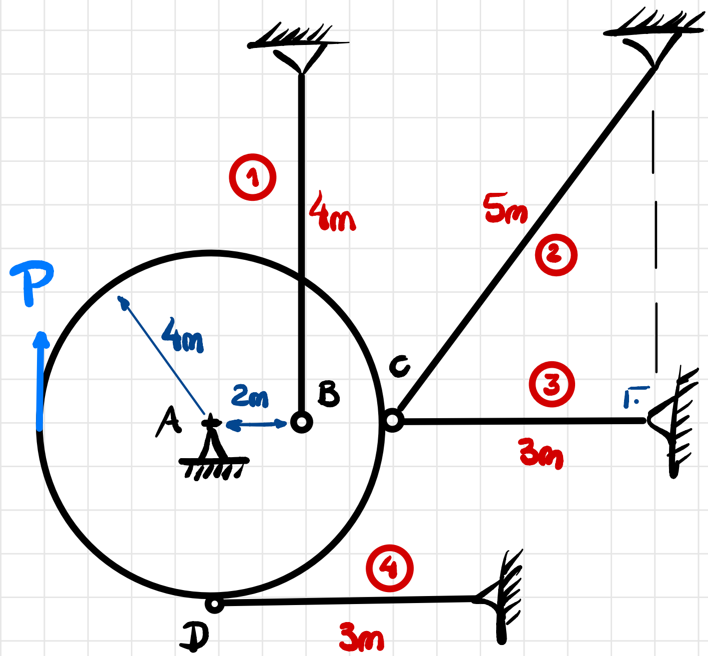
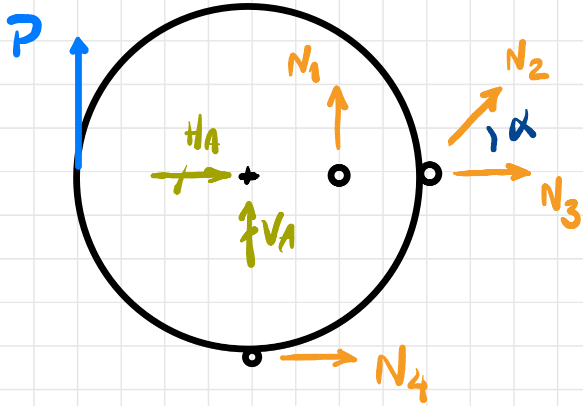
1



Determinar $P_{máx}$ sabendo que os fios são idênticos, tem mesmo material de módulo de elasticidade $E = 150\,GPa$ e área de seção transversal A de $5\,cm^2$ e que a tensão máxima de escoamento é $120\,MPa$, o fator de segurança a ser adotado é 2 e o giro máximo da estrutura é $0{,}001\,rad$. O disco é rígido.

Equilíbrio:



$\text{sen}\,\alpha = 4/5$

$\cos\alpha = 3/5$

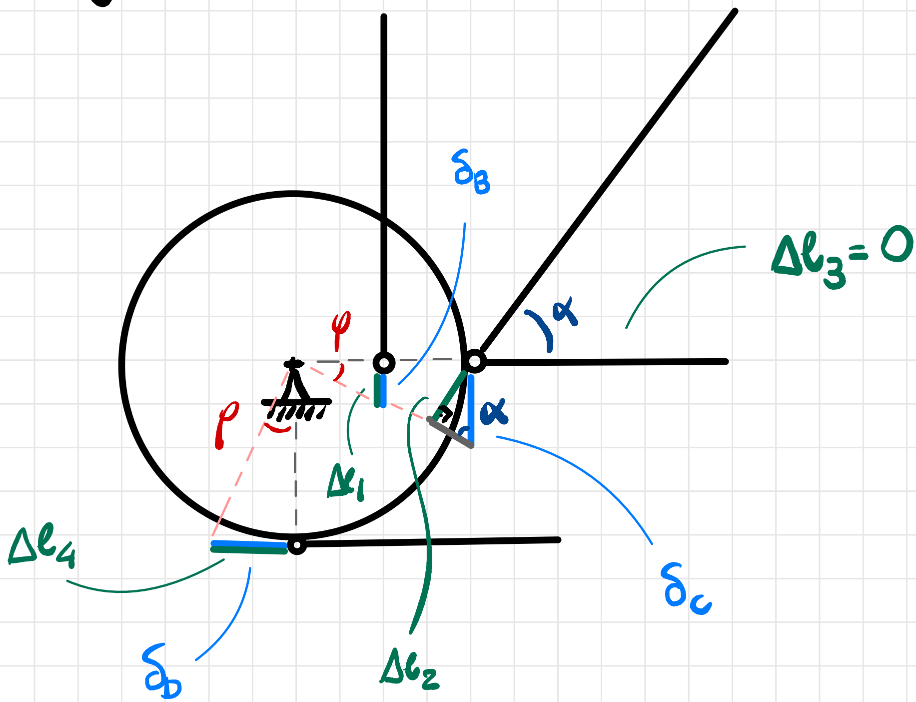
$$\sum F_H = 0: \quad H_A + N_2\cos\alpha + N_3 + N_4 = 0$$

$$\sum F_V = 0: \quad V_A + P + N_1 + N_2\,\text{sen}\,\alpha = 0$$

$$\circlearrowleft(+)\ \sum M_A = 0: \quad -P.4 + N_4.4 + N_2\,\text{sen}\,\alpha.4 + N_1.2 = 0$$

$$\boxed{N_1 + 8/5\,N_2 + 2N_4 = 2P}$$

Diagrama de Williot:



$$\delta_B = \Delta\ell_1$$

$$\operatorname{sen}\alpha = \frac{\Delta\ell_2}{\delta_C} \longrightarrow \delta_C = \frac{\Delta\ell_2}{\operatorname{sen}\alpha}$$

$$\delta_D = \Delta\ell_4$$

$$\operatorname{tg}\varphi = \frac{\delta_B}{2} = \frac{\delta_C}{4} = \frac{\delta_D}{4}$$

Assim:

$$\frac{\Delta \ell_1}{2} = \frac{\Delta \ell_2}{4 \operatorname{sen}\alpha} = \frac{\Delta \ell_4}{4}$$

$$\begin{cases} \Delta \ell_1 = \frac{1}{2} \Delta \ell_4 \\ \Delta \ell_2 = \frac{4}{5} \Delta \ell_4 \end{cases} \longrightarrow \begin{cases} \frac{N_1 . \ell_1}{\cancel{E}A} = \frac{N_4 \ell_4}{2 \cancel{EA}} \\ \frac{N_2 \ell_2}{\cancel{E}A} = \frac{4}{5} \frac{N_4 \ell_4}{\cancel{EA}} \end{cases}$$

$$\begin{cases} N_1 . 4 = \frac{1}{2} . N_4 . 3 \\ N_2 . 5 = \frac{4}{5} . N_4 . 3 \end{cases} \longrightarrow \begin{cases} N_1 = \frac{3}{8} N_4 \\ N_2 = \frac{12}{25} N_4 \end{cases}$$

Voltando ao equilíbrio:

$$\frac{3}{8} N_4 + \frac{8}{5} \left(\frac{12}{25} N_4 \right) + 2N_4 = 2P$$

$$\frac{3413}{1000} N_4 = 2P \longrightarrow N_4 = \frac{2000}{3143} P$$

$$(N_4 = 0{,}636 P)$$

$$N_1 = \frac{3}{8}\left(\frac{2000}{3143}P\right) \longrightarrow N_1 = \frac{750}{3143}P \quad (N_1 = 0{,}239P)$$

$$N_2 = \frac{12}{25}\left(\frac{2000P}{3143}\right) \longrightarrow N_2 = \frac{960}{3143}P \quad (N_2 = 0{,}305P)$$

As forças normais nas barras são:

BARRA	N [kN]
1	$\frac{750}{3143}P$
2	$\frac{960}{3143}P$
3	0
4	$\frac{2000}{3143}P$

Dimensionamento:

① Tensão máxima:

$$\text{máx}\left(\sigma_1, \sigma_2, \sigma_3, \sigma_4\right) \leqslant \bar{\sigma}$$

$$\sigma_4 \leqslant \bar{\sigma} \longrightarrow \frac{N_4}{A} \leqslant \frac{\sigma_e}{S}$$

$$\frac{2000P}{3143A} \leqslant \frac{\sigma_e}{S} \longrightarrow P \leqslant \frac{3143A\sigma_e}{2000S}$$

$$P \leqslant \frac{3143 \cdot 5 \cdot 10^{-4} \cdot 120 \cdot 10^{6}}{2000 \cdot 2}$$

$$P \leqslant 47{,}145 \text{ kN}$$

② Giro Máximo:

$$\varphi \leqslant \bar{\varphi} \rightarrow tg\,\varphi \leqslant \bar{\varphi} \rightarrow \frac{\delta_B}{2} \leqslant \bar{\varphi}$$

$$\frac{\Delta l_1}{2} \leqslant \bar{\varphi} \rightarrow \frac{N_1 l_1}{2EA} \leqslant \bar{\varphi} \rightarrow \frac{750P\,l_1}{6286EA} \leqslant \bar{\varphi}$$

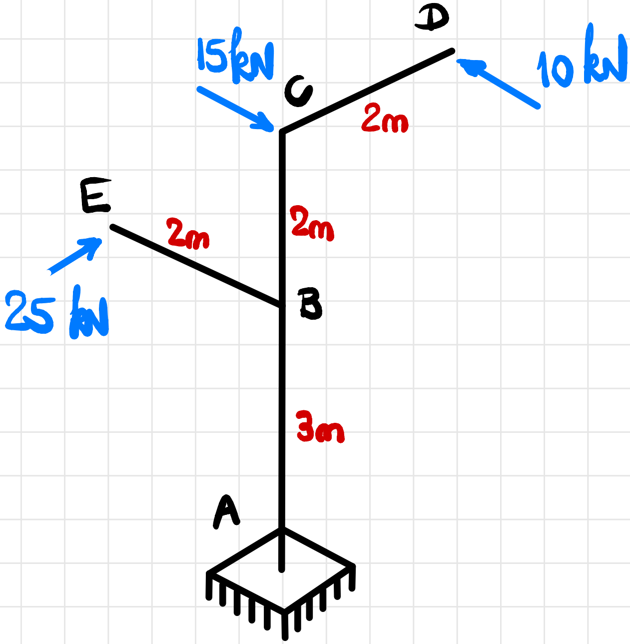
$$P \leqslant \frac{6286EA}{750\,l_1}\bar{\varphi}$$

$$P \leqslant \frac{6286 \cdot 150 \cdot 10^{9} \cdot 5 \cdot 10^{-4} \cdot 0{,}001}{750 \cdot 4}$$

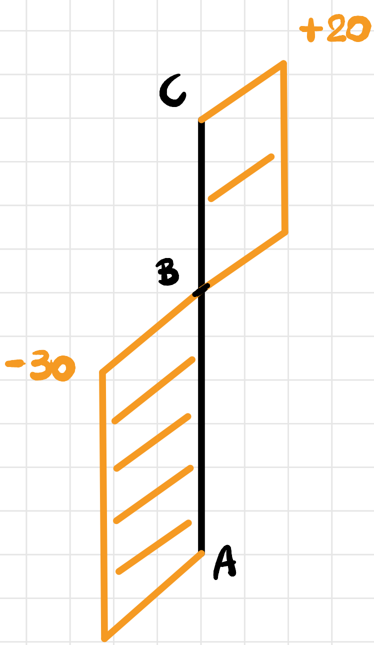
$$P \leqslant 157{,}15\ kN$$

Logo $P_{máx} = 47{,}145\,kN$.

②



Torção na barra AC:



Considerando que:

- $\tau_R = 150\,MPa$, $s = 2$
- $\overline{\theta}_c = 0{,}02\,rad$
- $G = 70\,GPa$

E que a barra é circular e cheia:

① Tensão de cisalhamento máxima:

$$\text{máx}\left(|\tau_{AB}|, \tau_{BC}\right) \leq \overline{\tau}$$

$$|\tau_{AB}| \leq \frac{\tau_R}{s} \longrightarrow \frac{|T_{AB}| \cdot R}{\frac{\pi R^4}{2}} \leq \frac{\tau_R}{s}$$

$$\frac{2\,|T_{AB}|}{\pi R^3} \leq \frac{\tau_R}{s} \longrightarrow R \geq \sqrt[3]{\frac{2\,|T_{AB}|\,s}{\pi\,\tau_R}}$$

$$R \geq \sqrt[3]{\frac{2 \cdot 30 \cdot 10^3 \cdot 2}{\pi \cdot 150 \cdot 10^6}} \longrightarrow R \geq 0{,}0634\,m$$

② Giro máximo em C:

$$|\theta_C| \leq \bar{\theta}_C \longrightarrow |\theta_{AB} + \theta_{BC}| \leq \bar{\theta}_C$$

$$\left| \frac{T_{AB}\, l_{AB}}{GJ} + \frac{T_{BC}\, l_{BC}}{GJ} \right| \leq \bar{\theta}_C$$

$$\frac{2}{\pi G R^4} |T_{AB}\, l_{AB} + T_{BC}\, l_{BC}| \leq \bar{\theta}_C$$

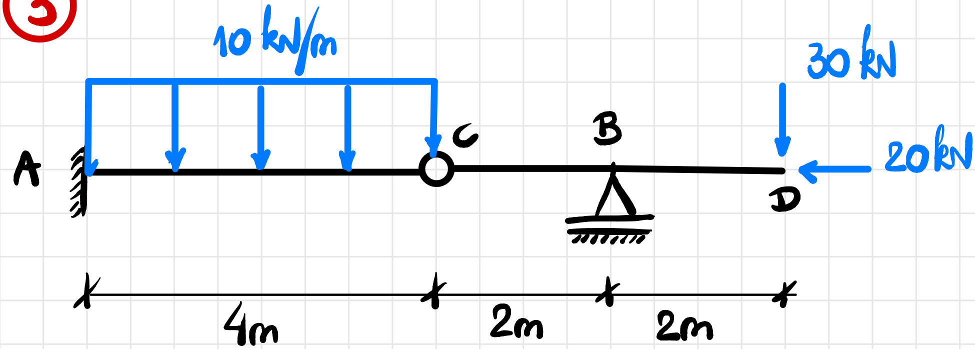
$$R \geq \sqrt[4]{\frac{2\,|T_{AB}\, l_{AB} + T_{BC}\, l_{BC}|}{\pi\, G . \bar{\theta}_C}}$$

$$R \geq \sqrt[4]{\frac{2 . |-30.10^3 . 3 + 20.10^3 . 2|}{\pi . 70.10^9 . 0{,}02}}$$

$$R \geq 0{,}069\, m$$

Logo $R_{min} = 6{,}9 cm$.

③



Reações de apoio:

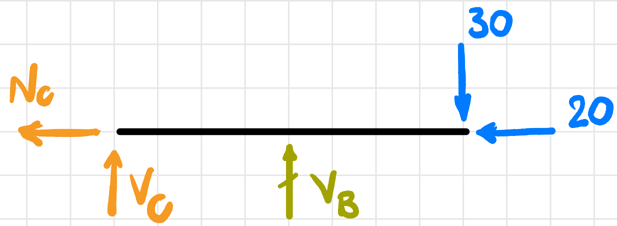
$$\sum F_H = 0: \; H_A - 20 = 0 \longrightarrow H_A = 20\,kN$$

$$\sum F_V = 0: \; V_A + V_B = 70$$

$$(+) \sum M_{(A)} = 0: \; -M_A - 40.2 + V_B.6 - 30.8 = 0$$

$$6V_B - M_A = 320$$

Corte em C:



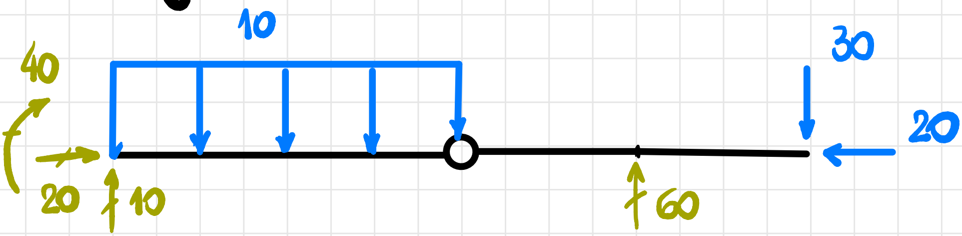
$\circlearrowleft + \sum M_C = 0: \quad V_B \cdot 2 - 30 \cdot 4 = 0$

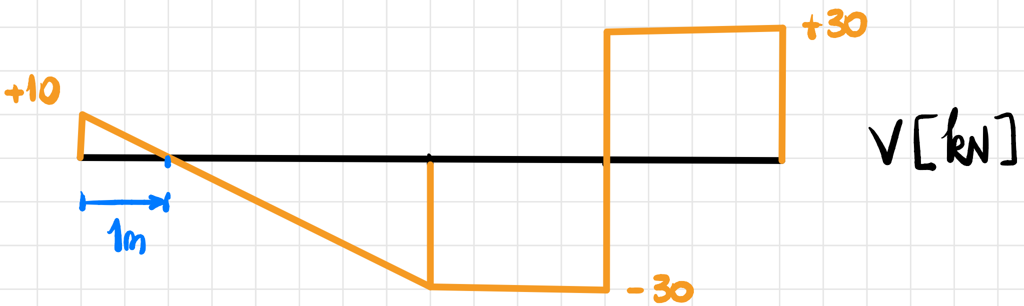
$\therefore \quad V_B = 60\ kN$

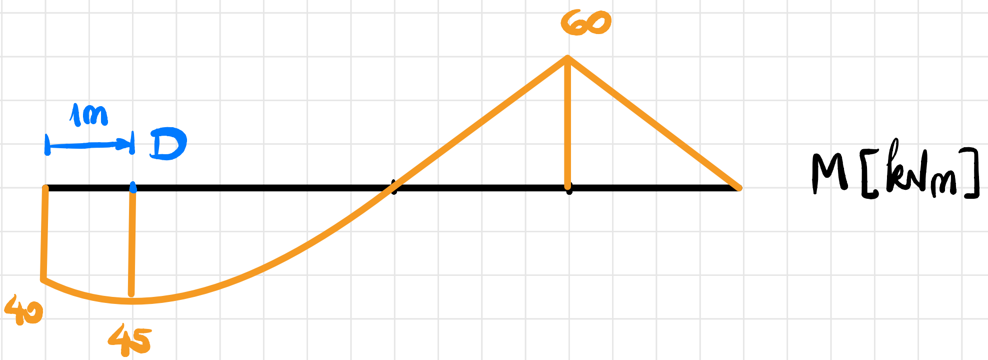
$V_A = 70 - V_B \longrightarrow V_A = 10\ kN$

$M_A = 6V_B - 320 \longrightarrow M_A = 40\ kN$

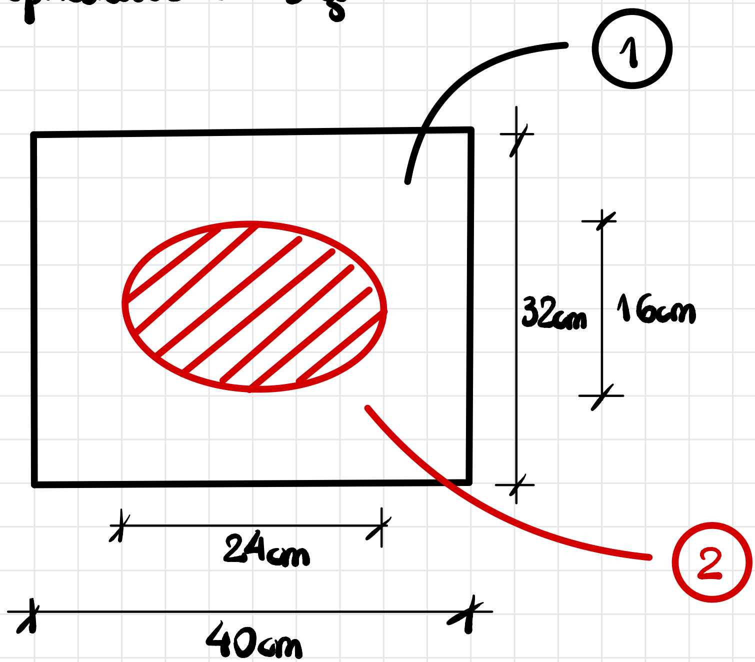
Diagramas:







Propriedades da seção:

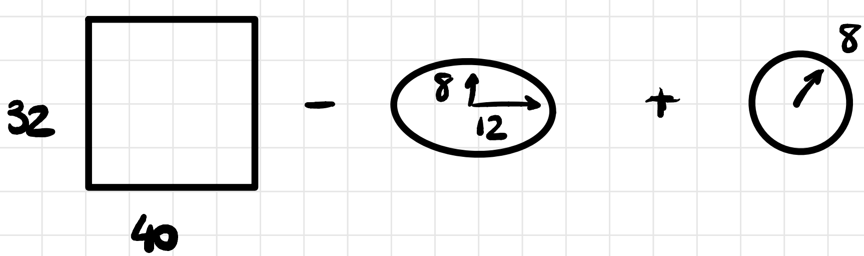


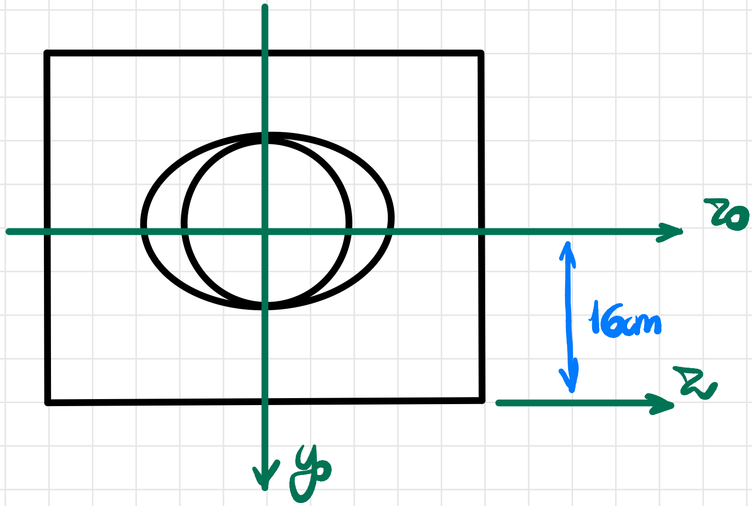
$$2E_1 = 3E_2$$

Usando 1 como base:

$$b_{eq} = \frac{E_2}{E_1} b = \frac{2}{3} b \longrightarrow b_{eq} = \frac{2}{3} \cdot 6 = 4\,cm$$

Logo:





Como a seç. é duplamente simétrica, $y_G = 4$cm.

Propriedades:

$$A = 32.40 - \pi.8.12 + \pi.8^2$$

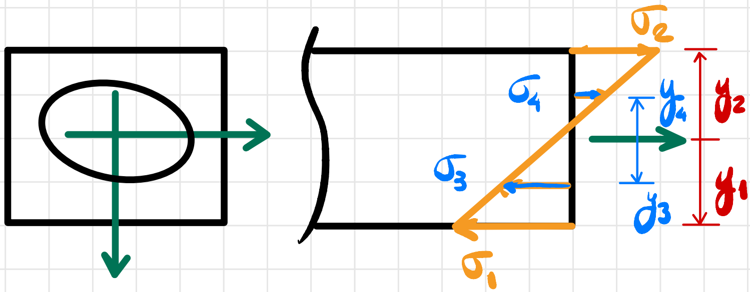
$$A = 1179{,}5\ cm^2$$

$$I_{z_0} = \frac{40.32^3}{12} - \frac{\pi.12.8^3}{4} + \frac{\pi.8^4}{4}$$

$$= \frac{327680}{3} - 1536\pi + 1024\pi$$

$$I_{z_0} = 107618{,}17\ cm^4$$

Para a seção B ($N = -20\ kN$, $M = -60\ kNm$):



$y_1 = 16cm$

$y_2 = -16cm$

$y_3 = 8cm$

$y_4 = -8cm$

$$\sigma_1 = \frac{N}{A} + \frac{M}{I_{z0}} y_1 = -\frac{20.10^3}{1179{,}5.10^{-4}} - \frac{60.10^3}{107618{,}17.10^{-8}} . 16.10^{-2}$$

$$\sigma_1 = -0{,}17.10^6 \ 55{,}75.10^6 . 16.10^{-2} \longrightarrow \sigma_1 = -9{,}09\ MPa$$

$$\sigma_2 = \frac{N}{A} + \frac{M}{I_{z0}} . y_2 = -0{,}17.10^6 - 55{,}75.10^6.(-16.10^{-2})$$

$$\sigma_2 = 8{,}74\ MPa$$

$$\sigma_3 = \frac{N}{A} + \frac{M}{I_{z0}} . y_3 = -0{,}17.10^6 - 55{,}75.10^6 . 8.10^{-2}$$

$$\sigma_3 = -4{,}63\ MPa$$

$$\sigma_4 = \frac{N}{A} + \frac{M}{I_{z0}} . y_4 = -0{,}17.10^6 - 55{,}75.10^6.(-8.10^{-2}) \rightarrow \sigma_4 = 4{,}29\ MPa$$

Linha neutra:

$$\sigma = 0 \rightarrow -0{,}17 \cdot 10^6 \quad 55{,}75 \cdot 10^6 \cdot \bar{y} = 0$$

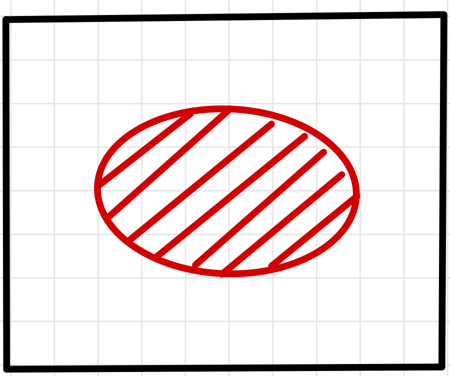
$$\bar{y} = -0{,}17 / 55{,}75 \rightarrow \bar{y} = -0{,}00305 \text{ m} \quad \text{ou} \quad \bar{y} = -3{,}05 \text{mm}$$

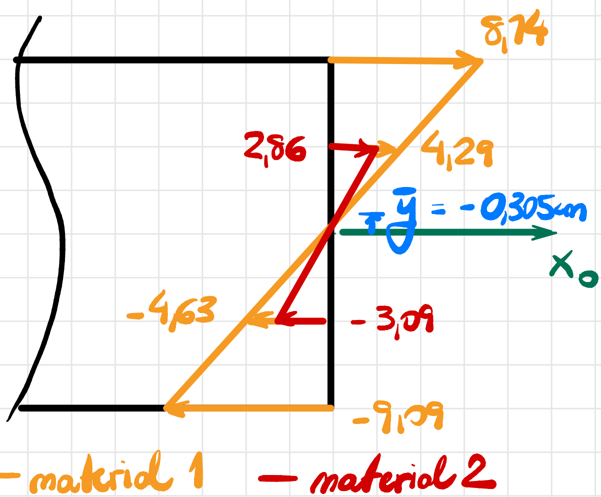
Tensões no material ②:

$$\sigma_3^{(2)} = \frac{E_2}{E_1}\,\sigma_3^{(1)} = \frac{2}{3} \, -4{,}63 = -3{,}09 \text{ MPa}$$

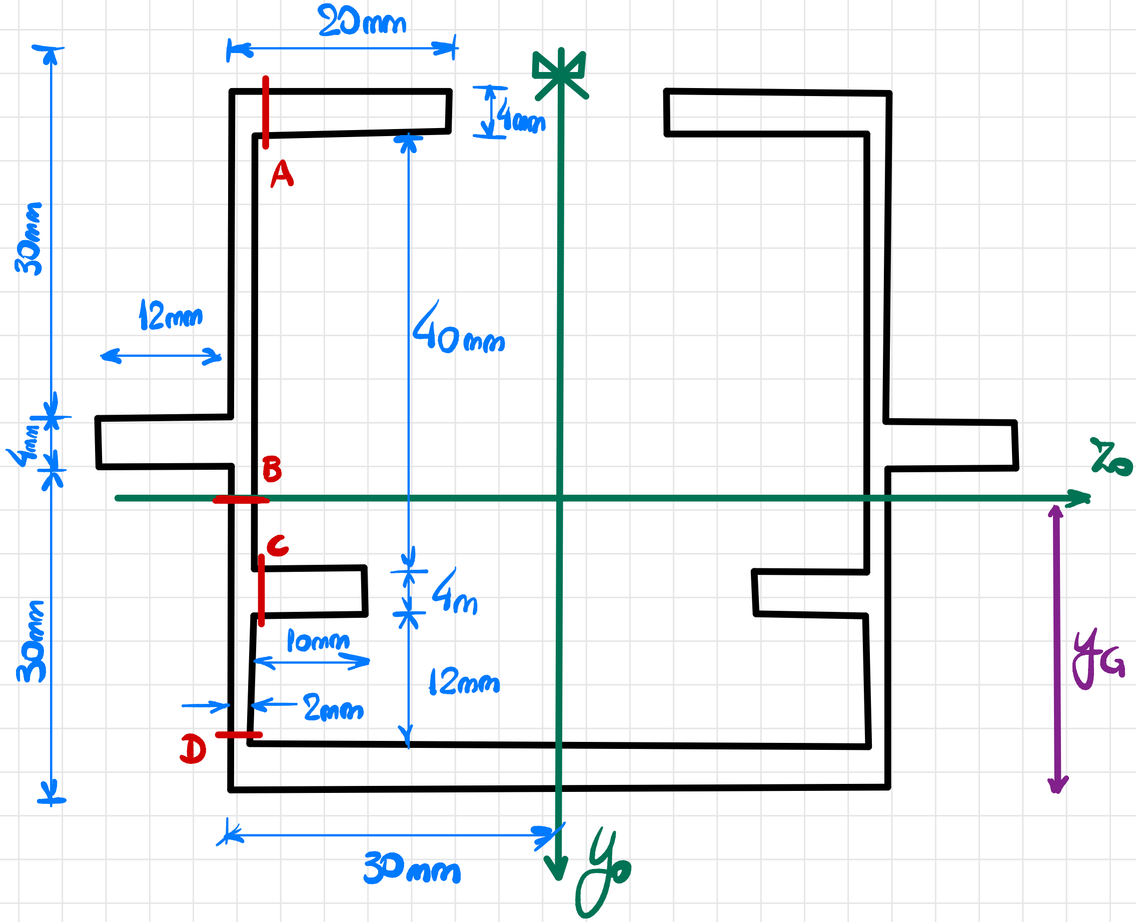
$$\sigma_4^{(2)} = \frac{E_2}{E_1}\,\sigma_4^{(1)} = \frac{2}{3} \cdot 4{,}29 = 2{,}86 \text{ MPa}$$

Diagrama de tensões (em MPa):





④



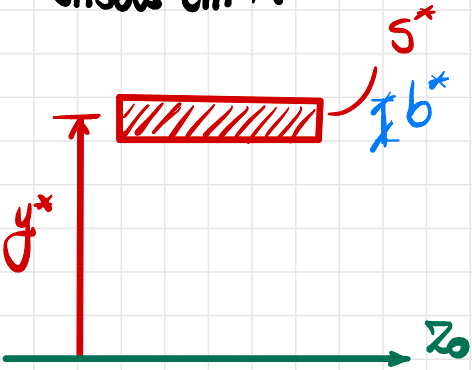
São dados:

$$y_G = 27{,}6\,mm$$

$$I_{z_0} = 4{,}195 \cdot 10^5\,mm^4$$

$$V = 8\,kN$$

Tensões em A:



$b^* = 4mm$

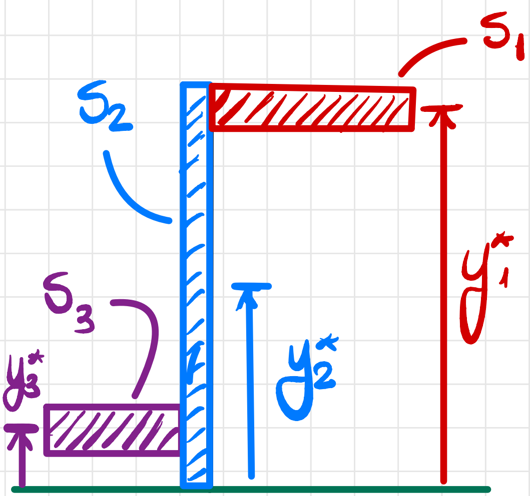
$S^* = 72mm$

$y^* = 62 - 27{,}6 = 34{,}4mm$

$M_S^* = S^* y^* = 2476{,}8\ mm^3$

$$\tau_A = \frac{8 \cdot 10^3 \cdot 2476{,}8}{4 \cdot 4{,}195 \cdot 10^5} \longrightarrow \tau_A = 11{,}81\ MPa$$

Tensões em B:



$M_{S1}^* = 2476{,}8\ mm^3$

$S_2^* = 72{,}8mm^2 \quad y_2^* = 18{,}2\ mm$

$M_{S2}^* = 1324{,}96\ mm^3$

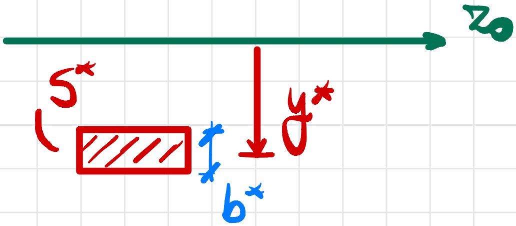
$S_3^* = 48mm^2 \quad y_3^* = 4{,}4mm$

$M_{S3}^* = 211{,}2\ mm^3$

$M_S^* = 4012{,}96\ mm^3$

$$\tau_B = \frac{8 \cdot 10^3 \cdot 4012{,}96}{2 \cdot 4{,}195 \cdot 10^5} \longrightarrow \tau_B = 38{,}26\ MPa$$

Tensões em C:



$b^* = 4\,mm$

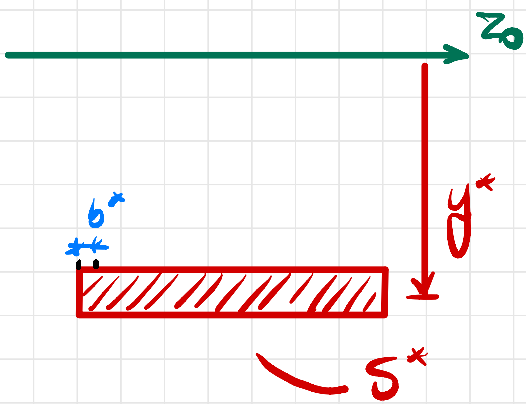
$S^* = 40\,mm^2$

$y^* = 27{,}6 - 18 = 9{,}6\,mm$

$M_S^* = 384\,mm^3$

$$\tau_C = \frac{8 \cdot 10^3 \cdot 384}{4 \cdot 4{,}195 \cdot 10^5} \longrightarrow \tau_C = 1{,}83\,MPa$$

Tensões em D:



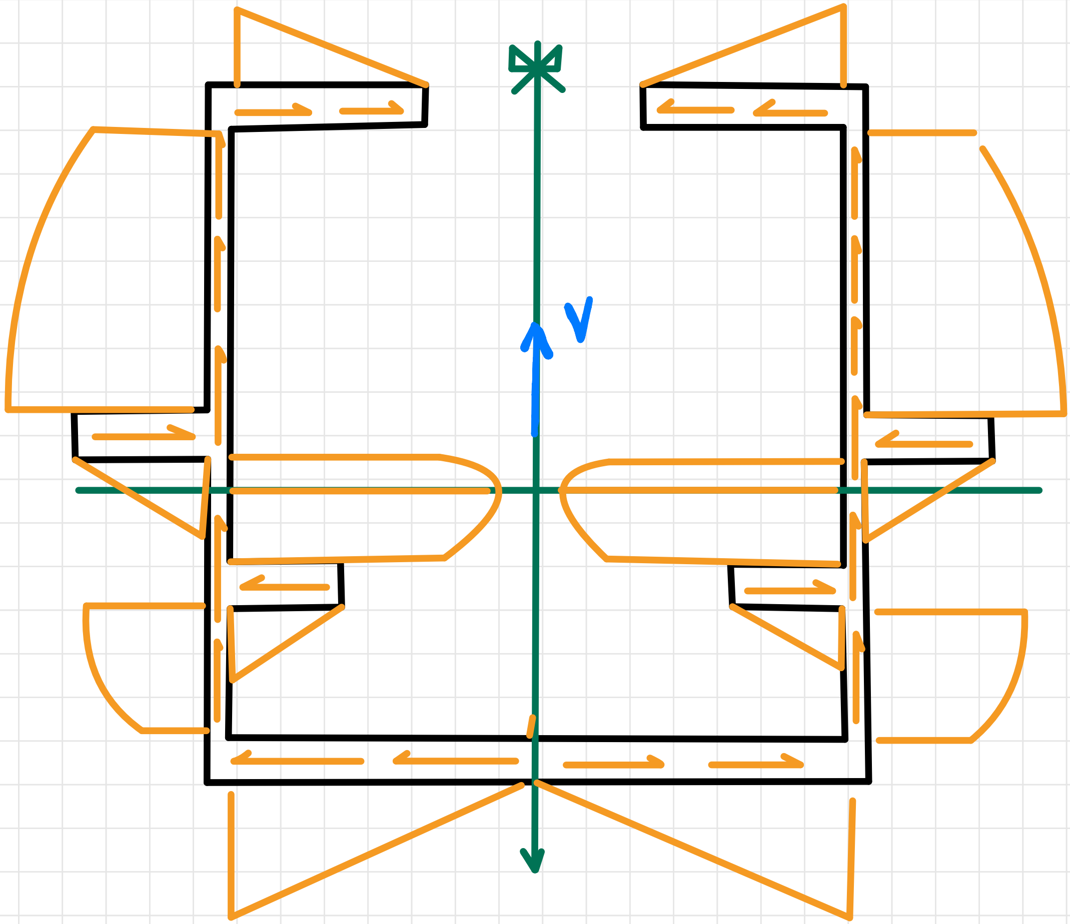
$b^* = 2\,mm$

$S^* = 120\,mm^2$

$y^* = 27{,}6 - 2 = 25{,}6\,mm$

$M_S^* = 3072\,mm^3$

$$\tau_D = \frac{8 \cdot 10^3 \cdot 3072}{2 \cdot 4{,}195 \cdot 10^5} \longrightarrow \tau_D = 29{,}29\,MPa$$



Ponto	Tensão [MPa]
A	11,81
B	38,26
C	1,83
D	29,29