Física IV

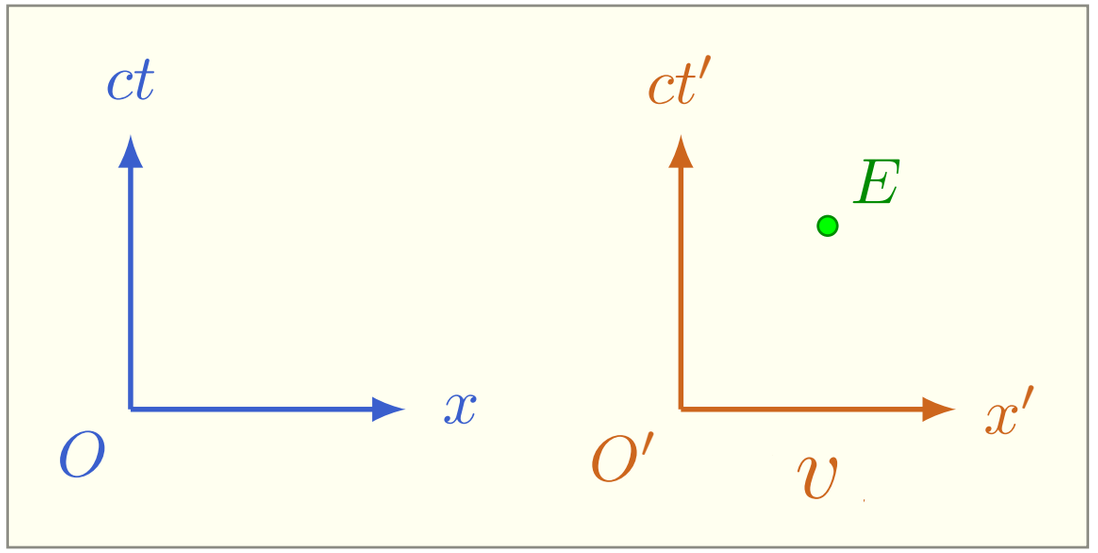
12 dezembro

Relatividade restrita

Relatividade restrita

Transformação de Lorentz

$$\begin{bmatrix} ct' \\ x' \end{bmatrix} = \gamma \begin{bmatrix} 1 & -\beta \\ -\beta & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix}$$

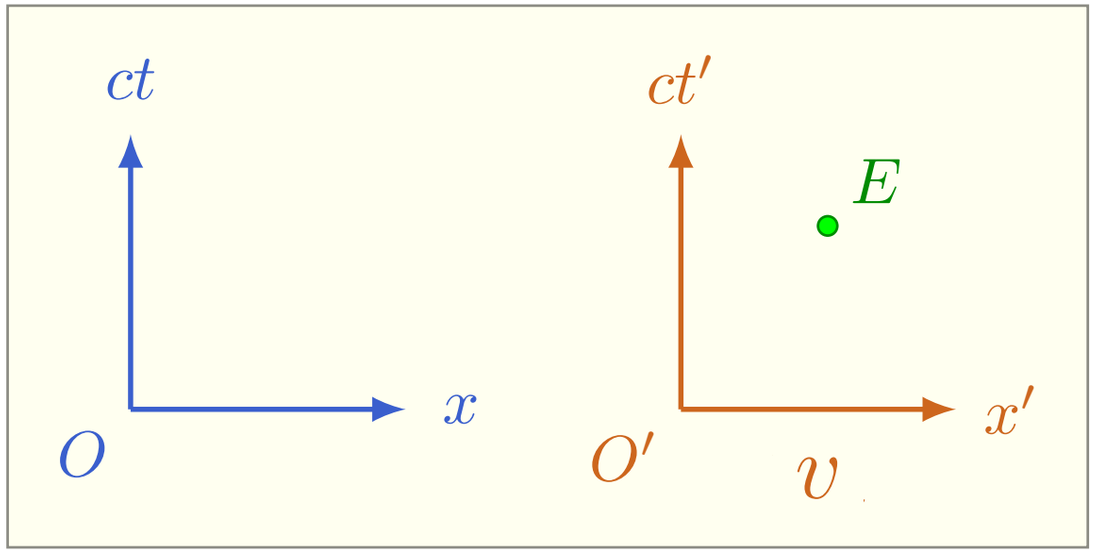


$$\gamma \equiv \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\beta \equiv \frac{v}{c}$$

Transformação de Lorentz

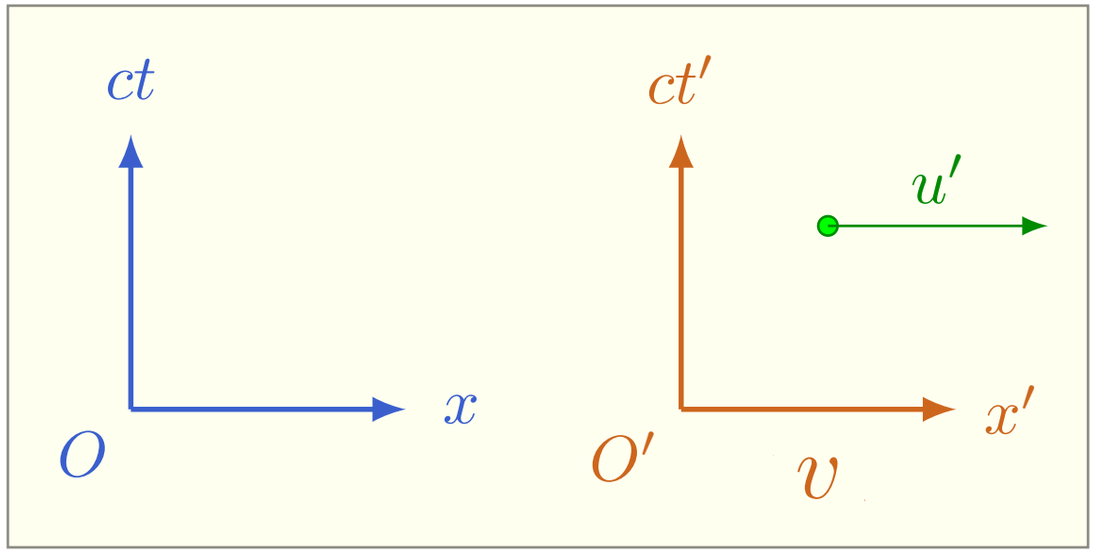
$$\begin{bmatrix} ct' \\ x' \end{bmatrix} = \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix}$$



Composição de velocidades

$$\begin{bmatrix} ct' \\ x' \end{bmatrix} = \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix}$$

$u = ?$



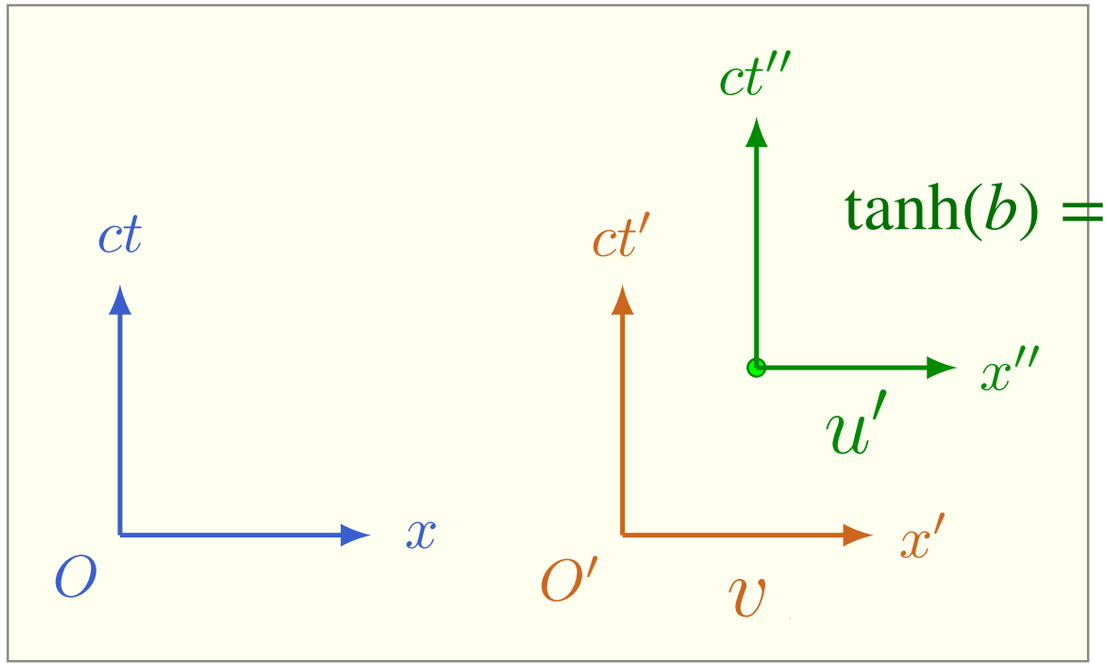
Composição de velocidades

$$\begin{bmatrix} ct' \\ x' \end{bmatrix} = \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix}$$

$$\tanh(a) = \frac{v}{c}$$

$$\begin{bmatrix} ct'' \\ x'' \end{bmatrix} = \begin{bmatrix} \cosh(b) & -\sinh(v) \\ -\sinh(b) & \cosh(b) \end{bmatrix} \begin{bmatrix} ct' \\ x' \end{bmatrix}$$

$$\tanh(b) = \frac{u'}{c}$$



Composição de velocidades

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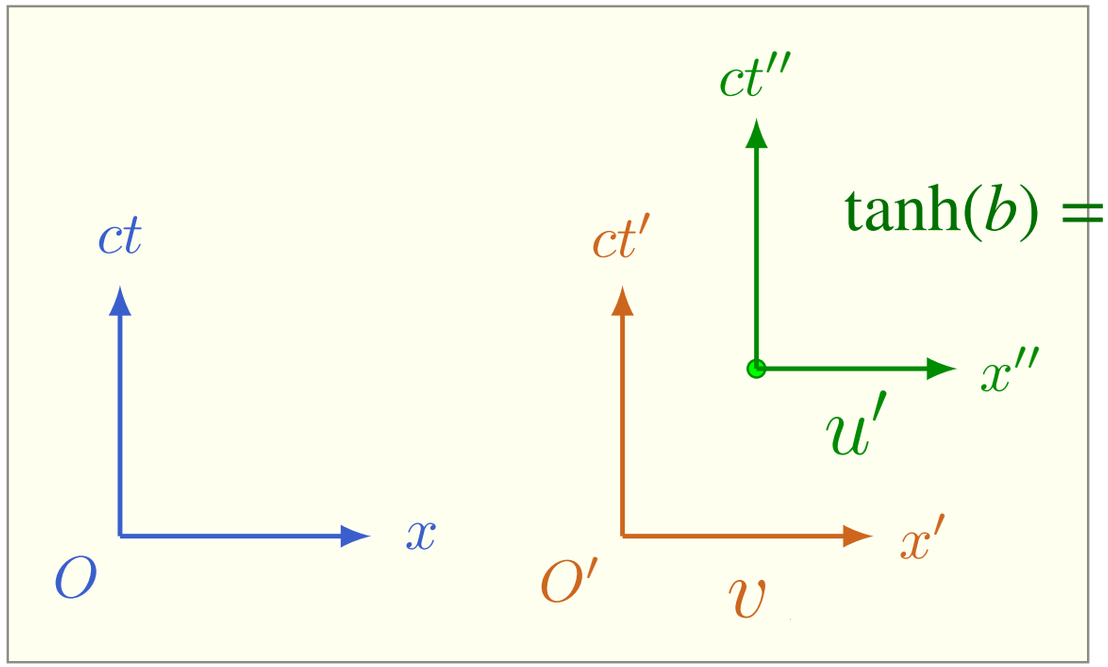

$$\begin{bmatrix} ct'' \\ x'' \end{bmatrix} = \begin{bmatrix} \cosh(b) & -\sinh(b) \\ -\sinh(b) & \cosh(b) \end{bmatrix} \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix}$$

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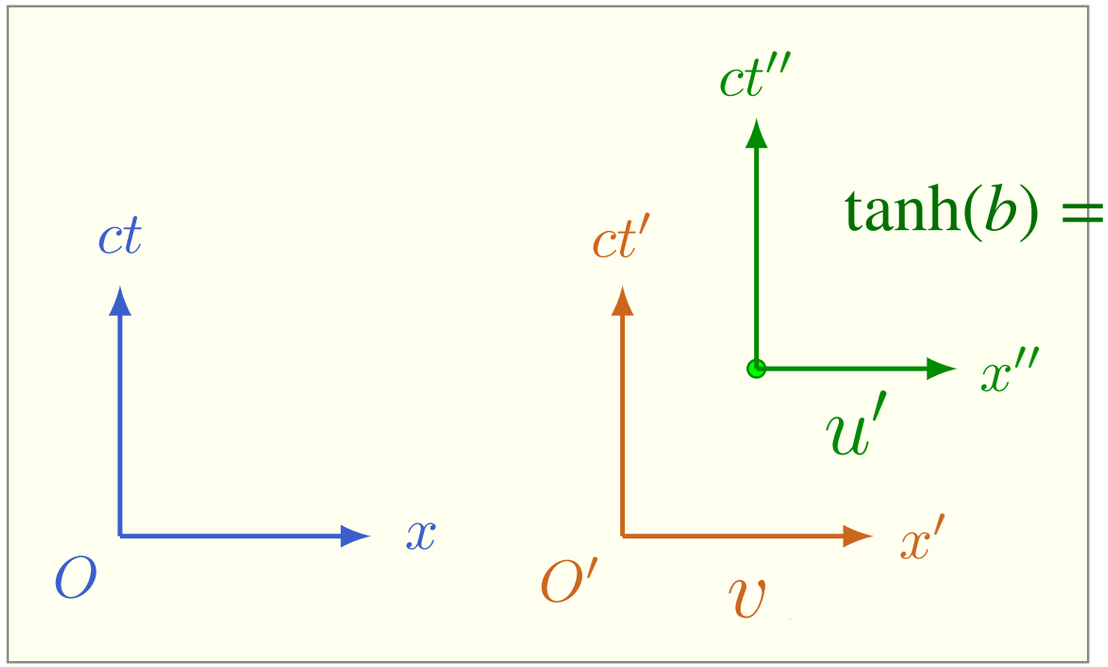

$$\tanh(b) = \frac{u'}{c}$$

$$\begin{bmatrix} ct'' \\ x'' \end{bmatrix} = \begin{bmatrix} \cosh(b) & -\sinh(b) \\ -\sinh(b) & \cosh(b) \end{bmatrix} \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix}$$

$$\begin{bmatrix} ct'' \\ x'' \end{bmatrix} = \begin{bmatrix} \cosh(b)\cosh(a) + \sinh(b)\sinh(a) & -\cosh(b)\sinh(a) - \sinh(b)\cosh(a) \\ -\sinh(b)\cosh(a) - \cosh(b)\sinh(a) & \sinh(b)\sinh(a) + \cosh(b)\cosh(a) \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix}$$

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$$\begin{bmatrix} ct' \\ x' \end{bmatrix} = \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix}$$

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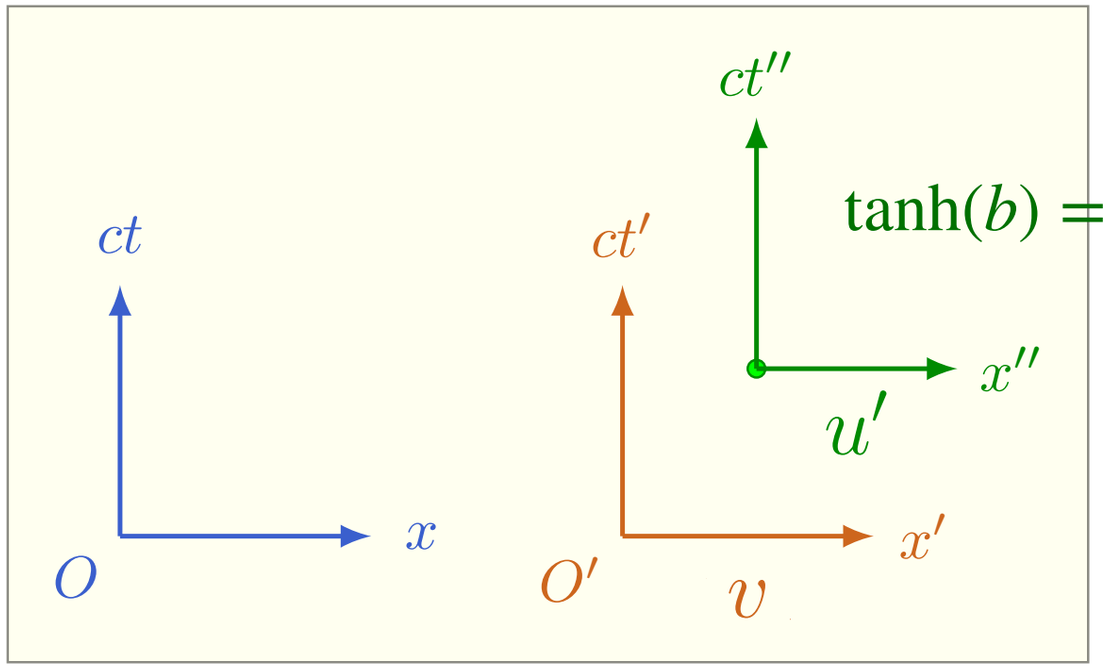

$$\begin{bmatrix} ct'' \\ x'' \end{bmatrix} = \begin{bmatrix} \cosh(b)\cosh(a) + \sinh(b)\sinh(a) & -\cosh(b)\sinh(a) - \sinh(b)\cosh(a) \\ -\sinh(b)\cosh(a) - \cosh(b)\sinh(a) & \sinh(b)\sinh(a) + \cosh(b)\cosh(a) \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix}$$

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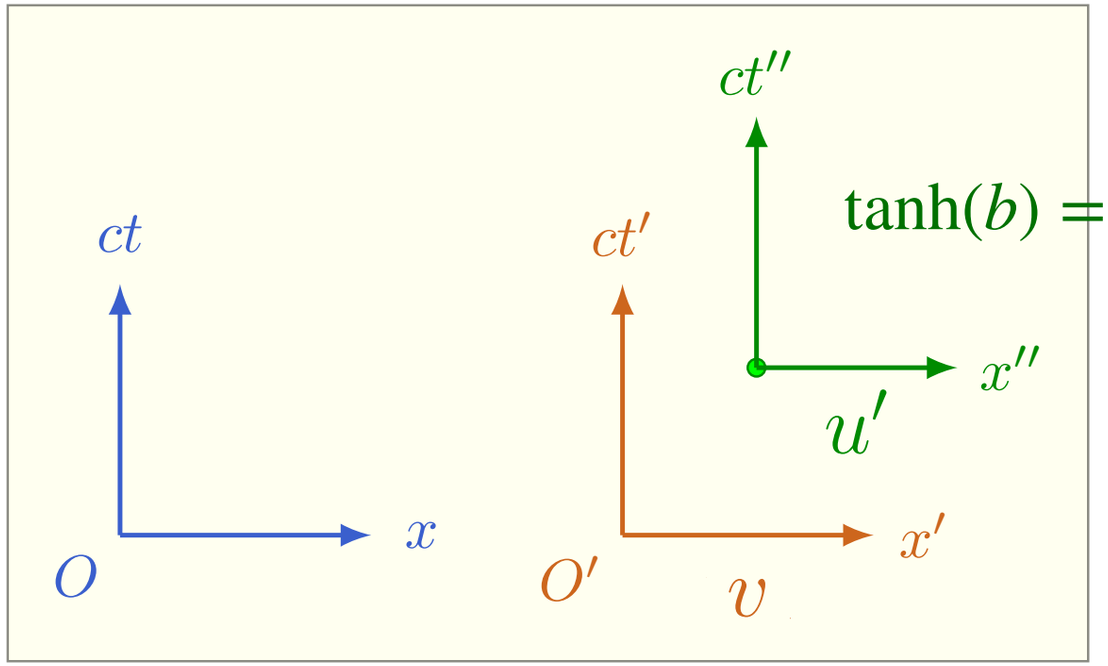

$$\begin{bmatrix} ct'' \\ x'' \end{bmatrix} = \begin{bmatrix} \cosh(b)\cosh(a) + \sinh(b)\sinh(a) & -\cosh(b)\sinh(a) - \sinh(b)\cosh(a) \\ -\sinh(b)\cosh(a) - \cosh(b)\sinh(a) & \sinh(b)\sinh(a) + \cosh(b)\cosh(a) \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix}$$

$$\begin{bmatrix} ct'' \\ x'' \end{bmatrix} = \begin{bmatrix} \cosh(a+b) & -\sinh(a+b) \\ -\sinh(a+b) & \cosh(a+b) \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix}$$

Composição de velocidades

$$\begin{bmatrix} ct' \\ x' \end{bmatrix} = \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix}$$

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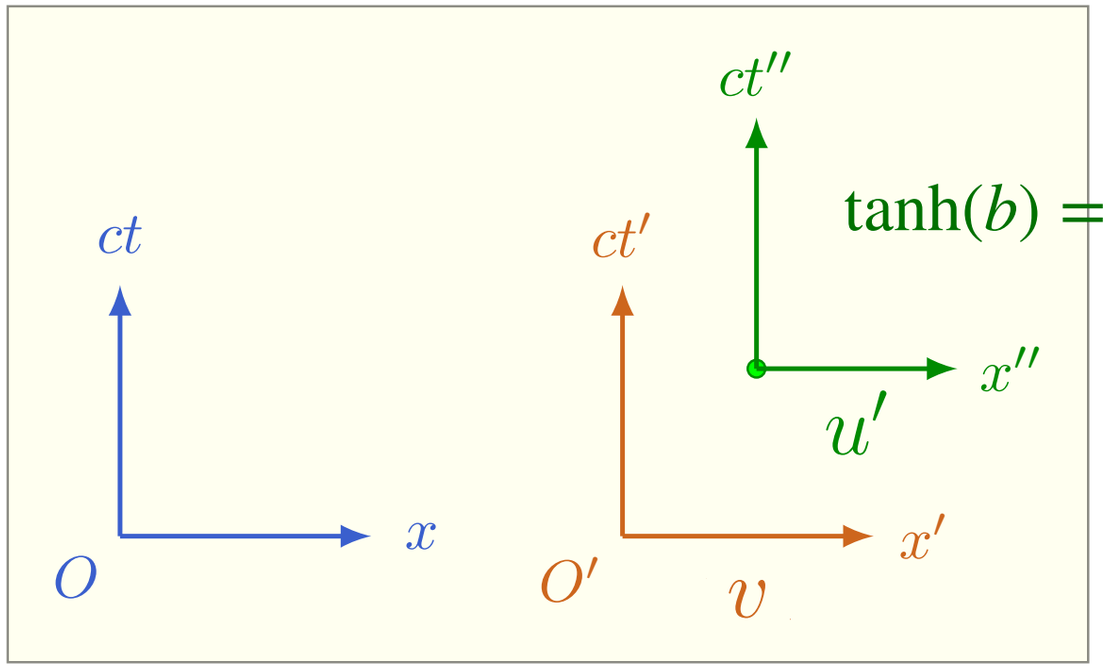

$$\begin{bmatrix} ct'' \\ x'' \end{bmatrix} = \begin{bmatrix} \cosh(b)\cosh(a) + \sinh(b)\sinh(a) & -\cosh(b)\sinh(a) - \sinh(b)\cosh(a) \\ -\sinh(b)\cosh(a) - \cosh(b)\sinh(a) & \sinh(b)\sinh(a) + \cosh(b)\cosh(a) \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix}$$

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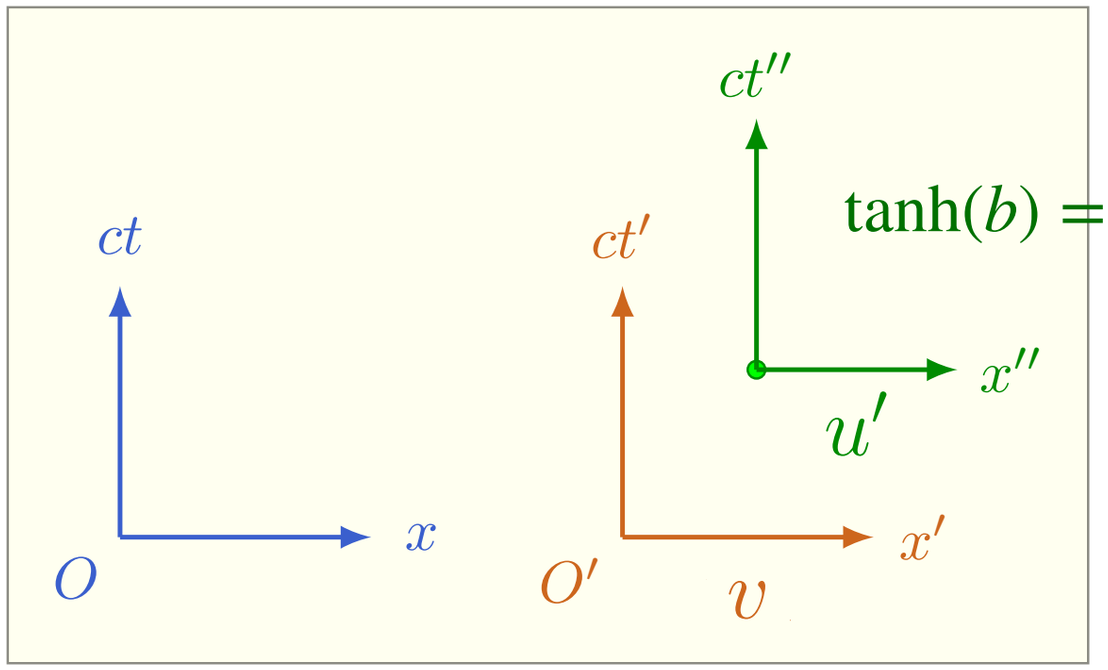

$$\tanh(b) = \frac{u'}{c}$$

$$\Rightarrow \tanh(a+b) = \frac{u}{c}$$

Composição de velocidades

$$\begin{bmatrix} ct' \\ x' \end{bmatrix} = \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix}$$

$$\tanh(a) = \frac{v}{c}$$

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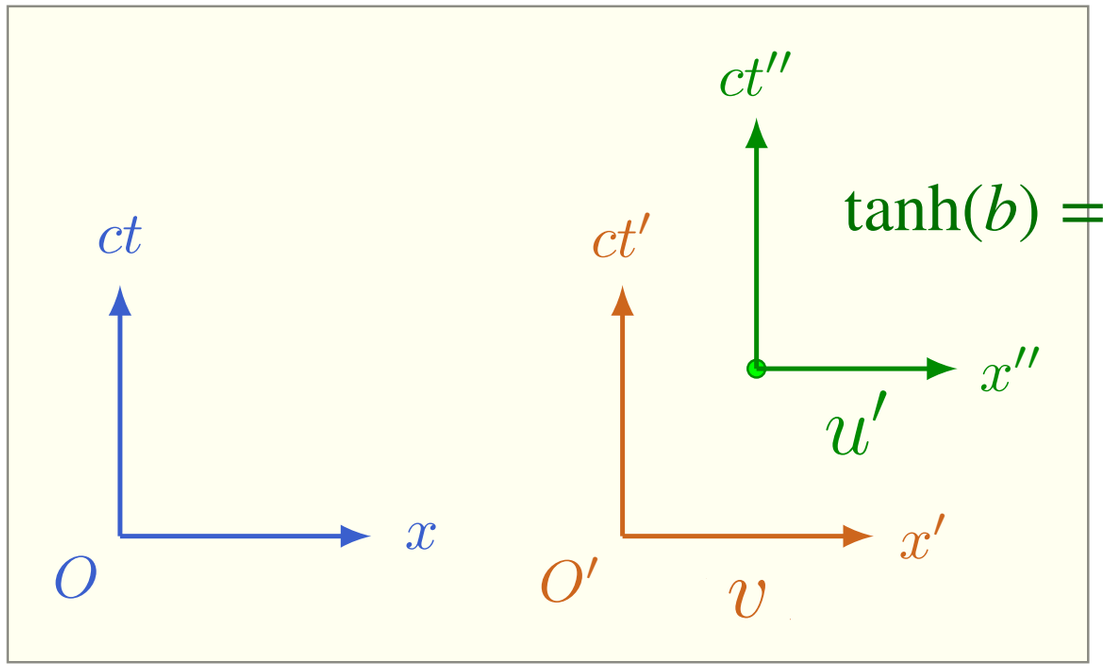

$$\tanh(b) = \frac{u'}{c}$$

$$\frac{u}{c} = \frac{\tanh(a) + \tanh(b)}{1 + \tanh(a)\tanh(b)}$$

Composição de velocidades

$$\begin{bmatrix} ct' \\ x' \end{bmatrix} = \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix}$$

$$\tanh(a) = \frac{v}{c}$$

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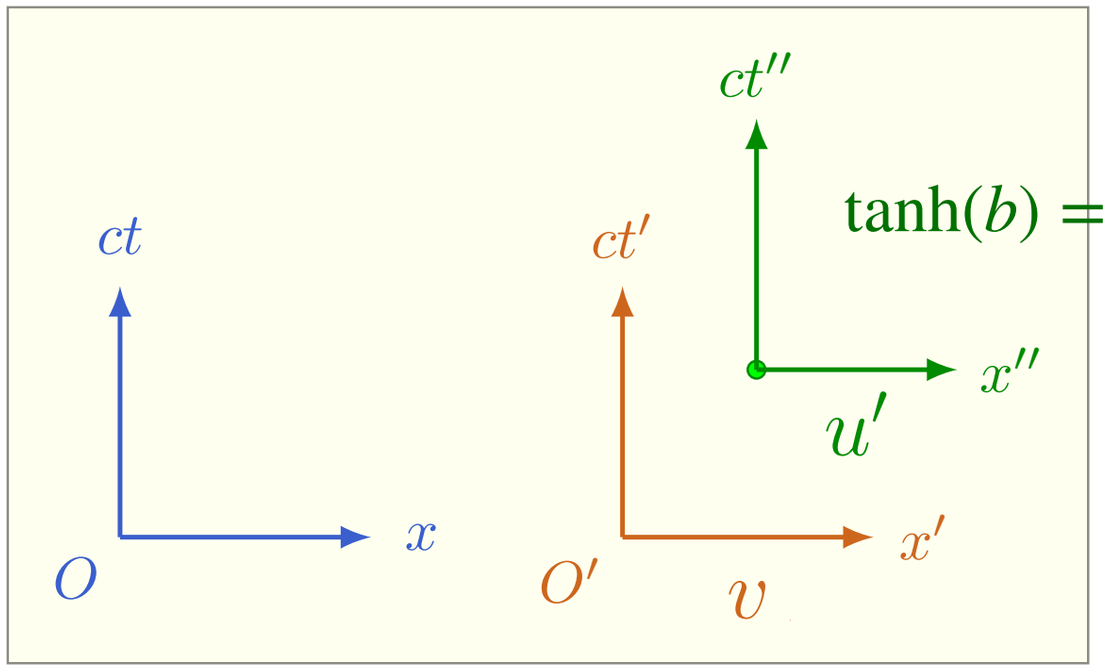

$$\tanh(b) = \frac{u'}{c}$$

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$$u = \frac{v + u'}{1 + \frac{v}{c}\frac{u'}{c}}$$

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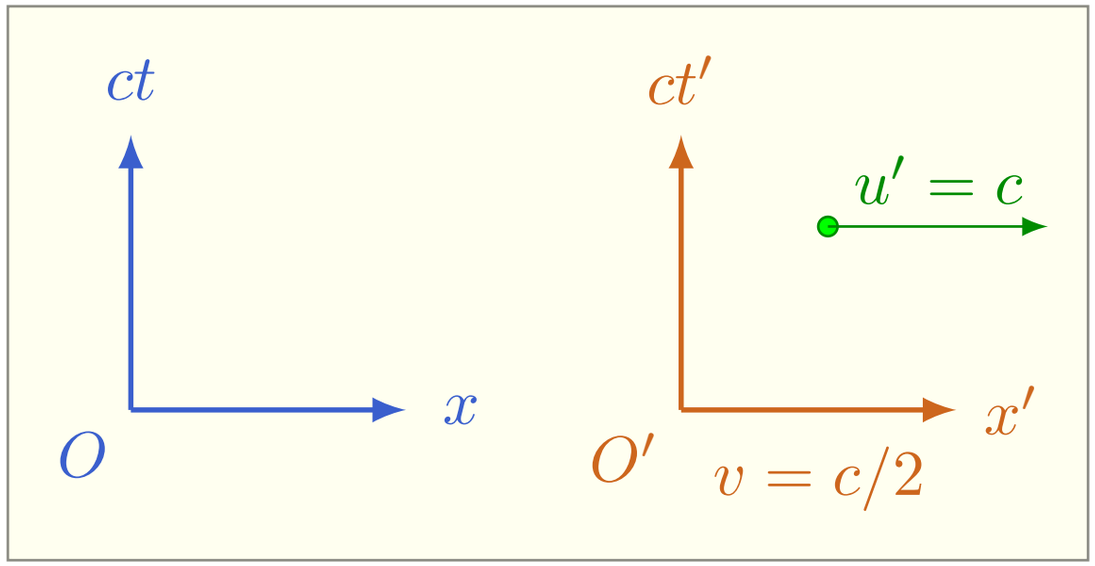
Pratique o que aprendeu

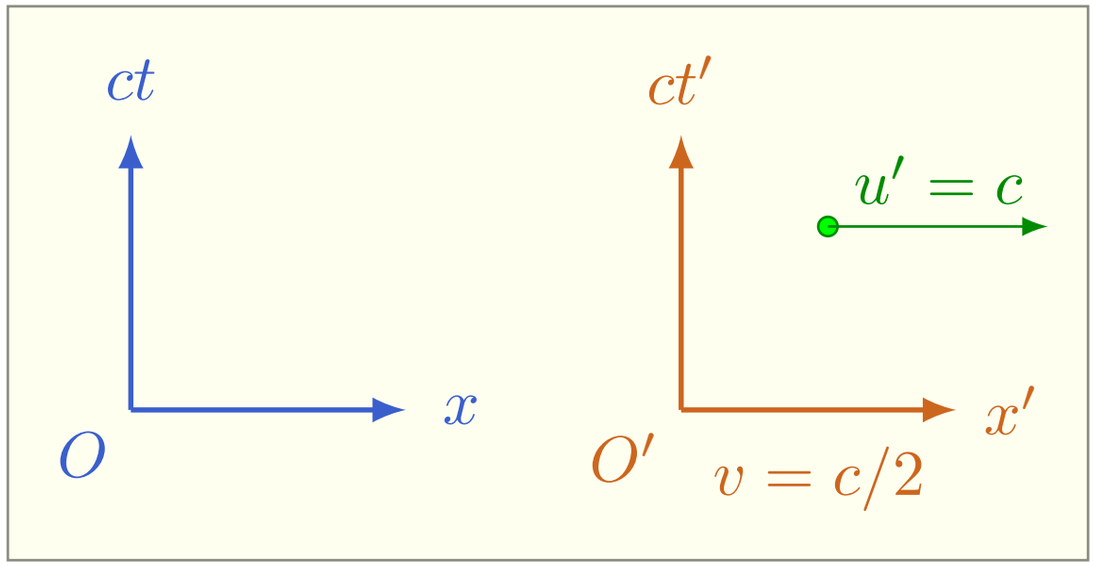
$u = ?$



$$u = \frac{v + u'}{1 + \frac{v}{c}\frac{u'}{c}}$$

Pratique o que aprendeu

$$u = \frac{\frac{c}{2} + c}{1 + \frac{1}{2}}$$



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Pratique o que aprendeu

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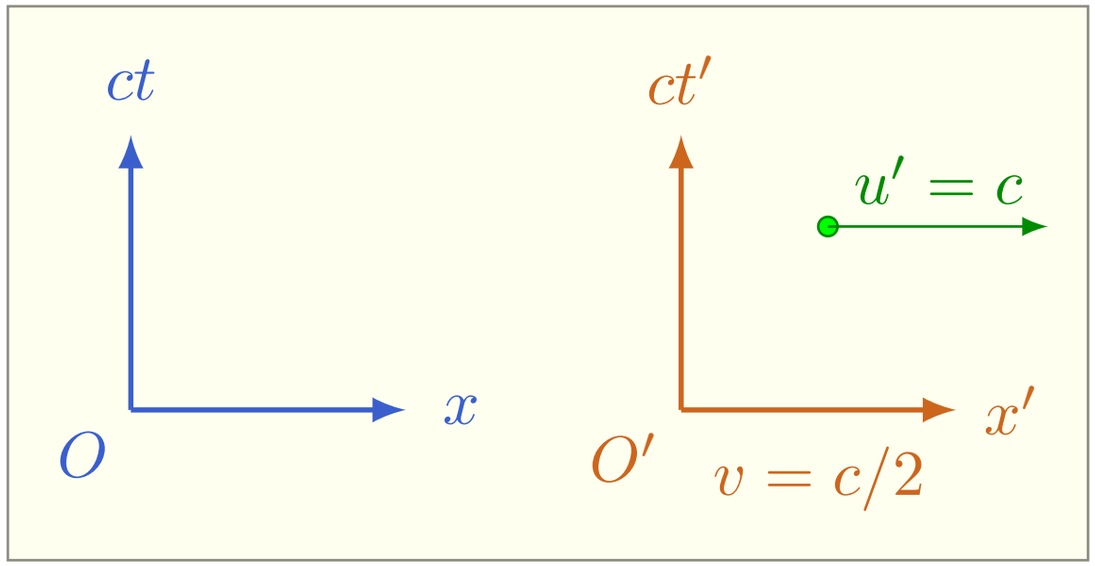


$$u = \frac{\frac{3c}{2}}{\frac{3}{2}} = c$$

Separação entre eventos

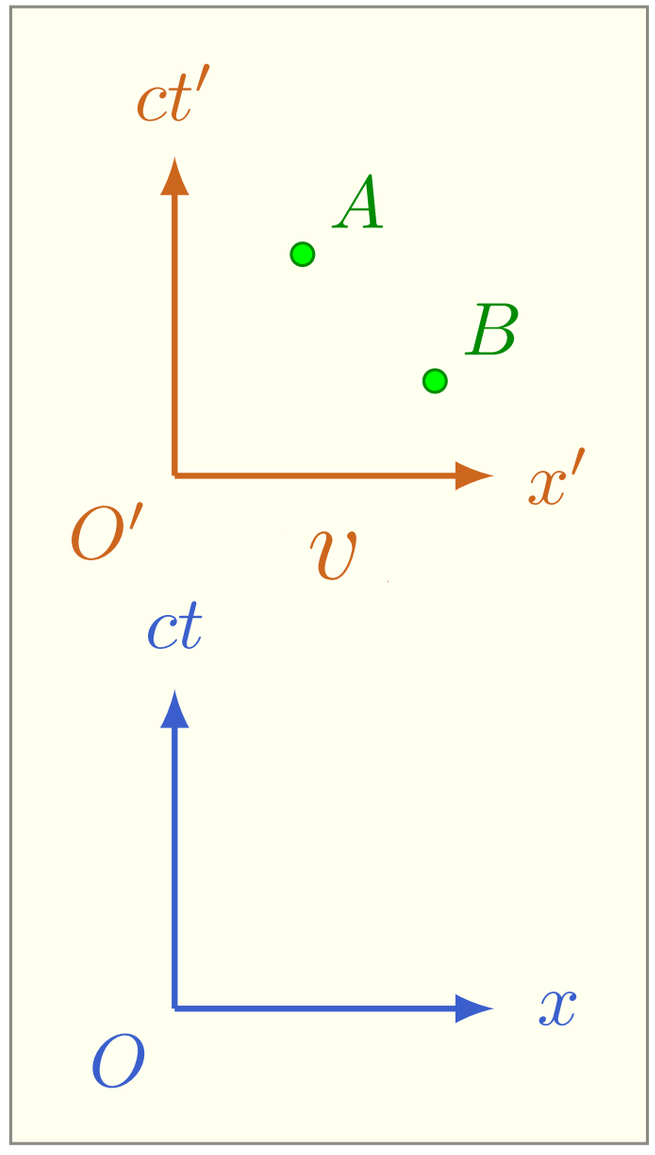
$$\begin{bmatrix} c\Delta t' \\ \Delta x' \end{bmatrix} = \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} c\Delta t \\ \Delta x \end{bmatrix}$$

$$\begin{bmatrix} -c\Delta t' \\ \Delta x' \end{bmatrix} = \begin{bmatrix} \cosh(a) & \sinh(a) \\ \sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} -c\Delta t \\ \Delta x \end{bmatrix}$$

$$\begin{bmatrix} -c\Delta t' & \Delta x' \end{bmatrix} = \begin{bmatrix} -c\Delta t & \Delta x \end{bmatrix} \begin{bmatrix} \cosh(a) & \sinh(a) \\ \sinh(a) & \cosh(a) \end{bmatrix}$$

$$\begin{bmatrix} -c\Delta t' & \Delta x' \end{bmatrix} \begin{bmatrix} c\Delta t' \\ \Delta x' \end{bmatrix} = \begin{bmatrix} -c\Delta t & \Delta x \end{bmatrix} \begin{bmatrix} c\Delta t \\ \Delta x \end{bmatrix}$$

$$s_{AB}^2 \equiv (\Delta x')^2 - (c\Delta t')^2 = (\Delta x)^2 - (c\Delta t)^2$$



Física IV

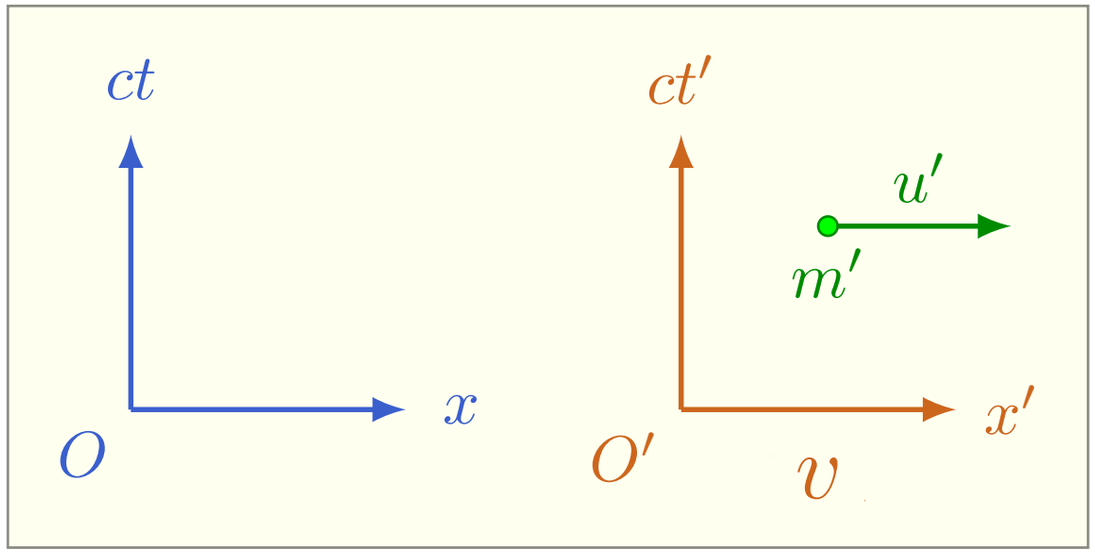
Dinâmica relativístico

Relatividade restrita

Dinâmica

$O \Rightarrow \vec{p}, E$

$O' \Rightarrow \vec{p}', E'$

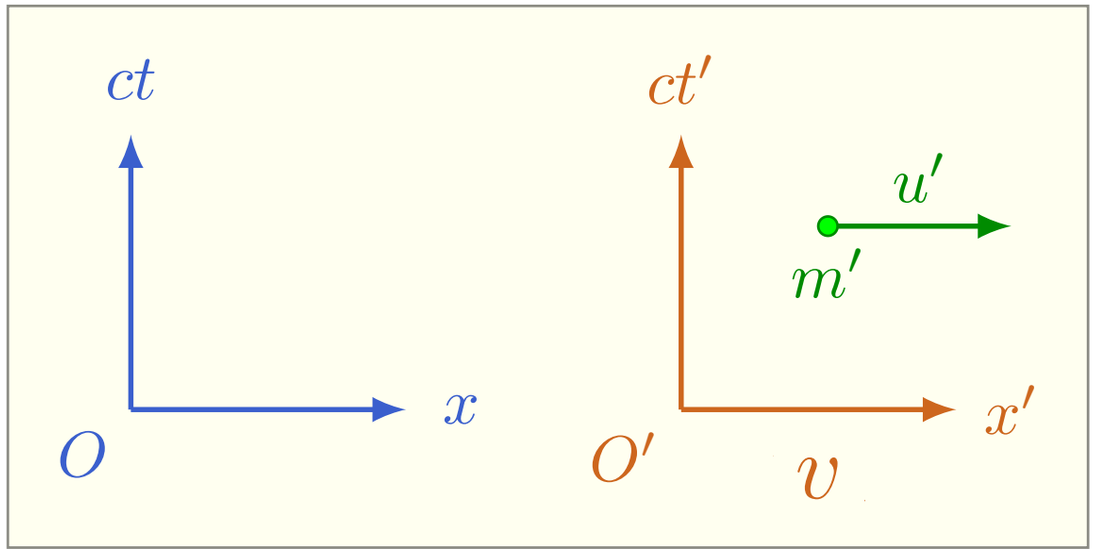


Relatividade restrita

Dinâmica

$O \Rightarrow \vec{p}, E$

$O' \Rightarrow \vec{p}', E'$



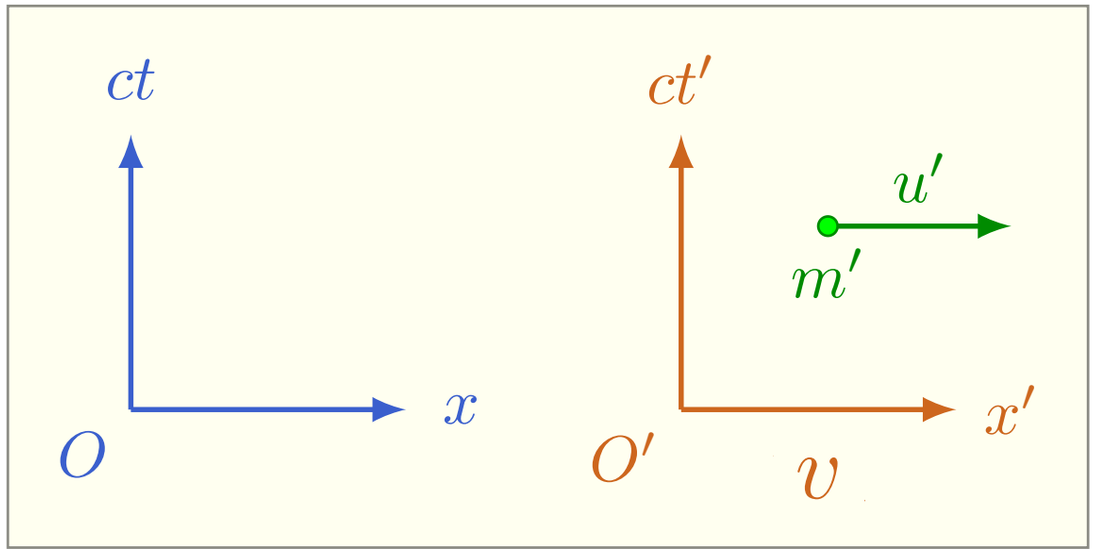
$$\begin{bmatrix} E' \\ p'c' \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} E \\ pc \end{bmatrix} \qquad \left(\tanh(a) = \frac{v}{c} \right)$$

$$\left(\tanh(a) = \frac{v}{c}\right)$$

Relatividade restrita

Dinâmica

$$\begin{bmatrix} E' \\ p'c' \end{bmatrix} = \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} E \\ pc \end{bmatrix}$$



$$\left(\tanh(a) = \frac{v}{c}\right)$$

Relatividade restrita

Dinâmica

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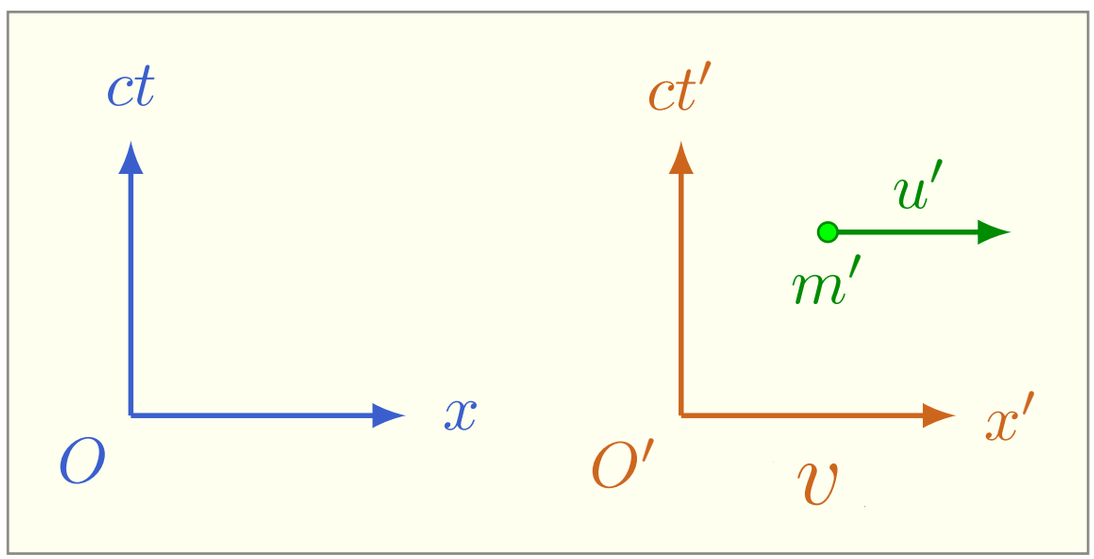
$$u' = 0 \Rightarrow \begin{cases} p = mv \\ p' = 0 \end{cases}$$



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Relatividade restrita

Dinâmica

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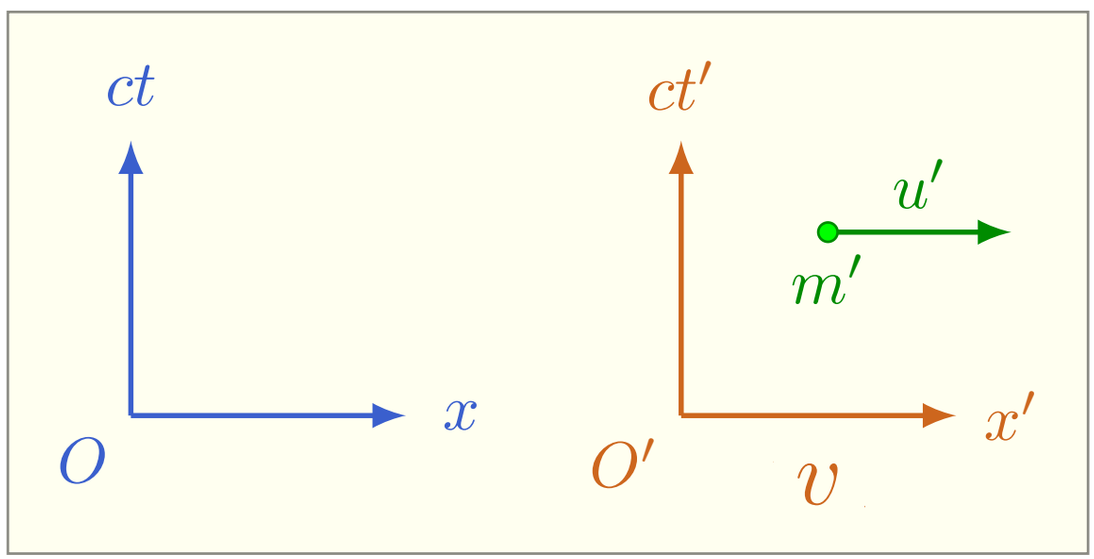

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Relatividade restrita

Dinâmica

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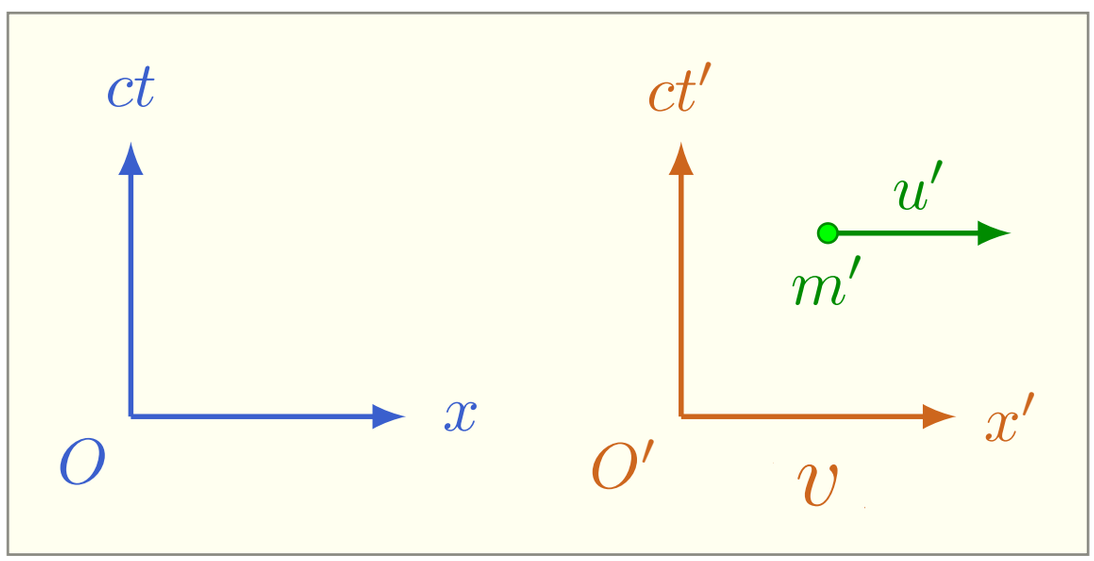


$$u' = 0 \Rightarrow \begin{cases} p = mv \\ p' = 0 \end{cases}$$

$$\begin{bmatrix} E' \\ 0 \end{bmatrix} = \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} E \\ mvc \end{bmatrix}$$

$$\left(\tanh(a) = \frac{v}{c}\right)$$

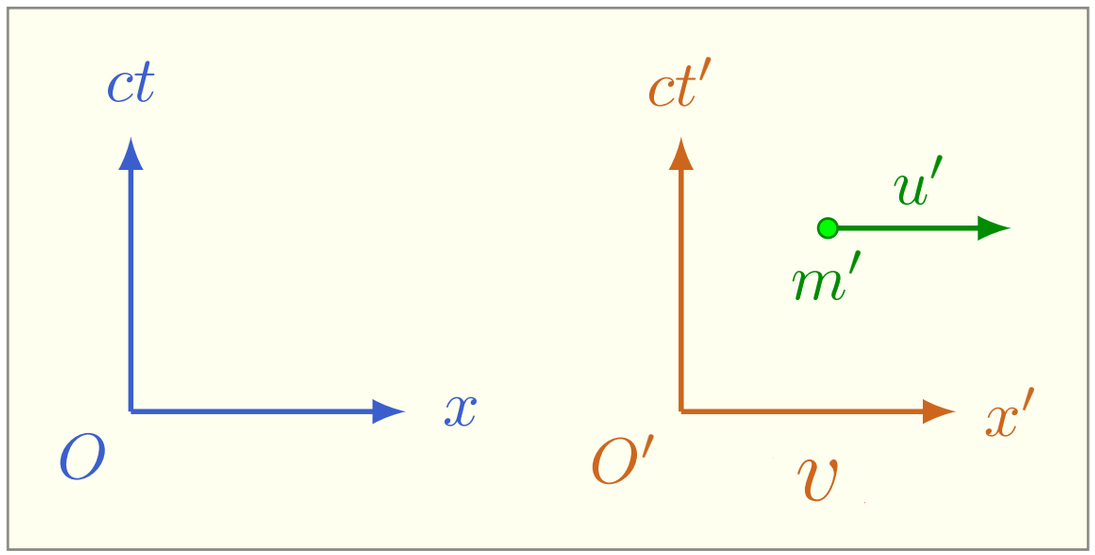
Relatividade restrita

Dinâmica

$$\begin{bmatrix} E' \\ p'c' \end{bmatrix} = \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} E \\ pc \end{bmatrix}$$

$$\begin{bmatrix} E' \\ 0 \end{bmatrix} = \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} E \\ mvc \end{bmatrix}$$

$$0 = -\sinh(a)\,E + mvc\cosh(a)$$



$$\left(\tanh(a) = \frac{v}{c}\right)$$

Relatividade restrita

Dinâmica

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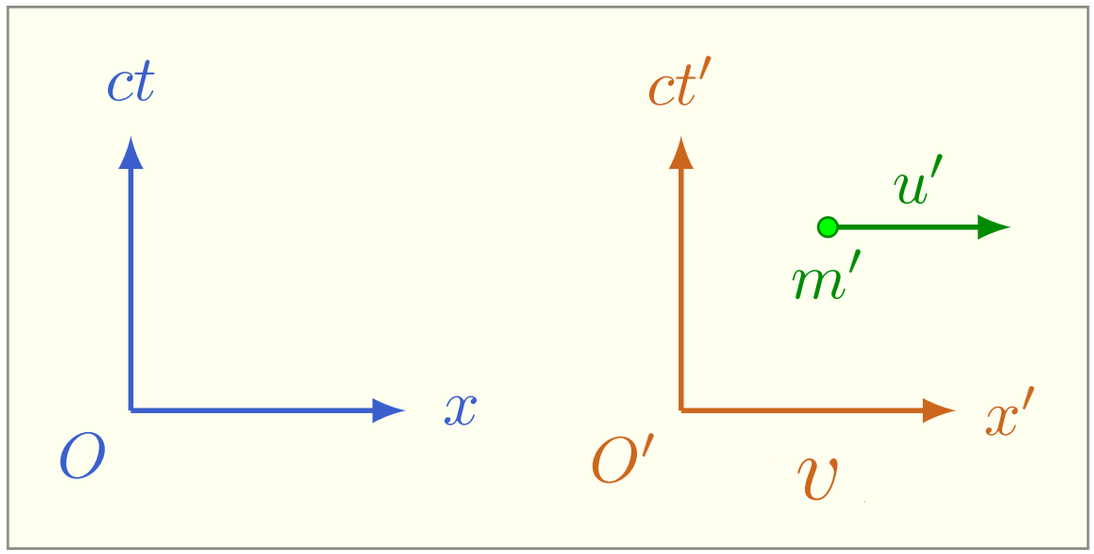

$$\begin{bmatrix} E' \\ 0 \end{bmatrix} = \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} E \\ mvc \end{bmatrix}$$

$$0 = -\sinh(a)\,E + mvc\cosh(a) \quad \Rightarrow \tanh(a)\,E = mvc$$

$$\left(\tanh(a) = \frac{v}{c}\right)$$

Relatividade restrita

Dinâmica

$$\begin{bmatrix} E' \\ p'c' \end{bmatrix} = \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} E \\ pc \end{bmatrix}$$

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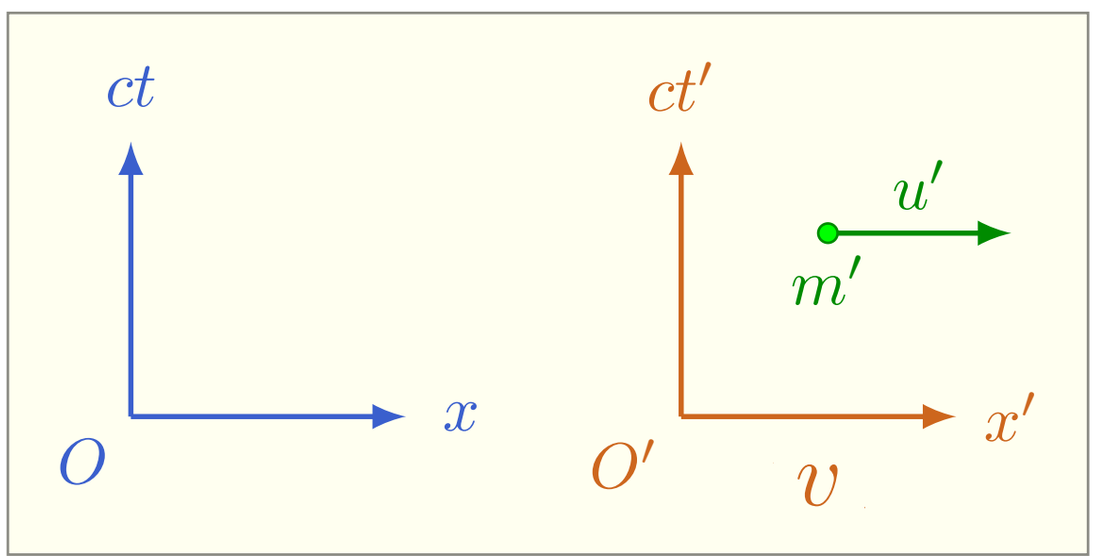

$$0 = -\sinh(a)\,E + mvc\cosh(a) \quad \Rightarrow \tanh(a)\,E = mvc \quad \Rightarrow E = mc^2$$

$$\left(\tanh(a) = \frac{v}{c}\right)$$

Relatividade restrita

Dinâmica

$$\begin{bmatrix} E' \\ p'c' \end{bmatrix} = \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} E \\ pc \end{bmatrix}$$

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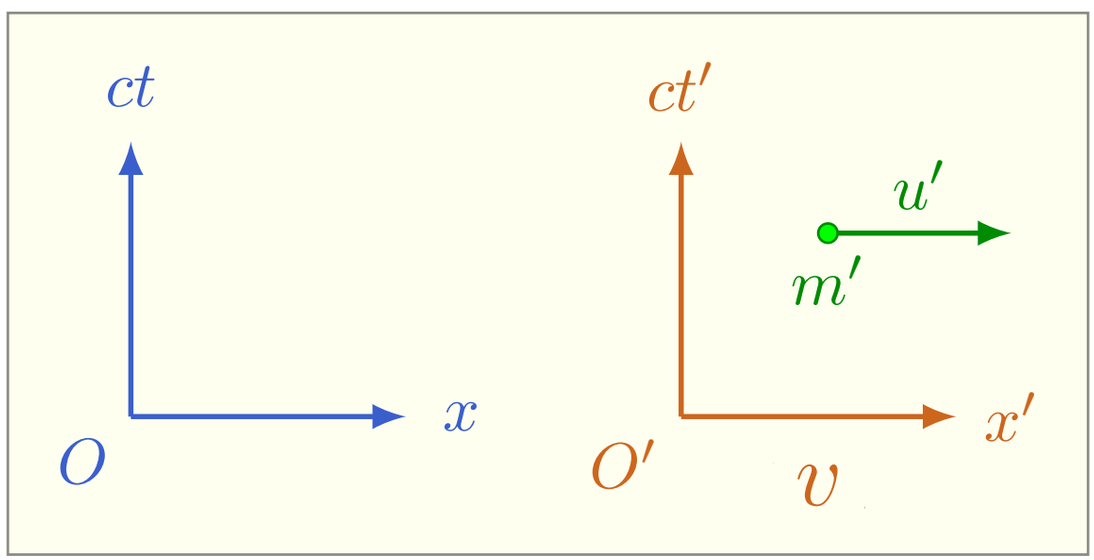


$$0 = -\sinh(a)\,E + mvc\cosh(a) \quad \Rightarrow \tanh(a)\,E = mvc$$

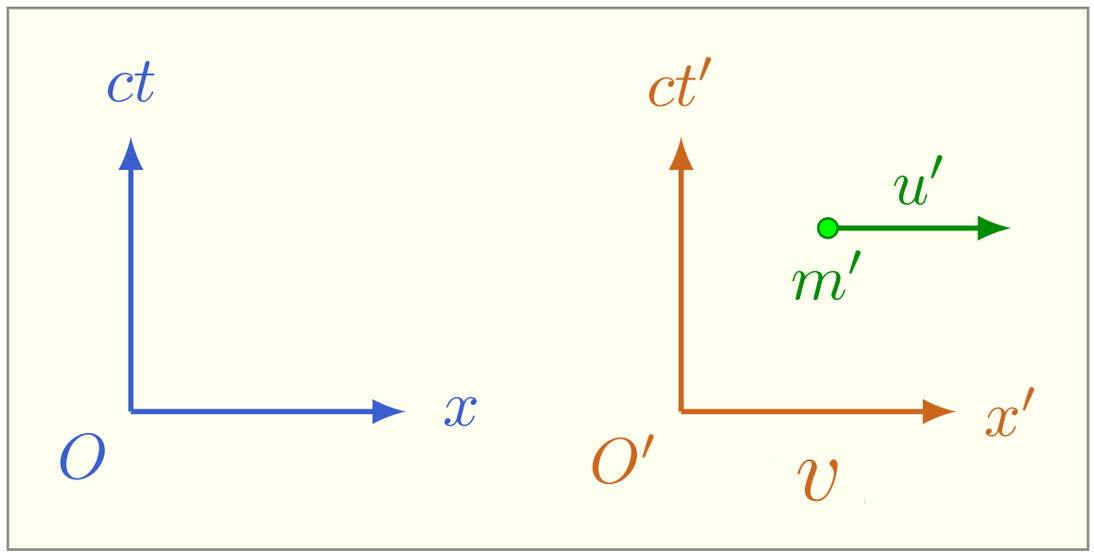
$$E = mc^2$$

$$\left(\tanh(a) = \frac{v}{c}\right)$$

Relatividade restrita

Dinâmica

$$\begin{bmatrix} E' \\ p'c' \end{bmatrix} = \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} E \\ pc \end{bmatrix}$$



$$\begin{bmatrix} E' \\ 0 \end{bmatrix} = \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} E \\ mvc \end{bmatrix}$$

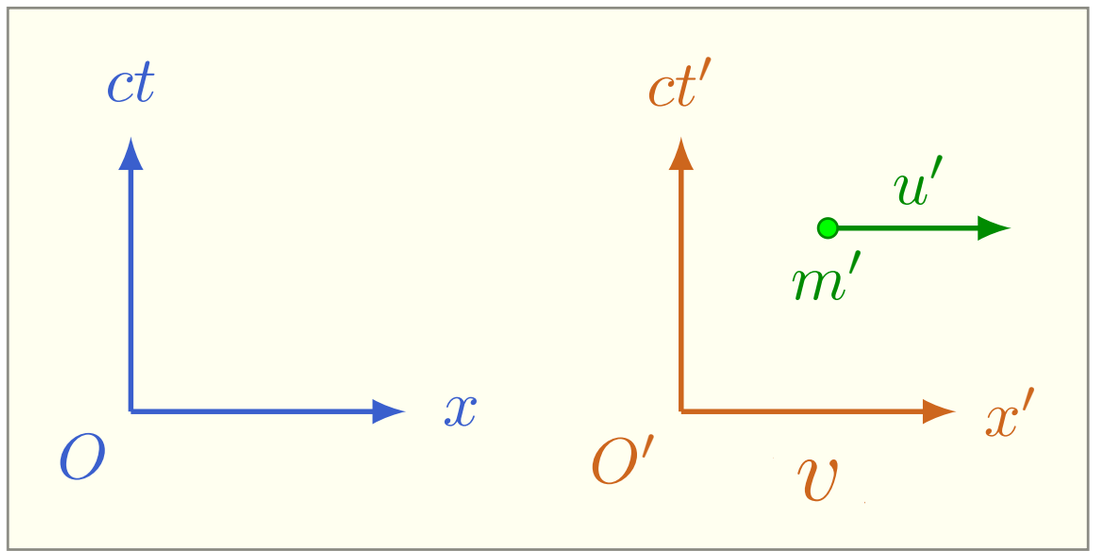
$$E' = \cosh(a)\, E - \sinh(a)\, mvc$$

$$\left(\tanh(a) = \frac{v}{c}\right)$$

Relatividade restrita

Dinâmica

$$\begin{bmatrix} E' \\ p'c' \end{bmatrix} = \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} E \\ pc \end{bmatrix}$$



$$\begin{bmatrix} E' \\ 0 \end{bmatrix} = \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} E \\ mvc \end{bmatrix}$$

$$E' = \cosh(a)\,E - \sinh(a)\,mvc \qquad \Rightarrow E' = \cosh(a)\left(E - \tanh(a)\,mvc\right)$$

$$\tanh(a) = \frac{v}{c}$$

$$E' = \cosh(a)\left(E - \tanh(a)\, mvc\right)$$

$$\tanh(a) = \frac{v}{c}$$

$$E' = \cosh(a)\left(E - \tanh(a)\, mvc\right)$$

$$m'c^2 = \cosh(a)\left(mc^2 - mv^2\right)$$

$$\tanh(a) = \frac{v}{c}$$

$$E' = \cosh(a)\left(E - \tanh(a)\, mvc\right)$$

$$m'c^2 = \cosh(a)\left(mc^2 - mv^2\right)$$

$$m' = \cosh(a)\, m\left(1 - \frac{v^2}{c^2}\right)$$

$$\tanh(a) = \frac{v}{c}$$

$$E' = \cosh(a)\left(E - \tanh(a)\, mvc\right)$$

$$m'c^2 = \cosh(a)\left(mc^2 - mv^2\right)$$

$$m' = \cosh(a)\, m\left(1 - \frac{v^2}{c^2}\right)$$

$$\cosh(a) = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$m' = m\sqrt{1 - \frac{v^2}{c^2}}$$

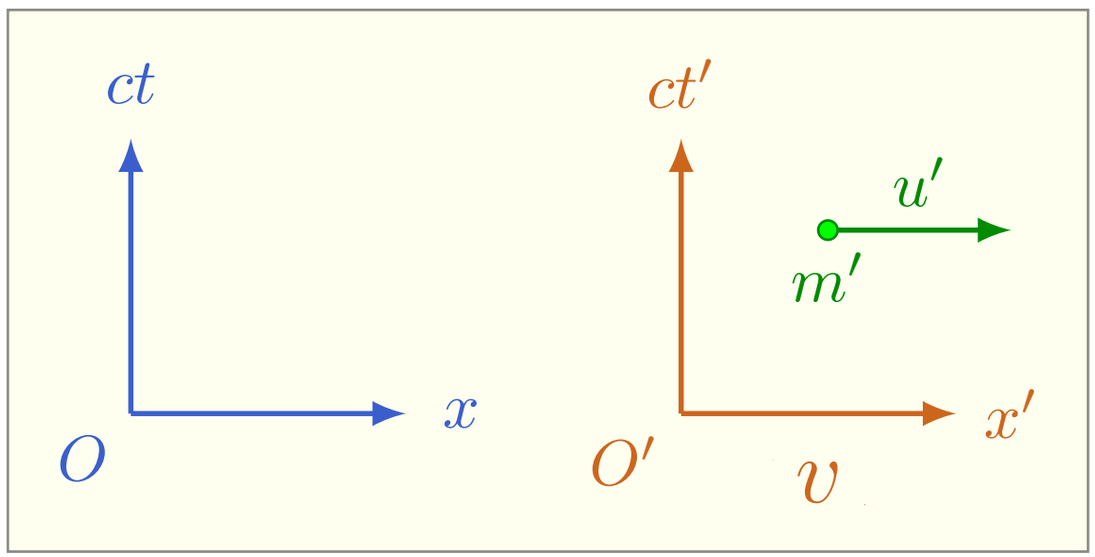
$$\left(\tanh(a) = \frac{v}{c}\right)$$

Relatividade restrita

Dinâmica

$$\begin{bmatrix} E' \\ p'c' \end{bmatrix} = \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} E \\ pc \end{bmatrix}$$

$$\begin{bmatrix} E' \\ 0 \end{bmatrix} = \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} E \\ mvc \end{bmatrix}$$



$$E' = \cosh(a)\,E - \sinh(a)\,mvc$$

$$m = \frac{m'}{\sqrt{1-\frac{v^2}{c^2}}}$$

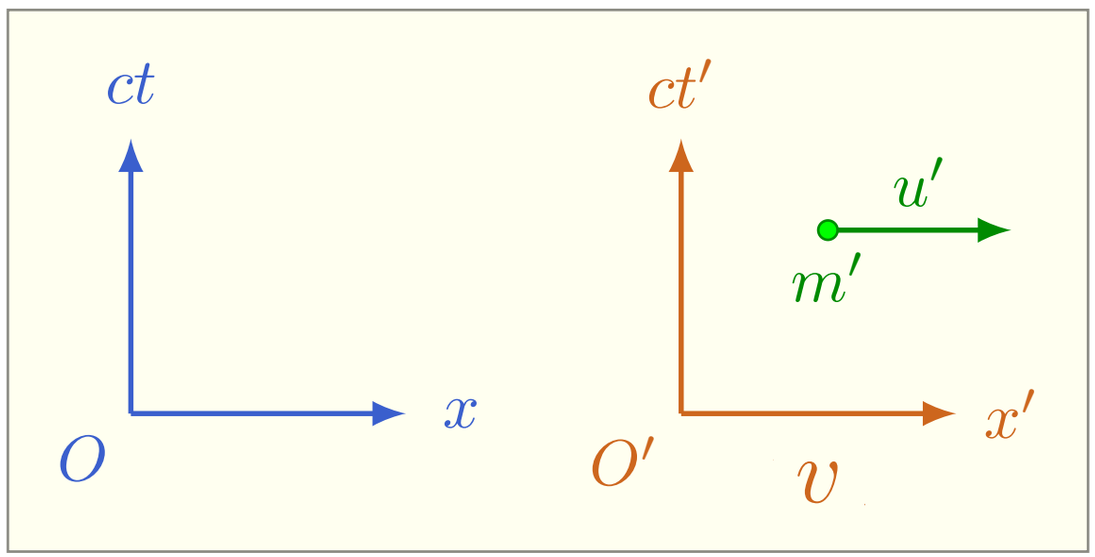
$$\left(\tanh(a) = \frac{v}{c}\right)$$

Relatividade restrita

Dinâmica

$$\begin{bmatrix} E' \\ p'c' \end{bmatrix} = \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} E \\ pc \end{bmatrix}$$

$$\begin{bmatrix} E' \\ 0 \end{bmatrix} = \begin{bmatrix} \cosh(a) & -\sinh(a) \\ -\sinh(a) & \cosh(a) \end{bmatrix} \begin{bmatrix} E \\ mvc \end{bmatrix}$$



$$E' = \cosh(a)\,E - \sinh(a)\,mvc$$

massa em repouso

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$