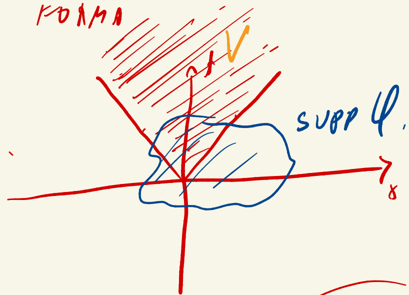
EX. 271 $\chi_V(x,t) = \begin{cases} 1, & t > 0, |x| \le t \\ 0, & \text{DE OUTRA FORMA} \end{cases}$



$(1 \leftrightarrow \frac{1}{2}$ PRECISA CORREÇÃO$)$.

MOSTRAR QUE $\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right)\chi_V = \delta_0$

$\downarrow$

T_{χ_V}

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right)T_{\chi_V}(\varphi) = T_{\chi_V}\left(\frac{\partial^2\varphi}{\partial t^2} - \frac{\partial^2\varphi}{\partial x^2}\right)$$

IMPORTANTE $\frac{\partial}{\partial x}\mu(\varphi) = -\mu\left(\frac{\partial\varphi}{\partial x}\right)$

t	0	∞
x	$-t$	t

x DE $-\infty$ A ∞

t DE $|x|$ A ∞.

$$= \int \chi_V(x,t)\left(\frac{\partial^2\varphi}{\partial t^2} - \frac{\partial^2\varphi}{\partial x^2}\right)dx\,dt = \int_V\left(\frac{\partial^2\varphi}{\partial t^2} - \frac{\partial^2\varphi}{\partial x^2}\right)dx\,dt$$

φ TEM SUP. COMPACTO

$$= \int_{-\infty}^{\infty}\left(\int_{|x|}^{\infty}\frac{\partial^2\varphi}{\partial t^2}dt\right)dx - \int_0^{\infty}\left(\int_{-t}^{t}\frac{\partial^2\varphi}{\partial x^2}dx\right)dt$$

$\varphi(\infty,\infty) = 0$

$$= -\underbrace{\int_{-\infty}^{\infty}\frac{\partial\varphi}{\partial t}(x,|x|)\,dx} - \int_0^{\infty}\left(\frac{\partial\varphi}{\partial x}(t,t) - \frac{\partial\varphi}{\partial x}(-t,t)\right)dt$$

$$= -\int_0^{\infty}\frac{\partial\varphi}{\partial t}(x,x)\,dx - \int_0^{\infty}\frac{\partial\varphi}{\partial t}(-x,x)\,dx - \int_0^{\infty}\left(\frac{\partial\varphi}{\partial x}(t,t) - \frac{\partial\varphi}{\partial x}(-t,t)\right)dt \quad (*)$$

$$-\int_{-\infty}^{\infty}\frac{\partial\varphi}{\partial t}(x,|x|)\,dt = -\int_0^{\infty}\frac{\partial\varphi}{\partial t}(x,x) - \underbrace{\int_{-\infty}^{0}\frac{\partial\varphi}{\partial t}(x,-x)dx}_{-\int_{\infty}^{0}\frac{\partial\varphi}{\partial t}(-x,x)dx}$$

$x \leftrightarrow -x$

$dx = -dx$ $\quad \int_{-\infty}^{0} \to -\int_{\infty}^{0}$

$$\frac{d}{ds}\varphi(s,s) = \frac{\partial\varphi}{\partial x}(s,s) + \frac{\partial\varphi}{\partial t}(s,s)$$

$$\frac{d}{ds}\varphi(-s,s) = -\frac{\partial\varphi}{\partial x}(s,s) + \frac{\partial\varphi}{\partial t}(s,s)$$

$$T_f(\varphi) = \int f(x)\,\varphi(x)\,dx$$

$$\partial_x^\alpha T_f(\varphi) = \underline{\int \partial^\alpha f(x)\,\varphi(x)\,dx} = (-1)^{|\alpha|}\int f(x)\,\partial^\alpha\varphi(x)\,dx \quad (*)$$

NESTE CASO χ_V NÃO É DERIVÁVEL.

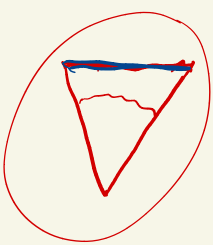
$\partial^\alpha T_{\chi_V}(\varphi)$ SÓ ESTÁ DEFINIDO NO SENTIDO DE DISTRIBUIÇÕES

$$\underline{\partial^\alpha T_{\chi_V}(\varphi) = (-1)^{|\alpha|}\int \chi_V\,\partial^\alpha\varphi\,dx}$$

$$(*) = -\int_0^\infty \underbrace{\left(\frac{\partial\varphi}{\partial t}(s,s) + \frac{\partial\varphi}{\partial x}(s,s)\right)}_{\frac{d}{ds}\left(\varphi(s,s)\right)} ds + \int_0^\infty \underbrace{\left(-\frac{\partial\varphi}{\partial t}(-s,s) + \frac{\partial\varphi}{\partial x}(-s,s)\right)}_{-\frac{d}{ds}\left(\varphi(-s,s)\right)} ds$$

$$= \underbrace{-\int_0^\infty \frac{d}{ds}\left(\varphi(s,s)\right)ds}_{=\varphi(0,0)} \underbrace{- \int_0^\infty \frac{d}{ds}\left(\varphi(-s,s)\right)ds}_{\varphi(0,0)} = 2\varphi(0,0).$$

<u>ERRO:</u> $\frac{1}{2}\chi_V$ É SOL. FUNDAMENTAL



APLICAR GREEN

$$\frac{\partial^2 u}{\partial r^2} + 2\frac{1}{r}\frac{\partial u}{\partial r} = \frac{\partial^2 u}{\partial t^2} \quad (*)$$

2) EX. 277

(FOLLAND 5.B2) a) $\frac{\partial^2 u}{\partial r^2} + (n-1)\frac{1}{r}\frac{\partial u}{\partial r} = \frac{\partial^2 u}{\partial t^2}$.

ACHE $u(r,t)$, $r>0$, $t>0$ QUE RESOLVE.

TRUQUE! DEFINIMOS $\tilde{u} = r u$.

(IMPORTANTE: MESMO TRUQUE USADO $U(x,r,t)$ $\tilde{U}(x,r,t) = r\,U(x,r,t)$)

$$\frac{\partial \tilde{u}}{\partial r} = r\frac{\partial u}{\partial r} + u.$$

$$\frac{\partial^2 \tilde{u}}{\partial r^2} = \frac{\partial u}{\partial r} + r\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{\partial r} = r\frac{\partial^2 u}{\partial r^2} + 2\frac{\partial u}{\partial r}.$$

$$\frac{\partial^2 \tilde{u}}{\partial t^2} = \frac{\partial^2 u}{\partial t^2} r \quad (*)$$

$$\frac{\partial^2 \tilde{u}}{\partial r^2} = r\frac{\partial^2 u}{\partial r^2} + 2\frac{\partial u}{\partial r} \overset{(*)}{=} r\frac{\partial^2 u}{\partial t^2} \overset{(*)}{=} \frac{\partial^2 \tilde{u}}{\partial t^2}$$

QUAL É A SOLUÇÃO?

$$\tilde{u}(r,t) = \phi(r+t) + \psi(r-t) \quad (273)$$

$$u(r,t) = \frac{1}{r}\phi(r+t) + \frac{1}{r}\psi(r-t)$$

b) $\frac{\partial^2 u}{\partial t^2} = \Delta u$

$u(x,0) = f(|x|)$

$\frac{\partial u}{\partial t}(x,0) = g(|x|)$

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} = \frac{\partial^2 u}{\partial t^2} \Rightarrow u(r,t) = \frac{1}{r}\left(\phi(r+t) + \psi(r-t)\right)$$

$u(r,0) = f(r)$

$\frac{\partial u}{\partial t}(r,0) = g(r)$

$$\boxed{\begin{aligned} u(r,0) &= \frac{1}{r}\phi(r) + \frac{1}{r}\psi(r) = f(r) \\ \frac{\partial u}{\partial t}(r,0) &= \frac{1}{r}\phi'(r) - \frac{1}{r}\psi'(r) = g(r) \end{aligned}}$$

$\tilde{u} = r u$

$$\frac{\partial^2 u}{\partial t^2} - \Delta u = 0 \qquad x \in \mathbb{R}^3,\ t \in \mathbb{R},$$

$x = r\cos\theta \sin\varphi$

$y = r \sin\theta \sin\varphi$

$z = r\cos\varphi$

$$\boxed{\Delta = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \text{TERMOS}\left(\frac{\partial}{\partial\theta}, \frac{\partial}{\partial\varphi}, \frac{\partial^2}{\partial\theta^2}, \frac{\partial^2}{\partial\varphi^2}\right)}$$

$g, f \quad (x)$

$u(r,t)$

$$\boxed{\begin{aligned}\phi(r) + \psi(r) &= r f(r)\\ \phi'(r) - \psi'(r) &= r g(r)\end{aligned}}$$

DERIVANDO

$$\boxed{\begin{aligned}\phi' + \psi' &= (rf)'(r)\\ \phi' - \psi' &= rg\end{aligned}}$$

$$2\phi' = (rf)' + rg \Rightarrow \phi = \frac{1}{2}\left(rf + \int_0^r s g(s)\,ds + C\right)$$

$$2\psi' = (rf)' - rg \Rightarrow \psi = \frac{1}{2}\left(rf - \int_0^r s g(s)\,ds - C\right)$$

$$u(r,t) = \frac{1}{2}\left(f(r+t) + f(r-t) + \frac{1}{r}\int_0^{r+t} s g(s)\,ds - \frac{1}{r}\int_0^{r-t} s g(s)\,ds\right)$$

$$= \frac{1}{2}\left(f(r+t) + f(r-t) + \frac{1}{r}\int_{r-t}^{r+t} s g(s)\,ds\right) \qquad s \leftrightarrow -s$$

$r < t$

$$\frac{1}{2}\left(f(r+t) + f(t+r) + \underbrace{\frac{1}{r}\int_0^{r+t} s g(s)\,ds + \frac{1}{r}\int_0^{t-r} -s g(s)\,ds}_{}\right)$$

$f = f(|x|)$

$$\frac{2}{r}\int_{t-r}^{t+r} s g(s)\,ds \xrightarrow[r\to 0]{} 2 r g(r).$$

$$\frac{\partial^2 \tilde{u}}{\partial t^2} = \frac{\partial^2 \tilde{u}}{\partial r^2}$$

$$\tilde{u}(r,0) = r f(r)$$

$$\frac{\partial \tilde{u}}{\partial t}(r,0) = r g(r)$$

273. Π_1 $\quad t \geq r > 0$

$$\tilde{u}(r,t) = \frac{1}{2}\left((r+t) f(r+t) - (t-r) f(t-r) + \frac{1}{2}\int_{t-r}^{t+r} s g(s)\, ds \right)$$

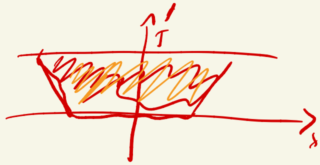
273

$$u(r,t) = \underbrace{\frac{1}{2}\left(f(r+t) + f(t-r) \right)}_{f(t)} + \underbrace{\frac{t}{2r}\left(f(r+t) - f(t-r) \right)}_{t f'(t)} + \underbrace{\frac{1}{2}\int_{t-r}^{t+r} s g(s)\, ds}_{+\, t g(t)}$$

$r \to 0$

Ex. 288

$$\frac{\partial^2 u}{\partial t^2} = \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}\frac{\partial u}{\partial x_j}\right)$$



a) $$E(t) = \int_{\mathbb{R}^n} \left(\frac{\partial u}{\partial t}\right)^2 + \sum_{i,j} a_{ij}\frac{\partial u}{\partial x_i}\frac{\partial u}{\partial x_j}\, dx$$

$$E'(t) = 2\int \frac{\partial u}{\partial t}\frac{\partial^2 u}{\partial t^2} + \underbrace{\sum a_{ij}\left(\frac{\partial^2 u}{\partial t\partial x_i}\frac{\partial u}{\partial x_j} + \frac{\partial u}{\partial x_i}\frac{\partial^2 u}{\partial t\partial x_j}\right)}$$ $\longrightarrow i \leftrightarrow j$

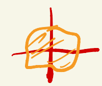
$$= \sum\left(a_{ij}\frac{\partial^2 u}{\partial t\partial x_i}\frac{\partial u}{\partial x_j} + a_{ji}\frac{\partial u}{\partial x_j}\frac{\partial^2 u}{\partial t\partial x_i}\right)$$ $\leftrightarrow$ $\underline{a_{ij} = a_{ji}}$

$$= 2\sum_{i,j} a_{ij}\frac{\partial^2 u}{\partial t\partial x_i}\frac{\partial u}{\partial x_j}$$

$$= 2\int_{\mathbb{R}^n}\left(\frac{\partial u}{\partial t}\frac{\partial^2 u}{\partial t^2} + \sum_{i,j}\frac{\partial}{\partial x_i}\left(\frac{\partial u}{\partial t}\right) a_{ij}\frac{\partial u}{\partial x_j}\right) dx$$

$\mathbb{R}^n \longleftrightarrow B(0,R)$

R GRANDE



$$= 2\int_{\mathbb{R}^n}\frac{\partial u}{\partial t}\frac{\partial^2 u}{\partial t^2}\,dx + 2\lim_{R\to\infty}\sum_{i,j}\int_{B(0,R)}\left(\frac{\partial}{\partial x_i}\left(\frac{\partial u}{\partial t}\right) a_{ij}\frac{\partial u}{\partial x_j}\right)dx$$

u TEM SUPORTE COMPACTO

$\Rightarrow \exists R>0$ T.Q. $u(x)=0$, $|x|\geq R$.

$$= 2\int_{\mathbb{R}^n}\frac{\partial u}{\partial t}\frac{\partial^2 u}{\partial t^2}\,dx + 2\lim_{R\to\infty}\sum_{i,j}\left[\underbrace{\int_{\partial B(0,R)}\frac{\partial u}{\partial t} a_{ij}\frac{\partial u}{\partial x_j}\nu_i\, dS(x)}_{\to 0} - \int_{B(0,R)}\frac{\partial u}{\partial t}\frac{\partial}{\partial x_i}\left(a_{ij}\frac{\partial u}{\partial x_j}\right)dx\right]$$

$u=0$ $|x|\geq R$ GRANDE

$$= 2\int_{\mathbb{R}^n}\frac{\partial u}{\partial t}\underbrace{\left(\frac{\partial^2 u}{\partial t^2} - \sum_{i,j}\frac{\partial}{\partial x_i}\left(a_{ij}\frac{\partial u}{\partial x_j}\right)\right)}_{=0} dx = 0.$$

b e c) $$E(t) = \int_U \left(\left(\frac{\partial u}{\partial t}\right)^2 + \sum_{i,j} a_{ij}\frac{\partial u}{\partial x_i}\frac{\partial u}{\partial x_j}\right) dx$$

DERIVADA CONORMAL

b) $u(x,t) = 0$, $x\in\partial U$

$\Rightarrow \frac{\partial u}{\partial t}(x,t) = 0$, $x\in\partial U$

c) $\sum_{i,j} a_{ij}\frac{\partial u}{\partial x_j}\nu_i = 0$, $x\in\partial U$

$$E'(t) = 2\int_U\left(\frac{\partial u}{\partial t}\frac{\partial^2 u}{\partial t^2} + \sum_{i,j} a_{ij}\frac{\partial^2 u}{\partial t\partial x_i}\frac{\partial u}{\partial x_j}\right)dx$$

$$= 2\int_U\frac{\partial u}{\partial t}\underbrace{\left(\frac{\partial^2 u}{\partial t^2} - \sum_{i,j}\frac{\partial}{\partial x_i}\left(a_{ij}\frac{\partial u}{\partial x_j}\right)\right)}_{=0}dx + \underbrace{\sum_{i,j}\int_{\partial U}\frac{\partial u}{\partial t} a_{ij}\frac{\partial u}{\partial x_j}\nu_i\, dS(x)}_{=0} = 0.$$

d) SE u_1 E u_2 SÃO SOLUÇÕES $u = u_1 - u_2$.

SATISFAZ:
$$\begin{cases} \dfrac{\partial u}{\partial t^2} = \displaystyle\sum_{i,j} \frac{\partial}{\partial x_i}\left(a_{ij}\frac{\partial u}{\partial x_j}\right) \\ u(x,0) = 0 \ (*) \\ \dfrac{\partial u}{\partial t}(x,0) = 0 \ (*) \end{cases}$$

$$E(t) = E(0) = \int \Big(\underbrace{\left(\frac{\partial u}{\partial t}(x,0)\right)^2}_{=0} + \underbrace{\sum_{i,j} a_{ij}(x)\frac{\partial u}{\partial x_i}(x,0)\frac{\partial u}{\partial x_j}(x,0)}_{=0}\Big)dx = 0$$

$\Downarrow$

$E'(t) = 0$

$u(x,0) = 0 \Rightarrow \dfrac{\partial u}{\partial x_i}(x,0) = 0$.

$$\Rightarrow E(t) = \int \left(\frac{\partial u}{\partial t}\right)^2(x,t) + \underbrace{\sum_{i,j} a_{ij}\frac{\partial u}{\partial x_i}\frac{\partial u}{\partial x_j}}_{\geq a_0 \sum_i \left(\frac{\partial u}{\partial x_i}\right)^2 = a_0|\nabla u|^2} dx = 0$$

$$0 \leq \int \left(\frac{\partial u}{\partial t}\right)^2 + a_0|\nabla u|^2 dx \leq 0$$

$$\Rightarrow \int \left(\frac{\partial u}{\partial t}\right)^2 + a_0|\nabla u|^2 dx = 0 \Rightarrow \begin{matrix}\dfrac{\partial u}{\partial t} = 0 \\ \nabla u = 0\end{matrix} \Rightarrow \nabla_{x,t} u = 0$$

$\Downarrow$ V conexo

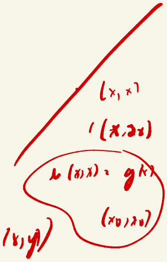
$u = cte$.

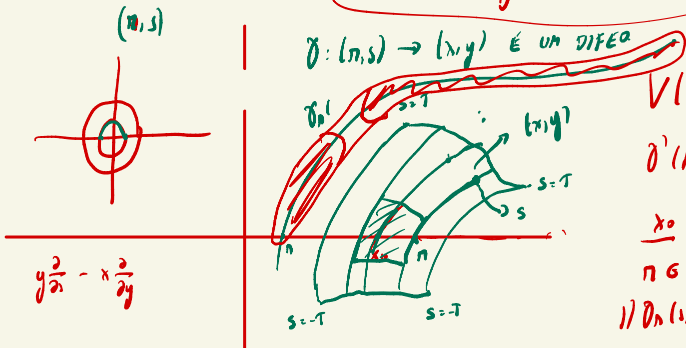
COMO $u(x,0) = 0 \Rightarrow u \equiv 0 \Rightarrow u_1 = u_2$.

CARACTERÍSTICA: SEMILINEAR

$$a(x,y)\frac{\partial u}{\partial x} + b(x,y)\frac{\partial u}{\partial y} = f(x,y,u(x,y)), \quad (x,y)\in\mathbb{R}^2$$

$$u(x,0) = g(x), \quad x\in\mathbb{R}.$$





$V(x,y) = (a(x,y), b(x,y))$

$\gamma'(t) = V(\gamma(t))$

x_0 $\quad b(x_0,0)\neq 0.$

$r \in \,]x_0-R, x_0+R[$

1) $\gamma_r(s)$.

$\underbrace{(x_0-R,x_0+R)}_{r}\times\underbrace{(-T,T)}_{s} \to V\subset\mathbb{R}^2.$ (x,y)

Dentro da região verde (x,y) corresponde a $\gamma_r(s)$

$$\exists\, \gamma_r \quad \begin{cases} \gamma_r' = V(\gamma_r) \\ \gamma_r(0) = r \end{cases}$$

Se u é solução $u\circ\gamma_r(s)$ resolve $\boxed{(u\circ\gamma_r)'(s) = f(\gamma_r(s), u\circ\gamma_r(s))}$

<u>Ideia:</u> 1) Dado ponto (x,y), acha r e s t.q.

$\gamma_r(s) = (x,y)$

2) Resolve $u\circ\gamma_r$. Logo $\boxed{u(x,y) = u(\gamma_r(s))}$

$$(x,y) \longmapsto (r,s) \longmapsto u(\gamma_r(s)).$$

$V\in C^1$, Lipschitz