and the enhanced intermediate solution term should be reported. Usually, the enhanced intermediate solution should be at the center of the substantive discussion.

11.1.6.3 Analysis of the negative outcome

The outcome and its negation should always be dealt with in two separate analyses. DeMorgan's law can only be meaningfully applied for generating the solution formula for the non-occurrence of the outcome if, and only if, the truth table does not contain any logical remainders and all truth table rows show perfect values for consistency (i.e., 0 or 1) – in other words, hardly ever in empirical research based on observational data (section 5.1). When analyzing both the occurrence and the non-occurrence of the outcome with the same conditions, care must be taken not to include the same truth table row(s) in both minimizations. This can either happen through contradictory assumptions on

logical remainders (section 8.2.2) or through the same empirically observed row passing the consistency thresholds in both the analysis of the outcome and its negation (section 9.2.2). Often, however, it is required to alter the selection of conditions and thus construct a new truth table when shifting the analysis from the occurrence to the non-occurrence of the outcome.

11.1.7 Presentation of results

More than one presentational form should be used. Collectively, they should convey information on three interrelated, yet different aspects of the analysis: (1) which conditions account for the outcome; (2) which cases are (not) accounted for by which part of the solution; and (3) how well does the solution fit to the underlying empirical evidence (Schneider and Grofman 2006)? In order to depict both the case- and the condition-oriented aspects of QCA, researchers should resort to the full repertoire of presentational forms. The most important *graphical* forms of presentation are Venn diagrams (e.g., Figure 3.2) and XY plots (e.g., Figure 3.5).[3] *Tabular* presentational forms are truth tables (section 4.1) and tables displaying each case's membership in all sufficient paths, in the overall solution term, and the outcome (e.g., Table 5.3). Each published QCA should provide access to the truth table(s) and each case's membership in the single sets and the outcome. Another indispensable presentational form is the *solution term*. In addition to stating the result in a Boolean expression, researchers should add the following information: for each path and the entire solution term, the parameters of fit (consistency and coverage) (Chapter 5); for each path, the uniquely covered cases (section 5.3) and the true logically contradictory cases (section 5.2); and for the entire solution, the uncovered cases that are more in than out of the outcome set. The solution formulas in section 11.2 provide an illustration of how to present these pieces of information.

11.1.8 Interpretation of results

11.1.8.1 Focus on cases

Solution formulas and high parameters of fit should not be seen as the ultimate goal of a QCA. Instead, they need to be related back to the individual

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[3] Two potentially useful, yet so far hardly ever used, graphical presentational forms are tree diagrams as suggested in Schneider and Grofman (2006), which might work particularly well in order to display sequence of conditions (section 10.3.3) or table 11.5 in Ragin and Fiss (2008), which distinguishes between core and contributing causal conditions.

cases. If cases disappear behind computer-based algorithms and parameters of fit, the method loses one of its major strengths. Researchers should make clear which cases – mentioned by their proper names – are (uniquely) covered by which of the paths in the solution formula (typical cases), and which are responsible for lower levels of consistency or coverage (deviant cases; see also section 11.3).

11.1.8.2 Focus on parts of the solution

Almost by default, set-theoretic solutions are composed of multiple conjunctural terms. Only paths that pass a pre-established consistency threshold should be interpreted. The consistency of the overall solution term is less relevant. Researchers should always give explicit justifications in situations where one (or more) of the paths towards the outcome are deemed more important for the substantial conclusions than others. Theoretical importance often deviates from empirical importance (coverage) and might be more interesting. Sometimes, an empirically less important path that covers only a few cases, or even only one, can be theoretically and substantively more interesting and important than paths with high coverage values. Focusing on single conditions in equifinal and conjunctural QCA solution terms usually runs counter to the logic of this method. In a causally complex solution, single conditions are INUS conditions which possess causal relevance only in combination with other conditions. If, however, in a given research field strong consensus prevails that a particular individual condition is indispensable for producing (or preventing) the outcome, then a researcher might exceptionally want to pay tribute to this prominence in the interpretation of the QCA results. The researcher might be able to conclude from a QCA that the single condition in question does not prevent or is not indispensable (necessary) for the occurrence of the outcome.

11.1.9 Reiteration of the research cycle

It cannot be reiterated enough that QCA is both a research approach and a data analysis technique. Once the data has been analyzed according to the guidelines above, a researcher might find reasons to go back to the beginning and redesign the research. Based on the evidence, the scope conditions of the study might be altered and cases thus added or dropped; conditions might be added, dropped, or reconceptualized, and thus some cases' membership in these sets might be changed (Rihoux and Lobe 2009; Schneider and Rohlfing in press). A good (but not the only) starting point for such a redefinition is

to have a look at the deviant cases from the QCA solution term. Due to the asymmetric nature of set relations, researchers need to distinguish between different types of deviant cases. There are deviant cases with regard to the statements of necessity and for sufficiency, respectively. Furthermore, for each of these two types of set relations, there are deviant cases with regard to consistency and coverage, respectively. Each of these types of deviant cases carries a different analytic meaning and suggests different changes to the QCA (section 11.4, and Schneider and Rohlfing in press).

Part of the notion of QCA as an approach implies that QCA-based research is inherently multi-method research. Findings generated by QCA as a technique alone are less convincing than those that are followed by other analyses, most likely, but not exclusively, within-case studies of cases identified as typical and deviant by the QCA.

11.1.10 The use of software

The data analysis should be performed with the help of adequate software. Sets need to be calibrated; data matrices displayed in a truth table format; consistency and coverage values calculated; logical remainders identified and classified; assumptions about these remainders made; information in truth tables logically minimized; solution terms displayed in a graphical form. Doing all this by hand is difficult, if not impossible. In the chapter-by-chapter online appendix (www.cambridge.org/schneider-wagemann), we therefore provide detailed instructions on the use of the different software packages.

Table 11.1 provides a summary of the analytic features that can currently be performed with the existing programs: fsQCA 2.5 (Ragin and Davey 2009), Tosmana 1.3.2 (Cronqvist 2011), ado file "fuzzy" in Stata (Longest and Vaisey 2008), and the packages "QCA" (Dusa and Thiem 2012), "QCAGUI" (Dusa 2012), and "QCA3" (Huang 2011) in R. We differentiate between essential, presentational, and convenient tools.

fs*QCA* 2.5 is the oldest and currently most widely used software, most probably due to several characteristics. It produces all the standard parameters of fit; indicates which cases are members of the different sets involved in a solution formula; and allows for a separate analysis of necessity. XY plots are easily produced; subset/superset analyses can be quickly performed, a particularly useful feature when hunches on specific conjunctions of conditions exist; it is freeware; performs the direct and indirect calibration; and it allows for the specification of directional expectations and thus produces the intermediate solution term. On the downside, fsQCA 2.5 performs less well in terms of

Table 11.1 Synopsis of software packages for performing set-theoretic analyses

Analytic features	fsQCA 2.5	Tosmana 1.3.2	Stata	R*
Essential				
Codable truth table	X		(X)	
Intermediate solution term	X			
Automatic exclusion of untenable assumptions				
Parameters of fit	X		X	X
Analysis of sufficiency in csQCA	X	X	X	X
Analysis of sufficiency in fsQCA	X		X	X
Performance of mvQCA		X		X
Analysis of necessity	X		X	X
Display of results				
Venn diagram		X		(X)
XY plot	X		X	X
Identification of cases in solutions	X	X	X	X
Listing cases in truth table rows		X		X
Listing simplifying assumptions		X		X
Convenience attributes				
Fuzzy-set calibrations	X		X	
Syntax-based			X	X
Freeware	X	X		X
Boolean calculator		X		(X)
Open platform for developers			X	X
PRI and PRODUCT	X			
Statistical measures of fit			X	
Compatible with all major operating systems		X	X	X

* refers to packages QCA (0.6–5), QCAGUI (1.3–7), and QCA3 (0.0–4)
(X) not foreseen in specific package but possible within the software environment

user-friendliness. It is not syntax–based; inferior in data handling (such as creating new or manipulating existing datasets); slow when the number of cases is high; prone to break down quite often; and incompatible with several important operating systems.

These partial shortcomings can be circumvented by resorting to other packages for specific tasks. Tosmana 1.3.2 is the only software that has a ready-made tool for producing Venn diagrams and a Boolean calculator. In addition, it lists all simplifying assumptions that went into producing the most parsimonious solution term; yields truth tables with case labels in the respective rows; and can handle crisp sets and multi-value data (section 10.2), though not fuzzy sets. The

biggest shortcoming is that Tosmana neither calculates the parameters of fit nor allows for the specification of directional expectation. This prevents Tosmana from being used as the only software when the Truth Table Algorithm and the Standard Analysis option should be applied, which is virtually always.

The ado files and packages in Stata and R, respectively, are the most recent additions to the software tools in set-theoretic analyses. As Table 11.1 shows, the fuzzy ado file provides several useful functions, yet the QCA packages in R provide even more and are kept more up-to-date. In fact, the recently updated package QCA 1.0-3 not only mimics all core features of the fsQCA 2.5 software. It is also designed for handling multi-value data; offers greater flexibility in the calibration functions; and provides a more encompassing strategy for the analysis of necessity (Thiem and Dusa 2012). In addition, being inserted in software environments that allow for a vast amount of operations, allows for more flexibility, at least to those familiar with R (or, to a lesser extent, Stata). Stata and R have advantages when large-N data are analyzed; when it is important to save the command syntax; when more information needs to be added to an XY plot; and/ or when probabilistic measures of set relations are employed (section 5.2).

In conclusion, several points are worth emphasizing. First, over the past years, set-theoretic software packages have been quickly evolving. This implies that ours is a snapshot of the current state of the art.[4] Second, no single package is capable of performing all the analytic tasks that are required for a good QCA. Currently, fsQCA 2.5 and R (package QCA) offer most functions. In fact, a good QCA cannot be performed without the use of at least one of these two packages. In applied QCA, most users still apply fsQCA, Tosmana, or both (Rihoux et al 2012). This use of multiple software packages will most likely continue in the foreseeable future. While among users the popularity of R is likely to increase due to the recently improved QCA package, it is improbable that it will soon become the modal software choice in set-theory based research – a status the R platform has not yet even gained within the statistical camp of the social sciences.

11.2 Robustness and uncertainty in QCA

Our recipe for a good QCA has already made reference to those moments during the process of performing a QCA where researchers are confronted

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[4] In the chapter-by-chapter How to sections of the online appendix, we discuss in detail the different functions on the various software packages (www.cambridge.org/schneider-wagemann).

with decisions for which a considerable margin of discretion exists. Cases and conditions are dropped and added, calibration functions changed, consistency thresholds altered, etc. Depending on these decisions, the result obtained may change. This raises the question of how robust QCA findings are, given that it is an inherent feature of this method that researchers end up retaining a large amount of leverage on all these decisions rather than outsourcing them to standards commonly agreed on in the scientific sub-communities, as is usually done in quantitative methods.

We do believe that QCA findings can and, in fact, should be subjected to robustness tests. These tests, however, need to stay true to the fundamental principles and nature of set-theoretic methods and thus cannot be a mere copy of robustness tests known to standard quantitative techniques. Despite some valuable attempts (Goldthorpe 1997; Lieberson 2004; Seawright 2005; Marx 2006; Skaaning 2011), the topic of robustness has so far not received enough systematic treatment in the QCA literature.

In this section, we focus on those decisions in the research process that are unique to QCA as a set-theoretic method and for which researchers usually possess enough discretion to warrant doubts about whether results would change substantively if (slightly) different, yet equally plausible decisions were taken. We are referring to the process of set-membership calibration (location of qualitative anchors and choice of functional form); the choice of consistency levels for truth table rows; and the choice to add and drop single cases.[5]

11.2.1 How do we see robustness in set-theoretic methods when it is there?

In multivariate regression, the notion of robustness is, by and large, uncontested. If the significance, direction, and strength of coefficient(s) remain unaffected across different model specifications and different samples, then the findings on the effect of the variable(s) of interest are deemed robust. In QCA, the notion of robustness needs to be extended. In QCA, solution terms can be deemed robust if they involve similar necessary and sufficient conditions and if consistency and coverage are roughly the same across different model specifications. By "similar" we mean solutions that are in a clear

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[5] The treatment of logical remainders is also consequential for the result obtained. However, we do not include this issue in this debate on robustness, because, first, we have provided an extensive treatment of it in Chapters 6 and 8. Second, as explained in those chapters, all solutions obtained through different assumptions about logical remainders are supersets of the empirical evidence at hand as expressed in the conservative solution term. This, in contrast, is not always true for different decisions on case selection, calibration, or consistency thresholds.

subset relation and parameters of fit that do not warrant different substantive interpretations.

We therefore suggest two set-theoretic-method-specific dimensions of robustness: the differences in the parameters of fit and the set-relational status of the different formulas. First, if different choices lead to differences in the parameters of fit that are large enough to warrant a meaningfully different substantive interpretation, then results are not robust. If, however, differences in consistency and coverage are too marginal to provide the basis for meaningfully different substantive interpretations, then the results can be considered robust. Second, if different choices lead to solution terms that are not in a subset relation with one another, then results are not robust. If, however, there is a clear subset relation between different solution terms, then results can be interpreted as robust, even if these solution terms look quite different on the surface. In the following, we discuss the consequences of changing consistency levels, case selection, and calibration functions on the subset relation of sufficiency solution terms[6] and their parameters of fit.

In order to illustrate our arguments, we use Emmenegger's (2011) analysis of job security regulation in Europe in 19 Western democracies. In the article,[7] the outcome of interest is high levels of job security regulations (JSR). Six conditions are specified: strong state–society relationships (S); dominant non-market coordination of the economy (C); strong labor movement (L); dominant Catholic religious denomination (R); strong religious parties (P); and strong presence of veto points in the political system (V). Emmenegger opts for theory-guided calibration for the outcome JSR.[8] He relies on quantitative information, without, however, using a mathematical transformation. His calibration alters the rank order of the original values, and he justifies this with in-depth knowledge of the recalibrated case combined with theoretical considerations, an issue we address in further detail in section 11.2.2.

In the analysis of sufficient conditions,[9] the decision on where to set the raw consistency threshold is difficult.[10] All rows but one with enough empirical evidence are either highly consistent or highly inconsistent as sufficient conditions for the outcome. One truth table row, however, denoting the

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[6] We do not offer a discussion on the robustness of an analysis of necessity, both for reasons of clarity and because it is less intriguing than the robustness of sufficiency analyses, mainly because the analysis of necessity almost exclusively focuses on single conditions rather than combinations thereof (section 3.2.1.2).

[7] See the online appendix for more details (www.cambridge.org/schneider-wagemann).

[8] See section 1.2.2 for the calibration of the condition high number of veto points.

[9] No necessary conditions are found.

[10] Another striking feature not dealt with here is the high number of logical remainders: 51 out of 64 truth table rows.

combination ~SCL~R~P~V has a raw consistency value of 0.84[11] and contains two cases with membership higher than 0.5, which, however, display qualitatively different membership scores in the outcome – Sweden (0.6; 0.86) and Denmark (0.6; 0.29). Consequently, including this row into the logical minimization gives rise to a true logically contradictory case (Denmark), whereas excluding the row means that Sweden will be an uncovered case.

We follow Emmenegger both in excluding this row from the logical minimization and his directional expectations.[12] This setup leads to the following intermediate solution:

	SR~V +	SRPL +	SRPC +	LCP~V → JSR
Raw coverage	0.402	0.354	0.277	0.297
Unique coverage	0.152	0.027	0.041	0.138
Consistency	0.990	1.000	0.965	0.964
Covered cases*	F, P, I	E, I, AU	BE, D, AU	N
Solution consistency	0.97			
Solution coverage	0.69			
Uncovered cases**	S, NL			

* Cases with membership in path > 0.5
** Cases with membership in solution < 0.5 and outcome > 0.5

The comparatively low coverage value is mainly due to two cases with high values in the outcome and low membership in the solution formula: Sweden and the Netherlands. In the following, we gauge how robust these findings are vis-à-vis changes in the calibration of the outcome and some conditions; the raw consistency threshold; and the dropping of specific cases.

11.2.2 The effects of changing calibration

Both in csQCA and in fsQCA, changing the threshold that determines whether a case is (more) in or (more) out of a set implies that some cases will also change the truth table row to which they belong (section 4.2). This, in turn, might change a row from a logical remainder into an empirically observed row (or vice versa), or from an inconsistent row into a consistent one (or vice versa). The choice of the 0.5 qualitative anchor is therefore the most important decision to be taken in the calibration process.

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[11] The neighboring truth table rows, ordered by raw consistency values, show values of 0.94 and 0.72, respectively.

[12] All conditions are assumed to contribute to JSR in their presence, only V in its absence.

While the precise effect on the solution of shifting the qualitative anchor of set membership depends on characteristics of the data at hand and other decisions taken during the analysis, some general implications can be formulated. If the threshold for membership in condition X is raised, fewer cases will be member of X. The coverage of all those sufficient paths in which X is an INUS conditions either decreases or stays the same, but will never increase. The consistency of such paths remains the same or increases, but it never decreases. Notice, however, that raising the threshold for membership in X automatically implies lowering it for ~X. Thus, for any path that involves ~X as an INUS condition, coverage will either increase or stay the same, but never decrease, while consistency will remain the same or decrease but never increase. In general, whether the change of the 0.5 qualitative anchor of INUS condition X (or ~X) has an effect on the parameters of fit of a sufficient conjunction depends on whether X (or ~X) provides the minimum score for that conjunction. If it does not provide the minimum, nothing will change.

Predicting the consequences of changing the qualitative anchor for an *equifinal solution term* is impossible. If both X and ~X are INUS conditions in some of the paths, then parameters of fit might increase or decrease, depending on the specific data at hand. The only statement that can be made is this: if the consistency of a truth table row involving either X or ~X decreases below the consistency threshold for sufficient conditions, then a change in qualitative anchor produces a different solution term. This solution, however, will be a subset of the previous solution. If the consistency of the truth table row instead increases beyond the threshold, then the new solution will be a superset of the previous one.

With fuzzy sets, next to establishing the cross-over point, two further calibration-related decisions have to be taken: first, the precise position of the two other qualitative anchors (full membership and full non-membership) and, second, the functional form with which the underlying raw data translate into set-membership scores given the qualitative anchors chosen. It is important to realize that, as long as the 0.5 anchor remains unchanged for conditions, none of these decisions can alter the truth table row to which a case belongs. As a consequence of this, these calibration decisions can be expected to hardly ever[13] have any substantively relevant effect on the solution term. They can, however, have an effect on the parameters of fit.[14]

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[13] We say, "hardly ever" because a change of the functional form could trigger a change in the raw consistency that is big enough to shift a truth table row from being consistent enough to being too inconsistent (or vice versa) to be included into the logical minimization. If the qualitative anchors are fixed, however, such an effect should rarely ever occur.

[14] Thiem's (2010) study on the interactive effect of different calibration functions, on the one hand, and various locations of the 0.5 qualitative anchors, on the other, on the coverage of *single consistent*

In sum, changes in the calibration do bring about changes in the parameters of fit. Most of the time, though, such changes are too small to be substantively meaningful. In addition, the set relation between the original and the altered solution is usually maintained. Only when major changes to the location of the cross-over point are made can it happen that solution terms are different. This sensitivity of QCA results to the choice of the 0.5 anchor must be put in perspective, though. Remember, the decision whether to consider a given case as a member of a set establishes a qualitative difference between cases (section 1.1.1). Since set-theoretic methods are all about qualitative differences and similarities, they ought to be sensitive to such decisions. In general, researchers are advised to run different analyses with slightly modified calibration criteria. Often, the change will only be moderate. If so, this message should be conveyed to the reader. If the change is more substantial, then the fine tuning and the subsequent justification of the calibration decisions are even more important.

Applied to the example by Emmenegger, we present the effect of three modifications to the calibration.

Recalibrating condition R: if we assign a fuzzy value of 0.55 instead of 0.4 to Denmark, Finland, Norway, and Sweden in condition religious denomination (R) – i.e., if we change the qualitative characteristic of these cases and consider them to be more in than out of the set of countries with a dominant Catholic religious denomination – then the solution formula is:[15]

	SR~C~V +	SRLP +	SRCP +	LCRP~V → JSR
Raw coverage	0.329	0.354	0.277	0.283
Unique coverage	0.152	0.027	0.041	0.124
Consistency	0.988	1.000	0.965	0.963
Covered cases*	F, P, I	E, I, AU	BE, D, AU	N
Solution consistency	0.96			
Solution coverage	0.67			
Uncovered cases**	S, NL			

* Cases with membership in path > 0.5
** Cases with membership in solution < 0.5 and outcome > 0.5

We see that two paths are identical to two paths of the original solution while the other two paths are slightly different. However, this solution is in a

sufficient conditions shows, in essence, that the choice of the calibration function is inconsequential as long as one is not dealing with highly skewed single-set-membership scores.

[15] For reasons of comparability with the original result, the threshold for raw consistencies is set at 0.9.

perfect subset relation with the previous one. Yet another observation is that all consistency and coverage parameters change only slightly.

Recalibrating New Zealand's membership in outcome JSR: Emmenegger mainly uses the OECD Employment Protection Legislation Indicator (EPLI) as the empirical evidence for calibrating set membership in outcome JSR. Respecting the rank order of the EPLI, New Zealand would have received a score of 0.29 but Emmenegger assigns a score of 0.14 to it by using additional empirical and theoretical arguments.[16] Let us therefore rerun the analysis with a score of New Zealand of 0.29 in JSR. Virtually nothing changes. The four paths of the intermediate solution remain identical, and there are only very marginal decreases in the solution consistency (0.96) and coverage (0.67). Most of the consistency values and raw and unique coverages only change after the second decimal, i.e., too small to be of any substantive meaning.

Recalibrating Denmark's membership in JSR: in Emmenegger's data, Denmark (0.29) scores low in the outcome set, whereas Sweden (0.86) scores high. This makes sense, given the Danish "flexicurity" system. Both cases are located in the same truth table row, giving rise to a true logical contradiction (section 5.2). What happens to the solution term if a researcher was misled by stereotypical thinking about uniformly rigid labor market regulations in Scandinavia and thus assigned the same high set membership in the outcome to Denmark as Sweden? Such a mistaken recoding of cases, which after all implies a qualitative change of Denmark because it crosses the 0.5 qualitative anchor, does indeed affect the solution formula:

	SR~V +	SRLP +	SRCP +	LC~S~V → JSR
Raw coverage	0.380	0.335	0.262	0.298
Unique coverage	0.144	0.026	0.039	0.298
Consistency	0.990	1.000	0.965	0.957
Covered cases*	F, P, I	E, I, AU	BE, D, AU	DK, N, S
Solution consistency	0.97			
Solution coverage	0.75			
Uncovered cases**	NL			

* Cases with membership in path > 0.5
** Cases with membership in solution < 0.5 and outcome > 0.5

Solution consistency is similar to the original formula, but the solution coverage is higher. In addition, three of the four paths remain identical; only

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[16] These refer to New Zealand's low level of regulation of collective dismissals, its low company-based protection, and its low collective bargaining coverage.

the original path LCP~V has changed to LC~S~V. It covers Norway (which had also been covered by the previous solution), but now also Denmark and Sweden, the cases hitherto uncovered by the original result. Hence, changing Denmark's outcome value increases the raw consistency of the truth table they belong to, which therefore is included in the logical minimization, thus producing a solution in which both cases are covered. Note that this revised solution is not in a perfect set relation with the original one.

These examples demonstrate that changes in the calibration tend not to trigger all-too-severe changes. This should give some pause to those who suspect that in QCA specific results can be produced at the discretion of the researcher by simply manipulating the data accordingly. This is neither a feasible strategy, nor a credible suspicion. First, the range of discretion in calibration is limited due to the substantive meaning of the concepts represented by sets. Absurd membership scores will be spotted by critical readers. Second, as long as it stays within a plausible range, the effects of recalibration are rather limited and small. And finally, these effects are largely unpredictable, so that researchers would have to engage in a time-consuming trial-and-error iteration of calibrating all their sets in order to produce a desired solution.

11.2.3 The effects of changing consistency levels

Throughout a QCA, another crucial decision to be taken by researchers is the choice of the raw consistency threshold for truth table rows (sections 5.2 and 7.2). Compared to changes in the calibration, changes to the raw consistency threshold have clear and predictable effects on the solution formulas obtained.

By increasing the raw consistency threshold, fewer truth table rows are used for logical minimization. As a consequence, the new solution will (a) be more consistent, (b) show lower coverage, and (c) be a perfect subset of the solution generated based on a lower raw consistency threshold. In contrast, by lowering the consistency thresholds, the solution term will (a) be less consistent, (b) show higher coverage, and (c) be a true superset of the solution term based on a higher threshold.[17] Note that both points (c) apply only to the conservative formula and not to the most parsimonious or any intermediate solution term. This is because for the latter two different logical remainders

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[17] Whether this new result is more complex than the previous one depends on the nature of the truth table row, which is either included in nor left out of the minimization process due to the change in consistency levels. Scenarios can be imagined in which raising the consistency thresholds adds a single truth table row to the result that cannot be further minimized, thus adding to the complexity of the solution.

might serve as simplifying assumptions once different truth table rows pass the consistency test.

Given that there is a rather straightforward effect of the raw consistency threshold on the parameters of fit and the set-relational features of the result obtained, researchers should always run their analyses with at least two different raw consistency thresholds and see if parameters of fit dramatically change. As in recalibrating sets, here the range of discretion is also limited, though. First, any raw consistency level should be above the lower-bound benchmark of 0.75. Second, if there is a clear gap in raw consistency levels across truth table rows which lends itself to choosing the raw consistency threshold (Ragin 2008b: ch. 7), then a robustness test requires the choice of one threshold above and another one below this gap.

When testing robustness against different raw consistency thresholds, researchers should always check which cases are affected by these changes: which of them become uncovered by the solution due to a higher consistency threshold and which contradict the statement of sufficiency due to a lower consistency threshold? Among the latter, does the lowering of the consistency threshold produce true logically contradictory cases (section 5.2)?

As mentioned when introducing Emmenegger's (2011) analysis (section 11.2.1), the status of one truth table row (~SCL~R~P~V) as a sufficient condition is subject to debate, with a consistency of 0.84 and the true logically contradictory case of Denmark (membership in row: 0.6; in JSR: 0.29). If we lower the raw consistency threshold from its original value of 0.9 to 0.8 – undoubtedly still an acceptable value – we now include this row in the logical minimization and obtain the following new intermediate solution:

	SR~V +	SRPL +	SRPC +	LC~S~V → JSR
Raw coverage	0.402	0.354	0.277	0.211
Unique coverage	0.152	0.027	0.041	0.178
Consistency	0.990	1.000	0.965	0.872
Covered cases*	F, P, I	E, I, AU	BE, D, AU	N, DK, S
Solution consistency	0.94			
Solution coverage	0.73			
Uncovered cases**	NL			

* Cases with membership in path of > 0.5
** Cases with membership in solution < 0.5 and outcome > 0.5

Three of the four paths are identical to the original solution (SR~V + SRPL + SRPC). The fourth path now is LC~S~V, whereas in the original analysis it

was LCP~V. In addition, the parameters of fit for most paths are the same as those reported in the original analysis (see section 11.2.1). As expected, the new solution is somewhat less consistent but achieves higher coverage.

Behind these rather small shifts in parameters of fit, some important differences are hidden, though. Denmark is now a true logically contradictory case with a membership in path LC~S~V of 0.67 and in outcome JSR of 0.29.

11.2.4 The effect of dropping or adding cases

One of the key points in a QCA where the researcher must use discretion is the selection of cases to be studied. As mentioned, this is not a one-way street, but rather a back-and-forth process during which cases are added and dropped based on preliminary empirical evidence and updated conceptual insights. While the effect of this case selection procedure on the parameters of fit follows some systematic patterns, its effect on the set relation of solution terms is impossible to generalize.

Dropping a case that contradicts a statement of sufficiency increases consistency and leaves coverage substantively unaffected.[18] Likewise, dropping a case that is (partially) uncovered, i.e., a case whose membership in the outcome exceeds that in the solution term, increases coverage and leaves consistency substantively unaffected.[19] Finally, dropping a typical case, i.e., a case with virtually equal (and relatively high) membership in both the solution and the outcome should leave both consistency and coverage substantively unaffected. Whether the dropping of deviant cases changes the subset relation of the solution formulas obtained, however, cannot be anticipated. Rather, it depends on whether the inclusion or exclusion of a case changes the raw consistency level of the truth table row to such an extent that it is (not) included in the logical minimization or whether a row turns from a logical remainder into a (consistent) row with enough empirical evidence, or vice versa.

Applied to the example by Emmenegger, if we exclude Denmark – one of the countries involved in the contradictory truth table row which was not included in the minimization in the original analysis – then the solution term becomes:

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[18] Here and in the following, we say "substantively unaffected" because a change in consistency always marginally affects coverage and vice versa. This is because the sum of all x values is a part of both the formula for consistency and for coverage (sections 5.2 and 5.3). When excluding or adding one or more cases, these sums change, even when the cases are typical.

[19] Below (section 11.4.1) we label these as deviant cases with regard to consistency and coverage, respectively.

	SR~V +	SRLP +	SRCP +	LC~S~V → JSR
Raw coverage	0.414	0.365	0.285	0.255
Unique coverage	0.157	0.028	0.042	0.187
Consistency	0.990	1.000	0.965	0.947
Covered cases*	F, P, I	E, I, AU	BE, D, AU	N, S
Solution consistency	0.96			
Solution coverage	0.75			
Uncovered cases**	NL			

*   Cases with membership in path of > 0.5
** Cases with membership in solution < 0.5 and outcome > 0.5

In sum, while it is difficult to formulate general laws of robustness in QCA (see also Skaaning 2011), researchers using QCA should be conscientious in keeping an eye on the robustness of their results while being assertive in claiming that QCA is not vastly inferior to other comparative methods in the social sciences both in terms of robustness and of adequately communicating the degree of robustness. The topic of robustness is intimately linked to the notion of uncertainty. Attaching some indication of uncertainty to one's findings is a must for any scientific research (King *et al.* 1994). In set-theoretic methods, uncertainty can be expressed in several different ways, of which statistical tests of significance are but one (imperfect) mode (see, e.g., Schrodt 2006). For instance, verbal qualifiers attached to fuzzy-set membership scores, such as "almost fully consistent" or "more in than out of a set" denote forms of uncertainty and should be read as such. Furthermore, reporting parameters of fit and verbalizing their meaning in the substantive interpretation of the findings is another way of reporting uncertainty. Finally, researchers should perform and report the results of different robustness tests along the lines discussed above.

At-a-glance: robustness and uncertainty in QCA

The robustness of results generated by QCA is a legitimate concern. Given the specific nature of set-theoretic approaches, the assessment of robustness has to follow a different logic than robustness tests in statistics-based research.

QCA results are sensitive to decisions made by the researcher about issues such as the calibration strategy, the selection of cases, and the choice of consistency thresholds (among others). This sensitivity is both an asset and a potential pitfall. If researchers have very good reasons to classify their cases in a given manner, it is a good thing that the results will then reflect these deliberate decisions made by the researchers.

Most of the discretional decisions taken by researchers have only small effects, if any, as long as they remain within a reasonable and plausible range. The precise consequences usually cannot be foreseen and depend on features of the data and other decisions made in the analysis. Only changes in the raw consistency threshold can be precisely predicted. Increasing (decreasing) the threshold makes the solution more (less) consistent, with lower (higher) coverage, and the conservative solution will be a true subset (superset) of the conservative solution based on a lower (higher) consistency threshold.

11.3 The evaluation of theories in set-theoretic methods

Testing theories is the bread-and-butter business of most quantitative social science research. In contrast, most qualitative research – and, within this group, QCA – is inductive in spirit. Based on some initial hunches, guided by established knowledge and theoretical considerations, conditions are selected that are expected to be relevant for producing the outcome of interest (Amenta and Poulsen 1994; Berg-Schlosser and De Meur 2008). After the analysis, the task usually consists of finding a plausible interpretation of the solution formulas obtained, ideally backed up with further empirical evidence provided through other methods, such as, for instance, within-case analysis (section 11.4). Despite this emphasis on developing rather than testing hypotheses as the goal of QCA, it still seems plausible to ask the extent to which the theoretical expectations prior to a QCA overlap with the empirical results generated by the QCA.

In the following, we first elaborate why the principles and practices of standard hypothesis testing cannot be meaningfully applied to set-theoretic methods (11.3.1). In a second step, we outline how it is possible to evaluate theories while staying true to the principles and practice of set-theoretic methods (11.3.2). Here, we follow ideas first suggested by Ragin (1987: 118–21). The main thrust is that, based on set-theoretic methods, hypotheses are not rejected or supported *tout court*. Rather, the evaluation of theory-guided hunches sheds light on which parts of existing theories are supported by empirical findings; in which direction they should be expanded; and which parts need to be dropped. In a third step (11.3.3), we refine the practice of theory evaluation by arguing that one needs to take stock of the fact that QCA findings are usually not fully consistent, nor do they fully cover the outcome (on parameters of fit, see especially Chapter 5). As we show, taking inconsistency and less-than-perfect coverage into account provides for a more subtle take on the process of theory evaluation in set-theoretic methods.

11.3.1 Why standard hypothesis testing does not fit into set-theoretic methods

Set-theoretic methods, in general, and QCA, in particular, have a strong affinity with classic qualitative research approaches. Qualitative researchers usually aim at formulating plausible accounts for the outcome of interest in precisely specified cases. These accounts can often be seen as hypotheses themselves. They are developed, however, on the basis of empirical findings and thus appear at the *end* of the research process rather than at the beginning. It therefore should come as no surprise that hypothesis testing as understood in the vast majority of applied quantitative methods does not feature among the primary goals of standard applications of set-theory-based methods.

In the process of elaborating plausible accounts of social reality, researchers using set-theoretic methods are required to move back and forth between ideas and evidence (Ragin 2000: ch. 11), a process also sometimes labeled upstream and downstream (Rihoux and Lobe 2009) or pre-QCA and post-QCA research phase (Schneider and Rohlfing in press and section 11.4). The standards of good QCA research practice (Schneider and Wagemann 2010 and section 11.1) dictate that preliminary findings are used as a justification for changing crucial elements of the data at hand: cases and conditions are dropped or added; conditions and the outcome reconceptualized and thus membership scores of cases are altered; or scope conditions (Walker and Cohen 1985) are shaped.

All this is in stark contrast to – and incompatible with – the principles and practices of research based on inferential statistics. They are the cornerstone of most mainstream hypothesis-testing statistical approaches and require, among other things, that the researcher has not screened the data prior to testing the hypothesis. In other words, tests of a hypothesis via statistical significance are valid only if researchers have no second chance to go back to the data and make those adjustments to their model that are purely motivated by the findings of previous statistical tests. All this implies that when using set-theoretic methods, one either adheres to the standards of good practice of carefully crafting the data and thus cannot engage in straightforward hypothesis testing,[20] or one performs proper hypothesis tests, which, however, can only be done by violating the standards of good practice of set-theoretic methods.[21]

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[20] Note that this is not an argument against using statistical inference when assessing claims about subset relations. The point is rather that the latter are a product of set-theoretic empirical research rather than clearly formulated theory-derived hypothesis at the beginning of the empirical research.

[21] Such violation might be justifiable and, if so, should be explicitly argued for. If, for instance, strong interest, strong expectations, or both exist in a particular set relation, this hypothesized relation can be subjected to a statistical test.

Another reason why a direct translation of standard notions of hypothesis testing into set-theoretic approaches is difficult is that set-theoretic scholars tend to subscribe to the notion of causal complexity. Among other things, this implies that they embrace equifinality and conjunctural causation. They are, thus, interested in the role played by various conjunctions of several conditions. While hypotheses about the interaction effect of different variables can be subjected to a statistical test, there are practical limits to how many variables can be interacted in a statistical model. Tests of third-order interaction effects are already quite rare, fourth-order interaction effects virtually absent in the literature.[22]

Yet, the even bigger problem when trying to subject solutions obtained from QCA to a standard hypothesis-testing procedure is that, in QCA, it is frequently postulated that alternative conjunctions are considered to be the causes for a given outcome. We have introduced this as the principle of equifinality (section 3.3.1). In multivariate statistical research rooted in the potential outcomes framework and the experimental template, by contrast, each model allows for only one inference (Morgan and Winship 2007; Brady 2008; Sekhon 2008; Dunning 2010; Gerring 2012). That is, only one variable (or one specific interaction of variables) can be the treatment, i.e., the cause. All other variables are control variables.

Given the incompatibility between the principles and practices of set-theory based research, on the one hand, and hypothesis testing, on the other, how can and should researchers find out whether the initial theoretical expectations are in line with the empirical findings, contradict them, or both?

11.3.2 The basics of theory evaluation in set-theoretic methods

Ragin (1987: 118–21) provides an extensive treatment of how to evaluate theories in set-theoretic approaches, suggestions which, however, have not been frequently used in the literature so far. To illustrate, imagine a hypothetical csQCA on 130 cases.[23] The outcome set is stable democracy (Y). Seventy-eight cases are members of Y. The three hypothetical conditions are a presidential system of government (A), economic prosperity (B), and a British colonial past (C).[24] Let us further assume that, from the existing literature, the following hypothesis can be derived: democratic stability is brought about by

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[22] See Braumoeller (2003) for an attempt at mimicking set-theoretic notions of causal complexity within a statistical framework.

[23] We choose csQCA for didactical purposes but the same principles apply to fsQCA.

[24] The data can be found in the online appendix (www.cambridge.org/schneider-wagemann).

economic prosperity, and/or through a combination of a British colonial past and an absence of presidential government. In Boolean notation, the theory-derived hunches (T) can be written as follows:

Theory (T): B + ~AC → Y.

The analysis of necessity reveals that no single condition or any of their complements is necessary for Y. This can be seen as being in line with the theoretical expectations, which also do not stipulate the presence of a necessary condition. The analysis of sufficiency yields the following solution term:[25]

Solution term (S): B~C + AC → Y.

Our data show that there are two sufficient paths towards the stability of democracies: either by economic prosperity combined with not having had a history of British colonialism (B~C), or by a combination of a presidential system of government and a British colonial past (AC).

Do these results confirm or reject our theoretical expectations? The answer is: both. In order to understand this counterintuitive answer, we separate the initial question into four: for which cases did the theoretical expectations (T) predict and the solution term (S) identify the outcome (TS)? For which cases did both T and S not predict and identify the outcome (~T~S)? For which cases did T predict the outcome while S did not identify it (T~S)? And for which cases did T not predict the outcome while S identified it (~TS)?

There are, thus, $2^2 = 4$ possible different intersections between sets T and S. Each of these intersections highlights a different aspect of the question of how far our empirical results confirm or contradict standing theory. For each area, we provide its Boolean expression, spell out its meaning for the theory at hand, and label cases that are found in these areas.

First, the intersection between theory and the empirical solution (TS) is the area where theory and empirical findings overlap. This is the part of the theory that is supported by empirical evidence. The Boolean expression of the area of overlap is this:

Intersection TS: (B + ~AC) * (AC + B~C)
            = ABC + B~C.[26]

Various insights can be gained from this. Most importantly, it shows that the expectation that condition B on its own is sufficient for Y was much too bold.

[25] No simplifying assumptions are made because, for the time being, our hypothetical truth table (online appendix www.cambridge.org/schneider-wagemann) does not contain any logical remainders.
[26] See section 2.4.2 for the rules of intersecting complex sets.

Rather than being sufficient, B is one among several INUS conditions. It needs to be combined with either AC or with ~C in order to imply Y. Furthermore, we see that also the role of condition A needs to be reassessed. The theoretical literature puts emphasis on the absence of A (i.e., ~A) being an INUS condition. Empirically, we can only confirm that the presence of A is an INUS condition, though. Cases covered by the intersection TS can be interpreted as confirmed most likely cases.

A second interesting area is where empirical findings overlap with those cases not expected by theory (~TS). The Boolean expression for this area is as follows:

Intersection ~TS: (A~B + ~B~C) * (AC + B~C)
             = A~BC.[27]

We thus find some cases with a presidential system (A) in non-prosperous countries (~B) that were formerly a British colony (C) which, contrary to our theoretical expectations, are members of the set of stable democracies (A~BC → Y). Cases in this area are a variant of discovered least likely cases. Results like this can be used to reformulate the existing theory so that it includes these hitherto overlooked cases. In general, the result of the intersection ~TS suggest an extension of existing theories.

Third, the area T~S captures those cases for which theory predicts the occurrence of Y but which our solution does not capture. In our example, the Boolean expression for these cases is:

Intersection T~S: (B + ~AC) * (~A~B + ~AC + ~B~C)
             = ~AC.

Contrary to our theoretical expectations, our cross-case model does not identify the combination of a non-presidential system in a former British colony as a sufficient condition for outcome Y. Cases in this area are also a form of unconfirmed most likely cases. In general, the results of the intersection T~S suggest a delimitation of existing theories.

Fourth, the intersection ~T~S denotes a configuration of conditions that neither theory nor our cross-case findings deem sufficient for the outcome. Applied to our example, the formula for ~T~S reads:

Intersection ~T~S: (A~B + ~B~C)*(~A~B + ~AC + ~B~C)
             = ~B~C.

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[27] One arrives at this formula by first applying DeMorgan's law (section 2.3) on the expression for T in order to obtain ~T and then to intersect ~T with S.

According to standing theory and our empirical evidence, cases that are both not economically prosperous (~B) and not a former British colony (~C) should not be stable democracies (~Y). Non-stable democracies that are members of set ~B~C are in line with theoretical expectations. Yet, due to the asymmetry inherent to set-theoretic methods, they should not be treated as typical cases rather than simply being consistent with the postulated set relation, yet substantively irrelevant cases (sections 3.3.3 and 11.4).

11.3.3 Extending theory evaluation by integrating consistency and coverage

What, however, if there are cases in the intersection ~T~S that are stable democracies (Y)? Or, what if some cases in the intersection TS are not stable democracies (~Y). In other words, how does theory evaluation look if different cases located in the same intersection between T and S display different membership scores in the outcome? The occurrence of such cases is possible whenever the solution term fails to achieve full consistency and coverage, which is virtually always in applied QCA. In the following, we refine the principles of theory evaluation by adding the notions of consistency and coverage to the procedure.

The parameters of fit for the solution term reported above are as follows:[28]

	B~C +	AC → Y
Consistency	0.89	0.88
Raw coverage	0.42	0.37
Unique coverage	0.42	0.37
Solution consistency	0.88	
Solution coverage	0.79	

The consistency of both paths are at an acceptable level (> 0.87) and each path covers a sizable amount of the outcome, without, however, explaining all cases. The solution consistency (0.79) implies that there are 8 cases that contradict the statement of sufficiency, and the solution coverage of 0.79 indicates that 62 of the 78 members of outcome Y are covered, i.e., 16 cases remain unexplained by this solution.

The four cells in Table 11.2 correspond to the four intersections between T and S. For each of them, we not only report the Boolean expression, but

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[28] For the Truth Table Algorithm, a frequency threshold of 1 and a raw consistency threshold of 0.85 were applied.

Table 11.2 Intersections of theory (T) and solution term (S) with types of cases

Theory \ Empirics	Outcome predicted by solution (S) Y: covered cases ~Y: inconsistent cases	Outcome not predicted by solution (~S) Y: uncovered cases ~Y: consistent cases
Outcome expected by theory (T) most likely case	ABC + B~C Y: covered most likely case (58)* ~Y: inconsistent most likely cases (8)	~AC Y: uncovered most likely cases (0) ~Y: consistent most likely cases (4)
Outcome not expected by theory (~T) least likely case	A~BC Y: covered least likely case (4) ~Y: inconsistent least likely cases (0)	~B~C Y: uncovered least likely cases (16) ~Y: consistent least likely case (40)

*   numbers in parenthesis indicate the number of cases found in the empirical example

also distinguish between cases that are members of Y and those that are not. There are, thus, eight different types of cases. Cases in row T are labeled *most likely cases*, whereas those in row ~T are *least likely* (Eckstein 1975; George and Bennett 2005). For cases in column S and ~S, we need to distinguish whether they are instances of Y or ~Y. Following the notions of consistency and coverage (Chapter 5), we label cases that are members of S and Y *covered cases*. Cases that are members of S but also of ~Y are *inconsistent cases*. Furthermore, cases in column ~S that are, however, members of Y are *uncovered cases*, whereas those that are members of both ~S and ~Y are labeled *consistent cases*.[29] With this terminology, we can label the eight different types of cases in the four cells of Table 11.2.

    First, for the intersection TS (ABC + B~C in our example), the claim was that it identifies theory-supporting cases. This, however, is true only for cases that are both members of this intersection and of the outcome Y (i.e., cases of TSY). These are the *covered most likely cases*. Of the 78 members of Y, 58 cases fall into this category. Each of them lends itself to within-case analysis with the aim of unraveling the causal mechanisms that link the sufficient conjunction to the outcome (section 11.4). In contrast, cases in this area that

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[29]   In section 3.3.3 we explained that cases of ~X and ~Y are not directly relevant for assessing set-relational claims involving X and Y. In section 11.4 we call them "individually irrelevant cases." Yet it remains true that these cases are consistent with the statements that X is sufficient for Y and/or that X is necessary for Y.

are not members of Y (TS~Y) strongly contradict both T and S. We suggest labeling them *inconsistent most likely cases*. Eight cases in our data qualify as such. Given that both theory and empirics predict the outcome which, however, does not materialize, within-case analysis of these cases is particularly interesting. It should aim at finding alternative explanations for Y that involve hitherto overlooked condition(s).

Another way of looking at the implications of the fact that cases in the same intersection between T and S do not share the same membership in the outcome is to use the parameters of fit. For each of the four intersections between T and S, we can calculate the consistency and coverage scores as sufficient conditions for Y and ~Y, respectively. The more the expression for TS is a consistent subset of Y, the stronger the support for T. In addition, the more cases are covered by TS, the stronger the support for T.

Applied to our example, calculating the overlap of ABC + B~C as a sufficient condition for Y reveals a consistency of 0.88 and a coverage of 0.74.[30] The consistency score indicates that most cases in TS are also members of Y. Behind the less than perfect consistency score are hiding the eight inconsistent most likely cases mentioned above. The coverage score shows that most members of Y (74 percent, or 58 out of 78) are both predicted by T and covered by S. This implies strong empirical support for that part of the theory that is described by ABC + B~C.

Second, for intersection ~TS (A~BC in our example) the claim was that it identifies cases that suggest the direction in which theoretical expectations should be extended. This, however, holds only for cases that are also members of Y (~TSY). Within-case analysis of one or more of the four covered least likely cases should reveal clues in which direction to extend theory. In contrast, cases that are members of ~TS and ~Y (~TS~Y) – the inconsistent least likely cases – are not puzzling at all from the theoretical perspective at hand and thus weaken the need for any modification of the theory. In our empirical example, no such case exists in the data. Using the perspective of parameters of fit, we can say that the higher the consistency of ~TS as a subset of Y, the stronger the support for extending T. In addition, the more cases of Y that fall into the area of ~TS, the more empirical evidence there is that supports the need for theory extension. Thus, the higher the coverage of ~TS for Y, the stronger the support for extending T.

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[30] These parameters can be easily calculated by using the Subset/Superset function in the fsQCA 2.5 software (see the How-to section for Chapter 5 in the online appendix, www.cambridge.org/schneider-wagemann).

In our example, the consistency of A~BC as a sufficient condition for Y is 1 and its coverage is 0.05. The perfect consistency means that all cases described by this conjunction are, indeed, members of Y. This provides good grounds for extending existing theory. The relatively low coverage of 5 percent (4 of 78 members of Y) puts the need for such a theory extension into perspective, though. An extension of the theory would help to explain only a few more cases. Of course, if these four cases are of particular substantive importance, then there are stronger incentives for a theory extension.

Third, the claim for intersection T~S (~AC in our example) was that it identifies cases that warrant a delimitation of T. This, however, is true only for cases that are members of ~Y (T~S~Y), i.e., so-called consistent most likely cases, of which there are four in our data. In contrast, members of T~S and Y (T~SY) – the uncovered most likely cases – represent support for T and a weakening of the plausibility of S. There are, however, no such cases in the data. Using the parameter of fit perspective, we can say that the higher the consistency of T~S as a sufficient condition for ~Y, the higher the need for delimiting T by dropping ~AC. Also, the higher the coverage of T~S as a sufficient condition for ~Y, the higher the empirical importance of delimiting T.

In our example, the consistency of ~AC as a sufficient condition for Y is 1 and coverage 0.08. The perfect consistency indicates that all (four) cases that are members of ~AC are also members of ~Y. They all provide empirical evidence against the theoretical expectations that postulates conjunction ~AC as a sufficient condition and warrant a reformulation of that part of the theory. The low coverage value, however, indicates that the need for such a reformulation of T is not too pressing.

Finally, for intersection ~T~S (~B~C in our example) we expect to find no cases that are members of Y. If all cases in ~T~S are also members of ~Y – i.e., so-called consistent least likely cases – then there is no empirical evidence that contradicts both T and S. There are, however, also some cases in ~T~S that are members of Y (~T~SY). These 16 uncovered least likely cases are truly puzzling, for neither theory predicts them, nor does the solution term cover them, yet they display the outcome. Their in-depth study should reveal which conditions are missing from both our theory and the empirical model. In general, the higher the consistency and coverage of ~T~S as a sufficient condition for Y, the higher the need for extending both the theory and empirical model by including hitherto overlooked condition(s).

In our hypothetical example, expression ~B~C has a consistency of 0.29 as a sufficient condition for Y and a coverage value of 0.21. The low consistency means that the majority of cases with ~B~C are, indeed, instances of

~Y, which do not challenge our theoretical expectation or empirical findings. Twenty-six percent of these cases do pose a challenge, though. In addition, the coverage value of 21 percent underlines this challenge. It shows that 16 out of the 78 cases with Y are not explained by either our theory or empirical findings. Within-case analysis of these cases is motivated by unraveling hitherto overlooked sufficient paths towards the outcome, rather than just single INUS conditions that should be added to already identified paths.

11.3.4   Summarizing set-theoretic theory evaluation

Theory evaluation in set-theoretic approaches aims at providing nuanced answers to the question of whether initial theoretical hunches are supported by the empirical findings. By creating different intersections between theoretical expectations and empirical findings, clues can be derived about which cases to study in further detail in the next round of the back-and-forth procedure between ideas and evidence. Such intersections can also help in reformulating a theory by either increasing or reducing its parsimony. It is important to point out that in this type of hypothesis testing, theories are not falsified in toto. Instead, it identifies areas, or parts, of the theory that are not consistent with the empirical findings. This makes theory evaluation quite different from hypothesis testing, where the emphasis is usually on rejecting or not rejecting the null hypothesis or a similar benchmark.

When engaging in theory evaluation, we believe it is essential to not only calculate the Boolean expression of the four different intersections that are logically possible between the sets denoted by T and S. In addition, it is of crucial importance to establish how many cases are members of the outcome and the non-outcome in each of these areas. Only once we know how many, if any, cases are members of Y and ~Y in each intersection, can we decide if a given intersection provides the evidence for supporting or challenging our initial theoretical beliefs.

Notice that the example above was devoid of any logical remainders. If logical remainders exist and the solution term (S) incorporates assumptions about some of them, then it is possible that one or more of the intersections between T and S also describe some logical remainders. If limited diversity is present, then, the researcher must not only distinguish between cases of Y and ~Y for each of the four intersections between T and S, but must also identify logical remainder cases. For instance, if the intersection TS almost exclusively covers logical remainders, then this obviously indicates very little empirical support for T. It would mean, in fact, that the solution term

S overlaps with T only because of the assumptions that the researcher has made about the logical remainders. Since these assumptions should be theory-guided, it would, of course, be flawed to claim that in such a scenario S provides empirical support for T.

At-a-glance: the evaluation of theories in set-theoretic methods

Hypothesis testing as understood in the vast majority of applied quantitative methods does not feature among the primary goals of standard applications of set-theory-based methods. However, hunches derived from theory can be evaluated in set-theoretic methods by creating intersections of the Boolean expression describing the theory (T) and the empirical solution (S).

The intersection TS describes the part of the theory that is supported by empirical evidence. In the intersection ~TS, empirical findings overlap with those cases not expected by theory. The result of this intersection suggests an extension of existing theories. T~S captures those cases for which theory predicts the occurrence of Y but which our solution fails to capture; it suggests a delimitation of existing theories. Finally, ~T~S denotes a configuration of conditions that neither theory nor the empirical findings deem sufficient for the outcome.

Integrating the notions of consistency and coverage, we can further refine the theory evaluation procedure and define: (a) covered most likely cases (cases in intersection TS which show Y); (b) inconsistent most likely cases (TS and ~Y); (c) covered least likely cases (~TS and Y); (d) inconsistent least likely cases (~TS and ~Y); (e) uncovered most likely cases (T~S and Y); (f) consistent most likely cases (T~S and ~Y); (g) uncovered least likely cases (~T~S and Y); and (h) consistent least likely cases (~T~S and ~Y).

11.4 Set-theoretic methods and case selection[31]

Much emphasis is put on the importance of intimate case knowledge for a successful QCA (Ragin 1987, 2000, 2008a; Rihoux and Ragin 2009). As a matter of fact, the idea of QCA as a research approach and of going back and forth between ideas and evidence (Ragin 2000) largely consists of combining (comparative) within-case studies and QCA as a technique. So far, the literature has mainly focused on how to choose cases prior to and during but not after a QCA, where by QCA we here refer to the analytic moment of analyzing a truth table. It is therefore puzzling that little systematic and specific

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[31] The ideas expressed in this sub-section draw heavily on the joint work of Ingo Rohlfing and Carsten Q. Schneider.

guidance has so far been provided on which cases to select for within-case studies based on the results of, i.e., after, a QCA (for some initial attempts, see Ragin 2000: 90, 2006: 19; Goertz 2008: 11–12; Rihoux and Lobe 2009). This is odd, for set-theoretic methods and (comparative) case studies share many properties – they largely agree in their epistemology by focusing on complex patterns of causation at the expense of parsimonious-yet-more-generalizable accounts of social phenomena – and multi-method research has become generally popular.

In an attempt to start bridging this gap, in this section, we focus on case selection principles *after* a QCA of sufficient conditions.[32] We show that while the notions of typical and deviant cases can also be applied when the cross-case evidence stems from a set-theoretic method rather than a regression-based analysis, their analytic meaning and their location in the empirical distribution differs in important ways. Depending on which type of case is selected, the aim of the post-QCA within-case analysis is to corroborate or to update the cross-case model (11.4.1). We also specify what researchers need to take into account when selecting cases for comparative process tracing (11.4.2). Finally, we summarize our argument in distinct set-theoretic case selection principles (11.4.3).

Note that in this section we define types of cases purely on their *empirical* properties – i.e., whether they are in line with general tendencies in the data (typical cases) or not (different types of deviant cases) – whereas in the previous section on theory evaluation in set-theoretic methods (11.3) we defined types of cases by intersection *theoretical* expectations about cases with their empirical properties.

11.4.1 Types of cases after a QCA

If we misinterpret an XY plot as a scatterplot between a dependent variable and a summary of independent variables, then we might be tempted to claim that all cases on the main diagonal are typical while all those off the diagonal are deviant. This is the standard (and appropriate) approach in the framework of regression-based case selection. In the framework of set-theoretic methods, however, it is over-simplistic, if not wrong. Crucial for a meaningful post-QCA case selection is the insight that set-theoretic methods impose

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[32] Issues in choosing cases at the very *beginning* of a QCA-based research project are discussed by, for instance, Rihoux and Lobe (2009). Case selection principles after an analysis of necessity follow a similar, yet not identical, logic to that of sufficiency. For a specific treatment of case selection principles after an analysis of necessity, see Schneider and Rohlfing (in press).

qualitative differences between cases and that set relations are asymmetric. Simply put, cases above or below the 0.5 set membership score are qualitatively different (section 1.1.3), and a statement of sufficiency has different implications with regard to membership in Y for cases that are members of X vis-à-vis non-members of X. As a consequence, in set-theory based research, not all cases on the main diagonal are typical, and cases off the diagonal denote different types of deviant cases, depending on which side of the diagonal they fall.

A graphical representation is helpful in demonstrating this point. Figure 11.1 displays an enhanced XY plot.[33] All cases above the main diagonal are consistent with the statement of sufficiency while those below are not (sections 3.1.2.1 and 5.2). Notice, though, that the notion of consistency in fuzzy sets, as represented by the main diagonal, does not take into account qualitative differences between cases. These qualitative differences are established by a case's membership above or below the 0.5 qualitative anchor in condition X and outcome Y, respectively. This is why a two-by-two table is superimposed in the XY plot in Figure 11.1. The latter graphically displays the qualitative difference between cases based on their membership scores in X and Y, respectively. Figure 11.1 is thus divided into six areas, most of them containing types of cases with different analytic meaning with regard to their status vis-à-vis the cross-case pattern. These qualitative differences are crucial for a meaningful selection of cases for within-case analysis.

Typical cases are those that are both in line with the statement of sufficiency (above the main diagonal) and good empirical instances of the outcome Y and condition X (area 1).[34] Cases that are more out of than in both X and Y (lower left quadrant of Figure 11.1, areas 4 and 5) are never directly relevant for assessing the claim of sufficiency (section 3.1.2.1). Cases in area 4 we label "individually irrelevant," for they carry analytic meaning not on their own, but only when used in a comparative within-case analysis (see section 11.4.2), whereas cases in area 5 are never relevant to study, for they are inconsistent with the statement of sufficiency and weak empirical instances of both X and Y. Cases with X > 0.5 and Y < 0.5 (area 3) are true logical contradictions (section 5.2). In the context of case selection principles, they can be labeled "deviant cases consistency." Cases in the upper left quadrant (area 6) are also deviant cases. However, they do not contradict the statement of sufficiency.

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[33] It is important to keep in mind that X is a placeholder for a conjunction of conditions, rather than a single set.

[34] Cases in area 2 are also members of X and Y but contradict the statement of sufficiency. They are therefore not appropriate choices when aiming at studying typical cases.
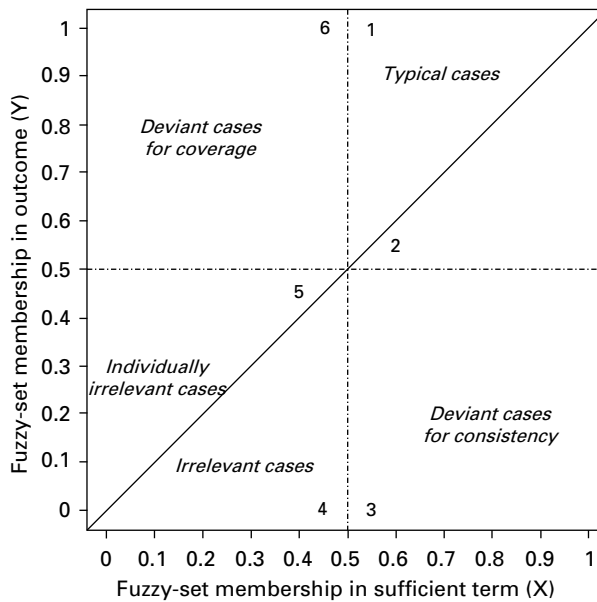
Figure 11.1   XY plot with two-by-two table and types of cases

Instead, they are left unexplained by the cross-case model, since they are good instances of the outcome (Y > 0.5) but not of the sufficient condition (X < 0.5). We therefore suggest the label "deviant cases coverage."[35]

11.4.2 Forms and aims of (comparative) within-case studies after a QCA

Within-case analyses of deviant cases, especially when geared toward updating the cross-case findings, are inherently comparative (Mahoney 1999; Tarrow 2010). There are good reasons for engaging in explicitly *comparative* within-case studies. When selecting cases for within-case studies, researchers must distinguish between the types of cases just introduced. With a statement of sufficiency, only three of the logically possible comparisons between two types of cases are meaningful, and each of these comparisons follows a different analytic aims.

First, a within-case comparative analysis of two typical cases should focus on unraveling the causal mechanisms that link the condition to the outcome

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[35] Within each area defining types of cases, differences in degree can be established based on each case's fuzzy-set membership score. The best-fitting typical case is located in the upper right corner; the most deviant consistency case in the lower right corner; and the most deviant coverage case in the upper left corner. See Rohlfing and Schneider (in press), who propose formulas for identifying the most typical and most deviant cases.

(Shively 2006; Gerring 2010). Comparing two typical cases represents a stronger basis for inference about causal mechanisms than the study of just one. This is especially true if researchers unravel the same causal mechanism in two typical cases that are located at different ends of the diagonal within area 1 in Figure 11.1 – aka two typical cases that differ most in their degree of membership in both X and Y.

Second, a within-case analysis of a deviant case for consistency with a typical case should focus on identifying a condition that is missing from the sufficient path under study. In this condition, the typical case must have high and the deviant case low membership.[36] To see this, let X stand for the conjunction A*B*C. Deviant cases with regard to consistency are those which, given their low membership in Y, have a too high a membership in A*B*C. By adding condition E to the path, a deviant case's membership in the new path is reduced and it is shifted to the left in the XY plot, making it an individually irrelevant case for the statement that A*B*C*E is sufficient for Y.

Third, a deviant case for coverage should be compared to those individually irrelevant cases that belong to the same truth table row. The first step, thus, consists of identifying the truth table row that best describes the deviant case for coverage and the individually irrelevant case (i.e., the one row in which each case has a membership greater than 0.5). By pure logic, this must be a row not implied by path A*B*C. For illustrative purposes, let us suppose it is truth table row A*~B*C*~D. This creates a comparative within-case analysis setup akin to that between a typical and a deviant case for consistency just presented. Hence, the within-case comparison should focus on identifying a condition that is missing from conjunction A*~B*C*~D and distinguishes the deviant case for coverage from the individually irrelevant case. Let us call this condition F. The result of this comparative within-case analysis is the addition of a new *path* – A*~B*C*~D*F – rather than of a single INUS condition to an already existing path of the cross-case model.[37]

Three comparisons are possible, in principle, but futile. First, a comparison between deviant case consistency and an individually irrelevant case is meaningless. Since none of the cases involved is a member of the outcome of interest, no point of reference exists that could inform us about the reasons for the occurrence of the outcome. Second, a comparison between a deviant case for consistency and a deviant case for coverage does not make much sense. There

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[36] A condition in which both types of cases have low membership does not qualify, as it would turn the typical case into a deviant case for coverage vis-à-vis the new path.

[37] Alternatively, condition E is added to the truth table and a new QCA run on that updated truth table.

is nothing puzzling about this pair of cases as they share neither the same membership in the path nor in the outcome.

Third, and perhaps surprisingly, a within-case comparison between a deviant case for coverage and a typical case is also logically flawed, and we discourage it for several reasons. First, adding a condition to path X – as is done in the comparison of a typical case with deviant case for consistency – would exacerbate rather than mitigate the deviance of this type of cases by moving it even further to the left in the XY plot. Second, dropping a condition from path X could shift deviant cases for coverage to the right and thus turn them into a typical case for the new, less complex path. For instance, a deviant case for coverage with regard to path A*B*C, which holds low membership in C, but high membership in A and in B, would become a typical case for path A*B. Notice, however, that this strategy has two flaws. One is that it is diametrically opposed to that suggested by the comparison of a typical case and a deviant case for consistency, where the clear guideline is to *add* a condition to path A*B*C rather than dropping a condition. The other is that such a comparative within-case analysis of a typical case and a deviant case for coverage mimics precisely what is done by the logical minimization procedure (section 4.3.1). This means that if condition C could be dropped from path A*B*C, it would have been dropped already during the QCA-based cross-case analysis. But, apparently, path A*B does not meet the consistency criteria as a sufficient condition.[38] A third argument against a comparison between a typical case and a deviant case for coverage is that path A*B*C simply is not a good point of reference for understanding why deviant cases for coverage display outcome Y, for these cases are, by definition, bad empirical instances of that path. And because just knowing that they are *not* members of path A*B*C is not a good enough start for within-case analysis, we suggest above identifying the truth table row that best describes deviant cases for coverage and comparing them to individually irrelevant cases from the same row.

11.4.3 Post-QCA case selection principles

The strategies for selecting cases after a QCA has been performed are governed by a list of principles (Schneider and Rohlfing in press). Some of them apply to both crisp-set and fuzzy-set QCA, others only to fuzzy-set QCA. Some of them apply to single and comparative within-case analysis, while others are

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[38] This is because by dropping condition C, not only the deviant cases for coverage but also some individually irrelevant cases might be shifted to the right in an XY plot, turning the latter into deviant cases for consistency for path A*B.

Table 11.3 Post-QCA case selection principles

	Crisp-set and fuzzy-set QCA	Fuzzy-set QCA
Single-case and comparative process tracing	*Principle of diverse case selection:* Choose at least one case for each term of the solution. *Principle of unique membership:* Choose cases that are covered by just one term. *Truth table principle for sufficiency:* For the choice of a deviant case for coverage, determine the truth table row to which the case belongs.	*Principle of maximum set membership:* The most typical case displays maximum set-membership scores in the subset and the superset. *Principle of maximum set-membership difference:* The most deviant case displays maximum difference in its set membership in the subset and the superset. *Principle of differences in degree:* Differences in degree should only be established among cases that are similar in kind and located on the same side of the secondary diagonal.
Comparative process tracing	*Positive outcome principle:* At least one case must be a member of the outcome in comparative process tracing. *Truth table principle for necessity:* When comparing a typical case and an individually irrelevant case, choose two cases that differ in their membership in the necessary condition and the outcome, but share the qualitative membership in all other conditions that constitute the truth table.	*Principle of deviance in kind:* Choose deviant cases that are qualitatively different from typical cases in their membership in the superset. *Principle of max–max difference:* When comparing two typical cases, or a typical case with an individually irrelevant case, maximize the difference of the cases' set membership in the superset and the subset. *Principle of maxi–min difference:* When comparing a typical case with a deviant case, maximize the difference of the cases' set membership in the superset and minimize the difference in the subset.

specific to comparative case studies. Unless explicitly mentioned otherwise, each principle applies to both analyses of necessity and of sufficiency.

Table 11.3 contains all 11 principles. In theory, they are not mutually exclusive. That means, under ideal conditions, that researchers are able to perform set-theoretic multi-method research that adheres to all principles. In practice, however, insufficient time and incomplete data might require the choice between adhering to some and not other principles. For instance, researchers simply might not have time to perform within-case analyses of all types of typical and deviant cases. Or the data might not contain cases that would be adequate for one of the comparative within-case strategies outlined above.

At-a-glance: set-theoretic methods and case selection

QCA results provide useful cues for subsequent case selection. We can define typical cases, individually irrelevant cases, and deviant cases for consistency and for coverage, respectively. Apart from within-case analyses of the various types of cases, we recommend comparative analyses of two typical cases in order to unravel the causal mechanisms that link the condition to the outcome; of a deviant case for consistency with a typical case in order to identify a sufficient path under study; and of a deviant case for coverage and those individually irrelevant cases that belong to the same truth table row in order to identify first a condition that is missing from the analysis and then a sufficient path that is missing from the solution formula. Several other types of comparison are futile: comparisons between a deviant case for consistency and an individually irrelevant case; between a deviant case for consistency and a deviant case for coverage; and between deviant case for coverage and a typical case.

This short outline has mapped some of the basic notions that should be kept in mind when selecting cases after a QCA. All the arguments we have presented equally apply to crisp-set QCA. Only when researchers want to identify not simply typical and deviant cases, but, beyond this, the *most* typical and the *most* deviant cases, is the use of fsQCA required (Rohlfing and Schneider in press). For the sake of simplicity, we have not discussed issues of case selection that arise from equifinality, or when the cross-case analysis is one of necessity rather than sufficiency (Schneider and Rohlfing in press). Also, we have exclusively focused on model-related reasons for deviance. Needless to say, within-case studies can also aid not only in the identification of other potential sources of deviance, such as concept misformation, measurement error, or misspecification of the population. The latter sources of deviance are usually dealt with in the pre-QCA phase of research, though, whereas the model-related reasons for deviance pertain more to the post-QCA phase that we have focused on here.

12   Looking back, looking ahead

12.1   Looking back: the main topics of this book

This book started off from the general observation that claims about set relations are pervasive in the social sciences. Set-theoretical methods are thus an important addition to the correlational approach. We have defined set-theoretic methods as those approaches to examining social reality that operate on sets, not variables; model relations between phenomena in terms of set relations rather than covariations; and that put emphasis on sufficient and necessary conditions and their derivates INUS and SUIN conditions, thus unraveling causally complex patterns in terms of equifinal, conjunctural, and asymmetric causation. Within the family of set-theoretic methods, QCA can be distinguished by its explicit use of truth tables; the application of the principle of logical minimization; and its interest in a causal interpretation of its results. We have focused on csQCA and fsQCA as the two main variants of QCA, with csQCA being a special case of fsQCA. mvQCA and tQCA are among the extensions of the main QCA variants.

In Chapter 1, we showed how sets are defined and how set membership is calibrated. Chapter 2 laid the basics for the analysis by introducing the main principles of set theory, Boolean algebra, and the logic of propositions – the three underpinnings on which QCA is built. In Chapter 3, we clarified in great detail the basic notions of sufficiency and necessity, which are at the core of any set-theoretic analysis. A major insight in this chapter was that necessity and sufficiency both denote subset relations, which apply in both crisp and fuzzy sets. Furthermore, we showed that those who stipulate the presence of necessary and/or sufficient conditions unavoidably embrace causal complexity, which in set-theoretic methods is defined in terms of equifinality, conjunctural causation, and asymmetric causality. In Chapter 4, we explained that three steps are needed in order to construct a truth table based on a set

membership data matrix: first, the truth table rows (i.e., the logically possible combinations of the conditions) have to be defined; second, cases have to be assigned to the single truth table row to which they best belong; and, third, for each truth table row the outcome value has to be determined by testing whether it is a subset of the outcome of interest. Rows that pass this test represent sufficient conditions. Subsequently, a QCA proceeds with the logical minimization of a truth table. The result of this procedure yields the sufficiency solution formula, one important goal of any QCA.

Chapter 5 discussed the problem of inconsistent truth table rows, i.e., the situation when some truth table rows are not perfect subsets of the outcome. We introduced the consistency parameter as a tool to assess whether a truth table row should count as a sufficient condition (and thus be included or excluded from the logical minimization). Once a set passes the formal test of consistency, the coverage parameter for sufficient conditions expresses how much of the outcome is explained by that condition. We showed how consistency and coverage are also calculated for necessary conditions and argued that the coverage parameter expresses the relevance of a necessary condition.

In Chapter 6, we dealt with another ubiquitous phenomenon in QCA – but also in other types of empirical social research – related to incomplete truth tables: limited diversity. This occurs when one or more truth table row does not contain enough empirical evidence, producing so-called logical remainders. In such a situation, the logical minimization of a truth table yields more than one solution formula, depending on the treatment of such remainder rows. We showed that none of these solutions ever contradicts the empirical evidence. Yet they differ in their degree of complexity and therefore sometimes look quite different and facilitate putting different emphases in the substantive conclusions. All this warrants transparent strategies for a plausible treatment of logical remainders. We explained that, in QCA, the strategy for such a conscious treatment consists of taking into consideration the three dimensions along which assumptions on logical remainders can vary: the dimension of set relations, that of complexity, and that of theoretical appropriateness. Three solution terms play a crucial role in QCA: the conservative solution, which is not based on any assumption on logical remainders; the most parsimonious solution, which is based on simplifying assumptions on many, sometimes even all logical remainders; and the intermediate solution, which is based on only easy counterfactuals and is in-between the previous two solutions, both in terms of set relations and in terms of complexity. Chapter 7 presented the so-called Truth Table Algorithm as the currently prevailing mode of analyzing data in QCA.

The remainder of the book looked beyond this default way of performing QCA. In Chapter 8, we identified various pitfalls in dealing with limited diversity when using the Standard Analysis procedure. We demonstrated that the most parsimonious solution, and even the intermediate solution, can be based on untenable assumptions. We also laid out various strategies for identifying and then for avoiding these assumptions, yielding what we called the Enhanced Standard Analysis procedure. We also encouraged researchers to engage more directly and consciously with logical remainders, and to dare to make counterfactuals based on theoretical reasoning and irrespective of whether such counterfactuals contribute to parsimony – a strategy we labeled Theory-Guided Enhanced Standard Analysis.

In Chapter 9, we discussed pitfalls that can occur when combining analyses and statements about both necessity and sufficiency. We showed that sufficiency solution formulas might suggest the presence of necessary conditions that are not actually necessary (false necessary conditions), and that some truly necessary conditions might disappear from the solution formula for sufficiency (hidden necessary conditions). Wrong conclusions can be avoided if necessary and sufficient conditions are analyzed in two separate analytical steps, preferably with the necessary conditions coming first. Also linked to the analysis of necessary and sufficient conditions are pitfalls triggered by skewed membership values. The literature has already discussed the problem of trivial necessary conditions. We demonstrated that the standard coverage formula for necessity captures only part of this problem and that current alternative measures of relevance work less well in the rather common scenario of less than perfectly consistent subset relations. As a remedy, we proposed a new formula for empirically assessing the relevance and trivialness of a necessary condition. As a further consequence of skewed set-membership scores, we discussed the phenomenon of simultaneous subset relations, a phenomenon that can afflict only fuzzy sets, and possible strategies for detecting it. In a final section, we showed that skewed set-membership scores trigger potential pitfalls that go well beyond what is currently discussed in the literature.

In Chapter 10, we introduced further variants of QCA. Multi-value QCA (mvQCA) distinguishes itself from crisp-set and fuzzy-set QCA by the use of a different type of set, whereas two-step QCA and temporal QCA (tQCA) can be used with crisp and fuzzy sets and even multi-value sets. In our discussion of mvQCA, we took issue with the argument that it leads to fewer logical remainders than a conventional QCA, an argument that is incompatible with the interpretation of mvQCA as a set-theoretic method. We presented tQCA as the currently most formalized strategy of integrating time into QCA. Other,

more informal strategies exist, but all face the conundrum of exponentially increasing complexity once time is considered as causally relevant.

The aim of Chapter 11 was to spell out various issues that arise when standards of good practice for QCA as a technique meet those for QCA as an approach. We started by providing a recipe for a good QCA, which also included an overview of the four software packages relevant for QCA: fsQCA 2.5, Tosmana 1.3.2, R, and Stata. Then we discussed from a set-theoretic perspective three core issues in any comparative social research. Robustness, theory evaluation, and case selection principles. With regard to robustness tests, we argued that set-theoretic methods are not exempted from them but that these tests look different than in non-set-theoretic techniques. We saw that changes in calibration, in the consistency threshold, and in the case selection are among the most consequential for the solution formulas obtained. While the precise effects depend very much on the specifics of the data at hand, it remains true that most modifications have only modest effects. With regard to theory evaluation, we argued that this is different from testing theories and that it consists in analyzing the different intersections that can be created between prior theoretical hunches and the empirical findings generated with QCA. In doing so, we deemed it important to take stock of the fact that solution terms tend to achieve less-than-perfect consistency and coverage. In our section on case selection, our major point was that meaningful case selection after a QCA must take into account the components of causal complexity – equifinality, conjunctural causation, and asymmetry – and the conceptual differences between consistency and coverage. This led to the definition of various types of deviant and typical cases whose selection for subsequent within-case analysis is governed by several set-theoretic case selection principles.

12.2 Myths and misunderstandings

Often implicitly, sometimes explicitly, our book responds to some of the prevailing myths and misunderstandings about QCA. We briefly list some of them in the hope of further promoting the critical debate on QCA in a fruitful manner. The list, while most likely not complete, contains those issues that we have most frequently encountered in our teaching activities and in discussions with colleagues.

QCA is *not a deterministic method* by default. With the introduction of parameters of fit, most importantly the consistency parameter, QCA precisely allows for deviations from perfect sufficiency and perfect necessity. Also, the

use of fuzzy values instead of only dichotomous crisp sets can be seen as a defense against determinism. And, of course, nothing prevents researchers from using tests of statistical significance when judging whether a given deviation from a perfect subset relation is significant enough to reject that relation (e.g., Ragin 2000: ch. 4; Braumoeller and Goertz 2003; Eliason and Stryker 2009). Beyond this, we second Mahoney's (2003: esp. 339–53) and Adcock's (2007) thoughtful defenses of the notion of determinism.

Crisp-set QCA and fuzzy-set QCA are not very different. All crucial analytic steps are equally valid for both variants of QCA. This has several implications. First, we discourage the artificial separation which is sometimes encountered in the literature. Second, it dispels the myth that the choice of the QCA variant is driven by the number of cases at hand. Third, it implies that whenever possible, fuzzy-set QCA should be used, for it allows for more fine-grained distinctions between cases, is more conservative in its assessment of subset relations, and can easily integrate some crisp-set conditions in the analysis.

Related to the number of cases, two further myths should be dispelled. First, *QCA is not a small-N method*. In fact, if the number of cases is very small, say below ten, then QCA loses most of its comparative advantage to traditional comparative case studies. The number of logical remainders will be high, barely any logical minimization of the truth table will be possible, and the paths identified will mostly cover only one case each. QCA, in short, does not resolve the universal "few cases, many variables" problem (Lijphart 1971: 687; King *et al.* 1994: ch. 6). We suggest that when the N is small, researchers should sort their empirical evidence in a truth table. But beyond this, there is little need to bring to the fore the whole technical arsenal of QCA. The second issue related to the number of cases is that *QCA can be a large-N method*. Applied to the very same large-N data set, QCA will, indeed, produce different results from a correlational statistical approach. If researchers have plausible reasons to be interested in causal complexity defined in terms of equifinality, conjunctural causation, and asymmetry, then these QCA results might unravel insights that would remain hidden if other methods were used (see, e.g., Grendstad 2007; Ragin and Fiss 2008; Glaesser and Cooper 2010). Of course, large-N QCA might require some adjustments to the standard protocol. Most importantly, notions of statistical significance might be applied when assessing subset relations and the focus on specific cases might be replaced by a focus on *types* of cases, especially if individual-level data are analyzed.

QCA results are not arbitrary. The fact that the same truth table allows for multiple solution terms does not testify against that claim. Different solution

terms mainly stem from different assumptions about logical remainders. These assumptions never contradict the empirical evidence at hand, and the solution formulas, different as they seem, are usually in a perfect subset relation to each other.

12.3 Looking ahead: tasks and developments in the coming years

This book, by necessity, has focused more on aspects of QCA as a technique, but QCA's second important pillar – that of also being a research approach – should not be downplayed (Rihoux and Lobe 2009; Rihoux and Ragin 2009). QCA would lose much of its strength and appeal if it was exclusively used as yet another off-the-shelf data analysis technique and if users were misled into thinking that the only thing that matters is getting all the technicalities straight and then going on the hunt for high values of consistency and coverage.

On several occasions, even our in-depth discussion of *technical* aspects made clear that the *research approach* part of QCA is crucial for success. The most straightforward examples are provided in Chapter 11 with our discussions of set-theoretic principles of case selection (section 11.4), of theory evaluation (section 11.3), and of robustness tests (section 11.2). All of these topics addressed issues that arise when linking QCA as a technique with QCA as a research approach, and could not just be solved by adding merely technical tweaks. Also, when spelling out our remedies for avoiding potential pitfalls in a QCA (Chapters 8 and 9), we have gone beyond the pure application of an algorithm and pointed to a more direct theoretical and substantial engagement of researchers with their cases – something which is also important in other, more fundamental steps, such as during the calibration (section 1.2). All this demonstrates that high-quality QCA must pay careful attention to research phases before and after the analytical moment and properly link these phases in "the dialogue between ideas and evidence" (Ragin 2000).

Several other issues raised in our book are based on seeing QCA predominantly as a research approach. For instance, we argued that the fundamental decision on whether to apply QCA, as opposed to a non-set-theoretic method, should be based on theoretical and substantive grounds, not the number of cases at hand. If the assumption of causal complexity makes sense in a given research area, then set-theoretic methods are a plausible methodological choice – regardless of the number of cases. If such an assumption is not plausible, then set-theoretic methods will not be a good choice – again, regardless of the N. Furthermore, we repeatedly emphasized that parameters

of fit, useful as they are in applied QCA, should not cause the researcher to lose sight of the cases. Simply reporting consistency and coverage values is inferior to identifying the cases that contribute to less-than-perfect consistency or coverage and subjecting them to within-case analyses. We have also made clear that in a good QCA, the data do not belong to the "untouchables" caste. Taking the requirement of a back and forth between ideas and evidence seriously, researchers must be allowed to alter their data during the research process. This is usually done through adjustments to the case selection, concept formation and calibration, and model specification. What is clear from the double nature of QCA as both a technique and an approach is that QCA is inherently a multi-method approach.

The framing of several social science research methods in terms of set relations is a rather recent phenomenon, and QCA has barely passed the stage of adolescence 25 years after the publication of Ragin's *The Comparative Method* (1987). No wonder that improvements and additions to this toolset are still ongoing.

With regard to the topics addressed in Chapter 11, we envisage the following developments. More insights about robustness checks will be developed, and eventually even standard routines be proposed that inform the standards of good QCA (publication) practice. The integration of time as a relevant analytic category will (hopefully) receive more attention, both by making better use of the already existing tools, such as tQCA, and by providing new ideas on how to make causally relevant claims about time-related features without loosing control of the property space thus created. Further specifications of set-theoretic case selection principles are already under way (e.g., Schneider and Rohlfing in press). And by resuscitating and developing Ragin's (1987) early thoughts on set-theoretic theory evaluation, we hope to stimulate users to think more about this crucial and general issue in social science methodology.

Other chapters also contained debates that require further reflection. For instance, our plea for a more conscious engagement with logical remainders, condensed into what we call the Enhanced Standard Analysis procedure (section 8.2.4), needs to be exposed to more practical tests. If it turns out that in the great majority of instances, the current Standard Analysis procedure correctly excludes all untenable assumptions, then our suggestion for improvement would be superfluous; we doubt that this will be true, though. Similarly, our debates on pitfalls related to skewed set-membership scores – relevant necessary condition and simultaneous subset relations of sufficiency – are of practical relevance only if enough researchers find that in their research these

pitfalls are indeed potential sources for flawed inference. We therefore hope that many researchers applying QCA will make use of the PRI and PRODUCT parameters, and of the various formulas for relevant necessary conditions, among them the formula developed by us in section 9.2.1.2.

Most of the pending methodological challenges are of a conceptual nature. Once settled, they need to be implemented in the relevant software packages for set-theoretic analyses. Much progress has been made on this front over the past years. The fsQCA 2.5 software being developed by Ragin is usually the front-runner in terms of innovations in set-theoretic analysis. The program has steadily improved over the past years and will remain an essential tool for good QCA. We foresee a trend, though, for an increasing number of scholars to turn their eyes to alternatives, and here especially to the R platform. It is freeware, tremendously flexible, powerful, reliable, stable, syntax-based, and open source. The more that R turns into a common tool for scholars using set-theoretic methods, the more important it becomes that packages be developed that allow for state-of-the-art analyses (Thiem and Dusa 2012). Tosmana (not open source) and Stata (not free) will remain useful additional tools, but we predict that they will not take over as the main software packages for QCA. Our impression is that the implementation of QCA in both Stata and R is linked to – both as a sign of and as a trigger for – an increased recognition of QCA among more quantitatively-minded scholars in the social sciences. As circumstantial evidence, we remember a colleague telling us (only half-jokingly, we suppose) that now that the "fuzzy" ado file in Stata exists, he is willing to consider QCA as a true method. We think that, overall, the expansion of software options will make a net contribution to the improvement of QCA. In addition to the above-mentioned general advantages of R (and Stata), we hope that an increased interest among advanced quantitative scholars will contribute fresh and constructive insights into the debate over the vices and virtues of QCA.

One aspect here might be the question of how set-theoretic methods, in general, and QCA, in particular, relate to the potential outcomes model (also known as the experimental template or the Neyman–Rubin–Holland causal model; see, e.g., Morgan and Winship 2007) as the current state of the art in quantitative social sciences. Some attempts in this direction have already been made (King and Powell 2008; Mahoney 2008; Goertz and Mahoney 2012; Yamamoto 2012), but more is needed. We suspect that if set-theoretic, method-specific concepts such as consistency and coverage, simplifying assumptions, and the like can be translated into the potential outcomes framework, the communication between scholars from different research traditions will be

facilitated – which is always a positive thing. Even if it turned out that some concepts cannot be translated and that, thus, QCA and other set-theoretic methods simply cannot be subsumed under the potential outcomes template, this would be a positive contribution, as it would clarify the differences and similarities and avoid future misunderstanding.

It might strike the reader as an odd final paragraph for a textbook on social science methodology, but we would like to use this last opportunity to caution those readers who are in danger of becoming obsessed with methods. It is helpful to remember that methods are simply a tool for good social science research. Methodological skills are but one characteristic of a good social scientist; boosting them at the expense of other competences – language skills, theoretical knowledge, and/or plain, simple curiosity about what is going on in the world out there – is not. Methodological consciousness defines high-quality scholars; methodological fetishism does not. Trying to apply methods in a correct manner is an obligation; doing so at the expense of other relevant aims of social research is not. No matter what, researchers will face tradeoffs at virtually every stage of their research, requiring hard decisions about which standards of perfection to sacrifice. Due to their reliance on a continuous dialogue between theoretical ideas and empirical evidence, set-theoretic methods in general, and QCA in particular, are well equipped to strike a more healthy balance between paying attention to technical matters, on the one hand, and focusing on the cases being studied, on the other. We hope that this book has helped to identify tradeoffs that exist when applying set-theoretic methods, and that it has helped to find strategies for making conscious and deliberate decisions. If so, then the threat of stultifying methodological perfectionism has been averted and a good service has been provided for the cause of both methodological and substantive progress.

Glossary

Addition, Boolean/fuzzy   See *logical OR*.

Arithmetic remainder   *Logical remainder* that occurs when the number of logically possible combinations of *conditions* (see also *configuration*) exceeds the number of cases at hand.

Associativity   The sequence in which single sets are combined (when the operator remains the same) is unimportant.
(A * B) * C = A * (B * C) = (A * C) * B
A + B) + C = A + (B + C) = (A + C) + B.

Assumption   Claim that a given *logical remainder* is sufficient for the *outcome*, which therefore is subsequently included into the *logical minimization* process. See also *counterfactual* and *simplifying assumption*.

Asymmetry   Implies that (a) a causal role attributed to a *condition* always refers to only one of the two qualitative states – presence or absence – in which the *condition* set can be found and (b) any solution term always refers to only one of the two qualitative states – presence or absence – in which the *outcome* set can be found. Both forms of asymmetry are the consequence of the fact that, in set-theoretic methods, the presence of a set and its negation denote two qualitatively different phenomena. *Sufficiency* and *necessity* are typical asymmetric relations.

Calibration   Process in which set *membership scores* are assigned to cases.

Causal complexity	Consists of *equifinality*, *conjunctural causation*, and *asymmetric causation*.
Clustered remainder	*Logical remainder* that occurs because social reality is structured by historical, social, cultural, and other processes.
Commutativity	The order in which two or more sets are connected through *logical AND* and *logical OR* is irrelevant. A * B = B * A A + B = B + A. This rule does not apply to the *negation* and the *implication*.
Complement	*Set* that contains all those cases that are not members in the original set. With *fuzzy sets*, one and the same case can have partial membership both in the set and its complement (see *Rule of the Excluded Middle*), but such partial *membership* will be above the *qualitative anchor* of 0.5 in only one of the two sets.
Complex solution term	Synonymous to *conservative solution* term. Attention: "complex" might be misleading, because the complex solution is not the most complex term, but the subset of all other possible solutions. See also *most parsimonious solution* and *superset solution*.
Condition	Factor which is used to explain the *outcome*. In set-theoretic methods, there are different types of conditions, such as *necessary*, *sufficient*, *SUIN*, and *INUS* conditions.
Configuration	Combination of *conditions* which describes a group of empirically observed or hypothetical cases (aka *logical remainders*).
Conjunction	A conjunction is "true" when all its components can be observed; otherwise it is "false." See also *logical AND* and *multiplication, Boolean/fuzzy*. In a different usage, this term is also often used as a synonym for a *sufficient* term or *path* which combines several conditions by a *logical AND*.

Conjunctural causality	Situation in which the effect of a single *condition* unfolds in combination with precisely specified other conditions.
Conservative solution term	Solution that rests on no *assumption* about *logical remainders*. It is based solely on *truth table* rows that are deemed *sufficient* for the *outcome* based on empirical evidence. It is the subset of all other possible solutions.
Consistency	Expresses the percentage of cases' *set-membership* scores in two sets that is in line with the statement that one of the two sets is a subset (or superset) of the other. It thus indicates to what degree the empirical data are in line with a postulated subset relation.
Contradictory easy counterfactual	An *easy counterfactual* that contradicts an *assumption* already made on the same *logical remainder* in the analysis of the *complement* of the *outcome*. Sub-type of *incoherent easy counterfactual*. See also *untenable assumption*.
Contradictory (simplifying) assumption	A *(simplifying) assumption* that contradicts an assumption already made on the same *logical remainder* in the analysis of the *complement* of the *outcome*. Contradictory assumptions are simplifying when they contribute to *parsimony*. They are one type of *incoherent assumptions*, which, in turn, are one form of *untenable assumption*.
Contradictory truth table rows/logical contradictions	With *crisp sets*, these are truth table rows or *sufficient conditions* (X) that contain cases with different membership in the *outcome* (see also *logical contradiction*). With *fuzzy sets*, X can be *inconsistent* without being a logical contradiction. The former are those in which one or more cases' membership in the row exceeds that in the outcome (so-called *inconsistent cases*). The latter are those rows where inconsistent cases are located on two different sides of the qualitative anchor (in row > 0.5; in outcome < 0.5). All logically contradictory cases are inconsistent, but not all inconsistent cases are also logically contradictory.

	The same applies to statements of *necessity*. They are inconsistent if some cases hold membership in X that is smaller than in Y. They are logically contradictory if, in addition, one or more cases hold membership in X < 0.5 and in Y > 0.5.
Counterfactual	Often used as synonym for *assumption* about *logical remainder*.
Coverage	Assesses the relation in size between the *condition set* and the *outcome set*. Coverage sufficiency expresses how much of the outcome is covered by the sufficient condition. "Coverage" necessity is better understood in terms of the *relevance* and *trivialness* of a necessary condition.
Crisp set	Set which allows only for full membership (1) and full non-membership (0). Can be perceived as special cases of *fuzzy sets*.
Crisp-set QCA (csQCA)	Version of QCA with which only *crisp sets* can be analyzed.
DeMorgan's law	Provides rules to calculate the *negation* of a complex set-theoretical expression.
Deviant case consistency	For *sufficiency*: a case with membership in X > 0.5 and Y < 0.5.
	For *necessity*: a case with membership in X < 0.5 and Y > 0.5.
Deviant case coverage	Exists only with regard to statements of *sufficiency*: a case with membership in X < 0.5 and Y > 0.5.
Difficult counterfactual	*Assumptions* on *logical remainders* that contribute to producing the *most parsimonious solution* term but that are not in line with directional expectations.
Direct (method of) calibration	*Calibration* procedure, based on a logit function which is established between the three *qualitative anchors* 0, 0.5, and 1 imposed by the researcher.
Directional expectation	Theoretically derived and justified argument that a single *condition* is expected to contribute to the occurrence of the *outcome* when it is present rather than absent (or vice versa). If such expectations are formulated for conjunctions of conditions, they are labeled conjunctural directional expectations.

Disjunction	A disjunction is "true" when all of its components can be observed (of which there must be at least one); if no component can be observed, it is "false."
Distributivity	If both *logical AND* and *OR* operators are used in the same logical expression, then single conditions that are shared by the various *conjunctions* can be factored out: A * B + A * C = A * (B + C).
Easy counterfactual	*Assumptions* on *logical remainders* that are in line with *directional expectations* and that contribute to *parsimony*.
Enhanced most parsimonious (intermediate) solution term	Variant of the *most parsimonious (intermediate) solution* that does not rest on *untenable assumptions*.
Enhanced Standard Analysis (ESA)	Produces the *enhanced most parsimonious solution* and the *enhanced intermediate solution*.
Equifinality	Allows for different, mutually non-exclusive *sufficient conditions*, or paths, for the *outcome*.
Excluded Middle (Rule of the)	Postulates that a case cannot belong both to a set and its *complement*. Does not apply to *fuzzy sets*.
False necessary condition	*Condition* that forms part of all sufficient paths, but fails the *consistency* test as a *necessary condition*.
Functional equivalent necessary condition	Two or more *conditions* represent the same overarching concept. Each condition on its own does not pass the *consistency* test of *necessity*, but their *logical OR* combination does.
Fuzzification	Sometimes used as synonym for the *calibration* of *fuzzy sets*.
Fuzzy set	Set which allows for partial membership, in addition to full membership and full non-memberships. Translated to the social sciences, it enables the researcher to work with concepts for which the establishing of differences in degree among qualitatively similar cases is both conceptually plausible and empirically feasible.

Fuzzy-set QCA (fsQCA)	Version of QCA with which *fuzzy sets* can be analyzed. Since *crisp sets* are nothing more than a special version of fuzzy sets, fsQCA can also be used for crisp sets.
Fuzzy-set membership score	Expresses the degree of *set membership* of a case in a *fuzzy set*.
Good counterfactual	Claim that an empirically non-observed combination of conditions is sufficient for the outcome. This claim is based on a set of criteria for good counterfactuals and irrespective of whether it contributes to *parsimony*. A good counterfactual cannot be an *implausible* or *incoherent assumption*.
Hidden necessary condition	*Condition* which is *consistent* as a *necessary condition*, but is not part of all sufficient paths.
Ideal type	A *configuration* of *conditions* (see also *truth table row*). With *crisp sets*, cases are either full members or full non-members of an ideal type. With *fuzzy sets*, cases can have partial membership in various ideal types.
Implausible assumption	Inclusion of an *impossible remainder* into the *logical minimization*.
Implausible easy counterfactual	*Easy counterfactual* made on an *implausible remainder*.
Impossible remainder	*Logical remainder* that describes a case whose existence defies pure formal logic (e.g., the rich poor country) or common-sense knowledge about the world (e.g., pregnant men).
Incoherent assumption	Inclusion into the *logical minimization* of a *logical remainder* that either contradicts statements about necessary *conditions* or an assumption made on the same remainder in the analysis of the complement of the outcome (aka *contradictory (simplifying) assumption*).
Inconsistent case	A case with membership in sets X and Y that is not in line with the statement of necessity or sufficiency made about *condition* X. With *crisp sets*, any inconsistent case is automatically also a *logical contradictory case*. With *fuzzy sets*, cases can be *inconsistent* without being *logically contradictory*.

Indirect (method of) calibration	Semi-automatic procedure of *calibration*, which establishes a fractional logit model between the preliminary *fuzzy set membership* scores imposed by the researcher.
Individually irrelevant case	Case with membership in X < 0.5 and Y < 0.5. Provides analytic insights to the analysis of *necessity* or *sufficiency* only when compared to a *deviant case consistency* (necessity) or *a deviant case coverage* (sufficiency).
Intermediate solution term	Solution term that is exclusively based on *easy counterfactuals*. It is a subset of, and more complex than, the *most parsimonious solution* term. It is a superset to, and less complex than, the *conservative solution* term.
Intersection	*Set* that contains those cases that are (partial) members in all the sets that are intersected. See also *logical AND*.
INUS condition	Single *condition* that is insufficient for producing the *outcome* on its own but which is a *necessary* part of a *conjunction* that, in turn, is unnecessary but *sufficient* for producing the outcome. Any sufficiency statement that consists of at least one *logical AND* and one *logical OR* operator contains at least one INUS condition. For instance, in the term A*B + C*D → Y, each single condition (A, B, C, D) is an INUS condition.
Limited diversity	Logically possible combinations of conditions used in an analysis, for which, however, not enough empirical evidence is at hand. In a *truth table*, limited diversity is represented by the set of all *logical remainder* rows.
Logical AND	Creates the *intersection* between two or more *sets*. Membership of cases in this intersection is determined by their minimum value across these sets.
Logical contradiction	In the framework of *sufficiency*, a situation in which cases that are members of the sufficient condition or *conjunction* are more out of than in the outcome. In the framework of *necessity*,

	a situation in which cases that are members of the *outcome* are more out of than in the necessary condition.
Logical minimization	Summary of the information contained in a *truth table*, applying the rules of Boolean algebra. Leads to the *solution formula* for *sufficiency*, but not *necessity*. See also *Quine–McCluskey algorithm*.
Logical OR	Creates the *union* between two or more *sets*. *Membership* of cases in the union is determined by their maximum value across these sets.
Logical remainder	*Truth table* rows for which not enough empirical evidence is at hand. "Enough empirical evidence" is defined by a minimum number of cases with full membership (*crisp sets*) and with membership higher than 0.5 (*fuzzy sets*) in a truth table row.
Logically redundant prime implicant	*Prime implicant* which can be omitted from the solution formula without leaving any *primitive expression* uncovered, i.e., without violating the truth value contained in the *truth table*.
Most parsimonious solution term	Solution formula among all logically possible solution terms that uses the lowest number of *conditions* and of the two operators *logical AND* and *logical OR*. In the presence of *logically redundant prime implicants*, two or more formulas can be equally most parsimonious.
Multifinality	One and the same single INUS condition can be causally relevant for producing both the occurrence of the outcome Y and its complement ~Y.
Multiplication, Boolean/fuzzy	See *logical AND*.
Multi-value QCA	Version of QCA which operates on multi-value variables.
Necessary condition	For *crisp sets* and in everyday language: a *condition* is necessary if, whenever the *outcome* is present, the condition is also present, but there can be cases that are members of the

condition but not the outcome. More generally (and for *fuzzy sets*): a condition might be interpreted as necessary if, across all cases, *set membership* in it is larger than or equal to each case's membership in the outcome.

Negation, logical   Determined by 1 – the membership value in the original *set*. See also *complement*.

Non-occurrence (of outcome or condition)   *Logical negation* of a *set* (*outcome* or *condition*). Sometimes also referred to as the absence of the set.

Outcome   Phenomenon to be studied in an analysis.

Parsimony   Applied to set-theoretic *solution terms*, it refers to the number of conditions and *logical AND* and *OR* operators. The fewer conditions and operators, the more parsimonious the solution term.

Path   *Logical AND* combination of *conditions* that is sufficient for the *outcome*. Often used as a synonym to *sufficient condition*.

PRI   Acronym for Proportional Reduction in Inconsistency. Expresses how much it helps to know that a given *condition* X is a subset of *outcome* Y rather than a subset of either Y, its *complement* ~Y, or the *intersection* between Y and ~Y.

Prime implicant   End product of the *logical minimization process* through pairwise comparisons of *conjunctions*.

Primitive expression   Synonym for truth table row that is *sufficient* for the *outcome*.

PRODUCT   Derived by multiplying the *raw consistency value* and the *PRI* score. Small values indicate that the truth table row is either not a subset of the *outcome* Y or that it is a subset of both Y and its *complement* ~Y. Such rows should not be considered as sufficient for Y and thus should not be included in the *logical minimization*.

Property space	The k number of conditions used in a set-theoretic analysis define a k-dimensional property space with 2^k corners. These corners correspond to the 2^k *truth table rows* (see also *ideal type*). With *crisp sets*, each case has full membership in one corner and full non-membership in all other corners. With *fuzzy sets*, cases can have partial membership in all corners but a membership of higher than 0.5 in only one.
Qualitative anchors	They identify qualitative differences of cases' membership in a set and need to be established during the *calibration* procedure by using criteria external to the empirical information at hand. The qualitative anchor of 0.5 describes the point of indifference, where it is impossible to say whether the case is more a member or a non-member of a set; the qualitative anchor of 0 describes full non-membership; and the qualitative anchor of 1 full membership. See also *qualitatively different set membership score.*
Qualitative Comparative Analysis (QCA)	Most formalized *set-theoretic method*, which uses formal logic and Boolean algebra in the analysis of *truth tables* and aims at establishing *necessary* or *sufficient conditions*, integrating parameters of fit (*consistency* and *coverage*). Variants exist: *crisp-set QCA*, *fuzzy-set QCA*, *multi-value QCA*, and *temporal QCA*.
Qualitatively different set membership score	*Set-membership* scores above or below the *qualitative anchor* 0.5 denote a qualitative difference. With *crisp sets*, cases with different membership scores in a set are always qualitatively different. In *fuzzy sets*, cases on the opposite side of the 0.5 anchor are qualitatively different. Cases on the same side of the 0.5 anchor are qualitatively identical but may differ in their degree of set membership.

Quine–McCluskey algorithm	Algorithm for the *logical minimization* of *truth tables*. It consists of first logically minimizing those *conjunctions* that are *sufficient* for the *outcome* and similar to each other and then of excluding *logically redundant prime implicants*.
Raw coverage	Percentage of all cases' *set membership* in the *outcome* covered by a single *sufficient path* of an *equifinal solution term*.
Raw consistency	*Consistency* of a single *truth table row*.
Relevance necessity	Measures how far a set X is not only a superset of Y (and thus denotes a necessary condition), but also to what degree it is not much bigger than Y nor ~X. If X is a superset of Y but much bigger than either Y, or ~X, or both, then X is not a relevant, but a trivial necessary condition for Y.
Set-membership score	Numerical expression for the belonging of a case to a set. With *crisp sets*, only full membership and full non-membership are possible. *With fuzzy sets*, degrees of membership can be expressed – yet the qualitative distinction between cases that are more in than out vis-à-vis those that are more out than in is maintained.
Set-theoretic methods	Approaches to analyzing social reality through the notion of sets and their relations. Can model causal complexity, expressed in terms of *equifinality*, *conjunctural causation*, and *asymmetry*. *Qualitative Comparative Analysis* is one, but not the only set-theoretic method.
Simplifying assumption	*Assumption* on *logical remainder* that yields a *solution term* that is less complex than the *conservative solution term*.
Solution coverage	Percentage of all cases' *set membership* in the *outcome* covered by the *solution term*.
Solution formula/term	The result of a *truth table* analysis (see also *logical minimization*). Usually consists of several *paths* (see also *equifinality*).

Standard Analysis	Produces the *most parsimonious solution*, the *intermediate solution*, and the *conservative solution*. See also *Enhanced Standard Analysis*.
SUIN condition	A single *condition*, which is unnecessary part of a *logical OR* combination that, in turn, is insufficient, but necessary for the outcome. Any statement of *necessity* that includes at least one *logical AND* and one logical OR operator contains at least one SUIN *condition*. In (A+B) * (C+D) ← Y, the conditions A, B, C, and D are SUIN conditions.
Sufficient condition	For *crisp sets* and in everyday language: a *condition* is sufficient if, whenever the condition is present, the *outcome* is also present, but there can be cases that are members of the outcome but not the condition. More generally (and for *fuzzy sets*): a condition can be interpreted as sufficient if, across all cases, *set membership* in it is smaller than or equal to each case's membership in the outcome.
Superset solution term	The result of a *logical minimization* that includes all *logical remainders*. This formula is the superset of all logically possible solution terms that can be derived from a truth table without violating its truth value.
Temporal QCA (tQCA)	Modification of a conventional QCA which includes *conditions* that express the temporal order of two or more single conditions. For example, A/B expresses that A occurred prior to B.
Tenable assumption	*Assumption* on *logical remainders* that is not *implausible* or *incoherent*, regardless of whether it contributes to *parsimony*.
Theory-Guided Enhanced Standard Analysis (TESA)	Extension of the *Enhanced Standard Analysis* (ESA): also allows for *good counterfactuals* that do not contribute to *parsimony*.
Tied redundant prime implicant	Situation in which two or more *prime implicants* are logically redundant and some of them, but not all, need to be kept in order to preserve the truth value of the solution formula.

Trivialness necessity	See *relevance necessity*.
True logically contradictory case	With *crisp sets*, all cases whose membership in X and Y is *inconsistent* with the postulated subset relation of *necessity* or *sufficiency*, respectively, are true logical contradictory cases. With *fuzzy sets*, only those cases with inconsistent *set-membership* scores in X and Y are true logical contradictory cases that possess *qualitatively different set-membership* scores in X and Y, respectively.
Truth table	At the core of any QCA. It contains the empirical evidence gathered by the researcher by sorting cases into one of the 2^k logically possible combinations, aka truth table rows, of k *conditions*. Each row linked to the outcome can be interpreted as a statement of *sufficiency*.
Truth Table Algorithm	Describes the sequence of the sufficiency analysis. First, the empirical information on cases is represented in a *truth table*. Then, rows are classified as either being *sufficient* for the outcome, not sufficient, or a *logical remainder*. Third, rows deemed as sufficient are included in the *logical minimization*. Works with both *crisp* and *fuzzy sets*.
Two-step QCA	Variant of (*crisp*, *fuzzy*, and *multi-value*) *QCA* in which conditions are grouped into remote and proximate factors and then analyzed in subsequent steps.
Typical case	For *sufficiency*: a case with membership in X > 0.5 and Y > 0.5 and with X < Y. For *necessity*: a case with membership in X > 0.5 and Y > 0.5 and with X > Y
Uncovered case	For *sufficiency*: a case with membership in Y > 0.5 and X < 0.5. See also *deviant case coverage*.
Union, Boolean/fuzzy	See *Logical OR*.
Unique coverage	Percentage of all cases' *set membership* in the *outcome* uniquely covered by a single *path* of an equifinal solution term (see *equifinality*).

Uniquely covered case	Case with membership in *outcome* > 0.5 and in only one *sufficient path* of > 0.5.
Untenable assumptions	Assumption on logical remainder that is either *implausible* or *incoherent*.
Venn diagram	Graphical representation of sets and their relations, using overlapping circles.
XY plot	Plot that displays each case's *fuzzy membership* in the (single or *conjunctural*) *condition* on the X axis and the membership in the *outcome* on the Y axis. Usually drawn with a diagonal at X = Y, which facilitates the detection of subset relations. Helps in the assessment of sufficient and necessary conditions and the identification of types of cases (see also *typical case*, *deviant case*, and *individually irrelevant case*).

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