EX 8

$$f(x) = \int_0^x e^{\cos y}\, dy$$

(a) $f(1)$, $f(2)$ E $f(3)$ USANDO n-TRAP COM ERRO $< 10^{-2}$

UMA MANEIRA DE SE GARANTIR ESTE ERRO NOS TRÊS PONTOS É DETERMINAR h QUE GARANTA ERRO $< 10^{-2}$ PARA

$$f(3) = \int_0^3 e^{\cos y}\, dy$$

INTEGRANDO $g(y) = e^{\cos y}$

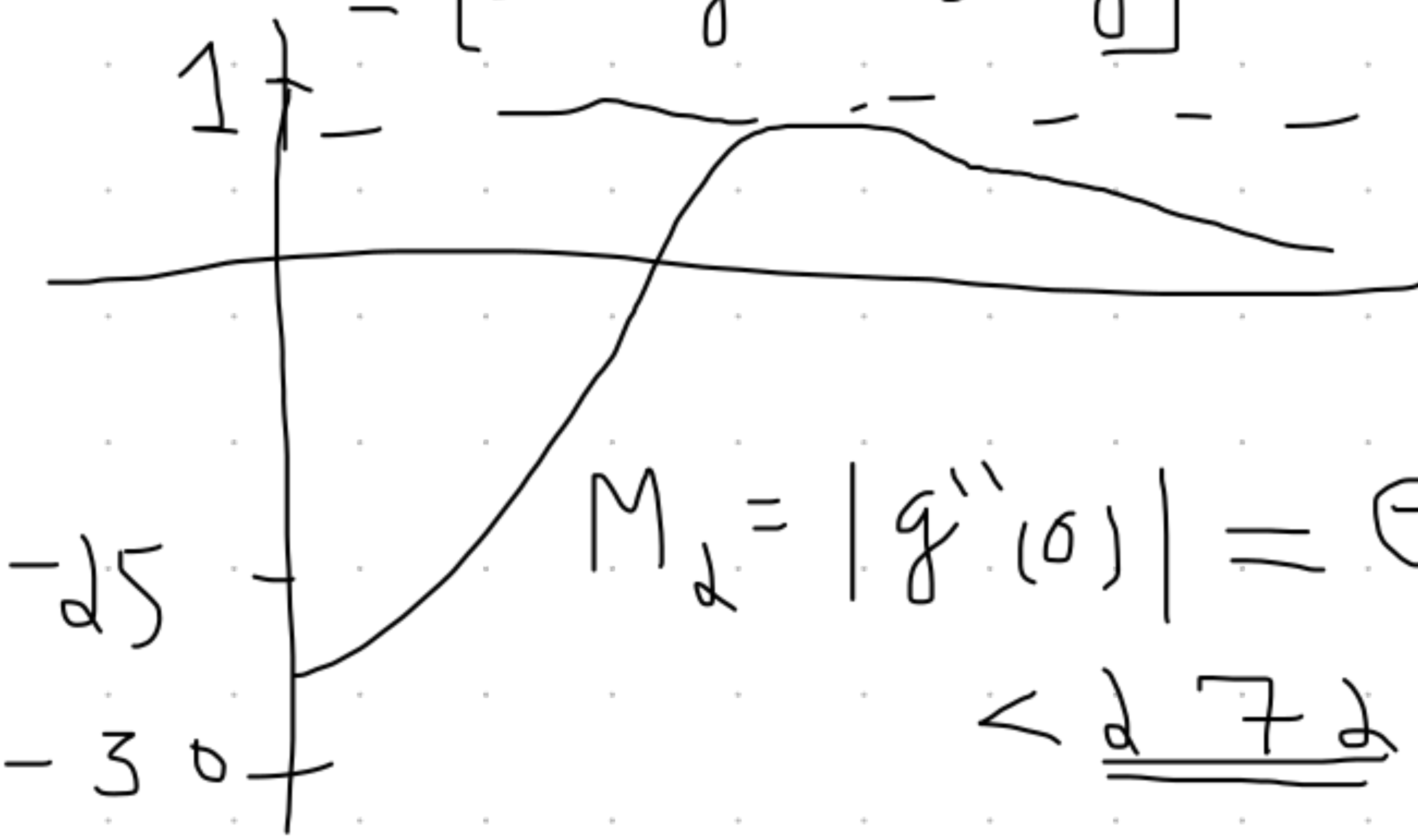
PRECISAMOS ESTIMAR

$$M_2 = \max_{y \in [0,3]} |g''(y)|$$

$$g'(y) = -\text{sen}\, y \; e^{\cos y}$$

$$g''(y) = -\left[\cos y \, e^{\cos y} - \text{sen}^2 y \, e^{\cos y}\right]$$

$$= \left[\text{sen}^2 y - \cos y\right] e^{\cos y}$$



$$M_2 = |g''(0)| = e$$

$$< 2.72$$

$$|E| \leq \frac{(3-0)h^2}{12} M_2 = \frac{2.72}{4} h^2$$

$$= 0.68\, h^2$$

$$0.68\, h^2 < 10^{-2}$$

$$h^2 < \frac{10^{-2}}{0.68} \Rightarrow h < 0.12$$

$$\boxed{h = 0.1}$$

OBS $f(1) = \int_0^1 g(y)\,dy$

$$f(2) = f(1) + \int_1^2 g(y)\,dy$$

$$f(3) = f(2) + \int_2^3 g(y)\,dy$$

$$f(1) = \int_0^1 g(y)\,dy$$

TRAP. COM $h = 0.1$ (1° RGPET.)

$$f(1) \simeq 0.1 \left[\frac{g(0)}{2} + g(0.1) + g(0.2) + \cdots + g(0.9) + \frac{g(1)}{2} \right]$$

$$g(y) = e^{\cos(y)}$$

$$\boxed{f(1) \simeq 2.34037}$$

$$f(2) \simeq f(1) + 0.1 \left[\frac{g(1)}{2} + g(1.1) + \cdots + g(1.9) + \frac{g(2)}{2} \right] \simeq 3.45386$$

$$f(3) \simeq f(2) + 0.1\left[\frac{g(2)}{2} + g(2.1) + g(2.2) + \quad + g(2.9) + \frac{g(3)}{2}\right] \simeq 3.92516$$

$f(1) \simeq 2.34037$

$f(2) \simeq 3.45386$

$f(3) \simeq 3.92516$

(b)

x	0	1	2	3
$f(x)$	0	2.3	3.45	3.9

$x_0 = 0,\ x_1 = 1,\ x_2 = 2,\ x_3 = 3$

$y_0 = 0;\ y_1 = 2.3,\ y_2 = 3.45,\ y_3 = 3.9$

$$L_0(x) = \frac{(x-1)(x-2)(x-3)}{(-1)(-2)(-3)} = -\frac{(x-1)(x-2)(x-3)}{6}$$

$$L_1(x) = \frac{x(x-2)(x-3)}{1(-1)(-2)} = \frac{x(x-2)(x-3)}{2}$$

$$L_2(x) = \frac{x(x-1)(x-3)}{2 \cdot 1 (-1)} = -\frac{x(x-1)(x-3)}{2}$$

$$L_3(x) = \frac{x(x-1)(x-2)}{3 \cdot 2 \cdot 1} = \frac{x(x-1)(x-2)}{6}$$

$$P_3(x) = \sum_{K=0}^{3} f(K) L_K(x)$$

TEOREMA

S_n: n-SIMPSONS

T_n n-TRAPÉZIOS

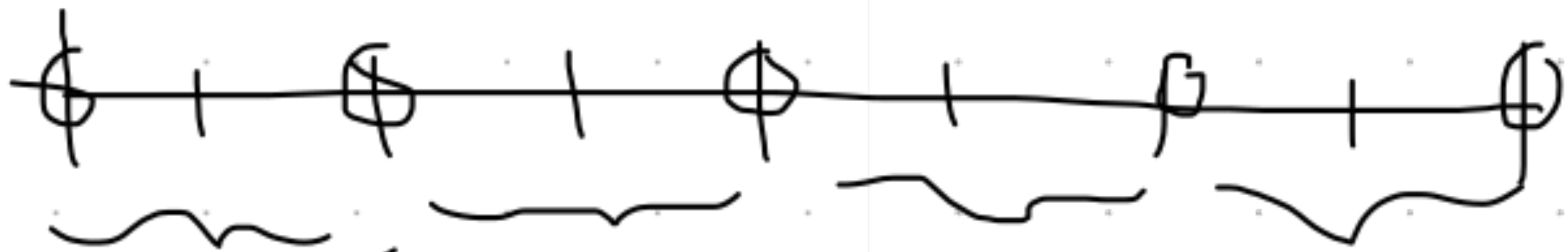
$$S_n = \frac{4T_{2n} - T_n}{3}$$

OBS

$S_n \rightarrow h = \frac{b-a}{2n}$

$T_{2n} \rightarrow h = \frac{b-a}{2n}$

$T_n \rightarrow h = \frac{b-a}{n}$



EXERCÍCIO 10

$$\int_a^b f(x)\,dx$$

SABE SE QUE

$$T_1 = \frac{4}{3}, \quad T_2 = \frac{7}{6}, \quad T_4 = \frac{67}{60}$$

$$S_1 = ?$$

$$S_2 = ?$$

$$S_1 = \frac{4T_2 - T_1}{3} = \frac{4 \times \frac{7}{6} - \frac{4}{3}}{3}$$

$$= \frac{4 \cdot 5}{18} = \frac{20}{18} = \frac{10}{9}$$

$$S_2 = \frac{4T_4 - T_2}{3} = \frac{4 \times \frac{67}{60} - \frac{7}{6}}{3}$$

$$= \frac{\frac{67}{15} - \frac{7}{6}}{3} = \frac{198}{180}$$

$$= \frac{11}{10}$$

$$I = \int_a^b f(x)\,dx$$

$h = \frac{b-a}{n}$ n-TRAPÉZIOS

$\frac{h}{2} = \frac{b-a}{2n}$, 2n-TRAPÉZIOS

COM $h = \frac{b-a}{n}$, QUAL É

A RELAÇÃO ENTRE

T_n E T_{2n} ?

$$h = \frac{b-a}{n}$$

$$T_n = h\left[\frac{f(a)}{2} + \sum_{j=1}^{n-1} f(a+jh) + \frac{f(b)}{2}\right]$$

$$T_{2n} = \frac{h}{2}\left[\frac{f(a)}{2} + \underbrace{\sum_{k=1}^{2n-1} f\left(a+k\frac{h}{2}\right)} + \frac{f(b)}{2}\right]$$

SEPARAR EM ÍNDICES PARES E IMPARES

$$\sum_{k=1}^{2n-1} f\left(a+k\frac{h}{2}\right) = \sum_{j=1}^{n-1} f\left(a+2j\frac{h}{2}\right) + \sum_{j=1}^{n} f\left(a+(2j-1)\frac{h}{2}\right)$$

PARES — IMPARES

$$T_{2n} = \frac{1}{2} T_n + \frac{h}{2} \sum_{j=1}^{n} f\left(a + (2j-1)\frac{h}{2}\right)$$

PONTOS "novos"