277. $$\phi(x) = \frac{1}{2}\left(x f(x) + \int_0^x s g(s)\,ds + C\right)$$

$$\psi(x) = \frac{1}{2}\left(x f(x) - \int_0^x s g(s)\,ds - C\right)$$

$$u(x,t) = \frac{1}{x}\left(\phi(x+t) + \psi(x-t)\right)$$

$$= \frac{1}{x}\left[\frac{1}{2}\left((x+t)f(x+t) + \int_0^{x+t} s g(s)\,ds\right) + \frac{1}{2}\left((x-t)f(x-t) + \int_0^{x-t} s g(s)\,ds\right)\right] \quad //$$

$$\frac{\partial^2 \tilde{u}}{\partial t^2} = \frac{\partial^2 \tilde{u}}{\partial x^2} \qquad (x > 0,\ t > 0)$$

$$w = x+t, \quad s = x-t \quad \Rightarrow \quad \frac{\partial^2 v}{\partial w \partial s} = 0 \quad \Rightarrow \quad \underline{v(w,s) = \phi(w) + \psi(s)}$$

...

---

b) $$\tilde{u} = x u \qquad \frac{\partial^2 \tilde{u}}{\partial t^2} = \frac{\partial^2 \tilde{u}}{\partial x^2}$$

$$\Rightarrow \tilde{u}(x,t) = \phi(x+t) + \psi(x-t)$$

$$u(x,t) = \frac{1}{x}\tilde{u}(x,t) = \frac{1}{x}\left(\phi(x+t) + \psi(x-t)\right)$$

$$u(x,0) = f(x)$$

$$\frac{\partial u}{\partial t}(x,0) = g(x)$$

NO ITEM a), PODEMOS USAR RESULTADO DA PÁGINA 69 DO EVANS. USANDO EXPANSÃO ÍMPAR DAS CONDIÇÕES INICIAIS g E h, DENOTADAS POR $\tilde{g}$ E $\tilde{h}$, O

ESCOLHEMOS $\tilde{H}$ T.Q. $\tilde{H}' = \tilde{h}$.

EVANS PROVA QUE A SOLUÇÃO EM $\mathbb{R}_+$ É DADA POR

$$\tilde{u}(x,t) = \frac{1}{2}\left(\tilde{g}(x+t) + \tilde{g}(x-t)\right) + \frac{1}{2}\int_{x-t}^{x+t} \tilde{h}(y)\,dy = \underbrace{\frac{1}{2}\left(\tilde{g}(x+t) + \tilde{H}(x+t)\right)}_{\phi(x+t)} + \underbrace{\frac{1}{2}\left(\tilde{g}(x-t) - \tilde{H}(x-t)\right)}_{\psi(x-t)}.$$

EXERCÍCIO 323. (FOLLAND 1.B.2)

$$\begin{cases} x^2 \dfrac{\partial u}{\partial x} + y^2 \dfrac{\partial u}{\partial y} = u^2 \\ u(x, 2x) = 1 \end{cases}$$

↳ $y = 2x$

$x_r'(s) = x_r^2(s)$, $x_r(0) = r$ $\longrightarrow$ $x_r(s) = \dfrac{r}{1 - rs}$

$y_r'(s) = y_r^2(s)$, $y_r(0) = 2r$ $\longrightarrow$ $y_r(s) = \dfrac{2r}{1 - 2rs}$

$z_r'(s) = z_r^2(s)$, $z_r(0) = 1$. $\quad z_r(s) = \dfrac{1}{1 - s}$

$f' = f^2$, $f(0) = C$.

$$\frac{f'}{f^2} = 1 \qquad \int_0^s \frac{f'(w)}{f^2(w)}\,dw = s \qquad \begin{aligned} t &= f(w) \\ dt &= f'(w)\,dw \end{aligned}$$

$$\int_{f(0)}^{f(s)} \frac{dt}{t^2} = s \Rightarrow -\frac{1}{t}\Big|_{f(0)}^{f(s)} = s \Rightarrow \frac{1}{f(0)} - \frac{1}{f(s)} = s$$

$$\Rightarrow \frac{1}{f(s)} = -s + \frac{1}{f(0)} \Rightarrow \boxed{f(s) = \frac{1}{-s + \frac{1}{f(0)}} = \frac{f(0)}{-f(0)s + 1}}$$

$$\boxed{f(s) = \frac{f(0)}{1 - s f(0)}}$$

$$u(x,y) = z(r(x,y), s(x,y)) = \underline{\frac{1}{1 - s(x,y)}}$$

$$x = \frac{r}{1-rs} \qquad (1-rs)x = r$$

$$y = \frac{2r}{1-2rs} \qquad (1-2rs)y = 2r$$

$$x = r + rsx \Rightarrow x = r(1+sx) \Rightarrow r = \frac{x}{1+sx}$$

$$\left(1 - \frac{2sx}{1+sx}\right) y = \frac{2x}{1+sx}$$

$$\frac{(1+sx-2sx)y}{1+sx} = \frac{2x}{1+sx}$$

$$(1-sx)y = 2x \Rightarrow y - sxy = 2x \Rightarrow y - 2x = sxy$$

$$\Rightarrow s = \frac{y-2x}{xy}$$

$$\mu(x,y) = \frac{1}{1-s} = \frac{1}{1 - \frac{y-2x}{xy}} = \frac{xy}{xy - y + 2x}$$

EX: 316 VASY 3.2

$$x \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = u^2$$

$u(x,0) = \cos(x)$, $|x|$ PEQUENO

$x_1'(s) = x_1(s)$, $x_1(0) = r$ $\Rightarrow$ $\boxed{x_1(s) = r e^s}$ $s = y$

$y_1'(s) = 1$, $y_1(0) = 0$ $\Rightarrow$ $\boxed{y_1(s) = s}$ $r = x e^{-y}$

$z_1'(s) = z_1^2(s)$, $z_1(0) = \cos(r)$

$$\hookrightarrow z_1(s) = \frac{z_1(0)}{1 - s\, z_1(0)} = \frac{\cos(r)}{1 - s \cos(r)}$$

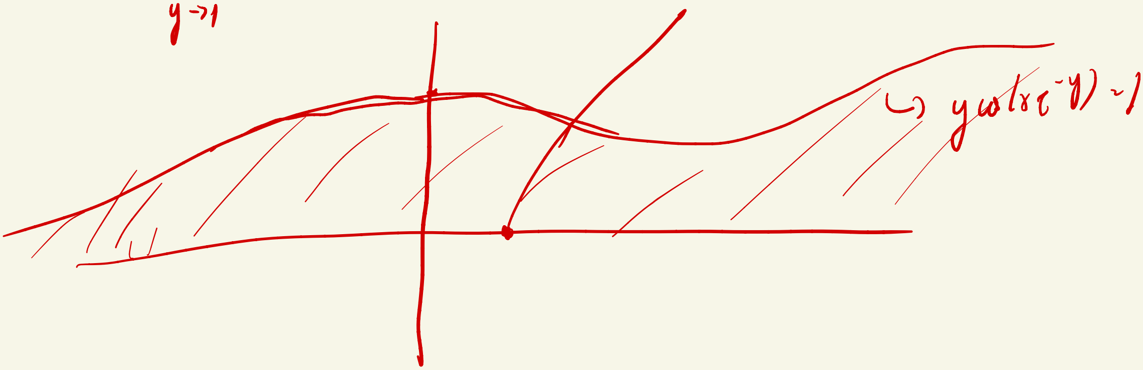
$$u(x,y) = \frac{\cos(x e^{-y})}{1 - y \cos(x e^{-y})}$$

$y \cos(x e^{-y}) = 1$

PRECISAMOS $1 - y\cos(x e^{-y}) \neq 0$, $|y|$ PEQUENO

$x = 0$ $y = 1$

$\lim_{y \to 1} u(0,y) = \infty$ PRECISAMOS $|y|$ PEQUENO



325 (FOLLAND 1.B.4)

$$u \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 1$$

$$u(x,x) = 0$$

$y = x$

a)
$x_r'(s) = z_r(s)$, $x_r(0) = r$

$y_r'(s) = 1$, $y_r(0) = r$

$z_r'(s) = 1$, $z_r(0) = 0$

$x_r'(s) = s$, $x_r(0) = r$

$y_r(s) = r + s$

$z_r(s) = s$

$$x_r(s) = r + \frac{s^2}{2}$$

$$y_r(s) = r + s \Rightarrow r = y - s$$

$$\boxed{x = y - s + \frac{s^2}{2}}$$

$$u(x,y) = z(r(x,y), s(x,y)) = s(x,y)$$

$$s^2 - 2s + 2(y-x) = 0 \qquad s = \frac{2 \pm \sqrt{4 - 8(y-x)}}{2}$$

$$s = 1 \pm \sqrt{1 - 2(y-x)}$$

SE $\boxed{s = 0, \text{ ENTÃO } x_r(0) = r, \ y_r(0) = r.}$

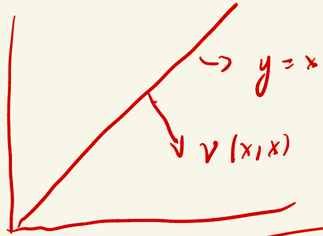
$$\boxed{0 = 1 \pm \sqrt{1 - 2(y_r(0) - x_r(0))}} = 1 \pm 1 = \begin{cases} 0 \\ 2 \end{cases}$$

LOGO O SINAL CORRETO É $-$

CONCLUSÃO $u(x,y) = 1 - \sqrt{1 - 2y + 2x}$

b) $u \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 1$

$u(x,x) = 1.$



NOTE QUE $\nu(x,x) = (1,-1).$

A NORMAL $\nu(x,x) = (1,-1).$

$\partial(1) = (1,1) \quad \partial'(1) = (1,1)$

OBSERVE QUE

$u(x,x)\, \nu_1(x,x) + 1\, \nu_2(x,x)$

$= 1 \times 1 + 1 \times -1 = 0.$

A CURVA (x,x) NÃO É CARACTERÍSTICA!

$x_r'(s) = z_r(s), \quad x_r(0) = r \qquad x_r(s) = r + s + \frac{s^2}{2}$

$y_r'(s) = 1, \quad y_r(0) = r \qquad y_r(s) = r + s$

$z_r'(s) = 1, \quad z_r(0) = 1 \qquad z_r(s) = 1 + s$

$(x_r(s), y_r(s)) = (r+s, r+s) + (\frac{s^2}{2}, 0)$

$u(x,y) = 1 + s(x,y)$

$r = y - s \qquad x = y - s + s + \frac{s^2}{2} \Rightarrow x - y = \frac{s^2}{2}.$

$s = \pm 2\sqrt{x-y}$

$s = 0 \Rightarrow x(0) = y(0) = r \Rightarrow s = \pm 2\sqrt{0}$ ou

NESTE CASO TEMOS 2 SOLUÇÕES $u(x,y) = \pm 2\sqrt{x-y}$, $x > y$

9.6.1

FOLLAND 1.D.2

$$\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} = f.$$

$$\boxed{\begin{aligned} U_0 &= u \\ U_1 &= \frac{\partial u}{\partial x_1} \\ U_2 &= \frac{\partial u}{\partial x_2} \end{aligned}}$$

$$\begin{cases} \dfrac{\partial U_0}{\partial x_2} = U_2 \\[2ex] \dfrac{\partial U_1}{\partial x_2} = \dfrac{\partial U_2}{\partial x_1} \\[2ex] \dfrac{\partial U_2}{\partial x_2} = f - \dfrac{\partial U_1}{\partial x_1} \end{cases}$$

360

d) $u'' = pu' + qu$

$$u(z) = \sum_{m=0}^{\infty} u_m z^m \qquad \sum_{m=0}^{\infty}(m+2)(m+1)u_{m+2}z^m \qquad \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} = \sum_{m=0}^{\infty}\sum_{k+l=m}$$

$$\underline{\sum_{m=0}^{\infty}(m+1)u_{m+1}z^m.}$$

m

$$\sum_{m=0}^{\infty}(m+2)(m+1)u_{m+2}z^m = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} p_k z^k (l+1)u_{l+1}z^l + \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} q_k u_l z^k z^l$$

$$\sum_{m=0}^{\infty}\underline{(m+2)(m+1)u_{m+2}}z^m = \sum_{m=0}^{\infty}\sum_{k+l=m}\left(\underline{p_k(l+1)u_{l+1} + q_k u_l}\right)z^m$$

$$u_{m+2} = \frac{1}{(m+2)(m+1)}\underbrace{\sum_{k+l=m}\left((l+1)p_k u_{l+1} + q_k u_l\right)}_{\sum_{j=0}^{m}\left[(j+1)p_{m-j}u_{j+1} + q_{m-j}u_j\right]}$$

b) $P_m \geq |p_m|$ e $Q_m \geq |q_m|$

$$P(z) = \sum_{m=0}^{\infty} P_m z^m \quad Q(z) = \sum_{n=0}^{\infty} Q_m z^m$$

$U(z)$ $\quad U'' = PU' + QU$, $C_0 \geq |c_0|$, $C_1 \geq |c_1|$

$$\Rightarrow U(z) = \sum_{n=0}^{\infty} U_n z^n \quad U_m \geq |u_m|$$

SOLUÇÃO:

$$U_{m+2} = \frac{1}{(m+2)(m+1)} \sum_{j=0}^{m} \left[(j+1) U_{j+1} P_{m-j} + U_j Q_{m-j}\right]$$

BASTA USAR INDUÇÃO. $U_0 \geq |u_0|$ e $U_1 \geq |u_1|$ POR HIPÓTESE ($C_0 \geq |c_0|$, $C_1 \geq |c_1|$)

$$U_{m+2} = \frac{1}{(m+2)(m+1)} \sum \ldots \geq \frac{1}{(m+2)(m+1)} \sum_{j=0}^{m} \left[(j+1)|u_{j+1}||p_{m-j}| + |u_j||q_{m-j}|\right]$$

$$\geq |u_{m+2}|$$

c) $P_n = K r^{-n}$, $Q_n = K(n+1) r^{-n}$.

$\sum_{n=0}^{\infty} p_n z^n$ CONVERGE P/ $|z| < R$.

LOGO SE $r < R$, ENTÃO $\sum_{n=0}^{\infty} |p_n| r^n < \infty$.

$\Rightarrow$ LOGO $\exists K$ T.Q. $|p_n| r^n \leq K, \forall n \Rightarrow |p_n| \leq \frac{K}{r^n}$.

$P_n = \frac{K}{r^n}$ $P_n \geq |p_n|$.

$$\begin{cases} \underline{P(z) = K\left(1 - \frac{z}{r}\right)^{-1}} = K \sum_{j=0}^{\infty} \left(\frac{z}{r}\right)^j \quad \text{OU} \quad P_n = K \frac{1}{r^n} \\ \underline{Q(z) = K\left(1 - \frac{z}{r}\right)^{-2}} = K \sum_{j=0}^{\infty} (j+1)\left(\frac{z}{r}\right)^j \quad Q_n = \frac{K(n+1)}{r^n} \end{cases}$$

$(1-x)^{-1} = \sum x^j$

$+\frac{1}{(1-x)^2} = \sum j x^{j-1} = \sum (j+1) x^j$

P E Q SÃO T.Q. $P_n \geq |p_n|$ $Q_n \geq |q_n|$.

COM ESSA ESCOLHA $\left(1 - \frac{z}{r}\right)^{\alpha}$ E $\left(1 - \frac{z}{r}\right)^{\beta}$ SÃO SOLUÇÕES.

d) Co

CONCLUSÃO FINAL:

1) ACHAMOS $u(z) = \sum u_m z^m$. COMO SOLUÇÃO

PROBLEMA: NÃO SABEMOS SE CONVERGE

2) MAJORAMOS p E q POR P E Q. E

ACHAMOS SOLUÇÃO ANALÍTICA DE

$$U'' = PU' + QU$$

3) MOSTRAMOS QUE $|u_m| \leq U_m$. $\Rightarrow$ u_m É MAJORADO

4) CONCLUSÃO $\sum u_m z^m$ CONVERGE E, PORTANTO,

É SOLUÇÃO