PGF5112 - Plasma Physics I

By

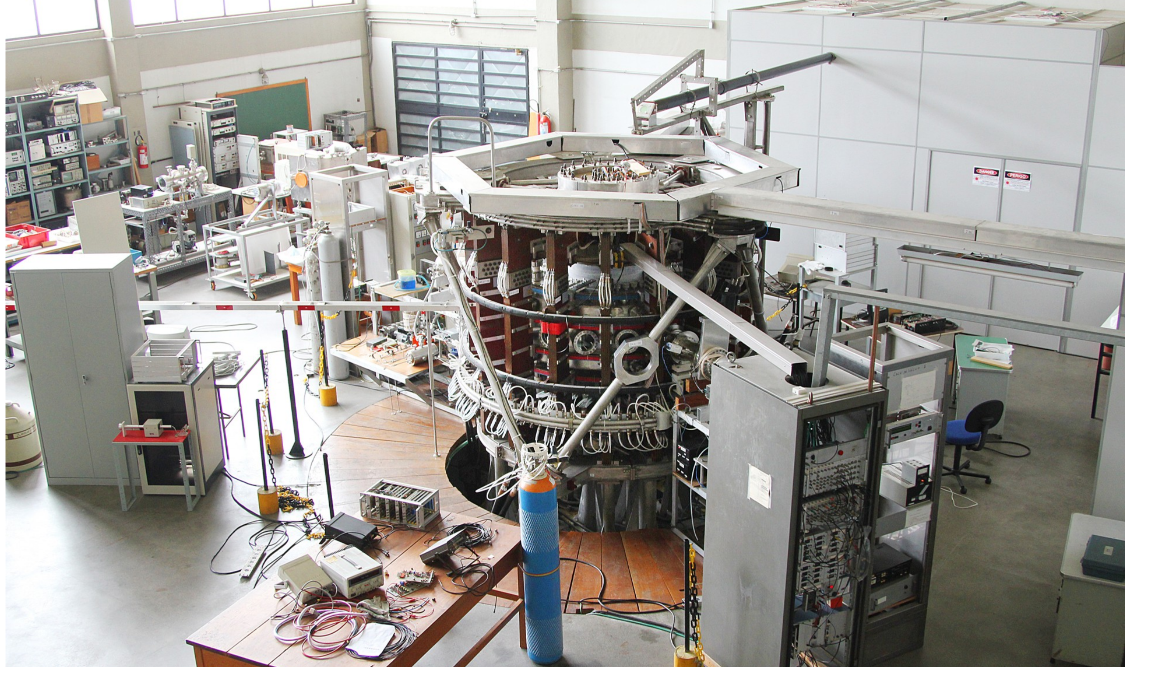
Prof. Gustavo Paganini Canal

Plasma Physics Laboratory

Department of Applied Physics

Institute of Physics

University of São Paulo



Postgraduate course ministered remotely from the

Institute of Physics of the University of São Paulo

IFUSP
Instituto de Física da USP

e-mail: canal@if.usp.br

São Paulo - SP, 25 March 2021

SCIENTIA VINCES

PGF5112 - Plasma Physics I

- **Particle interactions in plasmas**
  - *Collision cross section (previous lecture)*
  - *The Rutherford cross section (continuing from the previous lecture)*
  - *Collision parameters*
  - *Collisional processes*

- **Particle detailed balance**

- **Low pressure electrical discharges**
  - *The Townsend avalanche*
  - *The effect of secondary electrons*
  - *The Townsend criterion for breakdown*

PGF5112 - Plasma Physics I

- **Particle interactions in plasmas**
  - *Collision cross section (previous lecture)*
  - *The Rutherford cross section (continuing from the previous lecture)*
  - *Collision parameters*
  - *Collisional processes*

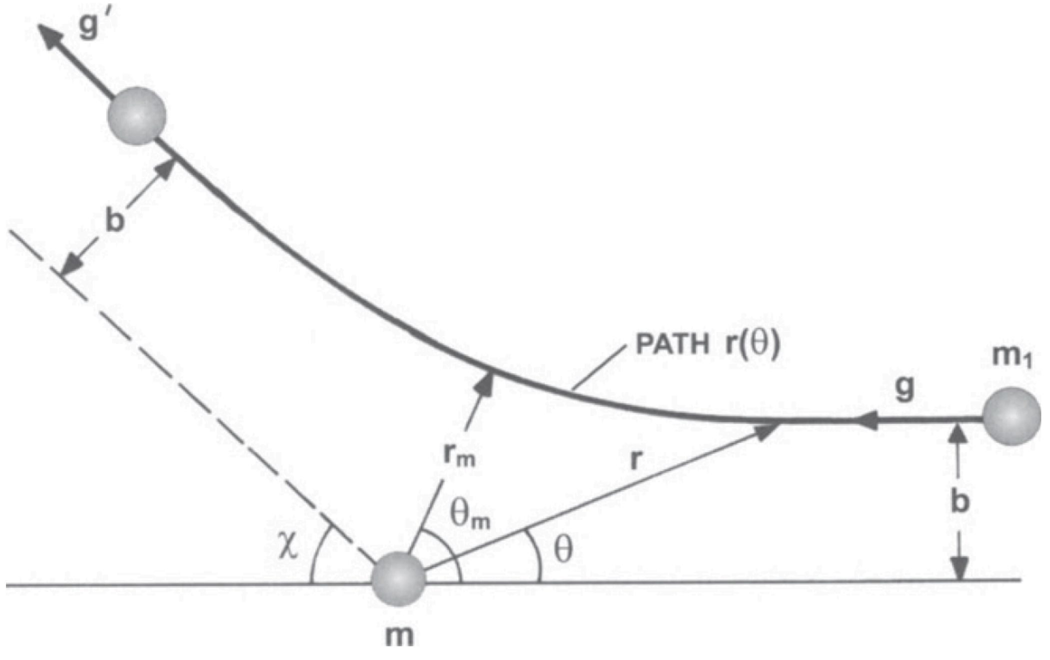
- **Particle detailed balance**

- **Low pressure electrical discharges**
  - *The Townsend avalanche*
  - *The effect of secondary electrons*
  - *The Townsend criterion for breakdown*

The scattering of particles via the Coulomb interaction

- **When a pair of particles interact via the Coulomb force, the scattering angle can be determined from (given as exercise):**

$$\tan\left(\frac{\chi}{2}\right) = \frac{b_0}{b}, \text{ with } b_0 = \frac{q\,q_1}{4\pi\epsilon_0 \mu g^2}$$



- **Differentiating this equation yields**

$$\left|\frac{db}{d\chi}\right| = \frac{b^2}{2b_0\cos^2(\chi/2)}$$

- **Therefore, the differential cross section for the Coulomb potential is**

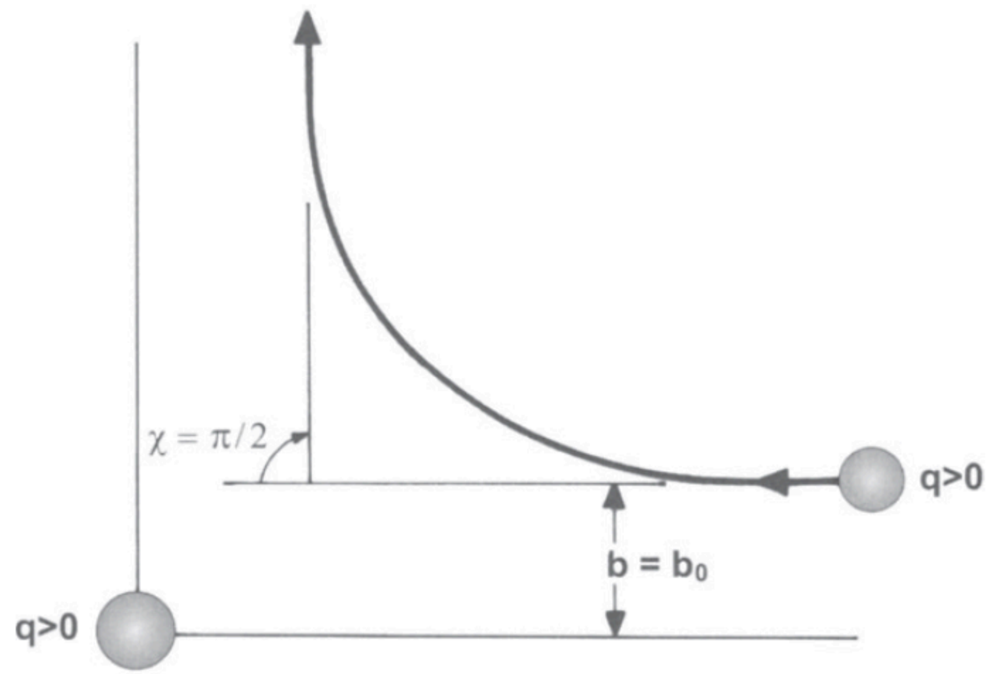
$$\sigma(\chi) = \frac{b}{\sin(\chi)}\left|\frac{db}{d\chi}\right| = \frac{b_0^2}{4\sin^4(\chi/2)} = \frac{b_0^2}{(1-\cos\chi)^2}$$

The Rutherford scattering

The scattering of particles via the Coulomb interaction

- **When a pair of particles interact via the Coulomb force, the scattering angle can be determined from (given as exercise):**

$$\tan\left(\frac{\chi}{2}\right) = \frac{b_0}{b}, \text{ with } b_0 = \frac{q\, q_1}{4\pi\epsilon_0 \mu g^2}$$



- **Differentiating this equation yields**

$$\left|\frac{db}{d\chi}\right| = \frac{b^2}{2b_0 \cos^2(\chi/2)}$$

- **Therefore, the differential cross section for the Coulomb potential is**

$$\sigma(\chi) = \frac{b}{\sin(\chi)}\left|\frac{db}{d\chi}\right| = \frac{b_0^2}{4\sin^4(\chi/2)} = \frac{b_0^2}{(1-\cos\chi)^2}$$

The Rutherford scattering

The total cross section due to the Coulomb interaction diverges

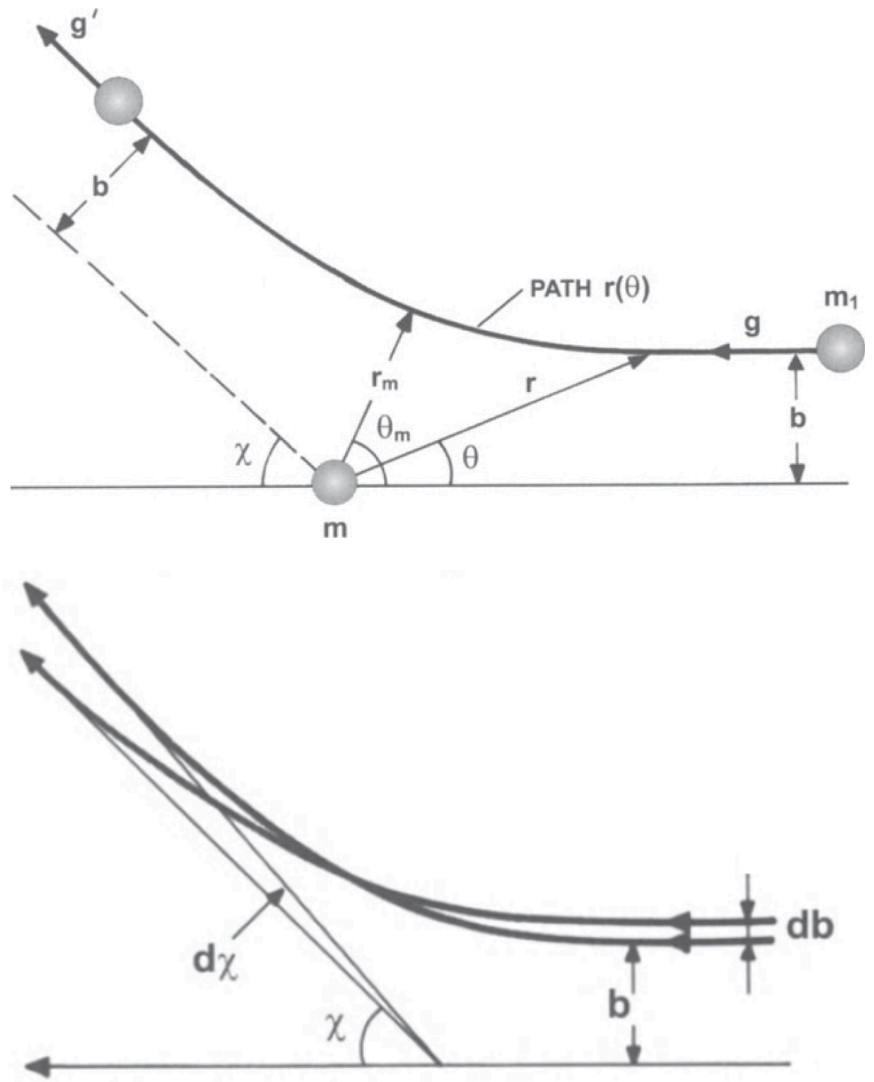
- **The total cross section can be found as**

$$\sigma_t = \int \sigma(\chi,\epsilon)\, d\Omega = \int_0^{2\pi}\int_{\chi_{\min}}^{\pi} \sigma(\chi,\epsilon)\, \sin\chi\, d\chi\, d\epsilon$$

$$\sigma_t = 2\pi \int_{\chi_{\min}}^{\pi} \sigma(\chi)\, \sin\chi\, d\chi$$

$$\sigma_t = 2\pi\, b_0^2 \int_{\chi_{\min}}^{\pi} \frac{\sin\chi}{(1-\cos\chi)^2}\, d\chi$$

$$\sigma_t = 2\pi\, b_0^2 \left[\frac{1}{\sin^2(\chi_{\min}/2)} - 1\right]$$



- **The total cross section becomes infinite for** $\chi_{\min} = 0$
  - Particles with very small deflection angles (very large value of b) contribute to make σ_t infinite ($1/r^2$ forces are of infinite range!)
  - In plasmas, there exists an upper limit for b (the so-called Debye length)

PGF5112 - Plasma Physics I

- **Particle interactions in plasmas**
  - *Collision cross section (previous lecture)*
  - *The Rutherford cross section (continuing from the previous lecture)*
  - *Collision parameters*
  - *Collisional processes*

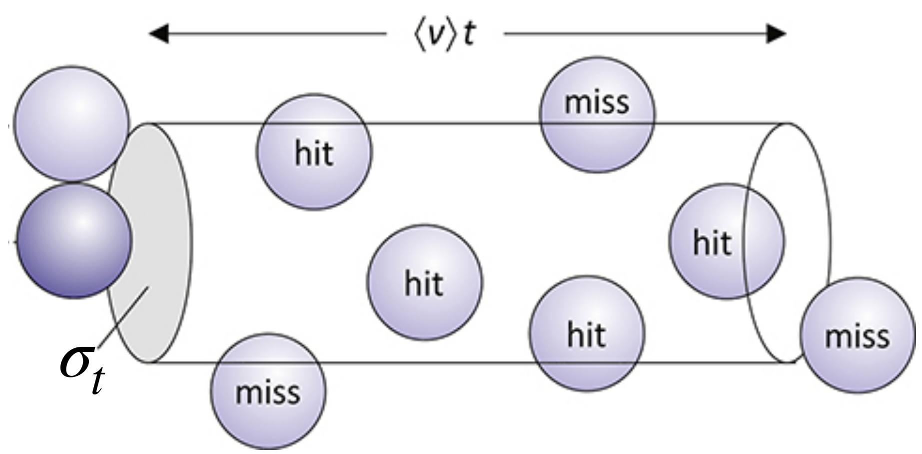
- **Particle detailed balance**

- **Low pressure electrical discharges**
  - *The Townsend avalanche*
  - *The effect of secondary electrons*
  - *The Townsend criterion for breakdown*

Collision frequency

- **The collision frequency corresponds to the number of collision per unit time**

- **Let's suppose that in a certain region of space (volume V) there exists N scattering centers: density of scattering centers n = N/V**



- **In a time interval Δt, a test particle travels a distance $d = v\,\Delta t$, where v is the particle velocity. Therefore, the "collisional volume" associated to this particle will be $V_{\rm coll} = \sigma\, v\,\Delta t$, with a number of encounters/collisions $N_{\rm coll} = nV_{\rm coll} = n\,\sigma\, v\,\Delta t$**

- **Therefore, the collision frequency will be** $\nu_{\rm coll}(v) = \dfrac{N_{\rm coll}}{\Delta t} = \dfrac{n\,\sigma\, v\,\Delta t}{\Delta t} = n\,\sigma(v)\, v$

Collision frequency

- **In plasmas, the velocities of the particles are not all the same**

- **One must take the average over the velocity distribution function**

$$\langle \nu_{\mathrm{coll}} \rangle = n \langle \sigma v \rangle$$

$$\langle \sigma v \rangle = \frac{1}{n} \int \sigma(v)\, v\, f(v)\, dv$$

- **Therefore, the average collision frequency is given by**

$$\langle \nu_{\mathrm{coll}} \rangle = \int \sigma(v)\, v\, f(v)\, dv$$

Reaction rate

- **Let's suppose that there are two species in the plasma, with densities n_1 and n_2**
- **Take the particles of type 2 as scattering centers (targets) and one single particle of type 1 as the scattered particle (projectile)**
- **The average collision frequency of each particle of type 1 will be**

$$\nu_1 = n_2 \langle \sigma v \rangle_{12}$$

- **The reaction rate is defined as the # of collisions per unit volume, per unit time**

$$R = n_1 \nu_1 = n_1 n_2 \langle \sigma v \rangle_{12}$$

The concept of reaction rate will be important when we calculate the power produced in fusion plasmas

Mean free path

- **The mean free path is defined as the average distance a particle travels in between collisions**

$$\lambda = \frac{d}{N_{\mathrm{coll}}}$$

- **Therefore,**

$$\lambda = \frac{\langle v \rangle \Delta t}{n \langle \sigma v \rangle \Delta t} = \frac{\langle v \rangle}{n \langle \sigma v \rangle}$$

$$\lambda = \frac{1}{n} \frac{\int v\, f(v)\, dv}{\int \sigma(v)\, v\, f(v)\, dv}$$

$$\lambda \approx \frac{1}{n \langle \sigma \rangle}$$

PGF5112 - Plasma Physics I

- **Particle interactions in plasmas**
  - *Collision cross section (previous lecture)*
  - *The Rutherford cross section (continuing from the previous lecture)*
  - *Collision parameters*
  - *Collisional processes*

- **Particle detailed balance**

- **Low pressure electrical discharges**
  - *The Townsend avalanche*
  - *The effect of secondary electrons*
  - *The Townsend criterion for breakdown*

Elastic collisions between electrons and neutral atoms

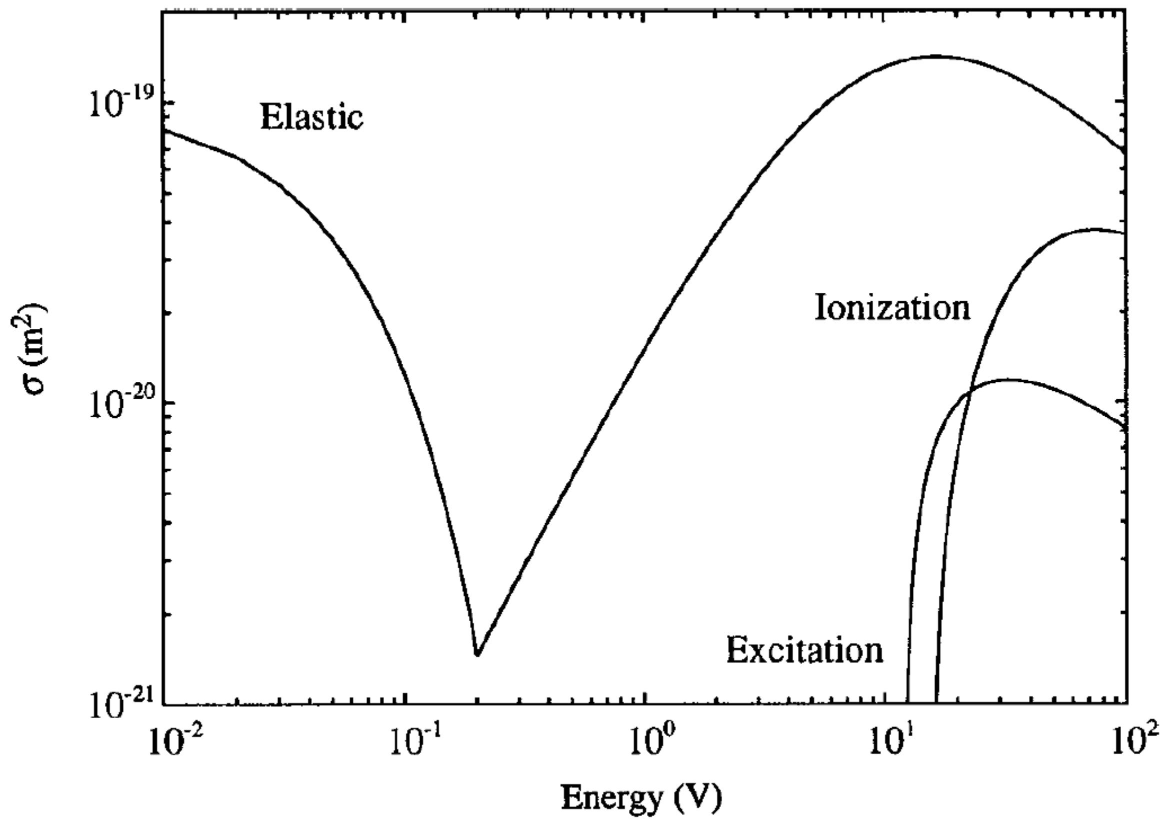
- **Due to the large mass difference between electrons and atoms, (almost) no energy is exchanged during an elastic electron-atom collision**

$$\Delta E = \frac{2m_e m_p}{(m_e + m_p)^2} \approx \frac{2m_e}{m_p} = 0.1\,\%$$

- **The cross section for elastic scattering of electrons in gases, in general, are significantly larger than in non-elastic collisions**

- **Elastic collisions between electrons and atoms/molecules is the main mechanism responsible for momentum transport**
  - Main effect responsible for the electric conductivity in weakly ionized plasmas

Argon Plasma



For E < 1 eV, the Ramsauer effect causes a decrease in the cross section. This effect does not occur in collision with molecules

Inelastic collisions: A large # of inelastic processes can occur in plasmas

- **Electrons (A: atoms or molecules)**
  - $e + A \rightarrow A^+ + 2e$ (Ionization by electronic impact)**
  - $e + A \rightarrow e + A^* \rightarrow e + A + h\nu$ (Excitation)
  - $e + A^* \rightarrow A^+ + 2e$ (Penning ionization)
  - $e + AB \rightarrow A + B + e$ (Molecular dissociation)*
  - $e + AB \rightarrow A^+ + B + 2e$ (Dissociative ionization)*
  - $e + AB \rightarrow A^- + B$ (Dissociative attachment)

- **Ions (A and B: atoms or molecules)**
  - $A^+ + B \rightarrow A + B^+$ (Charge exchange)*
  - $A^+ + B \rightarrow A^+ + B^+ + e$ (Ionization)
  - $A^+ + B \rightarrow A^+ + B^* \rightarrow A^+ + B + h\nu$ (Excitation)
  - $A^+ + e + B \rightarrow A + B$ (Three-body recombination)**
  - $A^+ + BC \rightarrow A^+ + B + C$ (Dissociation)
  - $A + BC \rightarrow C + AB$ (Chemical reaction)

***Most important processes in technological plasmas**

**** Dominat mechanism**

- **Particle interactions in plasmas**
  - *Collision cross section (previous lecture)*
  - *The Rutherford cross section (continuing from the previous lecture)*
  - *Collision parameters*
  - *Collisional processes*

- **Particle detailed balance**

- **Low pressure electrical discharges**
  - *The Townsend avalanche*
  - *The effect of secondary electrons*
  - *The Townsend criterion for breakdown*

The particle detailed balance

- The temporal evolution of electron density in a plasma depends on the reaction rate of each collisional processes that can happen in that plasma

$$\frac{dn_e}{dt} = \sum_j R^+_{e,j} - \sum_k R^-_{e,k}$$

- Suppose that the dominant processes occurring in a plasma are the ionization by electronic impact (σ_{ion}) and recombination (σ_{rec}). Therefore

$$\frac{dn_e}{dt} = n_e n_0 \langle \sigma_{\text{ion}} v \rangle_e - n_e n_i \langle \sigma_{\text{rec}} v \rangle_e = S_e \qquad \longrightarrow \qquad n_e(t) = n_{e0} e^{S_e t}$$

- Important equations used to model ionization and recombination are

$$\langle \sigma_{\text{ion}} v \rangle_e = \frac{2.0 \times 10^{-13}}{6.0 + \frac{T_e}{E_\infty}} \sqrt{\frac{T_e}{E_\infty}} \exp\left(-\frac{E_\infty}{T_e}\right) \quad \left(\frac{m^3}{s}\right)$$

$$\langle \sigma_{\text{rec}} v \rangle_e = 5.6 \times 10^{-39} \frac{n_e}{T_e^{9/2}} \quad \left(\frac{m^3}{s}\right)$$

E_∞ is the ionization energy (eV)
T_e is the electron temperature (eV)
n_e is the electron density (m^{-3})
n_0 is the neutral gas density (m^{-3})

The particle detailed balance

- **The plasma equilibrium is achieved when**

$$\frac{dn_e}{dt} = \sum_j R_{e,j}^{+} - \sum_k R_{e,k}^{-} = 0$$

- **For the case in which the dominant processes occurring in a plasma are the ionization by electronic impact and recombination, one obtain that**

$$n_e = \sqrt{\frac{3.6 \times 10^{25}\ n_0 T_e^5}{\left(6.0 + \frac{T_e}{E_\infty}\right)\sqrt{E_\infty}} \exp\left(-\frac{E_\infty}{T_e}\right)}$$

- **Example:** T_e **= 0.5 eV,** n_0 **= 1x10**^{15} **m**$^{-3}$ **and** E_∞ **= 13.6 eV (Hydrogen):** n_e **= 9x10**^{12} **m**$^{-3}$

Note that plasma transport was neglected in this very crude approximation

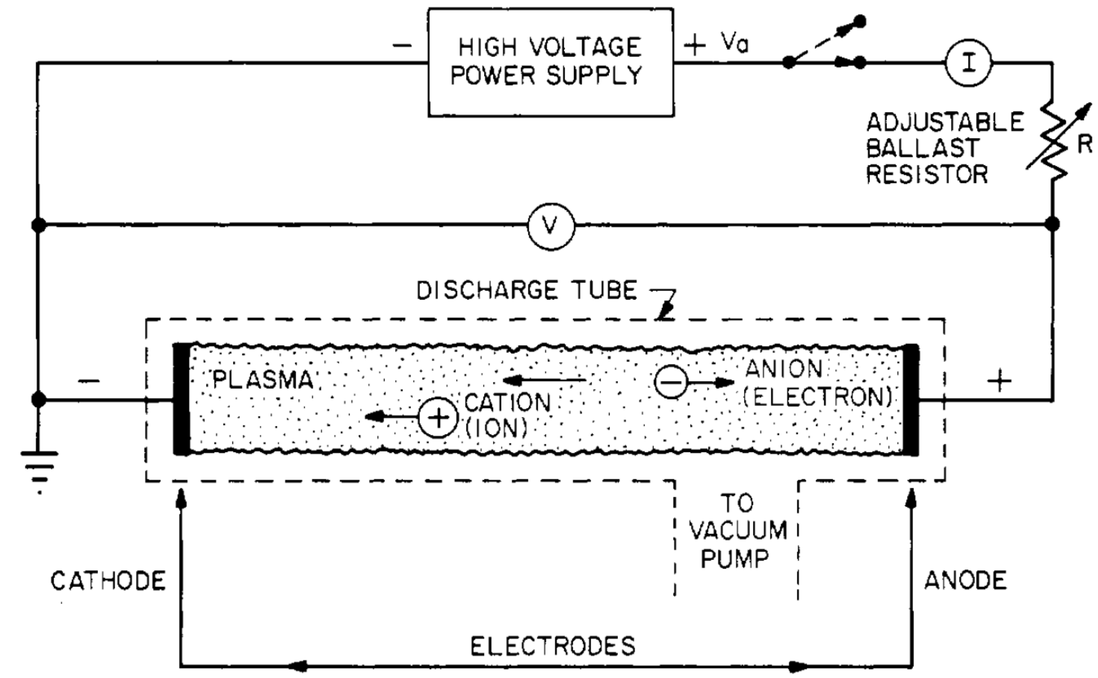
- **Particle interactions in plasmas**
  - *Collision cross section (previous lecture)*
  - *The Rutherford cross section (continuing from the previous lecture)*
  - *Collision parameters*
  - *Collisional processes*

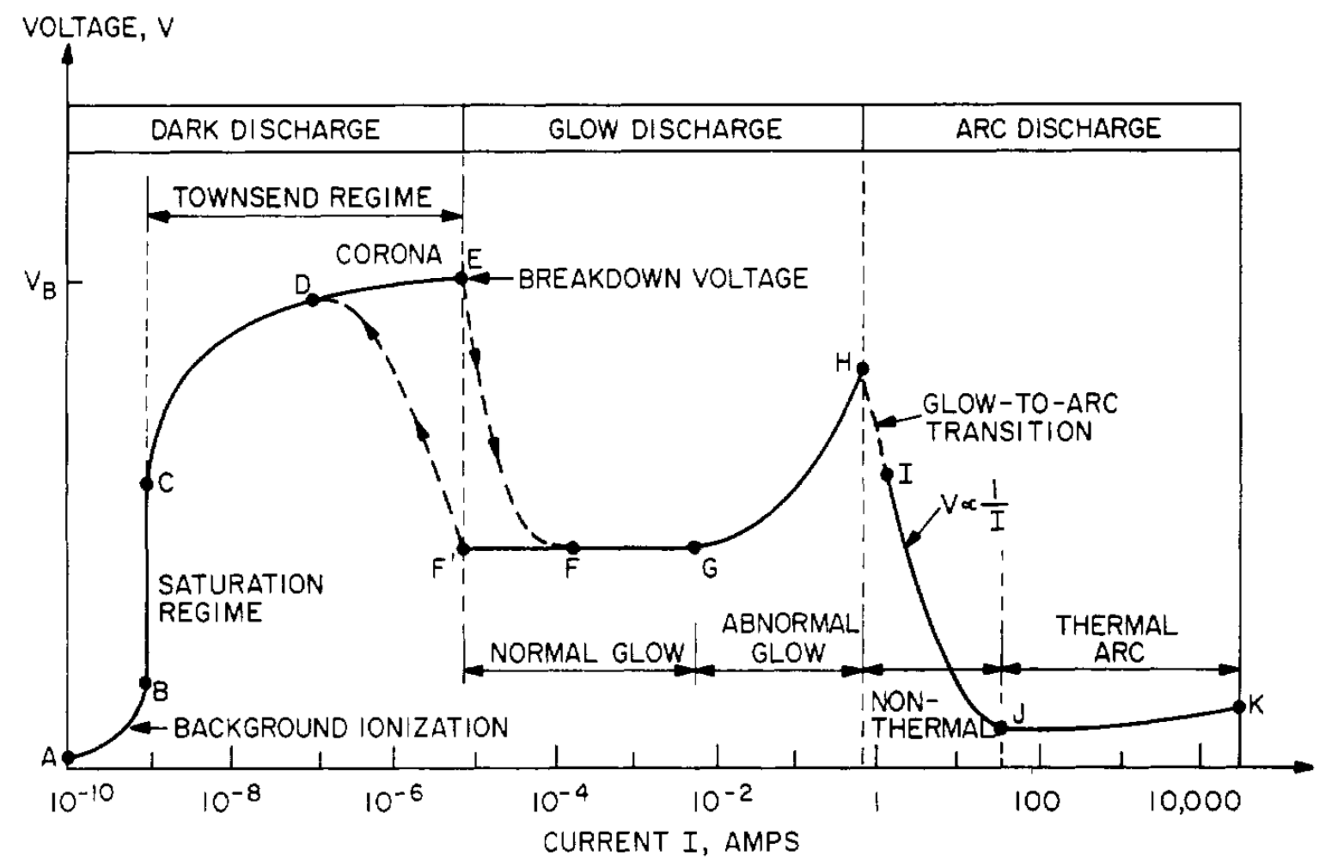
- **Particle detailed balance**

- **Low pressure electrical discharges**
  - *The Townsend avalanche*
  - *The effect of secondary electrons*
  - *The Townsend criterion for breakdown*

Depending on the conditions, different operation regimes are obtained in low pressure electrical discharges

- **Dark discharge**
  - Background ionization
  - Saturation regime
  - Townsend regime
    - *Corona*
    - *Breakdown*
- **Glow discharge**
  - Normal glow
  - Abnormal glow
- **Arc discharge**
  - Non-thermal
  - Thermal





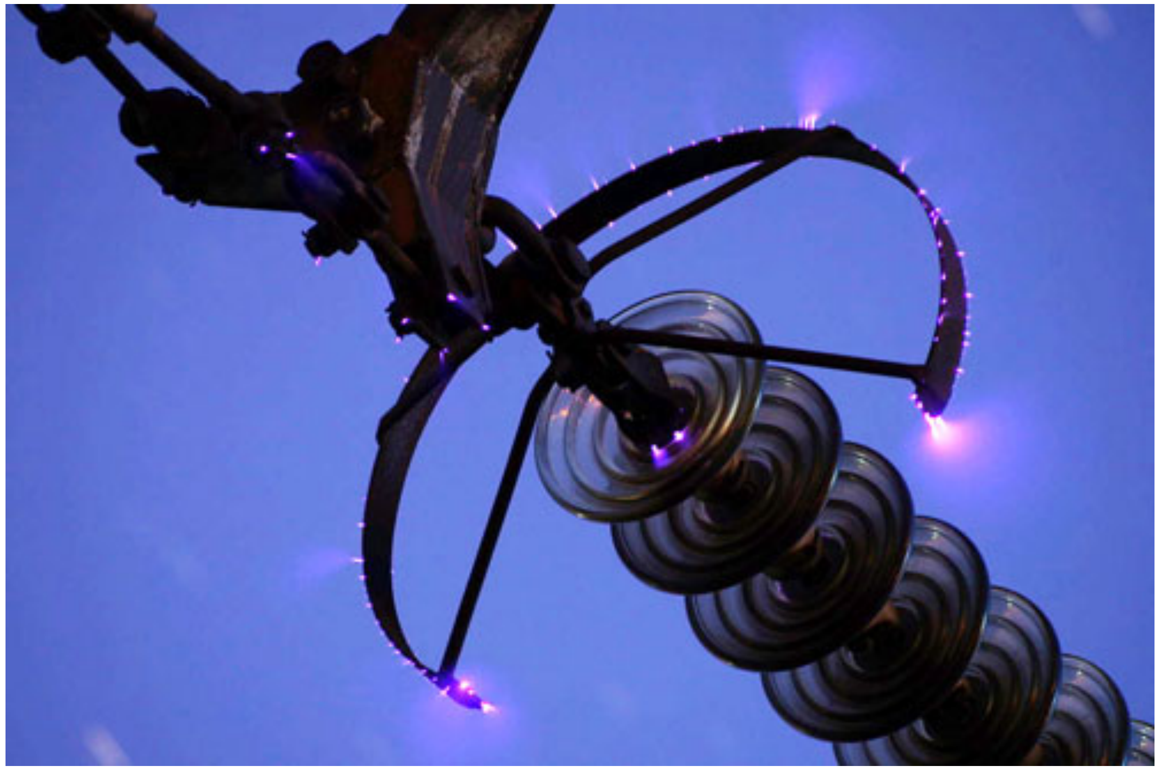
Depending on the conditions, different operation regimes are obtained in low pressure electrical discharges

- **Dark discharge**
  - Background ionization
  - Saturation regime
  - Townsend regime
    - + *Corona*
    - + *Breakdown*
- **Glow discharge**
  - Normal glow
  - Abnormal glow
- **Arc discharge**
  - Non-thermal
  - Thermal

This regime is called Dark because the plasma does not emit enough light to be seem by human eye

This regime ends with the corona effect and finally the breakdown of the gas

Corona due to high voltage

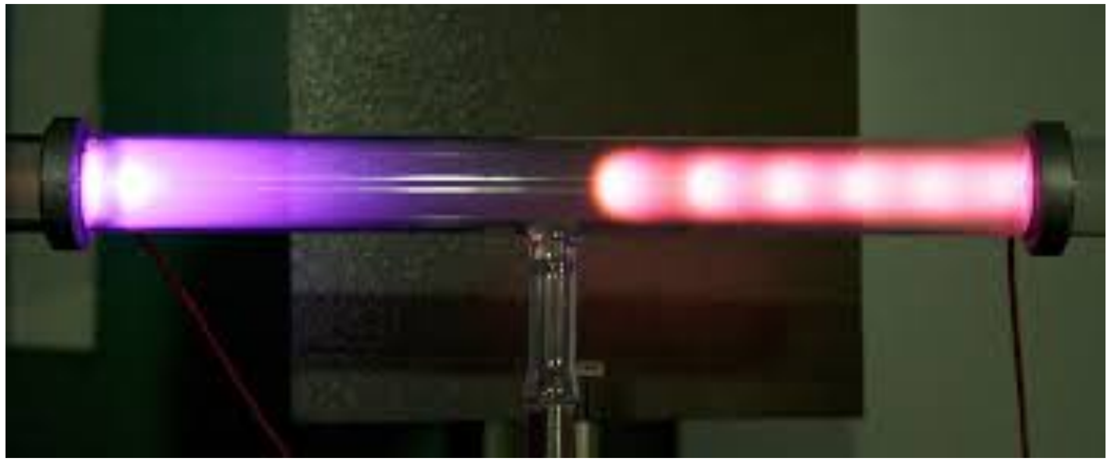


Depending on the conditions, different operation regimes are obtained in low pressure electrical discharges

- **Dark discharge**
  - Background ionization
  - Saturation regime
  - Townsend regime
    - *Corona*
    - *Breakdown*

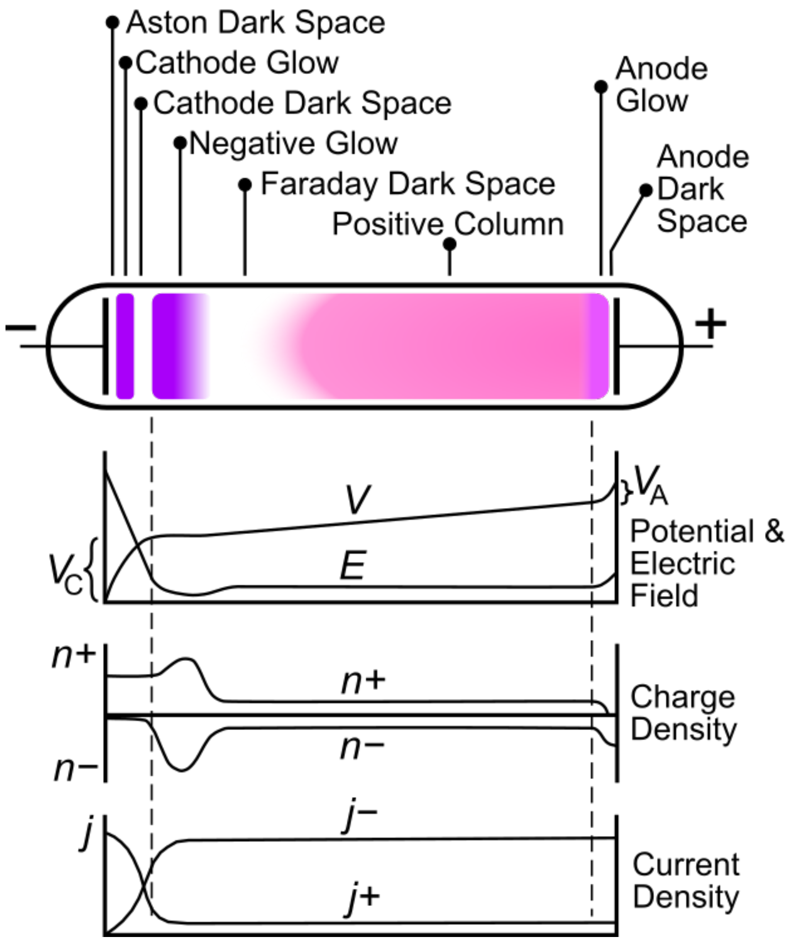
- **Glow discharge**
  - Normal glow
  - Abnormal glow

- **Arc discharge**
  - Non-thermal
  - Thermal



Very interesting phenomena appear in tube discharges (outside the scope of this course)

For those who are interested, see Ch. 9 of Industrial Plasma Engineering, Vol. 1



Depending on the conditions, different operation regimes are obtained in low pressure electrical discharges

- **Dark discharge**
  - Background ionization
  - Saturation regime
  - Townsend regime
    - *Corona*
    - *Breakdown*

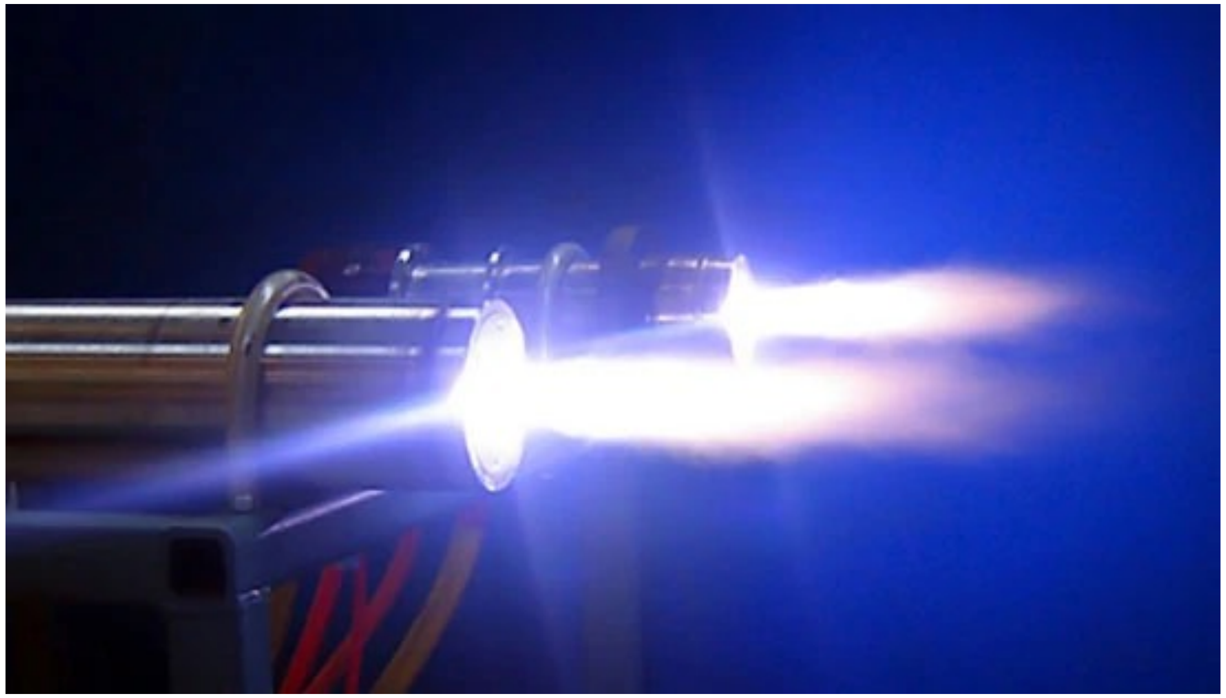
- **Glow discharge**
  - Normal glow
  - Abnormal glow

- **Arc discharge**
  - Non-thermal
  - Thermal

Important applications (outside the scope of this course)

Depending on the conditions, different operation regimes are obtained in low pressure electrical discharges

- **Dark discharge**
  - Background ionization
  - Saturation regime
  - Townsend regime
    - *Corona*
    - *Breakdown*
- **Glow discharge**
  - Normal glow
  - Abnormal glow
- **Arc discharge**
  - Non-thermal
  - Thermal



Important applications (outside the scope of this course)

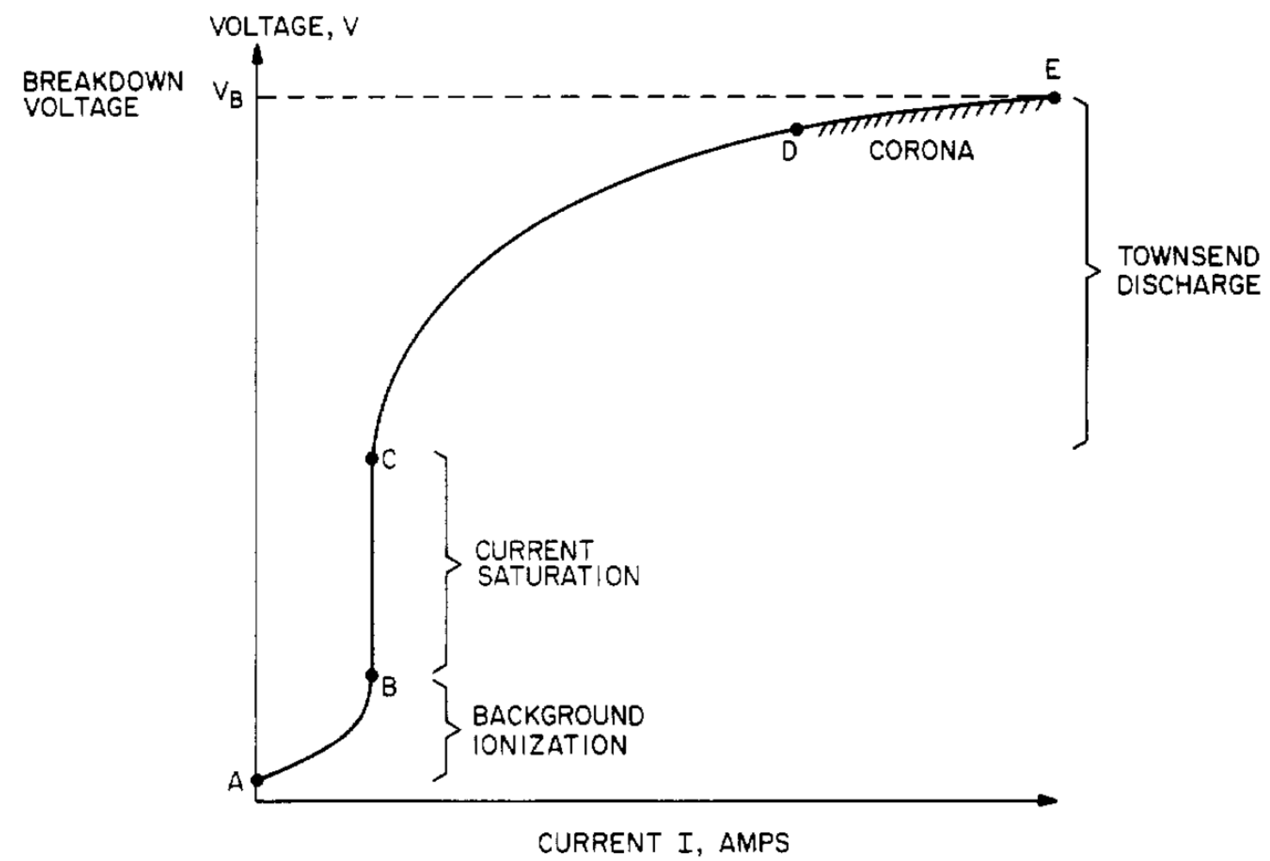
For those who are interested, see Ch. 10 of Industrial Plasma Engineering, Vol. 1

Modeling of the dark discharge regime, specially the Townsend regime, is important for plasma breakdown

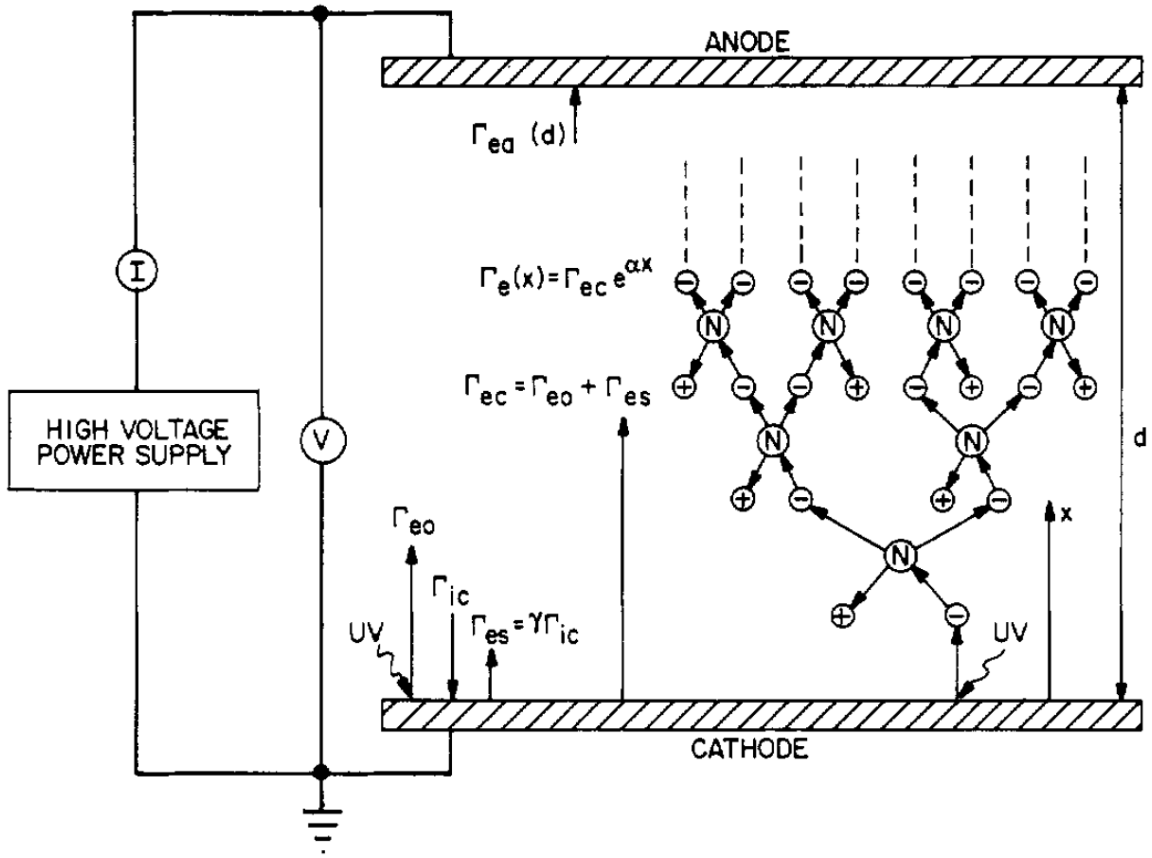
- **Background ionization**
  - Ions and electrons created by radiation, cosmic rays, radioactive mineral, etc.

- **Saturation current**
  - Eventually, all charged particles are contributing to the current
  - If $S = dn/dt$, the saturation current is given by $I_{\text{sat}} = eSAL$ (pA to nA)

- **Townsend discharge**
  - The current increases due to an avalanche process



Modeling of the dark discharge regime, specially the Townsend regime, is important for plasma breakdown



- **Electron-ion pair created by background ionization are accelerated by the applied electric field (E = V/d)**

- **At sufficiently high voltages, these electrons can eventually ionize the neutral gas**
  - α is the Townsend's first ionization coefficient: # of ionizations per meter

$$d\Gamma_e = \alpha\Gamma_e dx \qquad \int_{\Gamma_{e0}}^{\Gamma_e} \frac{d\Gamma_e}{\Gamma_e} = \int_0^x \alpha dx \qquad \Gamma_e(x) = \Gamma_{e0}e^{\alpha x}$$

- **If there exists a volumetric source of electrons**

$$d\Gamma_e = \alpha\Gamma_e dx + S_e dx \qquad \int_0^{\Gamma_e} \frac{d\Gamma_e}{\alpha\Gamma_e + S_e} = \int_0^x dx = \left.\frac{\ln(\alpha\Gamma_e + S_e)}{\alpha}\right|_0^{\Gamma_e} \qquad \Gamma_e(x) = \frac{S_e}{\alpha}\left(e^{\alpha x} - 1\right)$$

The Townsend first ionization coefficient

- **The probability of an electron ionize a neutral is**

$$\frac{n_e(x)}{n_{e0}} = e^{-\frac{x}{\lambda_i}}$$

- **The Townsend first coefficient can be estimated as**

$$\alpha = \frac{1}{\lambda_i}\frac{n_e(x)}{n_{e0}} = \frac{1}{\lambda_i}e^{-\frac{x}{\lambda_i}}$$

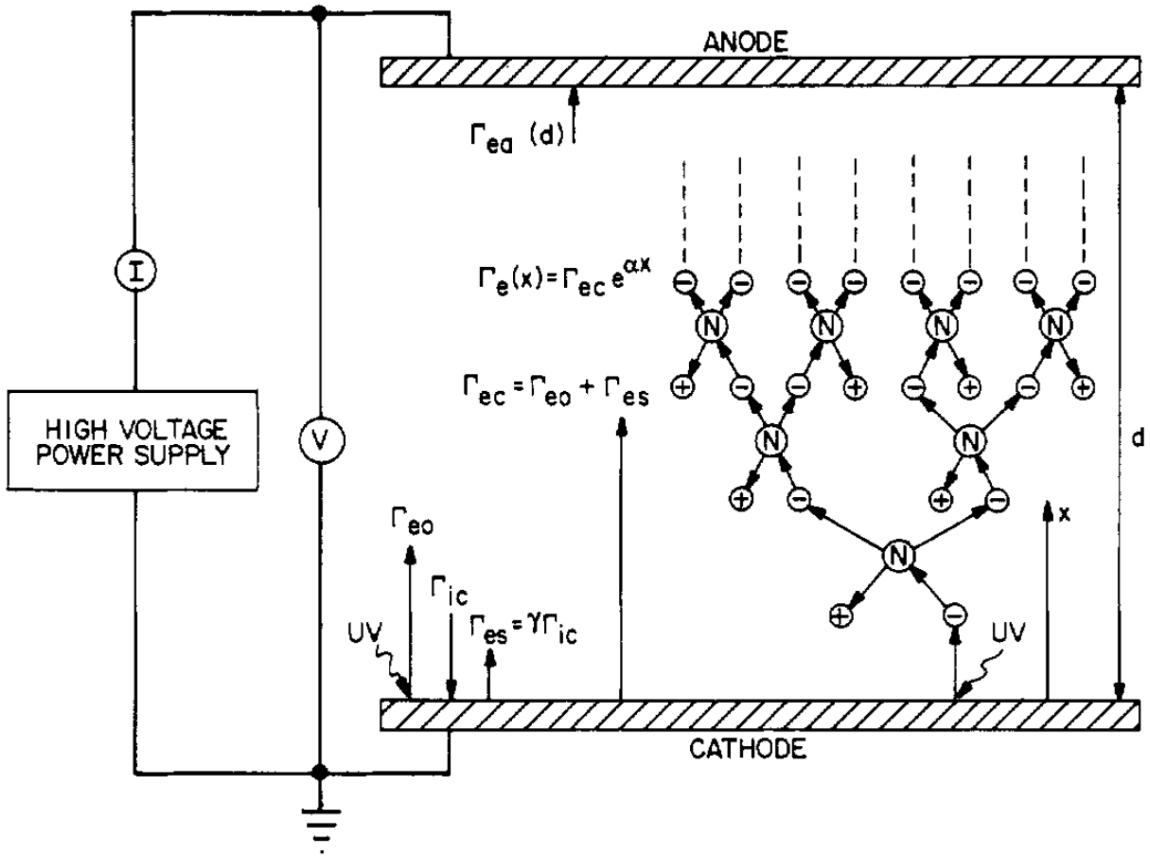
- **The mean free path for ionization is**

$$\frac{1}{\lambda_i} = \frac{n_0\langle\sigma_{\text{ion}}v_e\rangle}{\langle v_e\rangle} = Ap$$

- **If one sets $x = x_i = V_i/E$ as the distance an electron travel to gain enough energy to ionize an neutral, one has that**

$$\frac{\alpha}{p} = Ae^{-\frac{C}{E/p}}$$

A and C are constants that depends of the type of gas



Exercise: the Stoletow point

- **What is the pressure that maximizes the current in the plasma for a fixed electric field?**
  - Hint: It is the pressure that maximizes the Townsend first ionization coefficient

  - Answer: $p_{\max} = \dfrac{E}{C}$ (Eq. 8.33 of Industrial Plasma Engineering, Vol. 1)

- **Particle interactions in plasmas**
  - *Collision cross section (previous lecture)*
  - *The Rutherford cross section (continuing from the previous lecture)*
  - *Collision parameters*
  - *Collisional processes*

- **Particle detailed balance**

- **Low pressure electrical discharges**
  - *The Townsend avalanche*
  - *The effect of secondary electrons*
  - *The Townsend criterion for breakdown*

The effect of secondary electrons

- **The flux of electrons at the Anode is**

$$\Gamma_{ea} = \Gamma_{ec} e^{\alpha d}$$

- **The emission of secondary electrons is proportional to the ion flux at the cathode**
  - *γ is the secondary electron emission coefficient*

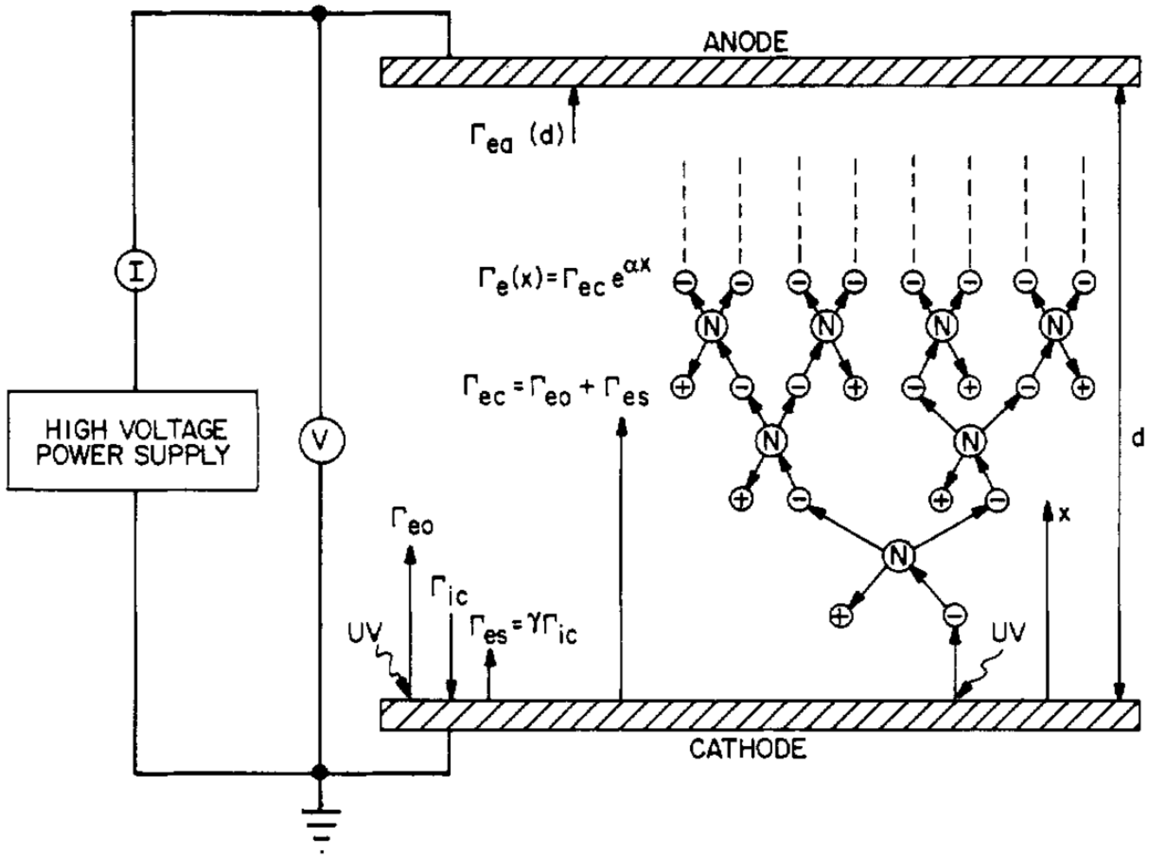
$$\Gamma_{ec} = \Gamma_{e0} + \Gamma_{es} = \Gamma_{e0} + \gamma \Gamma_{ic}$$

- **In the steady state, one must have**

$$\Gamma_{ea} - \Gamma_{ec} = \Gamma_{ic}$$

- **Therefore, after some algebra:**

$$\Gamma_{ea} = \Gamma_{e0} \frac{e^{\alpha d}}{1 - \gamma \left(e^{\alpha d} - 1 \right)}$$



PGF5112 - Plasma Physics I

- **Particle interactions in plasmas**
  - *Collision cross section (previous lecture)*
  - *The Rutherford cross section (continuing from the previous lecture)*
  - *Collision parameters*
  - *Collisional processes*

- **Particle detailed balance**

- **Low pressure electrical discharges**
  - *The Townsend avalanche*
  - *The effect of secondary electrons*
  - *The Townsend criterion for breakdown*

The Townsend criterion for breakdown

- **The Townsend criterion for breakdown is obtained from the condition for which the current between the two plates increases without limit**

$$\Gamma_{ea} = \Gamma_{e0}\frac{e^{\alpha d}}{1-\gamma\left(e^{\alpha d}-1\right)} \longrightarrow 1-\gamma\left(e^{\alpha d}-1\right)=0 \longrightarrow \gamma e^{\alpha d} = \gamma + 1$$

- **Defining the breakdown voltage as $V_b = E_b d$ and using the expression for α**

$$\ln\left(1+\frac{1}{\gamma}\right) = \alpha d = Apd\exp\left(-\frac{Cpd}{V_b}\right)$$

- **Manipulating this equation leads to the Paschen law**

$$V_b = \frac{Cpd}{\ln\left[Apd/\ln\left(1+\frac{1}{\gamma}\right)\right]} = f(pd)$$

Gas	*A* ion pairs/m-Torr	*C* V/m-Torr
A	1200	20 000
Air	1220	36 500
CO_2	2000*	46 600
H_2	1060	35 000
HCl	2500*	38 000
He	182	5 000
Hg	2000	37 000
H_20	1290*	28 900
Kr	1450	22 000
N_2	1060	34 200
Ne	400	10 000
Xe	2220	31 000

The Townsend criterion for breakdown

- **The Townsend criterion for breakdown is obtained from the condition for which the current between the two plates increases without limit**

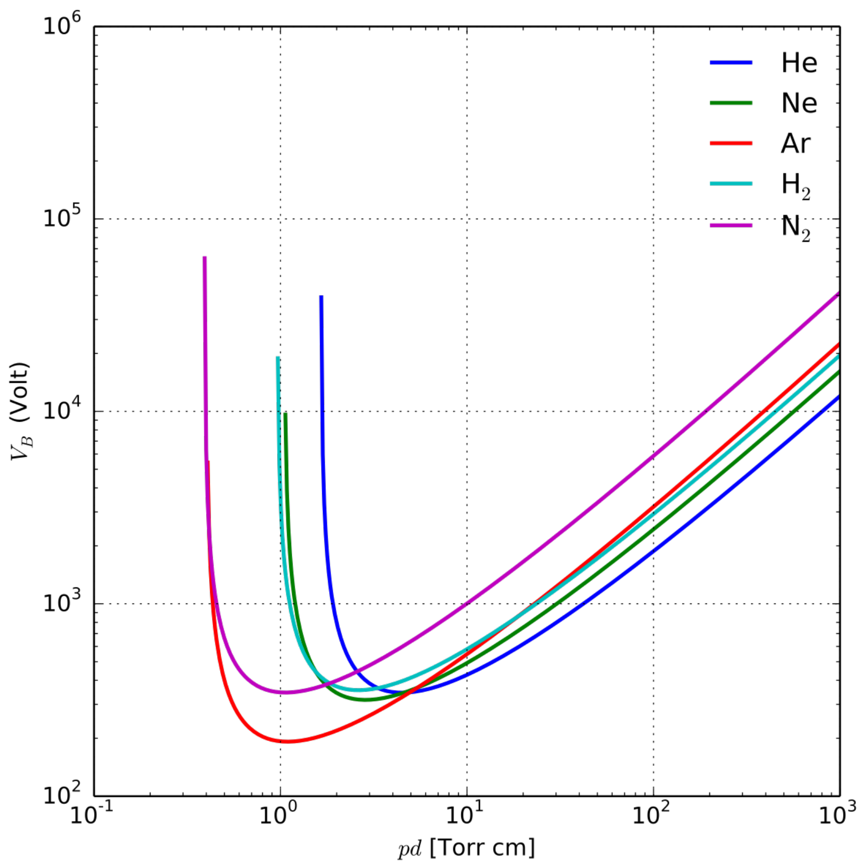
$$\Gamma_{ea} = \Gamma_{e0}\frac{e^{\alpha d}}{1-\gamma\left(e^{\alpha d}-1\right)} \quad \longrightarrow \quad 1-\gamma\left(e^{\alpha d}-1\right)=0 \quad \longrightarrow \quad \gamma e^{\alpha d} = \gamma + 1$$

- **Defining the breakdown voltage as $V_b = E_b d$ and using the expression for α**

$$\ln\left(1+\frac{1}{\gamma}\right) = \alpha d = Apd\exp\left(-\frac{Cpd}{V_b}\right)$$

- **Manipulating this equation leads to the Paschen law**

$$V_b = \frac{Cpd}{\ln\left[Apd/\ln\left(1+\frac{1}{\gamma}\right)\right]} = f(pd)$$



The minimum breakdown voltage

- **Differentiating the expression for V_b with respect to pd yield**

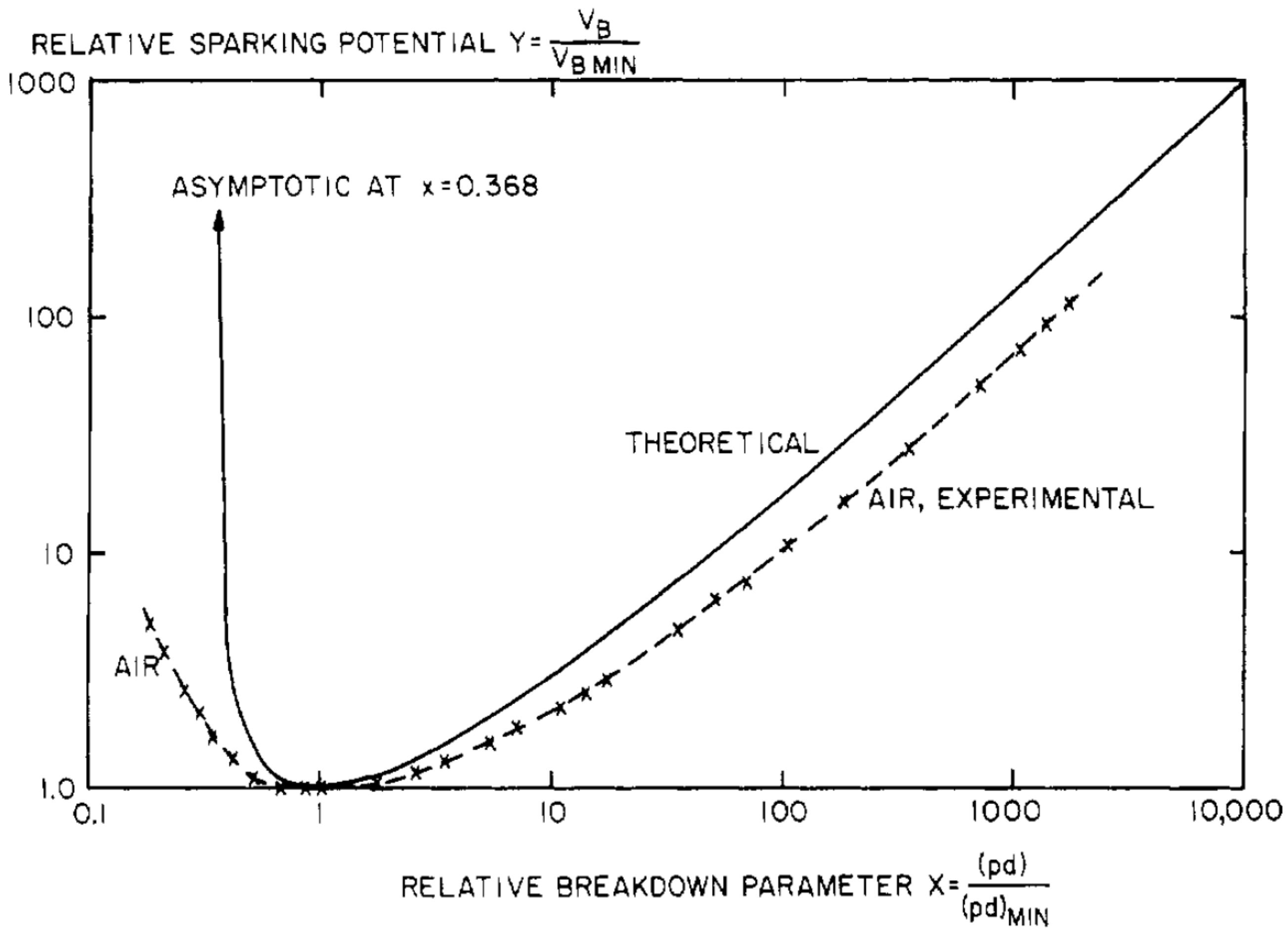
$$(pd)_{\min} = \frac{e^1}{A} \ln\left(1 + \frac{1}{\gamma}\right)$$

- **Let's define the parameters**

$$X = \frac{pd}{(pd)_{\min}} \qquad Y = \frac{V_\mathrm{b}}{V_{\mathrm{b,min}}}$$

- **The universal Paschen curve**

$$Y = \frac{X}{1 + \ln X}$$



References

- **Particle interactions in plasmas**
  - *Bittencourt: Ch. 20*

- **Collisional processes**
  - *M.A. Lieberman: Ch. 4*

- **Low pressure electrical discharges**
  - *Industrial Plasma Engineering, Vol. 1: Ch. 8 and Ch. 4 (Section 9)*
  - *Breakdown in low pressure gases: part I*
  - *Breakdown in low pressure gases: part II*