Chapter Five

Impedance Matching and Tuning

This chapter marks a turning point, in that we now begin to apply the theory and techniques of previous chapters to practical problems in microwave engineering. We start with the topic of *impedance matching*, which is often an important part of a larger design process for a microwave component or system. The basic idea of impedance matching is illustrated in Figure 5.1, which shows an impedance matching network placed between a load impedance and a transmission line. The matching network is ideally lossless, to avoid unnecessary loss of power, and is usually designed so that the impedance seen looking into the matching network is Z_0. Then reflections will be eliminated on the transmission line to the left of the matching network, although there will usually be multiple reflections between the matching network and the load. This procedure is sometimes referred to as *tuning*. Impedance matching or tuning is important for the following reasons:

- Maximum power is delivered when the load is matched to the line (assuming the generator is matched), and power loss in the feed line is minimized.
- Impedance matching sensitive receiver components (antenna, low-noise amplifier, etc.) may improve the signal-to-noise ratio of the system.
- Impedance matching in a power distribution network (such as an antenna array feed network) may reduce amplitude and phase errors.

As long as the load impedance, Z_L, has a positive real part, a matching network can always be found. Many choices are available, however, and we will discuss the design and performance of several types of practical matching networks. Factors that may be important in the selection of a particular matching network include the following:

- *Complexity*—As with most engineering solutions, the simplest design that satisfies the required specifications is generally preferable. A simpler matching network is usually cheaper, smaller, more reliable, and less lossy than a more complex design.
- *Bandwidth*—Any type of matching network can ideally give a perfect match (zero reflection) at a single frequency. In many applications, however, it is desirable to match a load over a band of frequencies. There are several ways of doing this, with, of course, a corresponding increase in complexity.

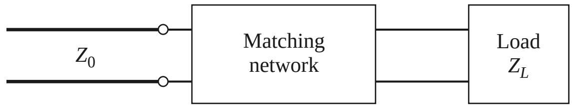


FIGURE 5.1 A lossless network matching an arbitrary load impedance to a transmission line.

- *Implementation*—Depending on the type of transmission line or waveguide being used, one type of matching network may be preferable to another. For example, tuning stubs are much easier to implement in waveguide than are multisection quarter-wave transformers.
- *Adjustability*—In some applications the matching network may require adjustment to match a variable load impedance. Some types of matching networks are more amenable than others in this regard.

5.1 MATCHING WITH LUMPED ELEMENTS (L NETWORKS)

Probably the simplest type of matching network is the *L-section*, which uses two reactive elements to match an arbitrary load impedance to a transmission line. There are two possible configurations for this network, as shown in Figure 5.2. If the normalized load impedance, $z_L = Z_L/Z_0$, is inside the $1 + jx$ circle on the Smith chart, then the circuit of Figure 5.2a should be used. If the normalized load impedance is outside the $1 + jx$ circle on the Smith chart, the circuit of Figure 5.2b should be used. The $1 + jx$ circle is the resistance circle on the impedance Smith chart for which $r = 1$.

In either of the configurations of Figure 5.2, the reactive elements may be either inductors or capacitors, depending on the load impedance. Thus, there are eight distinct possibilities for the matching circuit for various load impedances. If the frequency is low enough and/or the circuit size is small enough, actual lumped-element capacitors and inductors can be used. This may be feasible for frequencies up to about 1 GHz or so, although modern microwave integrated circuits may be small enough such that lumped elements can be used at higher frequencies as well. There is, however, a large range of frequencies and circuit sizes where lumped elements may not be realizable. This is a limitation of the L-section

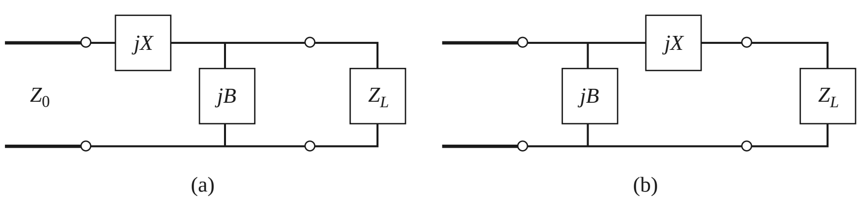


FIGURE 5.2 L-section matching networks. (a) Network for z_L inside the $1 + jx$ circle. (b) Network for z_L outside the $1 + jx$ circle.

matching technique. We will first derive analytic expressions for the matching network elements of the two cases in Figure 5.2, and then illustrate an alternative design procedure using the Smith chart.

Analytic Solutions

Although we will discuss a simple graphical solution using the Smith chart, it is also useful to have simple expressions for the L-section matching network components. These expressions can be used in a computer-aided design program for L-section matching, or when it is necessary to have more accuracy than the Smith chart can provide.

Consider first the circuit of Figure 5.2a, and let $Z_L = R_L + jX_L$. We stated that this circuit would be used when $z_L = Z_L/Z_0$ is inside the $1 + jx$ circle on the Smith chart, which implies that $R_L > Z_0$ for this case. The impedance seen looking into the matching network, followed by the load impedance, must be equal to Z_0 for an impedance-matched condition:

$$Z_0 = jX + \frac{1}{jB + 1/(R_L + jX_L)}. \tag{5.1}$$

Rearranging and separating into real and imaginary parts gives two equations for the two unknowns, X and B:

$$B(XR_L - X_L Z_0) = R_L - Z_0, \tag{5.2a}$$

$$X(1 - BX_L) = BZ_0 R_L - X_L. \tag{5.2b}$$

Solving (5.2a) for X and substituting into (5.2b) gives a quadratic equation for B. The solution is

$$B = \frac{X_L \pm \sqrt{R_L/Z_0}\sqrt{R_L^2 + X_L^2 - Z_0 R_L}}{R_L^2 + X_L^2}. \tag{5.3a}$$

Note that since $R_L > Z_0$, the argument of the second square root is always positive. Then the series reactance can be found as

$$X = \frac{1}{B} + \frac{X_L Z_0}{R_L} - \frac{Z_0}{BR_L}. \tag{5.3b}$$

Equation (5.3a) indicates that two solutions are possible for B and X. Both of these solutions are physically realizable since both positive and negative values of B and X are possible (positive X implies an inductor and negative X implies a capacitor, while positive B implies a capacitor and negative B implies an inductor). One solution, however, may result in significantly smaller values for the reactive components, or may be the preferred solution if the bandwidth of the match is better, or if the SWR on the line between the matching network and the load is smaller.

Next consider the circuit of Figure 5.2b. This circuit is used when z_L is outside the $1 + jx$ circle on the Smith chart, which implies that $R_L < Z_0$. The admittance seen looking into the matching network, followed by the load impedance, must be equal to $1/Z_0$ for an impedance-matched condition:

$$\frac{1}{Z_0} = jB + \frac{1}{R_L + j(X + X_L)}. \tag{5.4}$$

Rearranging and separating into real and imaginary parts gives two equations for the two unknowns, X and B:

$$BZ_0(X + X_L) = Z_0 - R_L, \tag{5.5a}$$

$$(X + X_L) = BZ_0R_L. \tag{5.5b}$$

Solving for X and B gives

$$X = \pm\sqrt{R_L(Z_0 - R_L)} - X_L, \tag{5.6a}$$

$$B = \pm\frac{\sqrt{(Z_0 - R_L)/R_L}}{Z_0}. \tag{5.6b}$$

Because $R_L < Z_0$, the arguments of the square roots are always positive. Again, note that two solutions are possible.

In order to match an arbitrary complex load to a line of characteristic impedance Z_0, the real part of the input impedance to the matching network must be Z_0, while the imaginary part must be zero. This implies that a general matching network must have at least two degrees of freedom; in the L-section matching circuit these two degrees of freedom are provided by the values of the two reactive components.

Smith Chart Solutions

Instead of the above formulas, the Smith chart can be used to quickly and accurately design L-section matching networks. The procedure is best illustrated by an example.

EXAMPLE 5.1 *L*-SECTION IMPEDANCE MATCHING

Design an L-section matching network to match a series RC load with an impedance $Z_L = 200 - j100\ \Omega$ to a $100\ \Omega$ line at a frequency of 500 MHz.

Solution

The normalized load impedance is $z_L = 2 - j1$, which is plotted on the Smith chart of Figure 5.3a. This point is inside the $1 + jx$ circle, so we use the matching circuit of Figure 5.2a. Because the first element from the load is a shunt susceptance, it makes sense to convert to admittance by drawing the SWR circle through the load, and a straight line from the load through the center of the chart, as shown in Figure 5.3a. After we add the shunt susceptance and convert back to impedance, we want to be on the $1 + jx$ circle so that we can add a series reactance to cancel jx and match the load. This means that the shunt susceptance must move us from y_L to the $1 + jx$ circle on the *admittance* Smith chart. Thus, we construct the rotated $1 + jx$ circle as shown in Figure 5.3a (center at $r = 0.333$). (A combined ZY chart may be convenient to use here, if it is not too confusing.) Then we see that adding a susceptance of $jb = j0.3$ will move us along a constant-conductance circle to $y = 0.4 + j0.5$ (this choice is the shortest distance from y_L to the shifted $1 + jx$ circle). Converting back to impedance leaves us at $z = 1 - j1.2$, indicating that a series reactance of $x = j1.2$ will bring us to the center of the chart. For comparison, the formulas (5.3a) and (5.3b) give the solution as $b = 0.29,\ x = 1.22$.

This matching circuit consists of a shunt capacitor and a series inductor, as shown in Figure 5.3b. For a matching frequency of 500 MHz, the capacitor has a value of

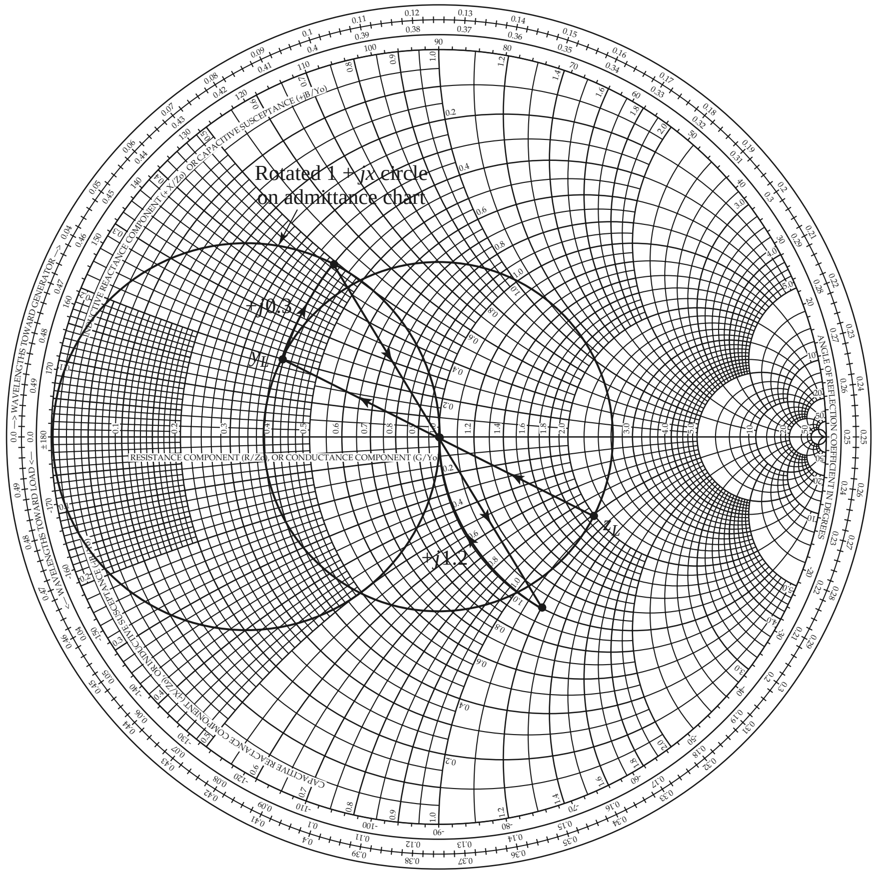
$$C = \frac{b}{2\pi f Z_0} = 0.92 \text{ pF},$$

and the inductor has a value of

$$L = \frac{x Z_0}{2\pi f} = 38.8 \text{ nH}.$$

It is also interesting to look at the second solution to this matching problem. If instead of adding a shunt susceptance of $b = 0.3$, we use a shunt susceptance of $b = -0.7$, we will move to a point on the lower half of the shifted $1 + jx$ circle, to $y = 0.4 - j0.5$. Then converting to impedance and adding a series reactance of $x = -1.2$ leads to a match as well. Formulas (5.3a) and (5.3b) give this solution as $b = -0.69$, $x = -1.22$. This matching circuit is also shown in Figure 5.3b, and is seen to have the positions of the inductor and capacitor reversed from the first matching network. At a frequency of $f = 500$ MHz, the capacitor has a value of

$$C = \frac{-1}{2\pi f x Z_0} = 2.61 \text{ pF},$$



(a)

FIGURE 5.3 Solution to Example 5.1. (a) Smith chart for the L-section matching networks.

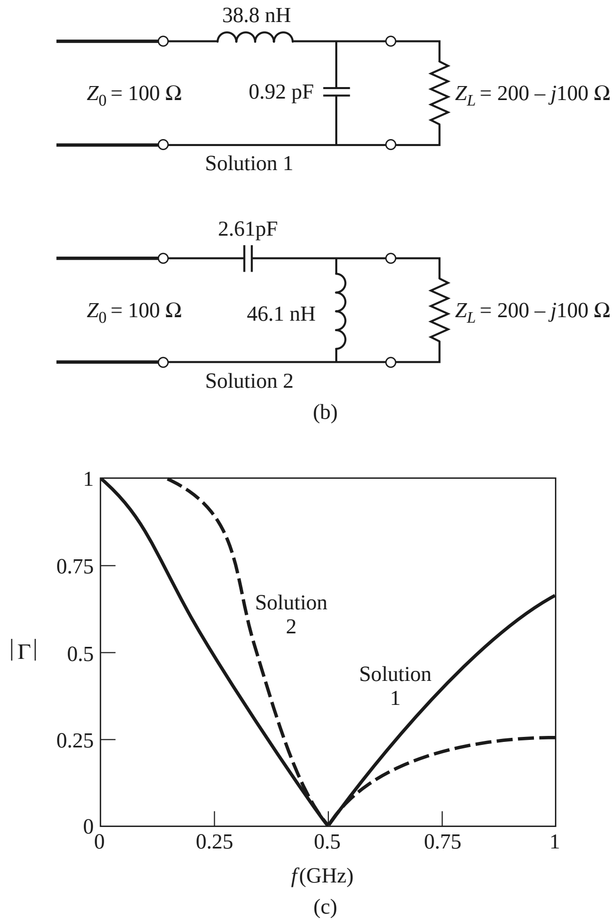
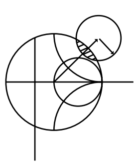


FIGURE 5.3 Continued. (b) The two possible L-section matching circuits. (c) Reflection coefficient magnitudes versus frequency for the matching circuits of (b).

while the inductor has a value of

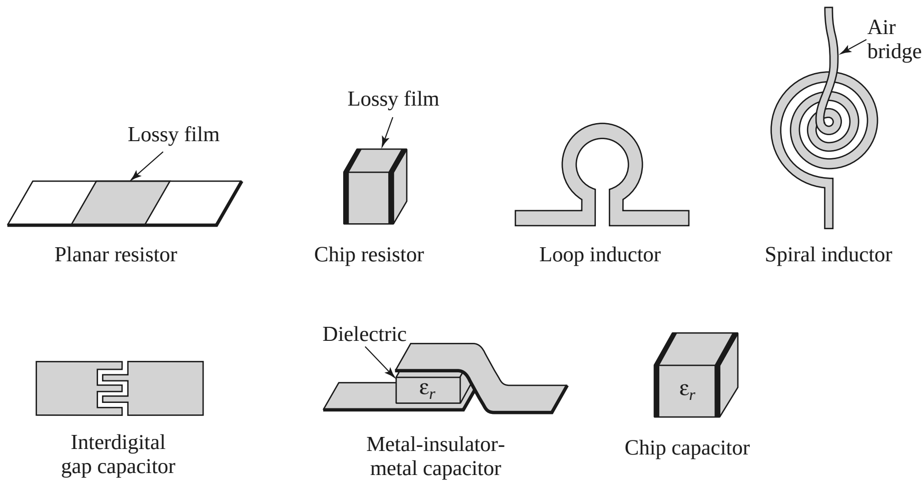
$$L = \frac{-Z_0}{2\pi f b} = 46.1 \text{ nH}.$$

Figure 5.3c shows the reflection coefficient magnitude versus frequency for these two matching networks, assuming that the load impedance of $Z_L = 200 - j100\ \Omega$ at 500 MHz consists of a 200 Ω resistor and a 3.18 pF capacitor in series. There is not a substantial difference in bandwidth for these two solutions. ■



POINT OF INTEREST: Lumped Elements for Microwave Integrated Circuits

Lumped R, L, and C elements can be practically realized at microwave frequencies if the length, ℓ, of the component is very small relative to the operating wavelength. Over a limited range of values, such components can be used in hybrid and monolithic microwave integrated circuits at frequencies up to 60 GHz, or higher, if the condition that $\ell < \lambda/10$ is satisfied. Usually, however, the characteristics of such an element are far from ideal, requiring that undesirable effects such as parasitic capacitance and/or inductance, spurious resonances, fringing fields, loss, and perturbations caused by a ground plane be incorporated in the design via a CAD model (see the Point of Interest concerning CAD).



Resistors are fabricated with thin films of lossy material such as nichrome, tantalum nitride, or doped semiconductor material. In monolithic circuits such films can be deposited or grown, whereas chip resistors made from a lossy film deposited on a ceramic chip can be bonded or soldered in a hybrid circuit. Low resistances are hard to obtain.

Small values of inductance can be realized with a short length or loop of transmission line, and larger values (up to about 10 nH) can be obtained with a spiral inductor, as shown in the following figures. Larger inductance values generally incur more loss and more shunt capacitance; this leads to a resonance that limits the maximum operating frequency.

Capacitors can be fabricated in several ways. A short transmission line stub can provide a shunt capacitance in the range of 0–0.1 pF. A single gap, or an interdigital set of gaps, in a transmission line can provide a series capacitance up to about 0.5 pF. Greater values (up to about 25 pF) can be obtained using a metal-insulator-metal sandwich in either monolithic or chip (hybrid) form.

5.2 SINGLE-STUB TUNING

Another popular matching technique uses a single open-circuited or short-circuited length of transmission line (a *stub*) connected either in parallel or in series with the transmission feed line at a certain distance from the load, as shown in Figure 5.4. Such a *single-stub tuning* circuit is often very convenient because the stub can be fabricated as part of the transmission line media of the circuit, and lumped elements are avoided. Shunt stubs are preferred for microstrip line or stripline, while series stubs are preferred for slotline or coplanar waveguide.

In single-stub tuning the two adjustable parameters are the distance, d, from the load to the stub position, and the value of susceptance or reactance provided by the stub. For the shunt-stub case, the basic idea is to select d so that the admittance, Y, seen looking into the line at distance d from the load is of the form $Y_0 + jB$. Then the stub susceptance is chosen as $-jB$, resulting in a matched condition. For the series-stub case, the distance d is selected so that the impedance, Z, seen looking into the line at a distance d from the load is of the form $Z_0 + jX$. Then the stub reactance is chosen as $-jX$, resulting in a matched condition.

As discussed in Chapter 2, the proper length of an open or shorted transmission line section can provide any desired value of reactance or susceptance. For a given susceptance or reactance, the difference in lengths of an open- or short-circuited stub is $\lambda/4$.

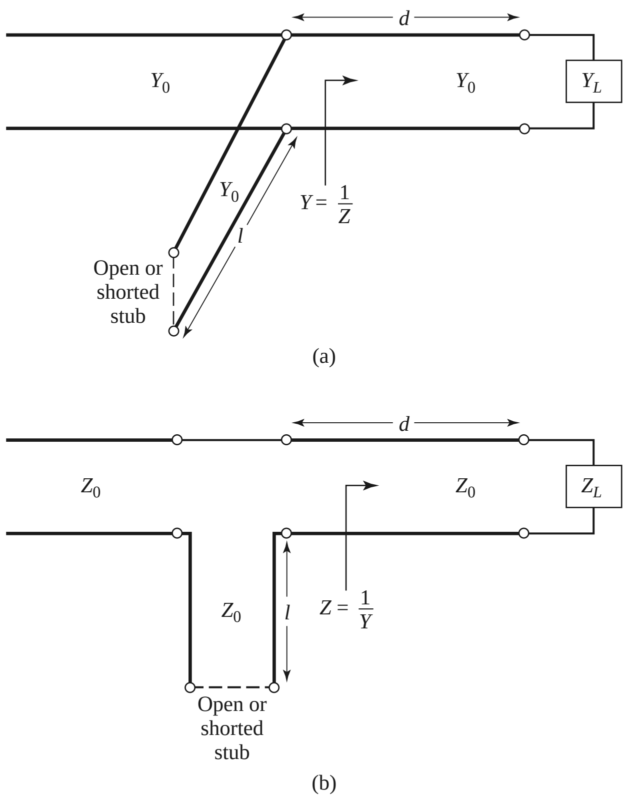


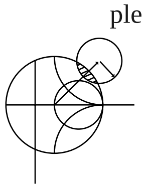
FIGURE 5.4 Single-stub tuning circuits. (a) Shunt stub. (b) Series stub.

For transmission line media such as microstrip or stripline, open-circuited stubs are easier to fabricate since a via hole through the substrate to the ground plane is not needed. For lines like coax or waveguide, however, short-circuited stubs are usually preferred because the cross-sectional area of such an open-circuited line may be large enough (electrically) to radiate, in which case the stub is no longer purely reactive.

We will discuss both Smith chart and analytic solutions for shunt- and series-stub tuning. The Smith chart solutions are fast, intuitive, and usually accurate enough in practice. The analytic expressions are more precise, and are useful for computer analysis.

Shunt Stubs

The single-stub shunt tuning circuit is shown in Figure 5.4a. We will first discuss an example illustrating the Smith chart solution and then derive formulas for d and ℓ.



EXAMPLE 5.2 SINGLE-STUB SHUNT TUNING

For a load impedance $Z_L = 60 - j80\ \Omega$, design two single-stub (short circuit) shunt tuning networks to match this load to a 50 Ω line. Assuming that the load is matched at 2 GHz and that the load consists of a resistor and capacitor in series, plot the reflection coefficient magnitude from 1 to 3 GHz for each solution.

Solution

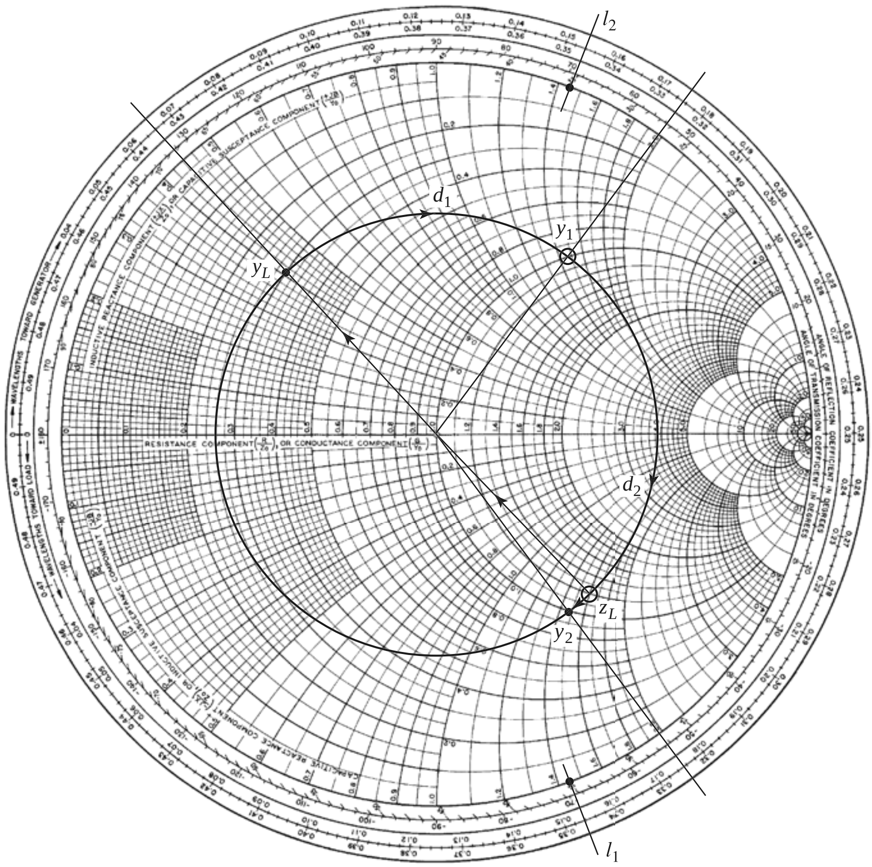
The first step is to plot the normalized load impedance $z_L = 1.2 - j1.6$, construct the appropriate SWR circle, and convert to the load admittance, y_L, as shown on the Smith chart in Figure 5.5a. For the remaining steps we consider the Smith chart as an admittance chart. Notice that the SWR circle intersects the $1 + jb$ circle at two points, denoted as y_1 and y_2 in Figure 5.5a. Thus the distance d from the load to the stub is given by either of these two intersections. Reading the WTG scale, we obtain

$$d_1 = 0.176 - 0.065 = 0.110\lambda,$$
$$d_2 = 0.325 - 0.065 = 0.260\lambda.$$

Actually, there is an infinite number of distances d around the SWR circle that intersect the $1 + jb$ circle. Usually it is desired to keep the matching stub as close as possible to the load to improve the bandwidth of the match and to reduce losses caused by a possibly large standing wave ratio on the line between the stub and the load.

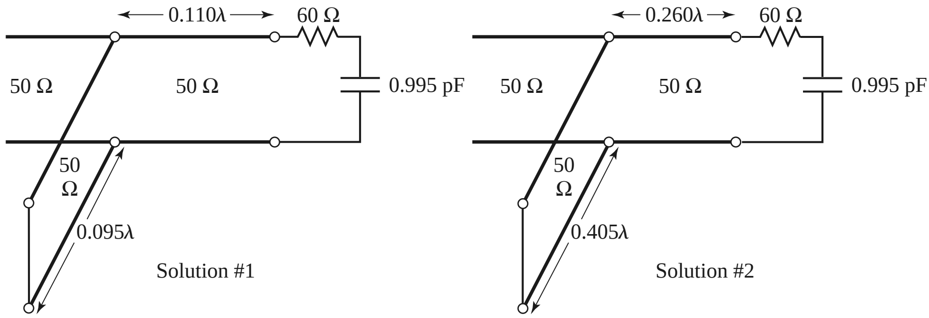
At the two intersection points, the normalized admittances are

$$y_1 = 1.00 + j1.47,$$
$$y_2 = 1.00 - j1.47.$$



(a)

FIGURE 5.5 Solution to Example 5.2. (a) Smith chart for the shunt-stub tuners.



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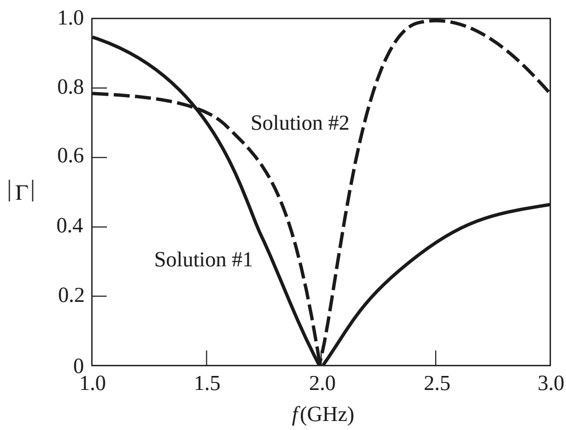


(c)

FIGURE 5.5 Continued. (b) The two shunt-stub tuning solutions. (c) Reflection coefficient magnitudes versus frequency for the tuning circuits of (b).

Thus, the first tuning solution requires a stub with a susceptance of $-j1.47$. The length of a short-circuited stub that gives this susceptance can be found on the Smith chart by starting at $y = \infty$ (the short circuit) and moving along the outer edge of the chart ($g = 0$) toward the generator to the $-j1.47$ point. The stub length is then

$$\ell_1 = 0.095\lambda.$$

Similarly, the required short-circuit stub length for the second solution is

$$\ell_2 = 0.405\lambda.$$

This completes the two tuner designs.

To analyze the frequency dependence of these two designs, we need to know the load impedance as a function of frequency. The series-*RC* load impedance is $Z_L = 60 - j80\ \Omega$ at 2 GHz, so $R = 60\ \Omega$ and $C = 0.995$ pF. The two tuning circuits are shown in Figure 5.5b. Figure 5.5c shows the calculated reflection coefficient magnitudes for these two solutions. Observe that solution 1 has a significantly better bandwidth than solution 2; this is because both d and ℓ are shorter for solution 1, which reduces the frequency variation of the match. ■

To derive formulas for d and ℓ, let the load impedance be written as $Z_L = 1/Y_L = R_L + jX_L$. Then the impedance Z down a length d of line from the load is

$$Z = Z_0 \frac{(R_L + jX_L) + jZ_0 t}{Z_0 + j(R_L + jX_L)t}, \tag{5.7}$$

where $t = \tan \beta d$. The admittance at this point is

$$Y = G + jB = \frac{1}{Z},$$

where

$$G = \frac{R_L(1 + t^2)}{R_L^2 + (X_L + Z_0 t)^2}, \tag{5.8a}$$

$$B = \frac{R_L^2 t - (Z_0 - X_L t)(X_L + Z_0 t)}{Z_0 \left[R_L^2 + (X_L + Z_0 t)^2\right]}. \tag{5.8b}$$

Now d (which implies t) is chosen so that $G = Y_0 = 1/Z_0$. From (5.8a), this results in a quadratic equation for t:

$$Z_0(R_L - Z_0)t^2 - 2X_L Z_0 t + \left(R_L Z_0 - R_L^2 - X_L^2\right) = 0.$$

Solving for t gives

$$t = \frac{X_L \pm \sqrt{R_L\left[(Z_0 - R_L)^2 + X_L^2\right]/Z_0}}{R_L - Z_0} \quad \text{for } R_L \neq Z_0. \tag{5.9}$$

If $R_L = Z_0$, then $t = -X_L/2Z_0$. Thus, the two principal solutions for d are

$$\frac{d}{\lambda} = \begin{cases} \dfrac{1}{2\pi} \tan^{-1} t & \text{for } t \geq 0 \\ \dfrac{1}{2\pi}(\pi + \tan^{-1} t) & \text{for } t < 0. \end{cases} \tag{5.10}$$

To find the required stub lengths, first use t in (5.8b) to find the stub susceptance, $B_s = -B$. Then, for an open-circuited stub,

$$\frac{\ell_o}{\lambda} = \frac{1}{2\pi} \tan^{-1}\left(\frac{B_s}{Y_0}\right) = \frac{-1}{2\pi} \tan^{-1}\left(\frac{B}{Y_0}\right), \tag{5.11a}$$

and for a short-circuited stub,

$$\frac{\ell_s}{\lambda} = \frac{-1}{2\pi} \tan^{-1}\left(\frac{Y_0}{B_s}\right) = \frac{1}{2\pi} \tan^{-1}\left(\frac{Y_0}{B}\right). \tag{5.11b}$$

If the length given by (5.11a) or (5.11b) is negative, $\lambda/2$ can be added to give a positive result.

Series Stubs

The series-stub tuning circuit is shown in Figure 5.4b. We will illustrate the Smith chart solution by an example, and then derive expressions for d and ℓ.

EXAMPLE 5.3 SINGLE-STUB SERIES TUNING

Match a load impedance of $Z_L = 100 + j80$ to a 50 Ω line using a single series open-circuit stub. Assuming that the load is matched at 2 GHz and that the load consists of a resistor and inductor in series, plot the reflection coefficient magnitude from 1 to 3 GHz.

Solution

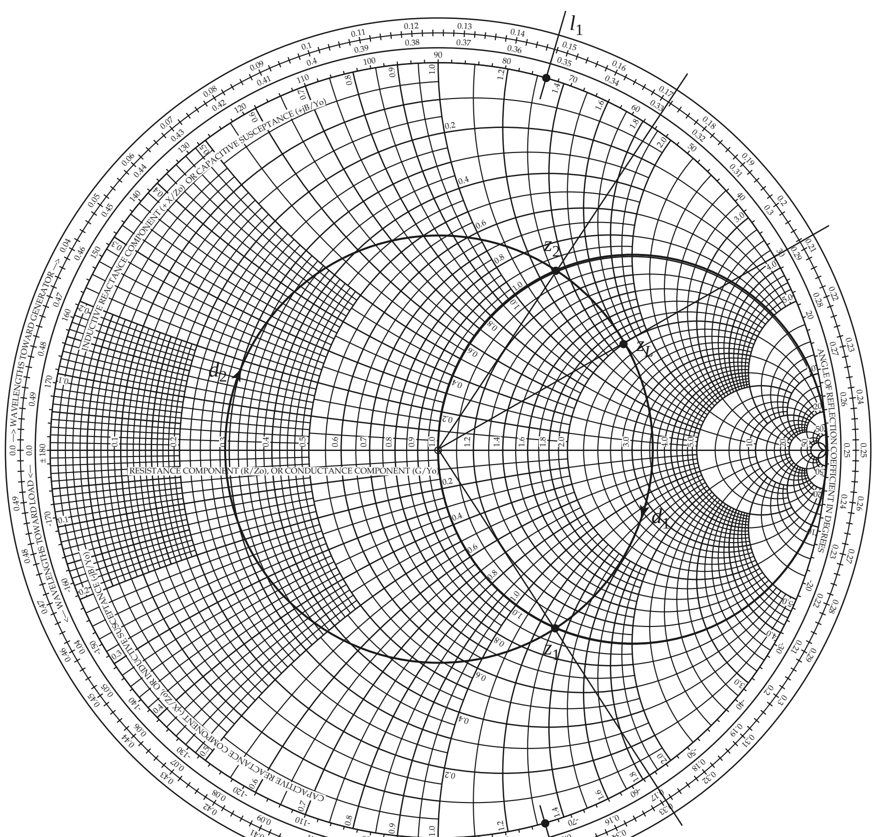
First plot the normalized load impedance, $z_L = 2 + j1.6$, and draw the SWR circle. For the series-stub design the chart is an impedance chart. Note that the SWR circle intersects the $1 + jx$ circle at two points, denoted as z_1 and z_2 in Figure 5.6a. The shortest distance, d_1, from the load to the stub is, from the WTG scale,

$$d_1 = 0.328 - 0.208 = 0.120\lambda,$$

and the second distance is

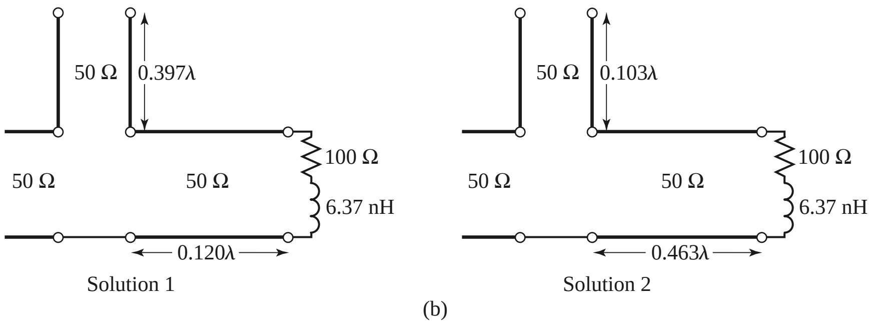
$$d_2 = (0.5 - 0.208) + 0.172 = 0.463\lambda.$$

As in the shunt-stub case, additional rotations around the SWR circle lead to additional solutions, but these are usually not of practical interest.



(a)

FIGURE 5.6 Solution to Example 5.3. (a) Smith chart for the series-stub tuners.



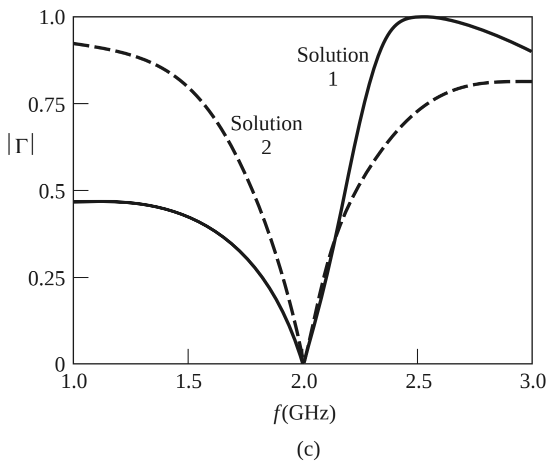


FIGURE 5.6 Continued. (b) The two series-stub tuning solutions. (c) Reflection coefficient magnitudes versus frequency for the tuning circuits of (b).

The normalized impedances at the two intersection points are

$$z_1 = 1 - j1.33,$$
$$z_2 = 1 + j1.33.$$

Thus, the first solution requires a stub with a reactance of $j1.33$. The length of an open-circuited stub that gives this reactance can be found on the Smith chart by starting at $z = \infty$ (open circuit), and moving along the outer edge of the chart ($r = 0$) toward the generator to the $j1.33$ point. This gives a stub length of

$$\ell_1 = 0.397\lambda.$$

Similarly, the required open-circuited stub length for the second solution is

$$\ell_2 = 0.103\lambda.$$

This completes the tuner designs.

If the load is a series resistor and inductor with $Z_L = 100 + j80\ \Omega$ at 2 GHz, then $R = 100\ \Omega$ and $L = 6.37$ nH. The two matching circuits are shown in Figure 5.6b. Figure 5.6c shows the calculated reflection coefficient magnitudes versus frequency for the two solutions. ■

To derive formulas for d and ℓ for the series-stub tuner, let the load admittance be written as $Y_L = 1/Z_L = G_L + jB_L$. Then the admittance Y down a length d of line from the load is

$$Y = Y_0 \frac{(G_L + jB_L) + jtY_0}{Y_0 + jt(G_L + jB_L)}, \tag{5.12}$$

where $t = \tan \beta d$ and $Y_0 = 1/Z_0$. The impedance at this point is

$$Z = R + jX = \frac{1}{Y},$$

where

$$R = \frac{G_L(1 + t^2)}{G_L^2 + (B_L + Y_0 t)^2}, \tag{5.13a}$$

$$X = \frac{G_L^2 t - (Y_0 - tB_L)(B_L + tY_0)}{Y_0\left[G_L^2 + (B_L + Y_0 t)^2\right]}. \tag{5.13b}$$

Now d (which implies t) is chosen so that $R = Z_0 = 1/Y_0$. From (5.13a), this results in a quadratic equation for t:

$$Y_0(G_L - Y_0)t^2 - 2B_L Y_0 t + \left(G_L Y_0 - G_L^2 - B_L^2\right) = 0.$$

Solving for t gives

$$t = \frac{B_L \pm \sqrt{G_L\left[(Y_0 - G_L)^2 + B_L^2\right]/Y_0}}{G_L - Y_0} \quad \text{for } G_L \neq Y_0. \tag{5.14}$$

If $G_L = Y_0$, then $t = -B_L/2Y_0$. Then the two principal solutions for d are

$$d/\lambda = \begin{cases} \dfrac{1}{2\pi}\tan^{-1} t & \text{for } t \geq 0 \\[2ex] \dfrac{1}{2\pi}(\pi + \tan^{-1} t) & \text{for } t < 0. \end{cases} \tag{5.15}$$

The required stub lengths are determined by first using t in (5.13b) to find the reactance X. This reactance is the negative of the necessary stub reactance, X_s. Thus, for a short-circuited stub,

$$\frac{\ell_s}{\lambda} = \frac{1}{2\pi}\tan^{-1}\left(\frac{X_s}{Z_0}\right) = \frac{-1}{2\pi}\tan^{-1}\left(\frac{X}{Z_0}\right), \tag{5.16a}$$

and for an open-circuited stub,

$$\frac{\ell_o}{\lambda} = \frac{-1}{2\pi}\tan^{-1}\left(\frac{Z_0}{X_s}\right) = \frac{1}{2\pi}\tan^{-1}\left(\frac{Z_0}{X}\right). \tag{5.16b}$$

If the length given by (5.16a) or (5.16b) is negative, $\lambda/2$ can be added to give a positive result.

5.3 DOUBLE-STUB TUNING

The single-stub tuner of the previous section is able to match any load impedance (having a positive real part) to a transmission line, but suffers from the disadvantage of requiring a variable length of line between the load and the stub. This may not be a problem for a fixed matching circuit, but would probably pose some difficulty if an adjustable tuner was

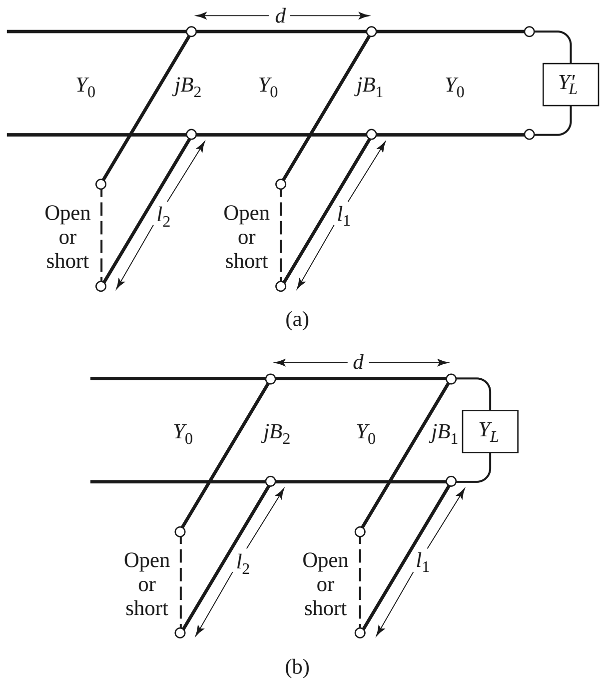


FIGURE 5.7 Double-stub tuning. (a) Original circuit with the load an arbitrary distance from the first stub. (b) Equivalent circuit with the load transformed to the first stub.

desired. In this case, the *double-stub tuner*, which uses two tuning stubs in fixed positions, can be used. Such tuners are often fabricated in coaxial line with adjustable stubs connected in shunt to the main coaxial line. We will see, however, that a double-stub tuner cannot match all load impedances.

The double-stub tuner circuit is shown in Figure 5.7a, where the load may be an arbitrary distance from the first stub. Although this is more representative of a practical situation, the circuit of Figure 5.7b, where the load Y_L' has been transformed back to the position of the first stub, is easier to deal with and does not lose any generality. The shunt stubs shown in Figure 5.7 can be conveniently implemented for some types of transmission lines, while series stubs are more appropriate for other types of lines. In either case, the stubs can be open-circuited or short-circuited.

Smith Chart Solution

The Smith chart of Figure 5.8 illustrates the basic operation of the double-stub tuner. As in the case of the single-stub tuner, two solutions are possible. The susceptance of the first stub, b_1 (or b_1', for the second solution), moves the load admittance to y_1 (or y_1'). These points lie on the rotated $1 + jb$ circle; the amount of rotation is d wavelengths toward the load, where d is the electrical distance between the two stubs. Then transforming y_1 (or y_1') toward the generator through a length d of line leaves us at the point y_2 (or y_2'), which must be on the $1 + jb$ circle. The second stub then adds a susceptance b_2 (or b_2'), which brings us to the center of the chart and completes the match.

Notice from Figure 5.8 that if the load admittance, y_L, were inside the shaded region of the $g_0 + jb$ circle, no value of stub susceptance b_1 could ever bring the load point to intersect the rotated $1 + jb$ circle. This shaded region thus forms a forbidden range of load admittances that cannot be matched with this particular double-stub tuner. A simple way

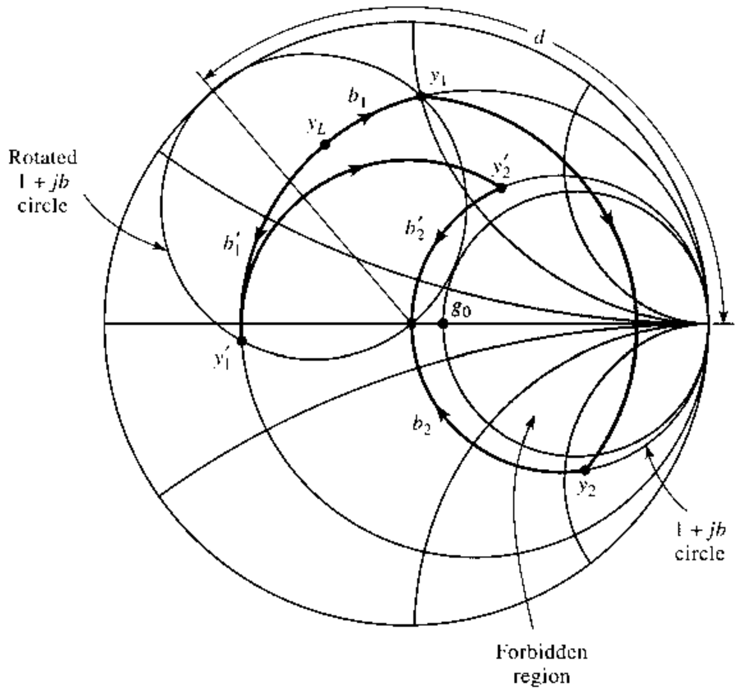


FIGURE 5.8 Smith chart diagram for the operation of a double-stub tuner.

of reducing the forbidden range is to reduce the distance d between the stubs. This has the effect of swinging the rotated $1 + jb$ circle back toward the $y = \infty$ point, but d must be kept large enough for the practical purpose of fabricating the two separate stubs. In addition, stub spacings near 0 or $\lambda/2$ lead to matching networks that are very frequency sensitive. In practice, stub spacings are usually chosen as $\lambda/8$ or $3\lambda/8$. If the length of line between the load and the first stub can be adjusted, then the load admittance y_L can always be moved out of the forbidden region.

EXAMPLE 5.4 DOUBLE-STUB TUNING

Design a double-stub shunt tuner to match a load impedance $Z_L = 60 - j80\ \Omega$ to a 50 Ω line. The stubs are to be open-circuited stubs and are spaced $\lambda/8$ apart. Assuming that this load consists of a series resistor and capacitor and that the match frequency is 2 GHz, plot the reflection coefficient magnitude versus frequency from 1 to 3 GHz.

Solution

The normalized load admittance is $y_L = 0.3 + j0.4$, which is plotted on the Smith chart of Figure 5.9a. Next we construct the rotated $1 + jb$ conductance circle by moving every point on the $g = 1$ circle $\lambda/8$ toward the load. We then find the susceptance of the first stub, which can be one of two possible values:

$$b_1 = 1.314 \quad \text{or} \quad b_1' = -0.114.$$

We now transform through the $\lambda/8$ section of line by rotating along a constant-radius (SWR) circle $\lambda/8$ toward the generator. This brings the two solutions to the following points:

$$y_2 = 1 - j3.38 \quad \text{or} \quad y_2' = 1 + j1.38.$$

Then the susceptance of the second stub should be

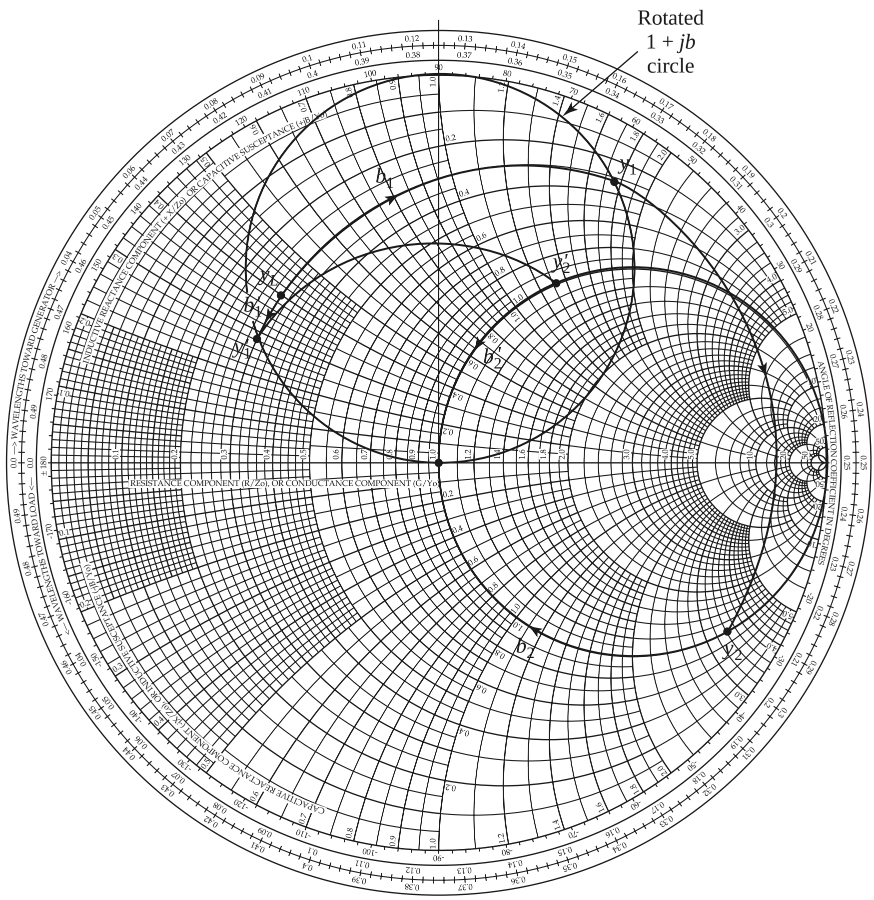
$$b_2 = 3.38 \quad \text{or} \quad b_2' = -1.38.$$

The lengths of the open-circuited stubs are then found as

$$\ell_1 = 0.146\lambda, \ell_2 = 0.204\lambda \quad \text{or} \quad \ell_1' = 0.482\lambda, \ell_2' = 0.350\lambda.$$

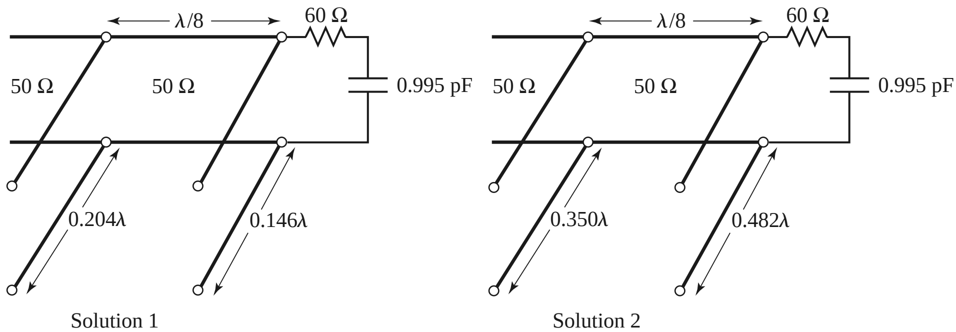
This completes both solutions for the double-stub tuner design.

At $f = 2$ GHz the resistor-capacitor load of $Z_L = 60 - j80\ \Omega$ implies that $R = 60\ \Omega$ and $C = 0.995$ pF. The two tuning circuits are then as shown in Figure 5.9b, and the reflection coefficient magnitudes are plotted versus frequency in Figure 5.9c. Note that the first solution has a much narrower bandwidth than the second (primed) solution due to the fact that both stubs for the first solution are somewhat longer (and closer to $\lambda/2$) than the stubs of the second solution. ■

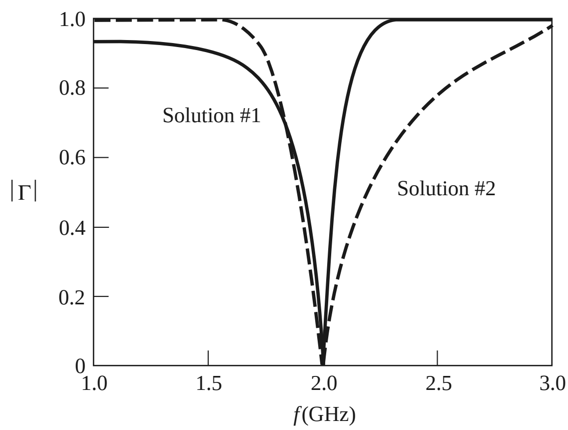


(a)

FIGURE 5.9 Solution to Example 5.4. (a) Smith chart for the double-stub tuners.



(b)



(c)

FIGURE 5.9 Continued. (b) The two double-stub tuning solutions. (c) Reflection coefficient magnitudes versus frequency for the tuning circuits of (b).

Analytic Solution

The admittance just to the left of the first stub in Figure 5.7b is

$$Y_1 = G_L + j(B_L + B_1), \tag{5.17}$$

where $Y_L = G_L + jB_L$ is the load admittance, and B_1 is the susceptance of the first stub. After transforming through a length d of transmission line, we find that the admittance just to the right of the second stub is

$$Y_2 = Y_0 \frac{G_L + j(B_L + B_1 + Y_0 t)}{Y_0 + jt(G_L + jB_L + jB_1)}, \tag{5.18}$$

where $t = \tan \beta d$ and $Y_0 = 1/Z_0$. At this point the real part of Y_2 must equal Y_0, which leads to the equation

$$G_L^2 - G_L Y_0 \frac{1 + t^2}{t^2} + \frac{(Y_0 - B_L t - B_1 t)^2}{t^2} = 0. \tag{5.19}$$

Solving for G_L gives

$$G_L = Y_0 \frac{1 + t^2}{2t^2} \left[1 \pm \sqrt{1 - \frac{4t^2 (Y_0 - B_L t - B_1 t)^2}{Y_0^2 (1 + t^2)^2}} \right]. \tag{5.20}$$

Because G_L is real, the quantity within the square root must be nonnegative, and so

$$0 \leq \frac{4t^2(Y_0 - B_L t - B_1 t)^2}{Y_0^2(1+t^2)^2} \leq 1.$$

This implies that

$$0 \leq G_L \leq Y_0 \frac{1+t^2}{t^2} = \frac{Y_0}{\sin^2 \beta d}, \tag{5.21}$$

which gives the range on G_L that can be matched for a given stub spacing d. After d has been set, the first stub susceptance can be determined from (5.19) as

$$B_1 = -B_L + \frac{Y_0 \pm \sqrt{(1+t^2)G_L Y_0 - G_L^2 t^2}}{t}. \tag{5.22}$$

Then the second stub susceptance can be found from the negative of the imaginary part of (5.18) to be

$$B_2 = \frac{\pm Y_0 \sqrt{Y_0 G_L (1+t^2) - G_L^2 t^2} + G_L Y_0}{G_L t}. \tag{5.23}$$

The upper and lower signs in (5.22) and (5.23) correspond to the same solutions. The open-circuited stub length is found as

$$\frac{\ell_o}{\lambda} = \frac{1}{2\pi} \tan^{-1}\left(\frac{B}{Y_0}\right), \tag{5.24a}$$

and the short-circuited stub length is found as

$$\frac{\ell_s}{\lambda} = \frac{-1}{2\pi} \tan^{-1}\left(\frac{Y_0}{B}\right), \tag{5.24b}$$

where $B = B_1$ or B_2.

5.4 THE QUARTER-WAVE TRANSFORMER

As introduced in Section 2.5, the quarter-wave transformer is a simple and useful circuit for matching a real load impedance to a transmission line. An additional feature of the quarter-wave transformer is that it can be extended to multisection designs in a methodical manner to provide broader bandwidth. If only a narrow band impedance match is required, a single-section transformer may suffice. However, as we will see in the next few sections, multisection quarter-wave transformer designs can be synthesized to yield optimum matching characteristics over a desired frequency band. We will see in Chapter 8 that such networks are closely related to bandpass filters.

One drawback of the quarter-wave transformer is that it can only match a real load impedance. A complex load impedance can always be transformed into a real impedance, however, by using an appropriate length of transmission line between the load and the transformer, or an appropriate series or shunt reactive element. These techniques will usually alter the frequency dependence of the load, and this often has the effect of reducing the bandwidth of the match.

In Section 2.5 we analyzed the operation of a quarter-wave transformer from both an impedance viewpoint and a multiple reflection viewpoint. Here we will concentrate on the bandwidth performance of the transformer as a function of the load mismatch; this

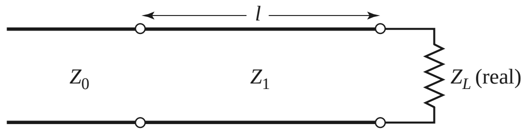


FIGURE 5.10 A single-section quarter-wave matching transformer. $\ell = \lambda_0/4$ at the design frequency f_0.

discussion will also serve as a prelude to the more general case of multisection transformers in the sections to follow.

The single-section quarter-wave matching transformer circuit is shown in Figure 5.10, with the characteristic impedance of the matching section given as

$$Z_1 = \sqrt{Z_0 Z_L}. \tag{5.25}$$

At the design frequency, f_0, the electrical length of the matching section is $\lambda_0/4$, but at other frequencies the length is different, so a perfect match is no longer obtained. We will derive an approximate expression for the resulting impedance mismatch versus frequency.

The input impedance seen looking into the matching section is

$$Z_{\text{in}} = Z_1 \frac{Z_L + jZ_1 t}{Z_1 + jZ_L t}, \tag{5.26}$$

where $t = \tan\beta\ell = \tan\theta$, and $\beta\ell = \theta = \pi/2$ at the design frequency f_0. The resulting reflection coefficient is

$$\Gamma = \frac{Z_{\text{in}} - Z_0}{Z_{\text{in}} + Z_0} = \frac{Z_1(Z_L - Z_0) + jt\left(Z_1^2 - Z_0 Z_L\right)}{Z_1(Z_L + Z_0) + jt\left(Z_1^2 + Z_0 Z_L\right)}. \tag{5.27}$$

Because $Z_1^2 = Z_0 Z_L$, this reduces to

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0 + j2t\sqrt{Z_0 Z_L}}. \tag{5.28}$$

The reflection coefficient magnitude is

$$\begin{aligned}
|\Gamma| &= \frac{|Z_L - Z_0|}{\left[(Z_L + Z_0)^2 + 4t^2 Z_0 Z_L\right]^{1/2}} \\
&= \frac{1}{\left\{(Z_L + Z_0)^2/(Z_L - Z_0)^2 + [4t^2 Z_0 Z_L/(Z_L - Z_0)^2]\right\}^{1/2}} \\
&= \frac{1}{\left\{1 + [4Z_0 Z_L/(Z_L - Z_0)^2] + [4Z_0 Z_L t^2/(Z_L - Z_0)^2]\right\}^{1/2}} \\
&= \frac{1}{\left\{1 + [4Z_0 Z_L/(Z_L - Z_0)^2]\ \sec^2\theta\right\}^{1/2}},
\end{aligned} \tag{5.29}$$

since $1 + t^2 = 1 + \tan^2\theta = \sec^2\theta$.

If we assume that the operating frequency is near the design frequency f_0, then $\ell \simeq \lambda_0/4$ and $\theta \simeq \pi/2$. Then $\sec^2\theta \gg 1$, and (5.29) simplifies to

$$|\Gamma| \simeq \frac{|Z_L - Z_0|}{2\sqrt{Z_0 Z_L}} |\cos\theta| \quad \text{for } \theta \text{ near } \pi/2. \tag{5.30}$$

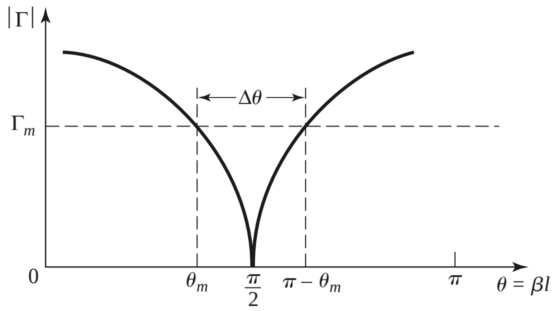


FIGURE 5.11 Approximate behavior of the reflection coefficient magnitude for a single-section quarter-wave transformer operating near its design frequency.

This result gives the approximate mismatch of the quarter-wave transformer near the design frequency, as sketched in Figure 5.11.

If we set a maximum value, Γ_m, for an acceptable reflection coefficient magnitude, then the bandwidth of the matching transformer can be defined as

$$\Delta\theta = 2\left(\frac{\pi}{2} - \theta_m\right), \tag{5.31}$$

since the response of (5.29) is symmetric about $\theta = \pi/2$, and $\Gamma = \Gamma_m$ at $\theta = \theta_m$ and at $\theta = \pi - \theta_m$. Equating Γ_m to the exact expression for the reflection coefficient magnitude in (5.29) allows us to solve for θ_m:

$$\frac{1}{\Gamma_m^2} = 1 + \left(\frac{2\sqrt{Z_0 Z_L}}{Z_L - Z_0}\sec\theta_m\right)^2,$$

or

$$\cos\theta_m = \frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}. \tag{5.32}$$

If we assume TEM lines, then

$$\theta = \beta\ell = \frac{2\pi f}{v_p}\frac{v_p}{4 f_0} = \frac{\pi f}{2 f_0},$$

and so the frequency of the lower band edge at $\theta = \theta_m$ is

$$f_m = \frac{2\theta_m f_0}{\pi},$$

and the fractional bandwidth is, using (5.32),

$$\begin{aligned}\frac{\Delta f}{f_0} &= \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{2 f_m}{f_0} = 2 - \frac{4\theta_m}{\pi}\\ &= 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}\right].\end{aligned} \tag{5.33}$$

Fractional bandwidth is usually expressed as a percentage, $100\Delta f/f_0\%$. Note that the bandwidth of the transformer increases as Z_L becomes closer to Z_0 (a less mismatched load).

The above results are strictly valid only for TEM lines. When non-TEM lines (such as waveguides) are used, the propagation constant is no longer a linear function of frequency, and the wave impedance will be frequency dependent. These factors serve to complicate

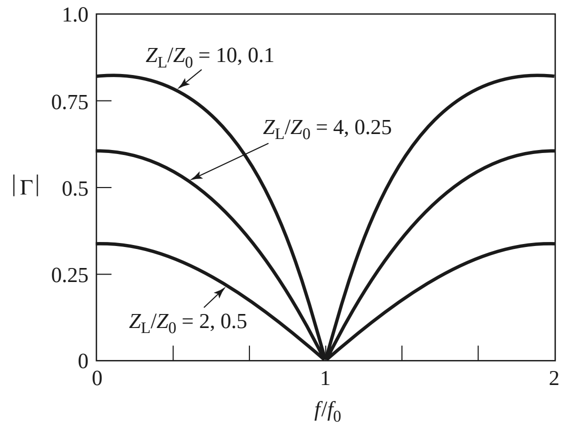
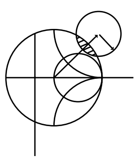


FIGURE 5.12 Reflection coefficient magnitude versus frequency for a single-section quarter-wave matching transformer with various load mismatches.

the general behavior of quarter-wave transformers for non-TEM lines, but in practice the bandwidth of the transformer is often small enough that these complications do not substantially affect the result. Another factor ignored in the above analysis is the effect of reactances associated with discontinuities when there is a step change in the dimensions of a transmission line. This can often be compensated by making a small adjustment in the length of the matching section.

Figure 5.12 shows a plot of the reflection coefficient magnitude versus normalized frequency for various mismatched loads. Note the trend of increased bandwidth for smaller load mismatches.



EXAMPLE 5.5 QUARTER-WAVE TRANSFORMER BANDWIDTH

Design a single-section quarter-wave matching transformer to match a 10 Ω load to a 50 Ω transmission line at $f_0 = 3$ GHz. Determine the percent bandwidth for which the SWR ≤ 1.5.

Solution

From (5.25), the characteristic impedance of the matching section is

$$Z_1 = \sqrt{Z_0 Z_L} = \sqrt{(50)(10)} = 22.36\ \Omega,$$

and the length of the matching section is $\lambda/4$ at 3 GHz (the physical length depends on the dielectric constant of the line). An SWR of 1.5 corresponds to a reflection coefficient magnitude of

$$\Gamma_m = \frac{\text{SWR} - 1}{\text{SWR} + 1} = \frac{1.5 - 1}{1.5 + 1} = 0.2.$$

The fractional bandwidth is computed from (5.33) as

$$\begin{aligned}\frac{\Delta f}{f_0} &= 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}\right] \\ &= 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{0.2}{\sqrt{1-(0.2)^2}}\frac{2\sqrt{(50)(10)}}{|10-50|}\right] \\ &= 0.29, \text{ or } 29\%.\end{aligned}$$

■

5.5 THE THEORY OF SMALL REFLECTIONS

The quarter-wave transformer provides a simple means of matching any real load impedance to any transmission line impedance. For applications requiring more bandwidth than a single quarter-wave section can provide, *multisection transformers* can be used. The design of such transformers is the subject of the next two sections, but prior to that material we need to derive some approximate results for the total reflection coefficient caused by the partial reflections from several small discontinuities. This topic is generally referred to as the *theory of small reflections* [1].

Single-Section Transformer

We will derive an approximate expression for the overall reflection coefficient, Γ, for the single-section matching transformer shown in Figure 5.13. The partial reflection and transmission coefficients are

$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1}, \tag{5.34}$$

$$\Gamma_2 = -\Gamma_1, \tag{5.35}$$

$$\Gamma_3 = \frac{Z_L - Z_2}{Z_L + Z_2}, \tag{5.36}$$

$$T_{21} = 1 + \Gamma_1 = \frac{2Z_2}{Z_1 + Z_2}, \tag{5.37}$$

$$T_{12} = 1 + \Gamma_2 = \frac{2Z_1}{Z_1 + Z_2}. \tag{5.38}$$

We can compute the total reflection, Γ, seen by the feed line using either the impedance method, or the multiple reflection method, as discussed in Section 2.5. For our present

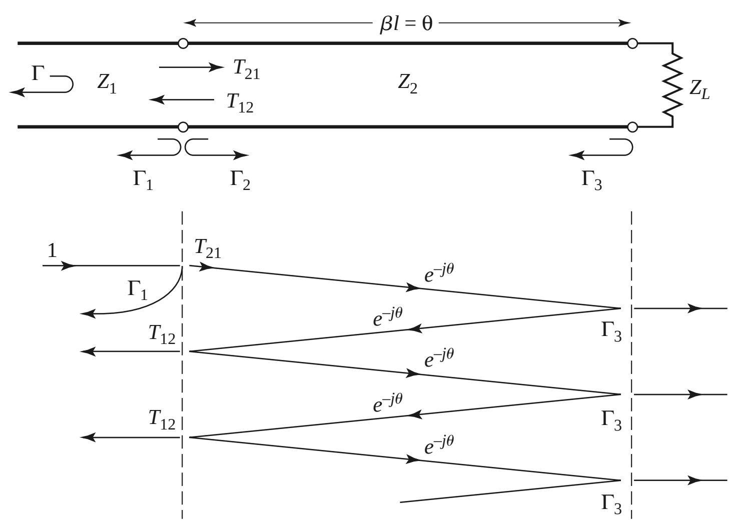


FIGURE 5.13 Partial reflections and transmissions on a single-section matching transformer.