Online channel strategies under different offline channel power structures

Xujin Pu\textsuperscript{a}, Shiming Zhang\textsuperscript{a}, Bowen Ji\textsuperscript{a}, Guanghua Han\textsuperscript{b,}\textsuperscript{*}

\textsuperscript{a} School of business, Jiangnan University, Wuxi, 214122, China
\textsuperscript{b} School of International and Public Affairs, Shanghai Jiao Tong University, Shanghai, 200030, China

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A B S T R A C T

This study explores the manufacturer’s marketing and pricing strategies for online channel under different offline channel power structures. Through these strategies, the manufacturer sells products through an offline retailer and an e-tailer. The manufacturer decides the cooperation mode with the e-tailer by the reselling or the agency selling mode and the pricing strategy on the basis of the power structures, i.e., vertical Nash structure (VN), manufacturer Stackelberg structure (MS), and retailer Stackelberg structure (RS). We find the manufacturer selects the online agency selling mode when the commission rate is less than the given threshold. As long as the commission rate is more than another threshold, the manufacturer selects the online reselling mode under the VN structure; however, the manufacturer selects the online agency selling mode under the other two structures. As well, the offline wholesale price is higher under the MS structure than those under the VN and RS structures. When the manufacturer selects the online agency selling mode, the offline retail price is highest under the MS structure, and the online retail price is highest under the VN structure. Meanwhile, consumers can always obtain a higher surplus in the online agency selling mode under all offline power structures.

1. Introduction

The rapid development of e-commerce and online retail markets has led consumers to recognize and use online shopping gradually. Therefore, more and more manufacturers are exploring online channel to sell their products. According to Statista.com, the value of online selling in the world reached $2.29 trillion in 2017 and is expected to reach $4.48 trillion in 2021 (Statista Inc., 2018). For instance, Euromonitor International, a consumer-spending research firm, indicates that there are approximately a million active sellers on Amazon’s platform. In 2017, 66% of the money spent by Amazon shoppers worldwide was for goods sold by active sellers (Resende et al., 2018). Meanwhile, according to the 42nd China Internet Development Statistics Report released by China Internet Network Information Center for the January–May 2018 period, the e-commerce platform revenue in China has reached 116.4 billion yuan, representing a 39.1% increase over the same period last year (State Internet Information Office, 2018). As e-commerce has witnessed strong growth over time, therefore more and more manufacturers try online channel to distribute their products.

Meanwhile, it is strategical to manufacturers to optimally select their online selling channels. Online selling modes have evolved into two typical modes: reselling and agency selling. The e-tailers buy at wholesale and sell at retail to consumers in the online reselling mode (Tian et al., 2018; Resende et al., 2018). For example, Hisense, Skyworth, TCL, PPTV, and other Chinese TV manufacturers have signed a 35 billion-yuan reselling order with Suning.com, a major e-tailer in China. Suning.com is responsible for selling these products to consumers. However, in the online agency selling mode, e-tailers provide manufacturers direct access to customers with their platform, while manufacturers retain the decision making for retail prices (Yan and Pei, 2018; Giwei et al., 2018). For instance, Random House, the largest English-language publisher, sells books online through e-tailers such as Books A Million and Barnes & Noble (Wang et al., 2018).

Online selling provide manufacturer an alternative channel to distribute products, which reduce the dependence on offline channels. As a result, manufacturer and offline retailer’s equilibrium decisions are likely to be different with and without online selling channel. The manufacturer and offline retailer cooperates under various power structures (El-Ansary and Stern, 1972), including vertical Nash structure (VN), manufacturer Stackelberg structure (MS), and retailer Stackelberg structure (RS). Differently from existing studies on selection of online selling mode (Tian et al., 2018; Abhishek et al., 2015; Tan et al., 2016, and others), we consider how offline supply chain power structure affects the manufacturer’s online selling decisions. To conduct the study,
we develop a strategic competition model among a manufacturer, a retailer, and an e-tailer in this study. The manufacturer sells homogeneous products to consumers through offline and online channels. In the offline channel, the manufacturer and the retailer have different degrees of offline channel power. As such, the sequence of the decision making correspondingly differs. In the online channel, the manufacturer can choose online selling mode (either reselling or agency selling mode) on the basis of the various offline channel power structures. We try to answer three questions in this study. First, how does the manufacturer choose between the online reselling mode and online agency selling mode under different offline channel power structures? Second, what is the impact of offline channel power structures and online channel selling modes on the pricing decisions and profitability of the manufacturer? Third, how do various offline channel power structures affect consumer surplus?

The rest of this study is organized as follows. In the next section, we summarize the relevant literature. Then, we describe our model and analysis in Sections 3 and 4, respectively. In Section 5, we discuss the major insights from our results. Finally, we conclude the paper and discuss future research directions in Section 6. All proofs can be found in the Online Appendix.

2. Literature review

Three streams of research are closely related to our work: channel power structures, distribution channel design, and online channel mode selection. In the following, we review studies relevant to each stream and highlight the research gap between this study and the existing literature.

First, our study is related to the literature on channel power structure. Wu et al. (2012) investigated the impact of power structure on pricing decisions in a dual-channel supply chain. They found that, when retail substitutability is low, vertical interaction has a stronger impact on the performance of the game models than the horizontal interaction. Luo et al. (2017) considered a supply chain with a retailer and two manufacturers with differentiated brands. Then, they developed multiple-stage game models to examine the impact of different power structures on the pricing decisions and profits of manufacturers and retailer; they found that intensified competition between the two manufacturers reduced their benefits but favored the retailer. Karray and Sigué (2018) analyzed a case in which an offline retailer’s expansion online channel is desirable and how it affects channel members’ strategies and profits in dual channels. They found that, when the online market is insufficient or excessive, the offline retailer that is expanding its online market can increase channel power and obtain additional profits. Luo et al. (2018) focused on the retailer’s product categories and pricing decisions on the basis of the power structures. They determined that the power structure has no effect on the retailer’s product choice but has a strong influence on supply chain members’ pricing policies and performances. Meng et al. (2018) used Nash game and Stackelberg game to analyze the effect of the power structure on the product selection strategies of two competitive firms. They reported that firms always adopt the same product strategy when they have equal power but select differentiated product strategy with a different channel power. Our study differs from the aforementioned studies in two key aspects. First, we consider the offline channel power structure in identifying an equilibrium online channel mode for the manufacturer. Second, we consider a manufacturer who sells the products through an e-tailer and a retailer.

Second, several studies focus on online channel mode selection. Abhishek et al. (2015) studied the impact of cross-channel spillovers in downstream competition on e-tailers’ mode selection. They determined that, when sales in the online channel have a positive effect on demand in the offline channel, e-tailers prefer the online reselling mode; otherwise, they prefer the online agency selling mode. In addition, as competition between e-tailers increases, they select the online agency selling mode. Tan et al. (2016) analyzed a supply chain with a supplier and two competing retailers. They found that the online agency selling mode facilitates coordination among retailers by dividing the coordinated profits into a pre-negotiated revenue sharing proportion. Moreover, all supply chain members prefer the online agency selling mode in the Pareto improving region. Dennis et al. (2017) analyzed the impact of market power and market size on the manufacturer’s mode selection. They found that, when the e-tailer has a relatively low market power, the online drop-shipping mode benefits both the manufacturer and the e-tailer. Yan et al. (2018) focused on the introduction of the marketplace mode under the condition of upstream disadvantage of sales efficiency and demand information. They found that the marketplace channel should be introduced under not only a low degree but also a high degree of upstream sales inefficiency, which also means that a weak direct channel would not necessarily become a burden for the two. Giwei et al. (2018) studied the dynamic selling strategies (direct selling or agency selling) for a firm under asymmetric market and product fit information in multiple periods. They showed that if the firm and retailer do not share the additional market information with each other, the agency selling mode is never optimal in the second period. Tian et al. (2018) discussed the impact of upstream competition and order-fulfillment costs on the manufacturer’s online channel optimal mode, a later study by Fu et al. (2016) considered trust in the similar decision problem. They found that, when the order-fulfillment costs and the competition intensity are large, the manufacturer prefers the reselling mode; when order-fulfillment costs and the competition intensity are small, the marketplace mode is the preferred choice. Our study on online channel mode strategies differ from the stream of online selling mode literature in three ways. First, we analyze the online channel mode selection from the manufacturer’s perspective. Second, we focus on the online agency selling mode and online reselling mode. Third, we illustrate the manufacturer’s online channel mode selection, the retailer’s online channel mode preference, and their common preferences.

In summary, this study contributes to the literature on online channel mode selection by considering offline channel power structures. This study divides offline channel power structure into three types: vertical Nash structure (VN), manufacturer Stackelberg structure (MS), and retailer Stackelberg structure (RS). The interaction between the offline channel power structures and the online selling modes is also investigated. In addition, we derive the key factors that influence the manufacturer’s preference for online selling modes.

3. Basic models

We consider a manufacturer (m) who sells the products through a retailer (r) and an e-tailer (e). In the offline channel, the manufacturer sells products at the wholesale price \( w \) to the retailer, who then sells to the consumers at the retail price \( p \). In the online channel, the manufacturer may choose to cooperate with an e-tailer via the reselling mode (R) or agency selling mode (A). In the online reselling mode, the manufacturer sells the products at the wholesale price \( w \) to the e-tailer, who then sells to the consumers at the retail price \( p \). In the online agency selling mode, the manufacturer sells its products directly to consumers at the price \( p \) and pays the e-tailer a fraction \( k \) of the revenues as fee for accessing customers. The above modes are illustrated in Fig. 1.

Throughout the analysis, we use the following notation: the superscript \( a/b \) represents the case where \( a \in \{ R, A \} \) denotes the online channel mode (“R” for reselling mode and “A” for agency selling mode) and the case where \( b \in \{ VN, MS, RS \} \) denotes the offline channel power structure (“VN” for vertical Nash structure, “MS” for manufacturer Stackelberg structure, and “RS” for retailer Stackelberg structure); the subscripts \( m, r, \) and \( e \) denote the manufacturer, retailer, and e-tailer, respectively.

To describe the demand functions of the offline and online channels, we adopt a linear demand substitution function. Let \( D_1 \) and \( D_2 \) represent the market demand in the offline and online channels, respectively.
Some consumers prefer to buy products via the offline channel because they can experience the products and the associated services. However, other consumers prefer to buy products in the online channel because they can save time and visit costs. Consumers also have different preferences for purchasing channels. Thus, we use the notations $a(0 < a < 1)$ and $(1 - a)$ to express the proportion of consumers who prefer the offline and online channels, respectively. On the basis of the consumers’ shopping channel preferences, the retail prices in the two channels can have an effect on consumers’ purchasing decisions. In this paper, we use $\beta(0 < \beta < 1)$ to represent the cross-price elasticity coefficient, with a high $\beta$ indicating a great intensity of channel competition (Dennis et al., 2017; Chen et al., 2012).

We focus on the effect of the offline channel power structure and intensity of channel competition on the selling mode strategies in the online channel. Thus, for simplicity, we assume that the production cost of the manufacturer and the sales costs of the retailer and e-tailer are zero (Kumar and Ruan, 2006; Yan and Pei, 2015). Without loss of generality, the total market potential is assumed to be 1. As proposed by Shang and Yang (2015), we assume the market demand follows linear functions (equations (1) and (2)).

$$D_r = a - p_r + \beta p$$  \hspace{1cm} (1)

$$D_e = 1 - a - p_r + \beta p$$  \hspace{1cm} (2)

Although demand function are often assumed to be deterministic (Kyparisis and Koulamas 2018; Mahmoodi 2019, and others), many existing studies believe the market may vary over time and denote demand to be random (Ozer et al., 2011; Li 2019, and others). Thus, we have an extending considering demand as a random variable in section 5.2.

When the manufacturer selects the reselling mode in the online channel, the manufacturer, retailer, and e-tailer’s profit functions are respectively given by

$$\pi^e_m = w_D + (1 - k)p_D$$  \hspace{1cm} (6)

$$\pi^e_r = m_D$$  \hspace{1cm} (7)

$$\pi^e_e = kp_D$$  \hspace{1cm} (8)

To explore supply chain decisions, we assume manufacturer’s wholesale price, the retailer’s selling price and online selling price are decision variables. Meanwhile, the basic market proportions of markets $\alpha$ and intensity of channel competition $\beta$ are assumed as exogenous variables.

4. Equilibrium analysis under various structures

Under the VN structure, manufacturers and retailers decide independently and simultaneously. They have completely symmetric and parallel powers when competing on product pricing. For example, Chinese companies Haier and Gome, a leading electrical appliance manufacturer and a giant household appliance retailer, have strong channel power and are in a balanced market position. Therefore, they are engaged in and cooperate under the VN structure. Under the MS structure (MS), manufacturers and retailers have asymmetric powers in the pricing decisions. Under this structure, manufacturers are the Stackelberg leader and anticipate the retailer’s response on sales margin before deciding on the wholesale price. For instance, in some electronic supply chains, Apple, Caterpillar, and Nike act as leaders with more power. As such, they play a dominant role over downstream retailers and cooperate with other retailers in the MS structure. Under the RS structure, retailers are the Stackelberg leader and manufacturers act as the follower, retailer can anticipate the manufacturer’s reaction before deciding on the sales margin and marketing effort level (Ma et al., 2013). For example, Wal-Mart, Best Buy, Carrefour, and Tesco are in a relatively strong competitive position and act as leaders with a dominant role compared with upstream manufacturers. Thus, these giant retailers cooperate with small and medium-sized manufacturers under the RS structure. In this research, we assume the manufacturer have two modes in online selling channel, i.e., reselling mode, agent selling mode. Meanwhile, the manufacturer can discard online selling channel without choose any modes.

4.1. Vertical Nash (VN) structure

4.1.1. Reselling mode

Under the VN structure, the manufacturer and retailer have equal offline channel power. Therefore, they make pricing decisions in offline
channel simultaneously. In the online channel, the reselling mode means that the e-tailer buys products from the manufacturer at a wholesale price and then sets the retail price. Accordingly, the sequence of events is as follows. Initially, the manufacturer and retailer simultaneously decide \( w_e \) and \( m_e \). The manufacturer then decides \( w_r \). Following this, the e-tailer decides \( m_e \). The sequence is illustrated in Fig. 2.

We solve this subgame using backward induction. For any given wholesale prices \( w_r \) and \( w_e \), and the retailer margin \( m_e \), we initially characterize the equilibrium e-tailer margin \( m_e \) that will maximize \( \pi_e^{VN} \). Then, we determine the wholesale price \( w_e \) that will maximize \( \pi_m^{VN} \). Finally, we determine the wholesale price \( w_r \) and retailer margin \( m_e \) by simultaneously maximizing \( \pi_m^{VN} \) and \( \pi_e^{VN} \). The equilibrium solutions under VN structure are presented in Table A1, based on which we have the manufacturer’s profit \( \pi_m^{RS} = w_r^{VN} D_r^{VN} + w_e^{VN} D_e^{VN} \). We observe that all decisions and market demand in the online and offline channels are directly influenced by the consumer’s channel preference and channel competition. Meanwhile, we find that the consumer’s preference of the offline channel has positive effects on the retailer’s profit margin and negative effects on the manufacturer’s wholesale price to the e-tailer. This observation seems logical and intuitive. It can explain how channel power derives additional advantages in supply chain transactions.

4.1.2. Agency selling mode

In the online channel, the agency selling mode means that the manufacturer can directly sell products to consumers and decide the retail price. However, the manufacturer needs to pay the e-tailer a certain portion of revenue as commission rate. In this case, the sequence of events is as follows. Initially, the manufacturer and retailer simultaneously decide \( w_e \) and \( m_e \). Then, the manufacturer decides \( p_e \). The sequence is illustrated in Fig. 3.

We solve this subgame using backward induction. For any given wholesale price \( w_e \) and retailer margin \( m_e \), we initially characterize the equilibrium retail price \( p_e \) that will maximize \( \pi_e^{VN} \). Then, we determine the wholesale price \( w_e \) and retailer margin \( m_e \) by simultaneously maximizing \( \pi_m^{VN} \) and \( \pi_e^{VN} \).

4.2. Manufacturer Stackelberg (MS) structure

4.2.1. Reselling mode

Under the MS structure, the manufacturer is the Stackelberg leader, whereas the retailer is the follower. Their interactions in the online reselling mode are modeled as a three-stage game. Initially, the manufacturer decides \( w_e \) and \( w_r \). The retailer then decides \( m_r \). Following this, the e-tailer decides \( m_e \). The sequence is illustrated in Fig. 4.

We solve this subgame using backward induction. For any given wholesale prices \( w_r \) and \( w_e \) and the retailer margin \( m_e \), we initially characterize the equilibrium e-tailer margin \( m_e \) that will maximize \( \pi_e^{MS} \). Then, we determine the retailer margin \( m_e \) that will maximize \( \pi_e^{VN} \). Finally, we determine the wholesale prices \( w_r \) and \( w_e \) by maximizing \( \pi_m^{MS} \).

4.2.2. Agency selling mode

We use a game theory framework to capture the strategic interactions between the manufacturer and the e-tailer in the online agency selling mode. In this case, the manufacturer decides the \( w_r \) and \( p_e \). The retailer then decides \( m_e \). The sequence of decisions between the manufacturer and retailer is illustrated in Fig. 5.

We solve this subgame using backward induction. For any given wholesale price \( w_r \) and retail price \( p_e \), we characterize the equilibrium retailer margin \( m_e \) that will maximize \( \pi_e^{MS} \). Then, we determine the wholesale price \( w_r \) and retail price \( p_e \) by maximizing \( \pi_m^{MS} \).

4.3. Retailer Stackelberg (RS) structure

4.3.1. Reselling mode

Under the RS structure, the retailer is the Stackelberg leader, whereas the manufacturer is the follower. The sequence of events is as follows. Initially, the retailer decides \( m_r \). Then, the manufacturer decides \( w_r \) and \( w_e \). Thereafter, the e-tailer decides \( m_e \). Similarly, the sequence of decisions between the manufacturer, retailer, and e-tailer is illustrated in Fig. 6.

We solve this subgame using backward induction. For any given wholesale prices \( w_r \), \( w_e \) and retailer margin \( m_r \), we characterize the equilibrium e-tailer margin \( m_e \) that will maximize \( \pi_e^{RS} \). Then, we simultaneously determine the wholesale prices \( w_r \) and \( w_e \) by maximizing \( \pi_m^{RS} \). Finally, we determine the retailer margin \( m_r \) by maximizing \( \pi_r^{RS} \).

4.3.2. Agency selling mode

In this configuration, anticipating the manufacturer’s pricing decisions in the online and offline channels, the retailer can announce the marginal profit. The sequence of events is as follows. Initially, the retailer decides \( m_r \). Then, the manufacturer simultaneously decides \( w_r \) and \( p_e \). The sequence of decisions is illustrated in Fig. 7.

We solve this subgame using backward induction. For any given retailer margin \( m_r \), we characterize the equilibrium wholesale price \( w_r \) and retail price \( p_e \) by maximizing \( \pi_m^{RS} \). Then, we determine the retailer margin \( m_r \) by maximizing \( \pi_r^{RS} \).

Sections 4.1-4.3 presents the supply chain partners’ decision process under different offline power structures. The equilibrium solutions under different offline power structures are summarized in Table A1 in appendix A.

5. Supply partners’ preference of mode selection

5.1. Mode selection with deterministic demand

5.1.1. Manufacturer’s online channel mode selection

Using the solutions of the online reselling mode and online agency selling mode under the three offline channel power structures, we compare the effect of competition intensity, commission rate, and proportion of preference for the offline channel on the manufacturer’s profit and online selling mode selection. The results established in the previous lemmas are useful in building the intuitions for the analytical results on profit comparisons.
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Fig. 3. Sequence of events in agency selling mode under the VN structure.

Fig. 4. Sequence of events in reselling mode under the MS structure.

Fig. 5. Sequence of events in agency selling mode under the MS structure.

**Proposition 1.** Under the three offline channel power structures, for any given $\alpha$, we define a set for the commission rate $k$. When $k \in k^\text{VN}$, $k \notin k^\text{MS}$, and $k \notin k^\text{RS}$, the manufacturer is indifferent toward the preference between the online reselling mode and online agency selling mode. However, when $k \notin k^\text{VN}$, $k \notin k^\text{MS}$, and $k \notin k^\text{RS}$, the manufacturer prefers different online selling modes, where

$$
k_{\text{VN}} = \left\{ k_{\text{VN}} \mid m^\text{VN}_m = m^\alpha, 0 < k < 2\sqrt{1-\beta_1}\sqrt{1\sqrt{1-\beta_1}}, \right\},$$

$$
k_{\text{MS}} = \left\{ k_{\text{MS}} \mid m^\text{MS}_m = m^\alpha, 0 < k < 2\sqrt{2(1-\beta_1)(1\sqrt{1-\beta_1})}, \right\},$$

$$
k_{\text{RS}} = \left\{ k_{\text{RS}} \mid m^\text{RS}_m = m^\alpha, 0 < k < 2\sqrt{1-\beta_1}^-1\sqrt{1-\beta_1}, \right\}.
$$

For the proof, please refer to the Appendix.

Proposition 1 suggests that, under different offline channel power structures, the manufacturer’s preference between the online reselling mode and online agency selling mode depends on the consumers’ channel preference, intensity of channel competition, and commission rate. In other words, if the commission rate is limited to a certain range, then a common boundary exists between the online agency selling mode and online reselling mode. Within this boundary, choosing between these two online selling modes causes no difference; however, outside the boundary, the manufacturer has to select either the online reselling mode or online agency selling mode.

Proposition 1 suggests that consumers’ preference of online and offline channel does not matter the manufacturer’s selection of selling modes. However, the manufacturer’s online selling model is affected by the cross-price elasticity. Since the manufacturer’s online selection is an optimization problem by comparing the benefit from different supply modes, it is reasonable that the online volume does affect the decisions of modes. Meanwhile, Figs. 8–10 also indicate a manufacturer’s decision process. At first, the manufacturer decides whether sells products by online channel, where the commission rate is a key impact factor. When commission rate exceeds a certain threshold, the manufacturer gives up the online channel. Once the manufacturer decides online sell, the next question is to decide the optimal online mode in selling. In industries, the commission rate comes under different names e.g., referral fee by Amazon, final value fees by eBay, transaction fees by Etsy etc. Commission rate internally means a percentage or fixed amount that the e-tailer or platform charge for each sale. Since commission rate matters manufacturer’s channel selection, it is an important issue for the e-tailers to offer more values. It is practical and strategic that the e-tailers alter commission rates based on the maturity of business. For example, the e-tailers attract more manufacturers when marketplace is not mature. When the marketplace expands the consumer base, it is reasonable to switch to a high commission rate. In this case, a sophisticated commission decision strategy help to improve the performance of the e-tailers.

Figs. 8–10 numerically illustrate how the interactions among these three parameters affect the manufacturer’s online channel mode preference.

As shown in Fig. 8, under the VN structure, the manufacturer prefers the online agency selling mode when the commission rate is low and the online reselling mode when the commission rate is high. Then, an increasing competition intensity between the online and offline channels results in a decreasing indifference threshold for the online agency selling mode and online reselling mode. Thus, the manufacturer is more likely to select the online reselling mode.

Figs. 9 and 10 illustrate the manufacturer’s online channel preference...
under the MS and RS structures, respectively. First, when the commission rate is low, and as the competition intensity increases, the manufacturer is more likely to select the online reselling mode. Second, when the commission rate is at a medium range, the online reselling mode remains the preferred mode. Third, when the commission rate is relatively high, the presence of many consumers preferring the offline channel leads to the preference for the online agency selling mode, and the presence of many consumers preferring the online channel leads to the preference for the online reselling mode. Meanwhile, as competition intensity increases, the manufacturer is likely to choose the online agency selling mode.

Interestingly, the results shown in Figs. 8–10 indicate that, if the commission rate is low, then the probability of the manufacturer selecting the online agency selling mode is highest under the MS structure, followed consecutively by the RS and VN structures. However, if the commission rate is high, then the probability of the manufacturer selecting the online reselling mode is highest under the VN structure, followed consecutively by the RS and MS structures.

5.1.2. Manufacturer’s and retailer’s online channel mode preference

Thus far, we have analyzed the manufacturer’s online selling mode preference under different offline channel power structures. In this section, we investigate whether and under which conditions the manufacturer’s and retailer’s online selling mode is consistent or in conflict with each other. We examine how the manufacturer’s and retailer’s online channel mode preference is affected by three key parameters: commission rate, preference of offline channel, and competition intensity.

Figs. 11–13 numerically illustrate the manufacturer’s and retailer’s online channel mode preference under different offline channel power structures. In this paper, regions ② and ③ indicate that the manufacturer’s and retailer’s online channel mode preferences are consistent; regions ④ and ⑤ indicate that the manufacturer’s and retailer’s online channel mode preferences are in conflict. Specifically, regions ② and ③ respect that the manufacturer and retailer prefer the online reselling mode and online agency selling mode, respectively. Regions ④ and ⑤ show that the manufacturer selects the online reselling mode but the retailer prefers the online agency selling mode, and the manufacturer selects the online agency selling mode but the retailer prefers the online reselling mode, respectively.

As shown in Figs. 11–13, when the commission rate is medium, there exists a phenomenon that the retailer’s online channel mode preference is consistent with the manufacturer’s online channel mode selection. Furthermore, intensifying competition between the online and offline channels leads to a decrease in the possibility of the manufacturer and retailer prefer the online reselling mode, however, the possibility of the manufacturer and retailer prefer the online agency selling mode.

![Stage One](#) The retailer decides $m_r$

![Stage Two](#) The manufacturer decides $w_r$ and $w_e$

![Stage Three](#) The e-tailer decides $m_e$

**Fig. 6.** Sequence of events in reselling mode under the RS structure.

![Stage One](#) The retailer decides $m_r$

**Fig. 7.** Sequence of events in agency selling mode under the RS structure.

![Fig. 8.](#) Manufacturer’s online channel mode selection under the VN structure.
Fig. 9. Manufacturer’s online channel mode selection under the MS structure.

Fig. 10. Manufacturer’s online channel mode selection under the RS structure.

Fig. 11. Manufacturer’s and retailer’s online channel mode preference under the VN structure.

Fig. 12. Manufacturer’s and retailer’s online channel mode preference under the MS structure.
increases. In particular, an indifference threshold $\alpha'$ exists under the VN structure. When $\alpha \leq \alpha'$, the common preference for the manufacturer and retailer is on-line reselling mode; meanwhile, the common preference for the manufacturer and retailer is online agency selling mode. In addition, the indifference threshold $\alpha'$ decreases with the increasing competition intensity between the online and offline channels.

In fact, supply chain partners aim to improve individual performance which leads some conflicts of interests among supply chain members. Double marginalization is one of results due to supply chain conflicts. However, the findings by Figs. 11–13 suggest that supply chain partners, i.e., retailer and manufacturer, can also achieve consistent supply chain decisions in many parameter situations which benefit all of them. Therefore, we highlight that supply chain partners potentially benefit from exploring more opportunities to get consistent and enlarge benefits of all supply chain partners.

5.1.3. Wholesale and retail prices in the online and offline channels

In general, e-tailers and retailers are always competing through price. They are willing to seize a large market share and obtain additional profits by adopting a competitive price. Therefore, we compare wholesale and retail prices under the three offline channel power structures and obtain the following propositions:

**Proposition 2.** (i) In the VN structure, $W^e_{V/N} \leq W^e_{R/V} \leq W^e_{M/S}$ when $\alpha \leq \alpha_0$, and $W^e_{R/RS} > W^e_{R/V} \geq W^e_{M/S}$ when $\alpha > \alpha_0$, where $\alpha_0 = \frac{\beta - \beta'_0}{\beta - \beta'_0 + \beta'}$. (ii) In the MS structure, $W^e_{R/MS} \leq W^e_{R/R/S}$ when $\alpha \leq \alpha_1$, and $W^e_{R/RS} > W^e_{R/M/S}$ when $\alpha > \alpha_1$, where $\alpha_1 = \frac{\beta - \beta'_0}{\beta - \beta'_0 + \beta'}$. (iii) In the RS structure, $W^e_{R/R/S} \leq W^e_{R/RS}$ when $\alpha \leq \alpha_2$, and $W^e_{R/RS} > W^e_{R/R/S}$ when $\alpha > \alpha_2$, where $\alpha_2 = \frac{\beta - \beta'_0}{\beta - \beta'_0 + \beta'}$.

For the proof, please refer to the Appendix.

Wholesale price indicates the allocation of benefits between manufacturer and its down-streamers; Proposition 2 presents the comparisons of wholesale prices with different product allocation channels. The analytical findings indicate that under three different offline channel power structures, if more consumers prefer the offline channel, then the wholesale price in the offline channel is higher than that in the online channel. Otherwise, the wholesale price in the offline channel becomes lower than that in the online channel. In other words, the manufacturer should implement differential pricing strategies according to the consumers’ channel preference. As well, Proposition 2 also reflects the effects of market share to manufacturer’s decisions on wholesale prices in different channels. For example, when the offline market share is low, the manufacturer support the offline market by providing a low wholesale price. This finding reflects the industrial practice of multiple sourcing strategy which helps to reduce reliance on one business partner and enhance capacity in transaction negotiations. Apple, as a famous global company, purchases its components from multiple channel of sources and its suppliers are diversifying their wholesale sources to reduce dependence on Apple. In the online reselling mode, comparing the wholesale prices $w_e$ and $w_r$ across the three offline channel power structures, we derive the following proposition:

**Proposition 3.** (i) $W^e_{R/RS} < W^e_{R/V/N} < W^e_{M/S}$; (ii) $W^e_{R/RS} = W^e_{R/V/N} = W^e_{M/S}$.

For the proof, please refer to the Appendix.

Proposition 3 indicates that the wholesale prices in the offline channel are highest under the MS structure. As the leader of the game, the manufacturer possesses greater bargaining power and tends to maximize the wholesale prices compared with the others. However, under the RS structure, the retailer’s attempt to maximize profits by suppressing the wholesale prices results in the lowest wholesale prices in the offline channel. This observation indicates that the companies with supply chain power have advantages in pricing, which emphasizes the importance to obtain powers in supply chain cooperation. As for the online wholesale price with different offline power structure, we find from Proposition 3 that offline power structure does not affect the online wholesale price. Proposition 3 suggests the wholesale prices in different situations with offline power structures and channels, we explore how the retailer and e-tailer price under different situations by Proposition 4.

**Proposition 4.** (i) $p^e_{R/V/N} < p^e_{R/M/S} = p^e_{R/R/S}$; (ii) $p^e_{R/V/N} < p^e_{R/M/S} = p^e_{R/R/S}$.

For the proof, please refer to the Appendix.

With reselling mode in the online channel, the consumers are charged the lowest price for products under the VN structure. Meanwhile, they pay the same price under the MS and RS structures. Therefore, the consumers receive additional surplus when the supply chain partners are equivalent in competition. Since the offline supply chain power structure matters the offline wholesale price from Proposition 3 and there exists competition between two retail channels (equations 1-2), online market demand is influenced by offline power structure.

**Proposition 5.** (i) In the VN structure, $p^r_{R/V/N} \leq p^e_{R/V/N} \leq p^e_{R/M/S} \leq p^e_{R/R/S}$ when $\alpha \leq \alpha_3$, where $\alpha_3 = \frac{18 - 40^2 - 13\beta_0 - 3\beta'_0}{34 \beta - 25 \beta'_0 - 3 \beta'}$. (ii) In the MS and RS structures, $p^r_{R/V/N} < p^e_{R/M/S} \leq p^e_{R/R/S}$ when $\alpha \leq \alpha_4$, and $p^r_{R/V/N} \leq p^e_{R/R/S}$ when $\alpha > \alpha_4$, where $\alpha_4 = \frac{12 \beta - 5 \beta'_0 - 3 \beta'}{34 \beta - 25 \beta'_0 - 3 \beta'}$ and $c \in \{MS, RS\}$.

For the proof, please refer to the Appendix.

We have Proposition 5 to explore the selling price in different channels with the reselling mode. The analytical result indicates that the price in the offline channel is lower than that in the online channel as long as the consumers’ preference for the online channel is under a certain threshold. In other words, the consumers’ preference directly affects their prices in dual channels. In general, market demand is directly influenced by the selling price, product reputation, recommendation, and others. Thus, pricing strategy is a frequently used approach to occupy the market, especially for newly launched products. The supply chain structures vary from time to time because of dynamic market competition. As a result, the analytical results by Proposition 5 suggest that the manufacturer promptly alternates the market...
price in different market competition situations. In the online agency selling mode, comparing the retail prices \( p^o \) and \( p^e \) across the three offline channel power structures, we derive the following proposition:

**Proposition 6.** (i) \( p^{AV/VN} < p^{AM/MS} < p^{RM/RS} \); (ii) \( p^{AM/MS} < p^{RM/RS} < p^{AV/VN} \).

For the proof, please refer to the Appendix.

Proposition 6 intuitively shows that in the online agency selling mode, the retail price in the offline channel is highest under the MS structure but lowest under the RS structure. Meanwhile, the retail price in the online channel is highest under the VN structure but lowest under the MS structure. Therefore, the manufacturer and retailer always adopt differential pricing strategies to set prices for the online and offline channels, respectively. Thus, the offline channel power structures and online selling mode can influence the pricing decisions in both channels.

### 5.1.4 Consumer surplus under different structures

In addition to the online channel selling mode preference and retailer price in the supply chain, we also explore the impact of the commission percentage and preference of offline channel on consumer surplus. From the consumers’ perspective, Figs. 14–16 compare the online reselling mode and online agency selling mode across the three offline channel power structures. Figs. 14–16 are generated by setting \( \beta = 0.6 \).

These figures illustrate that, with the increase of consumer preference for the offline channel, the consumer surplus initially decreases and then increases. In addition, regardless of the VN structure, MS structure or RS structure, the consumers always can obtain higher surplus in the online agency selling mode. In particular, when the manufacturer selects the online reselling mode, the consumer surplus retains the same level under the MS and RS structures. Interestingly, in the online agency selling mode, when the proportion of consumers who prefer the offline channel is low, the consumer surplus is reduced with the commission rate; however, when the proportion of consumers who prefer the offline channel is high, the consumer surplus increases with the increase in commission rate.

### 5.2 Mode selection with stochastic demand

In the basic model, we assume that market demand is linear deterministic referring to some existing studies (Kyparisis and Koulamas 2018; Mahmoodi 2019, and others). Because demand sometimes is random with uncertainty (Özer et al., 2011; Li 2019, and others), we consider the demand to be a random variable and examine how supply partners make decisions. Hereby, we focus on developing the random demand and numerically investigate whether the main contribution of the manufacturers’ and retailers’ online channel model preferences from the previous sections remain valid. By expanding the demand function in Section 3, we assume that the demand for offline stores \( D_i \) and online stores \( D_e \) are as follows:

\[
\tilde{D}_i = \tilde{a} - \tilde{P}_i + \beta \tilde{P}_o \quad \text{and} \quad \tilde{D}_e = 1 - \tilde{a} - \tilde{P}_o + \beta \tilde{P}_e
\]

The parameters \( a \) and \( 1 - a \) denote the base level demands for the online and offline stores, respectively. \( a \) is a random variable and we assume \( a = a + e \). Here \( e \) represents the mean of the potential intrinsic demand, and \( e \) follows a normal distribution such that \( E(e) = 0 \quad \text{Var}(e) = \sigma^2 \) (Yue and Liu, 2006; Tang, 2006).

According to the conclusion above, the random profits of the manufacturer, the retailer, and e-tailer in online reselling mode are formulated as \( \tilde{\Pi}_m = \tilde{w}_i \tilde{D}_i + \tilde{w}_e \tilde{D}_e \), \( \tilde{\Pi}_e = \tilde{m}_e \tilde{D}_e \), and \( \tilde{\Pi}_r = \tilde{m}_e \tilde{D}_e \).

The random profits of the manufacturer, the retailer, and e-tailer in online agency selling mode are as follows, \( \tilde{\Pi}_m = \tilde{w}_i \tilde{D}_i + (1 - k) \tilde{w}_p \tilde{D}_o \tilde{\Pi}_e = \tilde{m}_e \tilde{D}_e \) and \( \tilde{\Pi}_r = k \tilde{w}_p \tilde{D}_o \).

Because demand is stochastic, participants are exposed to financial risk. We assume that the manufacturer, the retailer, and e-tailer assess their utilities using the following mean–variance value function of their expected profits under uncertainty (Agrawal and Seshadri, 2000; Gan et al., 2005; Lee and Schwarz, 2007; Xiao and Yang, 2008),

\[
u_m = E(\tilde{u}_m) - \lambda_m Var(\tilde{u}_m) \quad \text{and} \quad \nu_e = E(\tilde{u}_e) - \lambda_e Var(\tilde{u}_e)
\]

where the second term is the participants’ risk cost, and \( \lambda_m(\lambda_m > 0) \) and \( \lambda_e(\lambda_e > 0) \) reflect the respective attitudes of the manufacturer, the retailer, and e-tailer toward uncertainty. Above equation suggest the manufacturer, the retailer, and e-tailer will all weigh the mean and variance of their random profits.

When the demand follows some linear function parameter without uncertainty, we propose an analytical solution. However, due to the complexity of mathematical expressions, it is impossible to propose an analytical solution when the demand is stochastic. To obtain some comparable results for the randomly distributed demand and the linear function demand, we observe how the manufacturer’s and retailer’s online channel mode preference is affected by three key parameters: commission rate, preference of offline channel, and competition intensity. The subsequent analysis includes numerous experiments to demonstrate the previous conclusions are still valid in the case of random demand. We let \( \sigma = 0.4 \lambda_m = 0.1 \lambda_e = 0.2, \lambda_e = 0.2 \). The decision process in this game is similar to the previous sections. This dynamic game can be solved by using backward induction.

#### 5.2.1. Manufacturer’s online channel mode selection

We focus on the effect of commission rate, preference of offline channel, and competition intensity. Figs. 17–19 numerically illustrate how the interactions among these three parameters affect the manufacturer’s profit and online selling mode selection. As shown in Fig. 17, under the VN structure, the manufacturer prefers the online agency

![Fig. 14. Consumer surplus under the VN structure.](image-url)
selling mode when the commission rate is low and the online reselling mode when the commission rate is high. The same occurs when we deem demand as a linear function parameter without uncertainty.

Figs. 18 and 19 illustrate the manufacturer’s online channel preference under the MS and RS structures, respectively. Under the MS structure, when the commission rate is at a medium range and the channel competition intensity is small, the online reselling mode remains the preferred mode. However, when the commission rate is at a medium range and the channel competition intensity is high, the presence of many consumers preferring the offline channel leads to the preference for the online agency selling mode, and the presence of many consumers preferring the online channel leads to the preference for the online reselling mode. In addition, in Figs. 18 and 19, when the commission rate is small and large, the manufacturer’s online channel mode preference is consistent with the demand as determined. Therefore, the previous conclusions are still valid in the case of random demand.

5.2.2. Manufacturer’s and retailer’s online channel mode preference

In this section, we investigate whether and under which conditions the manufacturer’s and retailer’s online selling mode is consistent or in conflict with each other when demand is stochastic. Similar to the results for deterministic demand (see Figs. 11–13), there are similar
observations for randomly distributed demand (see Figs. 20–22). Regions ② and ③ respect that the manufacturer and retailer prefer the online reselling mode and online agency selling mode, respectively. Regions ④ show that the manufacturer selects the online reselling mode but the retailer prefers the online agency selling mode. Regions ⑤ show that the manufacturer selects the online agency selling mode but the retailer prefers the online reselling mode.

When the commission rate is medium, there exists a phenomenon that the retailer’s online channel mode preference is consistent with the manufacturer’s online channel mode selection. This is consistent with the conclusion when demand is determined. Under the MS structure, when the commission rate is high, with the increase of competition intensity, the online channel choice of manufacturer and retailer will change from conflict to consistency, and the online reselling mode will be uniformly chosen. Therefore, when demand is stochastic, the effects of commission rate, preference of offline channel, and competition intensity on the online channel model selection of manufacturer and retailer are still valid.
6. Conclusions

Online commerce provide manufacturer more options to attribute the products. However, the manufacturers must select the appropriate online channel selling mode when they cooperate with other supply chain members with different channel power structures. We classified offline channel power structures into three types: the vertical Nash structure (VN), manufacturer Stackelberg structure (MS), and retailer Stackelberg (RS). We discussed the relationship between the offline channel power structure and online channel selling mode selection. Based on our results, the following novel insights are obtained. First, when the commission coefficient is low, the manufacturer selects the online agency selling mode under the different power structures; when the commission coefficient is high, the manufacturer selects the online reselling mode in the VN structure but selects the online agency selling mode in the other two structures. Second, when the commission coefficient is medium, the increasing levels of the consumers who prefer offline channel can result in a transition of the manufacturer’s and retailer’s common preference from the online reselling mode to the online agency selling mode. Third, regardless of the VN, MS, and RS structures, the consumers always obtain higher surplus in the online agency selling mode.

By comparing the profitability of different online selling modes under different offline power structures, our study helps shed light on how the manufacturer should adopt different online channel selling modes under different offline channel power structures and shows the common preference of online selling mode among the manufacturer and retailer under different offline channel power structures. From the economic and strategic perspectives, with different offline channel power structures, the manufacturer must be able to select the suitable online selling mode to coordinate their online and offline sales, thereby obtaining additional benefits. We note, however, a few limitations in this study. For example, we only consider the decisions between two supply chain partners; although a supply chain might contain more members, such as the suppliers and competitors. Future studies may incorporate more supply chain partners into our decision framework.

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Because the first leading principal minor is negative and the second one is positive, the Hessian matrix is negative definite, so there is a unique optimal solution $w$. By solving the combined equations, we obtain the Hessian matrix: $H(w, m) = \begin{pmatrix} -2(1 - \rho^2) & -1 + \beta^2 \\ -1 + \beta^2 & -2 + 3\beta^2 \end{pmatrix}$, and its leading principal minors are $-2(1 - \rho^2) < 0$, and $|H(w, m)| = (3 - 2\beta^2)(1 - \rho^2) > 0$.

Because the first leading principal minor is negative and the second one is positive, the Hessian matrix is negative definite, so there is a unique optimal solution $w$ and $m$. By solving the combined equations, we obtain the manufacturer’s optimal $w$ and retailer’s optimal $m$. Accordingly, the retail prices for retailer and e-tailer are $p^R_{VN} = \frac{(8 - 7k\beta^2 - 6k\rho^2\beta + (1 - \beta)(12 - 2k - k^2\beta - 2(6 - 7k + 2k^2)\beta^2 - (2 - k^2)\beta^2)\alpha}{4(1 - \rho^2)(1 - k)(2 - \rho^2)}$ and $p^E_{VN} = \frac{(8 - 7k\beta^2 - 6k\rho^2\beta + (1 - \beta)(12 - 2k - k^2\beta - 2(6 - 7k + 2k^2)\beta^2 - (2 - k^2)\beta^2)\alpha}{4(1 - \rho^2)(1 - k)(2 - \rho^2)}$.

The solutions under Vertical Nash (VN) structure with agent selling mode. As $\frac{\partial^2 \pi}{\partial m^2} = -2(1 - k) < 0$, there is a unique optimal solution $m$. By solving equation $\frac{\partial^2 \pi}{\partial m^2} = 0$, we obtain $m^*$, optimal function $m^*_{VN} = \frac{1 - \alpha - \beta w}{2 \alpha m^*}$. Substituting $m^*_{VN}$ into $w_m^*$ and $\frac{\partial \pi}{\partial w} = -2 < 0$, so there is a unique optimal solution $w$.

By solving equation $\frac{\partial^2 \pi}{\partial w^2} = 0$, we obtain $w$, optimal function $w^*_{VN} = \frac{1 - (1 - k)(12 - 2k - k^2\beta - 2(6 - 7k + 2k^2)\beta^2 - (2 - k^2)\beta^2)\alpha}{4(1 - \rho^2)(1 - k)(2 - \rho^2)}$.
matrix: \( H(m_r, w_r) = \begin{pmatrix} 2 + \beta^2 & (2 - k)\beta^2 - 2(1 - k) \\ 2 & 2(1 - k) \end{pmatrix} \). Its leading principal minors are \(-2 + \beta^2 < 0\), and when

\[ 0 < k < 2\sqrt{\frac{(1 - \beta^2)(3 - \beta^2)}{\beta^2 + \frac{3 - \beta^2}{2}}} \quad \text{and} \quad |H(m_r, w_r)| > 0. \]

Because the first leading principal minor is negative and the second one is positive, the Hessian matrix is negative definite, so there is a unique optimal solution \( w_r \) and \( m_r \). By solving the combined equations \( \frac{\partial^2 R}{\partial w_r^2} = 0 \) and \( \frac{\partial^2 M}{\partial m_r^2} = 0 \), we obtain the manufacturer’s optimal \( w_r^{\text{RS}} \), retailer’s optimal \( m_r^{\text{RS}} \), and the retail prices for retailer (i.e., \( p_r^{\text{RS}} \)) and e-tailer (i.e., \( p_e^{\text{RS}} \)).

Similarly as the proofs above, we have the equilibrium solutions of manufacturer and retailer under different power structures.

**Proof of Proposition 1.**

First. Because \( \Delta x_m^{\text{VN}} = \Delta x_m^{\text{MS}} - \Delta x_m^{\text{RS}} \), we introduce the solutions in Table A1 and have \( \Delta x_m^{\text{VN}} = \frac{A_0 + 2B + C}{A_0 + B + C} \), where

\[ A = -9\beta^8 - 28\beta^5 + 56\beta^6 + 208\beta^4 - 201\beta^3 - 492\beta^2 + 426\beta + 360\beta - 324 \]

\[ B = 18\beta^8 + 28\beta^5 + 184\beta^4 - 208\beta^3 + 690\beta^2 + 492\beta^3 - 1116\beta - 360\beta + 648 \]

\[ C = -9\beta^8 + 92\beta^8 - 345\beta^4 + 558\beta^2 - 324 \]

When \( k = 0 \), we have \( \Delta x_m^{\text{VN}}(k) = 0 \). Meanwhile, we have \( \Delta x_m^{\text{VN}}(k = 1) > 0 \) when \( 1 - \beta^2 \).

Second. Since \( \Delta x_m^{\text{MS}} = \Delta x_m^{\text{MS}} - \Delta x_m^{\text{RS}} \), we introduce the solutions in Table A1 and have \( \Delta x_m^{\text{MS}} = \frac{A_0 + 2B + C}{A_0 + B + C} \), where

\[ A = (4 - 4a - 3a\beta - 3\beta^2 + 3\beta^2 + 2a\beta^2) \]

\[ B = (4 - 4a + 3a\beta + 4a\beta^2 + 4a\beta^2 - 4a\beta^2 - 2a\beta^2) - (4 - 4a + 4a\beta)(-4 + 4a - 3a\beta - 3\beta^2 + 3\beta^2 + 2a\beta^2) \]

\[ C = (4 - 4a + 4a\beta)(-4 + 4a - 4a\beta + 4a\beta^2 + 4a\beta^2 + 2a\beta^2) \]

Let \( f(k) = Ak^2 + Bk + C \), it is suffice to prove that \( f(k) \) decrease when \( k \in (0, k_1) \) and increase when \( k \in \left(k_1, \frac{2\sqrt{2\beta^2(1 - \beta^2)} + 4-\beta^2}{\beta^2} \right) \), where \( k_1 = \frac{4 - 4a + 3a\beta - 3\beta^2 + 3\beta^2 + 2a\beta^2}{4 - 4a + 3a\beta + 4a\beta^2 + 4a\beta^2 - 4a\beta^2 - 2a\beta^2} < 2\sqrt{2\beta^2(1 - \beta^2)} + 4-\beta^2 \).

It means that \( \Delta x_m^{\text{MS}} \) increase when \( k \in (0, k_1) \) and decrease when \( k \in \left(k_1, \frac{2\sqrt{2\beta^2(1 - \beta^2)} + 4-\beta^2}{\beta^2} \right) \).

When \( k = 0 \), we find that \( \Delta x_m^{\text{MS}}(k) = 0 \). Meanwhile, \( \Delta x_m^{\text{MS}}(k = 1) = \frac{-6\beta^2 + 4\beta^4 + 4\beta^3 - 12\beta^2 + 8\beta^4 - 3\beta^2 - 12\beta^2 + 16\beta^4 - 4\beta^2}{16\beta^2(1 - \beta^2)} \). We find that \( \Delta x_m^{\text{MS}} > 0 \) when

\[ k = 2\sqrt{2\beta^2(1 - \beta^2)} + 4-\beta^2 \approx 4\beta^2 \quad \text{and} \quad \left(\frac{1}{\beta^2} - 1\right) \text{is an integer}. \]

Third. We let \( \Delta x_m^{\text{RS}} = \Delta x_m^{\text{RS}} - \Delta x_m^{\text{RS}} \) and we have \( \Delta x_m^{\text{RS}} = \left(-\frac{5\beta^2 + 12\beta^2 - 8\beta^4 - 3\beta^2 - 12\beta^2 + 16\beta^4 - 4\beta^2}{32\beta^2(1 - \beta^2)} \right) \). We find \( \Delta x_m^{\text{RS}}(k) < 0 \) and \( \Delta x_m^{\text{RS}}(k = 1) > 0 \).

Thus, we can conclude that a set of \( k^{\text{RS}} \) exist. This completes the proof.

**Proof of Proposition 3:**

(1) Under the VN structure:

\[ w_r^{\text{VN}} - w_e^{\text{VN}} = \frac{-6\beta^2 + 4\beta^4 + 4\beta^3 - 12\beta^2 + 8\beta^4 - 3\beta^2 - 12\beta^2 + 16\beta^4 - 4\beta^2}{4\beta^2 - 16\beta^2} \]

(2) Under the MS structure:

\[ w_r^{\text{MS}} - w_e^{\text{MS}} = \frac{6-\beta^2-\beta^2}{6-\beta^2} \]

(3) Under the RS structure:

\[ w_r^{\text{RS}} - w_e^{\text{RS}} = \frac{5\beta^2 + 12\beta^2 - 8\beta^4 - 3\beta^2 - 12\beta^2 + 16\beta^4 - 4\beta^2}{32\beta^2} \]

This completes the proof.
Therefore, \( w_r^{R/RS} < w_e^{R/VN} < w_e^{R/MS} \).

(2) it is apparent that \( w_e^{R/RS} = w_e^{R/VN} = w_e^{R/MS} = \frac{1}{2} \left( \frac{\beta}{1-\beta} \right) \).

This completes the proof.

**Proof of Proposition 4:**

(1) \( p_r^{R/VN} - p_r^{R/MS} = \frac{1-\beta^2}{2(1-\beta)^2} < 0 \). According to the proof of Lemma 3 and Lemma 5, we can find that \( p_r^{R/MS} = p_r^{R/RS} \). Therefore, \( p_r^{R/VN} < p_r^{R/RS} \).

(2) \( p_r^{R/VN} - p_r^{R/RS} = \frac{1-\beta^2}{2(1-\beta)^2} < 0 \). According to the proof of Lemma 3 and Lemma 5, we can find that \( p_r^{R/RS} = p_r^{R/MS} \). Therefore, \( p_r^{R/VN} < p_r^{R/MS} \).

This completes the proof.

**Proof of Proposition 5:**

(1) \( p_e^{R/VN} - p_e^{R/MS} = \frac{1-\beta^2}{2(1-\beta)^2} < 0 \). Therefore, \( p_e^{R/VN} - p_e^{R/MS} = 0 \). Therefore, \( p_e^{R/VN} - p_e^{R/MS} < 0 \).

This completes the proof.

**Proof of Proposition 6:**

(1) It is suffice to prove that \( p_e^{R/MS} - p_e^{R/RS} = \frac{k^2 \beta^2 (1-\beta)^2}{2(1-\beta)^2} < 0 \). Let \( f(\beta) = (k-2)^2 \beta^2 + 2 - 2k \), it shows that \( f(\beta) \) decrease when \( \beta \in (0, 1) \) and \( f(1) = 0 \), thus implying that \( f(\beta) > 0 \).

(2) \( p_e^{R/VN} - p_e^{R/RS} = \frac{k^2 \beta^2 (1-\beta)^2}{2(1-\beta)^2} < 0 \), which means that \( p_e^{R/VN} - p_e^{R/RS} < 0 \). So we obtain that \( p_e^{R/VN} < p_e^{R/RS} < p_e^{R/MS} \).

This completes the proof.

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