

$$\underline{34} \quad \int_0^1 f(x) dx \approx \underbrace{a f\left(\frac{1}{4}\right) + b f\left(\frac{1}{2}\right) + c f(1)}_Q$$

EXATA P/POL DE GRAU ≤ 2

$$Q = \int_0^1 P_2(x) dx$$

ONDE P_2 É O POLINÔMIO

INTERPOLADOR DA TABELA

x	$\frac{1}{4}$	$\frac{1}{2}$	1
$f(x)$	$f\left(\frac{1}{4}\right)$	$f\left(\frac{1}{2}\right)$	$f(1)$

$$P_2(x) = f\left(\frac{1}{4}\right)L_0(x) + f\left(\frac{1}{2}\right)L_1(x) + f(1)L_2(x)$$

$$L_0(x) = \frac{(x - \frac{1}{2})(x - 1)}{(\frac{1}{4} - \frac{1}{2})(\frac{1}{4} - 1)} = \frac{16}{3} \left(x - \frac{1}{2}\right)(x - 1)$$

$$L_1(x) = \frac{(x - \frac{1}{4})(x - 1)}{(\frac{1}{2} - \frac{1}{4})(\frac{1}{2} - 1)} = -8 \left(x - \frac{1}{4}\right)(x - 1)$$

$$L_2(x) = \frac{(x - \frac{1}{4})(x - \frac{1}{2})}{(1 - \frac{1}{4})(1 - \frac{1}{2})} = \frac{8}{3} \left(x - \frac{1}{4}\right)\left(x - \frac{1}{2}\right)$$

$$a = \int_0^1 L_0(x) dx = \frac{16}{3} \int_0^1 \left(x^2 - \frac{3}{2}x + \frac{1}{2}\right) dx$$

$$= \frac{16}{3} \left(\frac{1}{3} - \frac{3}{4} + \frac{1}{2}\right) = \frac{4}{9} //$$

$$b = \int_0^1 L_1(x) dx = -8 \int_0^1 (x - \frac{1}{4})(x - 1) dx$$

$$= -8 \int_0^1 (x^2 - \frac{5}{4}x + \frac{1}{4}) dx$$

$$= -8 \left(\frac{1}{3} - \frac{5}{8} + \frac{1}{4} \right) = \frac{1}{3} //$$

$$c = \int_0^1 L_2(x) dx = \frac{8}{3} \int_0^1 (x - \frac{1}{4})(x - \frac{1}{2}) dx$$

$$= \frac{8}{3} \int_0^1 (x^2 - \frac{3}{4}x + \frac{1}{8}) dx$$

$$= \frac{8}{3} \left(\frac{1}{3} - \frac{3}{8} + \frac{1}{8} \right) = \frac{2}{9} //$$

$$a = \frac{4}{9}, \quad b = \frac{1}{3}, \quad c = \frac{2}{9}$$

$$\int_0^1 f(x) \approx \frac{4}{9} f\left(\frac{1}{4}\right) + \frac{1}{3} f\left(\frac{1}{2}\right) + \frac{2}{9} f(1)$$

QUANDO $f(x) = \frac{\text{SEN}(x)}{x}$

$$f\left(\frac{1}{4}\right) = 4 \text{SEN}\left(\frac{1}{4}\right) \approx 0.989616$$

$$f\left(\frac{1}{2}\right) = 2 \text{SEN}\left(\frac{1}{2}\right) \approx 0.958851$$

$$f(1) = \text{SEN}(1) \approx 0.841471$$

$$\begin{aligned} \int_0^1 \frac{\text{SEN}(x)}{x} &\approx \frac{4}{9} \times 0.989616 + \frac{1}{3} \times 0.958851 \\ &\quad + \frac{2}{9} \times 0.841471 \\ &\approx 0.946440 \end{aligned}$$