Statement of the Problem

The major goal of this mini-project is to formulate a discrete model of a distributed parameter system, as well as to compare the dynamic responses of both, the discrete and distributed parameter model. You can use any software available in developing your solutions (Matlab, Octave, Sig-Lab, etc)

The system under investigation is shown in Fig. 1 and consists of a shaft that is clamped at one end and presents different boundary conditions at the other end. Figure 1a shows the shaft with a free end, while Figs. 1b and 1c show the right end of the shaft connected to a lumped mass M_c and a linear translational spring k_c, respectively. For all three cases shown you are asked to formulate discrete models and Fig. 1d presents a suggestive way of conducting the discretization procedure. You will have to chose an appropriate number of degrees of freedom N for your discrete models by calculating the discrete parameters $k_i, m_i, i = 1...N$. Assuming that you will solve the problem with the aid of a computer language, the idea is to start with a minimum value for N and increase it such that the discrete model's dynamic behavior compares with the solution of the corresponding distributed parameter model. Each degree of freedom specified for the discrete model must be associated to an absolute displacement $u_i(t)$ that can be later compared with the longitudinal displacement $u(x,t)$ from the solution of the partial differential equation of motion for the system.

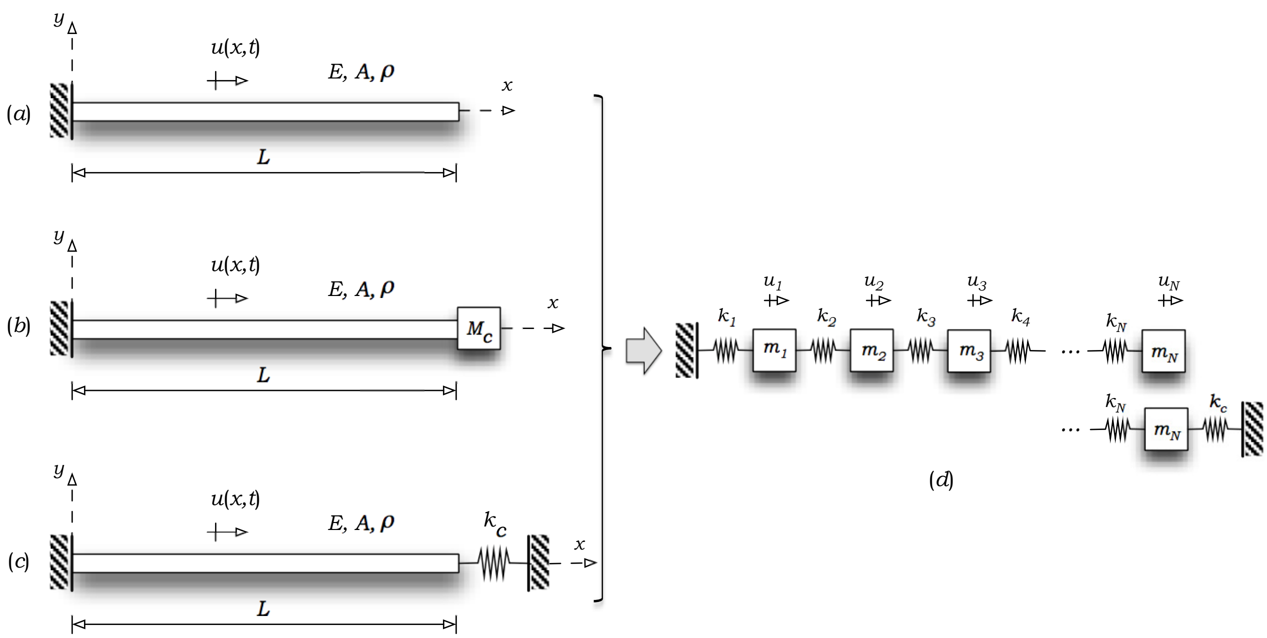

Figura 1: Distributed parameter and discrete models

A major task expected from you is to obtain a representative discrete model for each situation shown in Fig. 1. There are different ways to define the representativity of a discrete model. In the present case, you can, for example, take the first M undamped natural frequencies of the discrete model of N degrees of freedom (with $M < N$) and compare them to the corresponding values of the distributed parameter model. The following steps are suggeste in the process solution:

- Formulate a distributed parameter model for each one of the systems shown in Fig. 1. Obtain for each model the undamped natural frequencies and eigenfunctions.
- Formulate a discrete model for each case, choosing a number of degrees of freedom N. Do not worry in answering the fundamental question "what do I take for N ?". Just pick a number e assemble the solution for the undamped eigenproblem, and calculate the corresponding eigenproperties.

- Establish a comparison between your discrete model solution and the corresponding solution for the distributed model. Try to optimize your model such that the discrete and continuous solutions converge. This is clearly an interative process, but it is a very good way to get a good feel for modeling more complicated structures.

- Once you have defined the final order for your discrete models (N) determine their free undamped and damped response, comparing once again with the solution for the distributed parameter model.

- You are also asked to calculate the frequency response functions FRF for the continuous and discrete models considering a harmonic force $p(t) = P_0 e^{i\omega t}$, where P_0 is a unit amplitude and ω is the varying excitation frequency. Plot the resulting FRF for $H_{ij}(\omega)$ where i and j are coincident with the right end of the shaft.

- Put some additional effort to discuss different ways of defining the discrete model and how they influence the final results.

Feel free to adopt numerical values for the geometric and material properties parameters. Turn in your solutions and routines in a .pdf file through the moodle page of the course.