



FTs x Termo

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linguagem

$$\rho \frac{D\varphi}{Dt} = -\operatorname{div} \vec{j}_{\Phi} + \dot{\sigma}_{\nabla\Phi} = \frac{\partial \rho\varphi}{\partial t} + \operatorname{div} \rho \vec{v}\varphi$$

$$\begin{aligned} \Phi = & m; m_i; m\vec{v}; \frac{m}{2}\vec{v}\cdot\vec{v}; mgh; H; \frac{m}{2}\vec{v}'\cdot\vec{v}'; \dot{\sigma}_{K'}; \varepsilon; S; \dots \\ \varphi = & 1; x_i; \vec{v}; \frac{v^2}{2}; gz; c_p T; K'; \dot{\varepsilon}'; \rho\varepsilon; s; \dots \end{aligned}$$

$$\frac{\partial \rho \varphi}{\partial t} + \operatorname{div} \rho \vec{v} \varphi = -\operatorname{div} \vec{j}_{\Phi} + \dot{\sigma}_{\nabla \Phi}$$

$$\dot{\sigma}_{\nabla \Phi}$$

$$\frac{\partial \rho \varphi}{\partial t}$$

$$\operatorname{div} \vec{j}_{\Phi}$$

$$\operatorname{div} \rho \vec{v} \varphi$$

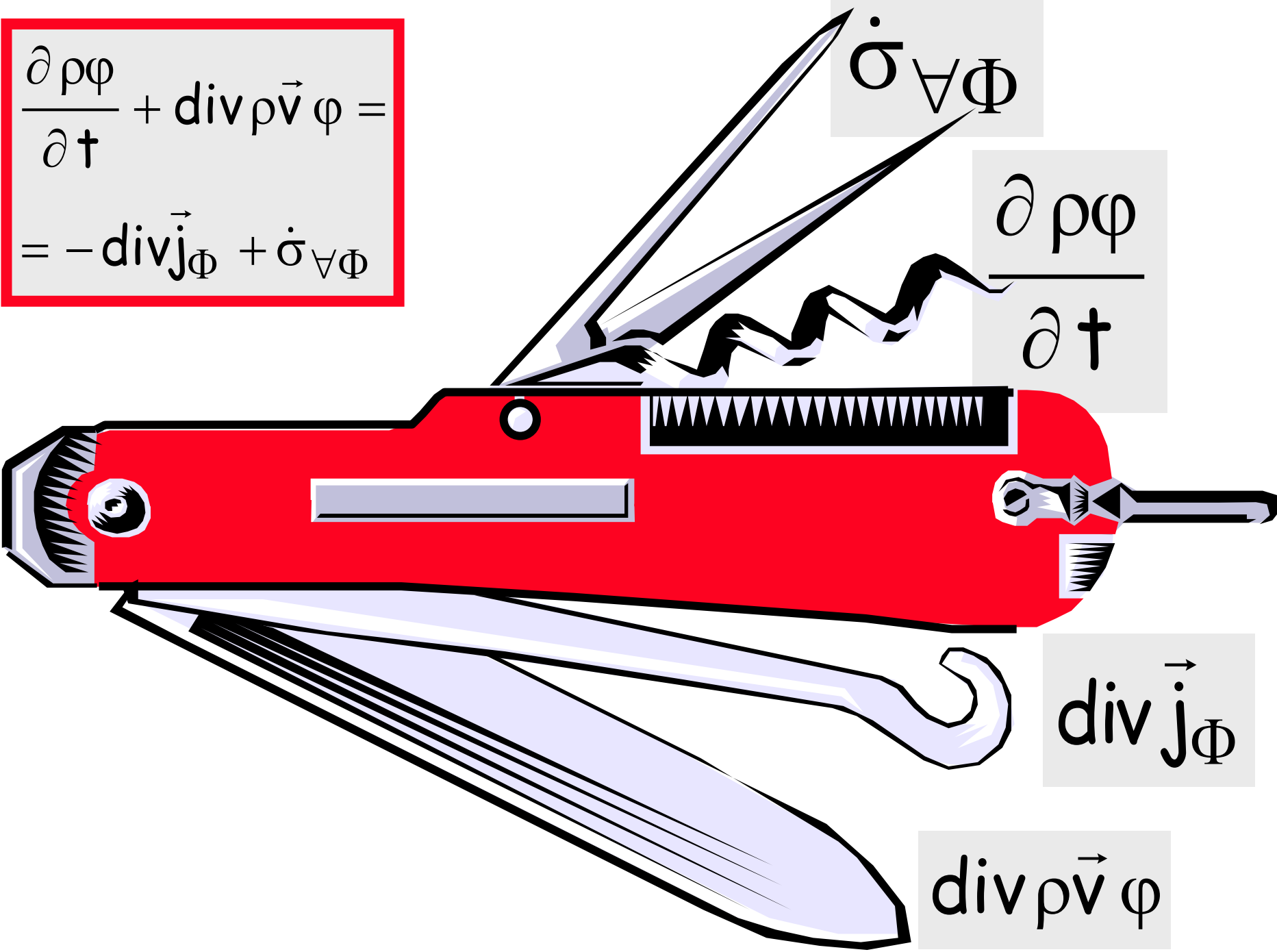


Table 11.4-1 Equations of Change for Pure Fluids in Terms of the Fluxes

Eq.	Special form	In terms of D/Dt		Comments
Cont.	—	$\frac{D\rho}{Dt} = -\rho(\nabla \cdot \mathbf{v})$	Table 3.5-1 (A)	For $\rho = \text{constant}$, simplifies to $(\nabla \cdot \mathbf{v}) = 0$
Motion	General	$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p - [\nabla \cdot \boldsymbol{\tau}] + \rho \mathbf{g}$	Table 3.5-1 (B)	For $\boldsymbol{\tau} = 0$ this becomes Euler's equation
	Approximate	$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p - [\nabla \cdot \boldsymbol{\tau}] + \bar{\rho} \mathbf{g} - \bar{\rho} \mathbf{g} \beta (T - \bar{T})$	11.3-2 (C)	Displays buoyancy term
Energy	In terms of $\hat{K} + \hat{U} + \hat{\Phi}$	$\rho \frac{D(\hat{K} + \hat{U} + \hat{\Phi})}{Dt} = -(\nabla \cdot \mathbf{q}) - (\nabla \cdot p\mathbf{v}) - (\nabla \cdot [\boldsymbol{\tau} \cdot \mathbf{v}])$	— (D)	Exact only for Φ time independent
	In terms of $\hat{K} + \hat{U}$	$\rho \frac{D(\hat{K} + \hat{U})}{Dt} = -(\nabla \cdot \mathbf{q}) - (\nabla \cdot p\mathbf{v}) - (\nabla \cdot [\boldsymbol{\tau} \cdot \mathbf{v}]) + \rho(\mathbf{v} \cdot \mathbf{g})$	— (E)	
	In terms of $\hat{K} = \frac{1}{2}v^2$	$\rho \frac{D\hat{K}}{Dt} = -(\mathbf{v} \cdot \nabla p) - (\mathbf{v} \cdot [\nabla \cdot \boldsymbol{\tau}]) + \rho(\mathbf{v} \cdot \mathbf{g})$	Table 3.5-1 (F)	From equation of motion
	In terms of \hat{U}	$\rho \frac{D\hat{U}}{Dt} = -(\nabla \cdot \mathbf{q}) - p(\nabla \cdot \mathbf{v}) - (\boldsymbol{\tau} : \nabla \mathbf{v})$	11.2-2 (G)	Term containing $(\nabla \cdot \mathbf{v})$ is zero for constant ρ
	In terms of \hat{H}	$\rho \frac{D\hat{H}}{Dt} = -(\nabla \cdot \mathbf{q}) - (\boldsymbol{\tau} : \nabla \mathbf{v}) + \frac{Dp}{Dt}$	11.2-3 (H)	$\hat{H} = \hat{U} + (p/\rho)$
	In terms of	$\rho \hat{C}_v \frac{DT}{Dt} = -(\nabla \cdot \mathbf{q}) - T \left(\frac{\partial p}{\partial T} \right) (\nabla \cdot \mathbf{v}) - (\boldsymbol{\tau} : \nabla \mathbf{v})$		

BIRD table 11.4-1

$$\rho \frac{D\phi}{Dt} = -\text{div } \vec{j}_\phi + \dot{\sigma}_{\nabla \phi}$$

$$\frac{\partial \rho \phi}{\partial t} + \text{div} \rho \vec{v} \phi = - \text{div} \vec{j}_{\phi} + \dot{\sigma}_{\nabla \phi}$$

	Approximate	$\frac{\partial}{\partial t} \rho v = -[\nabla \cdot \rho v v] - \nabla p - [\nabla \cdot \tau] + \bar{\rho} g - \bar{\rho} g \beta (T - \bar{T})$	— (M)	Displays buoyancy term
Energy	In terms of $\hat{K} + \hat{U} + \hat{\Phi}$	$\frac{\partial}{\partial t} \rho(\hat{K} + \hat{U} + \hat{\Phi}) = -(\nabla \cdot \rho(\hat{K} + \hat{H} + \hat{\Phi})v) - (\nabla \cdot q) - (\nabla \cdot [\tau \cdot v])$	11.1-9 (N)	Exact only for Φ time independent
	In terms of $\hat{K} + \hat{\Phi}$	$\frac{\partial}{\partial t} \rho(\hat{K} + \hat{\Phi}) = -(\nabla \cdot \rho(\hat{K} + \hat{\Phi})v) - (v \cdot \nabla p) - (v \cdot [\nabla \cdot \tau])$	3.3-2 (O)	Exact only for Φ time independent From equation of motion
	In terms of $\hat{K} + \hat{U}$	$\frac{\partial}{\partial t} \rho(\hat{K} + \hat{U}) = -(\nabla \cdot \rho(\hat{K} + \hat{H})v) - (\nabla \cdot q) - (\nabla \cdot [\tau \cdot v]) + \rho(v \cdot g)$	11.1-7 (P)	
	In terms of $\hat{K} = \frac{1}{2}v^2$	$\frac{\partial}{\partial t} \rho \hat{K} = -(\nabla \cdot \rho \hat{K} v) - (v \cdot \nabla p) - (v \cdot [\nabla \cdot \tau]) + \rho(v \cdot g)$	3.3-1 (Q)	From equation of motion
	In terms of \hat{U}	$\frac{\partial}{\partial t} \rho \hat{U} = -(\nabla \cdot \rho \hat{U} v) - (\nabla \cdot q) - p(\nabla \cdot v) - (\tau : \nabla v)$	11.2-1 (R)	Term containing $(\nabla \cdot v)$ is zero for constant ρ
	In terms of \hat{H}	$\frac{\partial}{\partial t} \rho \hat{H} = -(\nabla \cdot \rho \hat{H} v) - (\nabla \cdot q) - (\tau : \nabla v) + \frac{Dp}{Dt}$	— (S)	$\hat{H} = \hat{U} + (p/\rho)$
Entropy	—	$\frac{\partial}{\partial t} \rho \hat{S} = -(\nabla \cdot \rho \hat{S} v) - \left(\nabla \cdot \frac{q}{T} \right) - \frac{1}{T^2} (q \cdot \nabla T) - \frac{1}{T} (\tau : \nabla v)$	11D.1-1 (T)	Last two terms describe entropy

BIRD eq 24.1-1

modelos

BIRD cap 24

$$\vec{j}_\Phi = -\rho \lambda_\Phi \text{grad} \varphi$$

$$\vec{j}_\Phi = -\rho \vec{\lambda}_\Phi \bullet \text{grad} \mu_\Phi$$

compósitos

$$\vec{j}_\Phi = -\rho \sum_{\Psi} \vec{\lambda}_{\Phi\Psi} \bullet \text{grad} \mu_\Psi$$

acoplamentos

$$\vec{j}_\Phi = -\rho \sum_{\Psi} \left[\vec{\lambda}_{\Phi\Psi} \bullet \text{grad}^n \mu_\Psi + \vec{\lambda}_{\Phi\Psi} \bullet \text{grad}^m \mu_\Psi + \dots \right]$$

?

unificação

$$\rho \frac{D\varphi}{Dt} = -\text{div } \vec{j}_\Phi + \dot{\sigma}_{\nabla\Phi}$$

$$\rho \frac{De_c}{Dt} = -\vec{v} \cdot \text{div } \vec{P} + \rho \vec{v} \cdot \vec{f}$$

$$\rho \frac{De_p}{Dt} = -\rho \vec{v} \cdot \vec{f} \quad +$$

$$\rho \frac{De_m}{Dt} = -\vec{v} \cdot \text{div } \vec{P}$$

$$+ \rho \frac{Du}{Dt} = T\rho \frac{Ds}{Dt} + \frac{p}{\rho} \frac{D\rho}{Dt} + \sum_i \mu_i \rho \frac{Dx_i}{Dt} = \rho \frac{De}{Dt} = -\text{div} \left[\sum_i \mu_i \vec{j}_i + \mu_s \vec{j}_s + \vec{P} \cdot \vec{v} \right]$$

$$\vec{j}_e = \sum_{\psi} \mu_{\psi} \vec{j}_{\psi}$$

$$-\text{div} \vec{P} \cdot \vec{v} - \text{div } T \vec{j}_s - \text{div} \sum_i \mu_i \vec{j}_i = -\vec{v} \cdot \text{div } \vec{P} + T\rho \frac{Ds}{Dt} + \frac{p}{\rho} \frac{D\rho}{Dt} + \sum_i \mu_i \rho \frac{Dx_i}{Dt}$$

$$A_j = -\sum_i M_i v_{ij} \mu_i$$

$$\frac{D\rho}{Dt} = -\rho \text{div } \vec{v}$$

$$\rho \frac{Dx_i}{Dt} = -\text{div } \vec{j}_i + \sum_j v_{ij} M_i \frac{D\xi_j}{Dt}$$

$$T\rho \frac{Ds}{Dt} = T \text{div } \vec{j}_s - \vec{\zeta} : \text{grad } \vec{v} + \vec{j}_s \cdot \text{grad } T + \sum_i \vec{j}_i \cdot \text{grad } \mu_i - \sum_j A_j \frac{D\xi_j}{Dt}$$

$$\rho \frac{Ds}{Dt} = -\text{div } \vec{j}_s + \dot{\sigma}_{\nabla s}$$

$$\dot{\sigma}_{\nabla s} = -\frac{\text{grad } \mu_\varepsilon}{T} \cdot \vec{j}_\varepsilon - \frac{\text{grad } \vec{v}}{T} : \vec{\zeta} - \frac{\text{grad } T}{T} \cdot \vec{j}_s - \sum_i \frac{\text{grad } \mu_i}{T} \cdot \vec{j}_i - \sum_j \frac{A_j}{T} \frac{D\xi_j}{Dt}$$

$$\rho \frac{Ds}{Dt} = -\operatorname{div} \vec{j}_s + \dot{\sigma}_{\nabla s}$$

$$\dot{\sigma}_{\nabla s} = -\frac{\operatorname{grad} \vec{v}}{T} : \vec{\zeta} - \frac{\operatorname{grad} T}{T} \cdot \vec{j}_s - \sum_i \frac{\operatorname{grad} \mu_i}{T} \cdot \vec{j}_i - \sum_j \frac{A_j}{T} \frac{D\xi_j}{Dt}$$

$$-\frac{\operatorname{grad} \mu_\varepsilon}{T} \cdot \vec{j}_\varepsilon$$

Irreversibilidades

condução elétrica não resistida

escoamento não resistido (atrito)

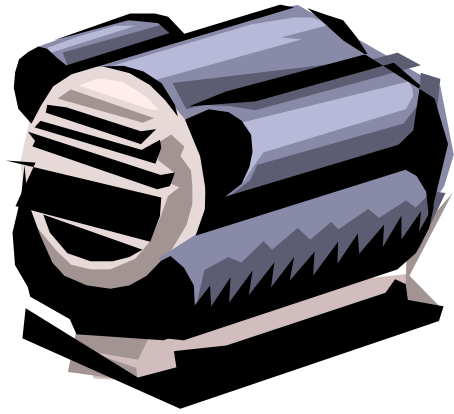
calor não resistido

difusões não resistidas

reações químicas não resistidas

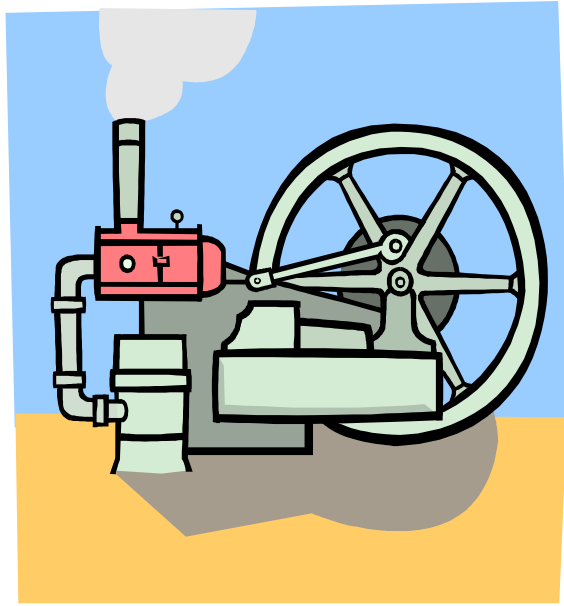
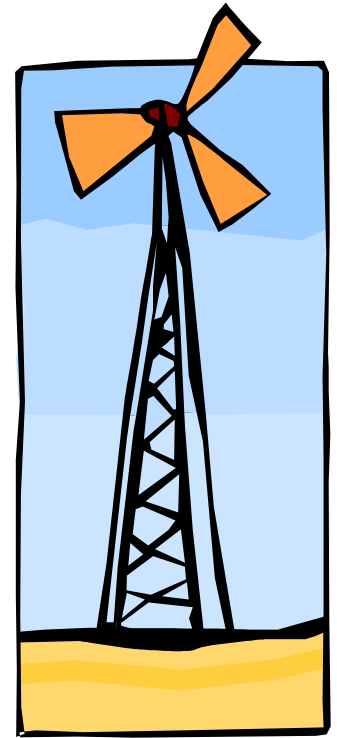
$$\begin{aligned} \dot{\sigma}_{\Delta S} = & \\ - \frac{g \vec{\text{grad}} \mu_{\varepsilon}}{T} \bullet \vec{j}_{\varepsilon} & \\ - \frac{g \vec{\text{grad}} \vec{v}}{T} : \vec{\xi} & \\ - \frac{g \vec{\text{grad}} T}{T} \bullet \vec{j}_S & \\ - \sum_i \frac{g \vec{\text{grad}} \mu_i}{T} \bullet \vec{j}_i & \\ - \sum_j \frac{A_j}{T} \frac{D \xi_j}{Dt} & \end{aligned}$$

resistir -> reversir

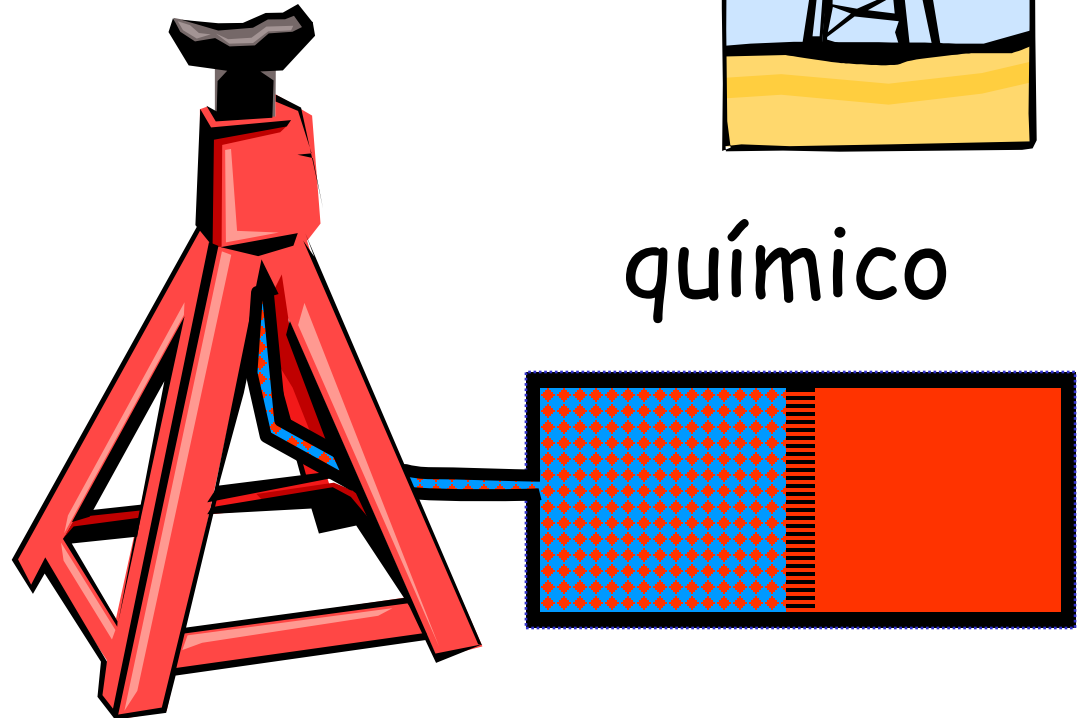


eléctrico

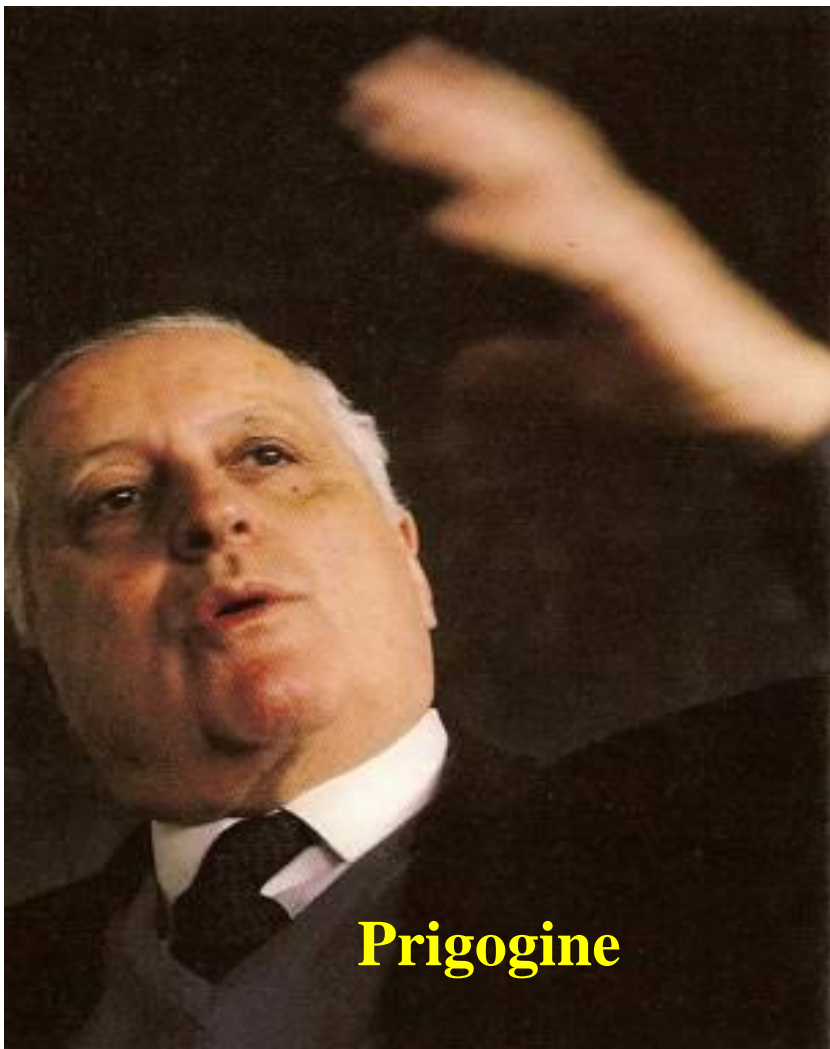
momentum



térmico



químico



Prigogine

1^a General ização

$$\dot{\sigma}_{\Delta S} = - \sum_{\Phi} \vec{j}_{\Phi} \cdot \frac{\text{grad} \mu_{\Phi}}{T} > 0$$



Onsager

$$\dot{\sigma}_{\nabla S} = - \sum_{\Phi} \vec{j}_{\Phi} \cdot \frac{\text{gr} \vec{a} d \mu_{\Phi}}{T} > 0$$

2ª Generalização

$$\vec{j}_{\Phi} = -\rho \sum_{\Psi} \vec{\lambda}_{\Psi\Phi} \cdot \text{gr} \vec{a} d \mu_{\Psi}$$

$$\vec{j}_{\Phi} = \sum_{\Psi} L_{\Phi\Psi} \text{gr} \vec{a} d \mu_{\Psi}$$

$$T \dot{\sigma}_{\nabla S} = - \sum_{\Phi} \sum_{\Psi} L_{\Phi\Psi} \text{gr} \vec{a} d \mu_{\Psi} \cdot \text{gr} \vec{a} d \mu_{\Phi} \approx L_{\Phi} (\text{gr} \vec{a} d \mu_{\Phi})^2$$

Dissipações



Maxwell

$$\begin{aligned}\dot{P}_\varepsilon &= \tau \dot{\sigma}_{\nabla_\varepsilon} = \\ &= \frac{1}{R_\varepsilon} \text{grad } \mu_\varepsilon \bullet \text{grad } \mu_\varepsilon = \\ &= \frac{U_\varepsilon^2}{R_\varepsilon}\end{aligned}$$



Rayleigh

$$\begin{aligned}\Phi_v &= \tau \dot{\sigma}_{\nabla_{\vec{v}}} = \\ &= \mu \text{grad } \vec{v} : \text{grad } \vec{v} = \\ &= \mu \left(\frac{\partial v_x}{\partial y} \right)^2\end{aligned}$$

sistema composto

vínculos



restrições
adiabática,
impermeável
indeformável

$$\rho \frac{Du}{Dt} = T\rho \frac{Ds}{Dt} + \frac{p}{\rho} \frac{D\rho}{Dt} + \sum_i \mu_i \rho \frac{Dx_i}{Dt}$$

$$DU = TDS - pDV + \sum_i \mu_i DN_i$$

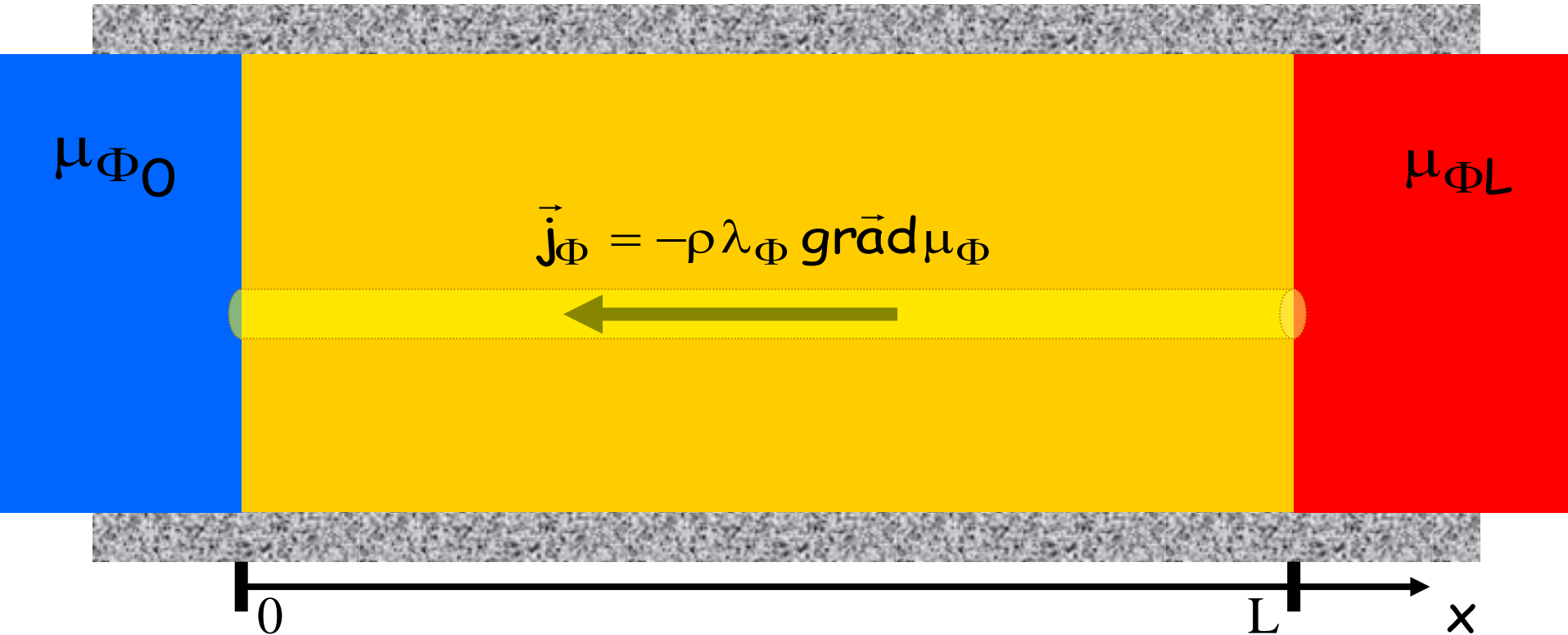
$$dS = \frac{dU}{T} + \frac{p}{T} dV - \sum_i \frac{\mu_i}{T} dN_i$$

$$\mu_1 > \mu_2$$

$$dS = \left(\frac{1}{T_1} - \frac{1}{T_2} \right) dU_1 + \left(\frac{p_1}{T_1} - \frac{p_2}{T_2} \right) dV_1 - \left(\frac{\mu_{A1}}{T_1} - \frac{\mu_{A2}}{T_2} \right) dN_{A1} - \left(\frac{\mu_{B1}}{T_1} - \frac{\mu_{B2}}{T_2} \right) dN_{B1}$$

um exemplo de

FT



$$\dot{\sigma}_{\Delta S} = - \sum_{\Phi} \vec{j}_{\Phi} \cdot \frac{\text{grad} \mu_{\Phi}}{\tau}$$

$$\vec{j}_{\Phi} = -\rho \sum_{\Psi} \vec{\lambda}_{\Psi\Phi} \cdot \text{grad}^m \mu_{\Psi}$$

$$\dot{\sigma}_{\Delta S} = - \vec{j}_{\Phi} \cdot \frac{\text{grad} \mu_{\Phi}}{\tau}$$

$$\vec{j}_{\Phi} = -\rho\lambda_{\Phi} \text{grad}^m \mu_{\Phi}$$

$$\dot{\sigma}_{\Phi S} = - \int_0^L -\frac{\rho\lambda_{\Phi}}{\tau} (\text{grad} \mu_{\Phi})^{m+1} dx$$

variational

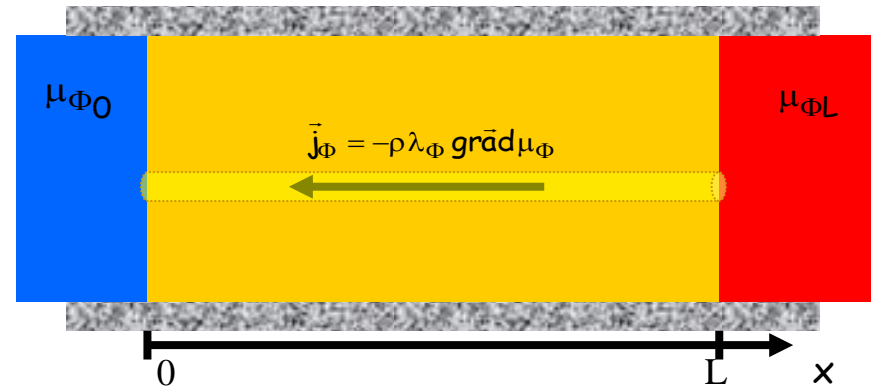
$$\dot{\sigma}_{\$S} = - \int_0^L - \frac{\rho \lambda_{\Phi}}{\tau} (\text{grād} \mu_{\Phi})^{m+1} dx$$

$$\frac{\dot{\sigma}_{\$S}}{\rho} = \int_0^L \frac{\lambda_{\Phi}}{\tau} \left(\frac{\partial \mu_{\Phi}}{\partial x} \right)^{m+1} dx$$

$$I = \int_0^L \Lambda \left(f, \frac{\partial f}{\partial x} \right) dx$$

$$\frac{d}{dx} \frac{\partial \Lambda}{\partial f / \partial x} - \frac{\partial \Lambda}{\partial f} = 0$$

$$T(x) = ax + b$$



$$T(x) = f = ?$$

outro exemplo

acoplamento de fts

$\mu_{\Phi 0}$

$$\vec{j}_{\Phi} = -\rho(\lambda_{\Phi\Phi} \text{grad}\mu_{\Phi} + \lambda_{\Phi\Psi} \text{grad}\mu_{\Psi})$$



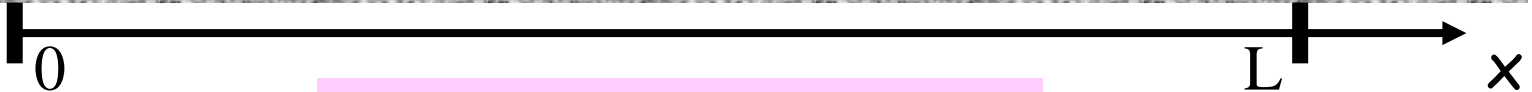
$$\vec{j}_{\Psi} = -\rho(\lambda_{\Psi\Psi} \text{grad}\mu_{\Psi} + \lambda_{\Psi\Phi} \text{grad}\mu_{\Phi})$$



$\mu_{\Psi 0}$

$\mu_{\Phi L}$

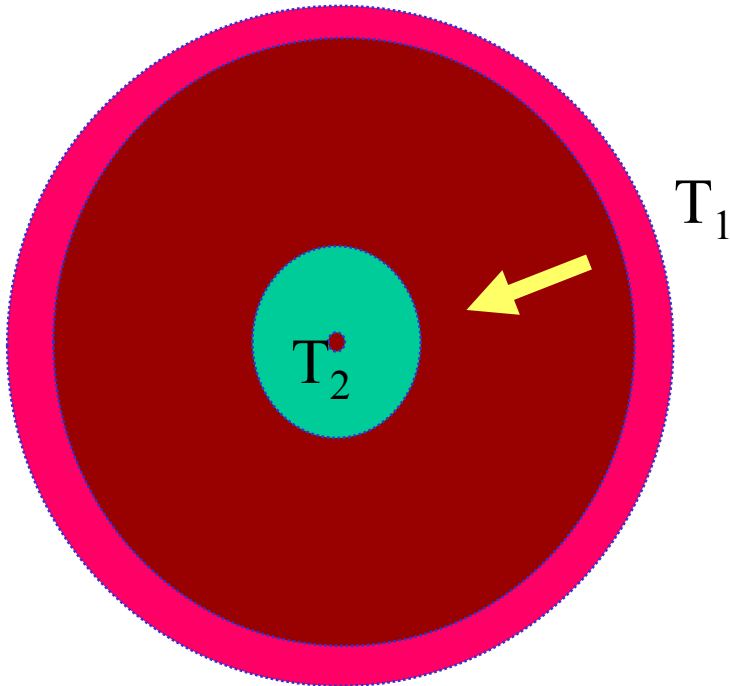
$\mu_{\Psi L}$



$$\dot{\sigma}_{\Delta S} = - \sum_{\Phi} \vec{j}_{\Phi} \cdot \frac{\text{grad}\mu_{\Phi}}{\tau}$$

$$\frac{\tau \dot{\sigma}_{\Delta S}}{\rho} = \left\{ \begin{array}{l} (\lambda_{\Phi\Phi} \text{grad}\mu_{\Phi} + \lambda_{\Phi\Psi} \text{grad}\mu_{\Psi}) \cdot \text{grad}\mu_{\Phi} + \\ + (\lambda_{\Psi\Psi} \text{grad}\mu_{\Psi} + \lambda_{\Psi\Phi} \text{grad}\mu_{\Phi}) \cdot \text{grad}\mu_{\Psi} \end{array} \right\}$$

picanha

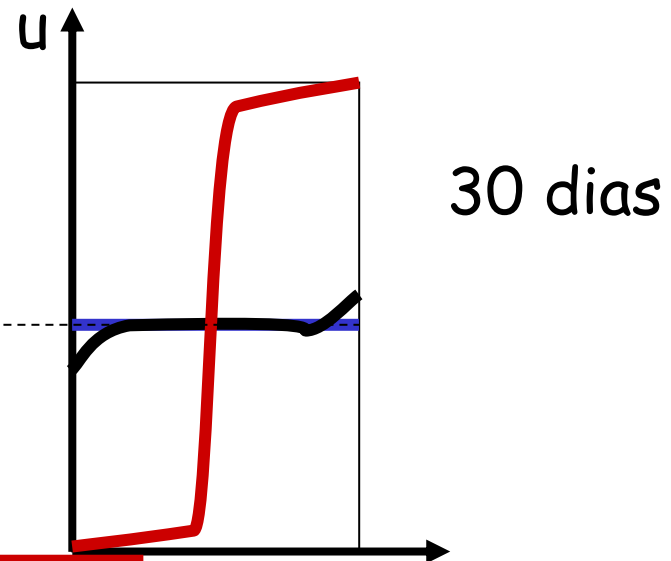
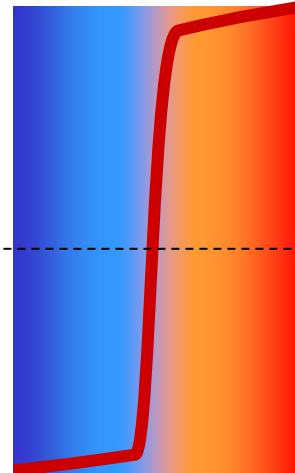
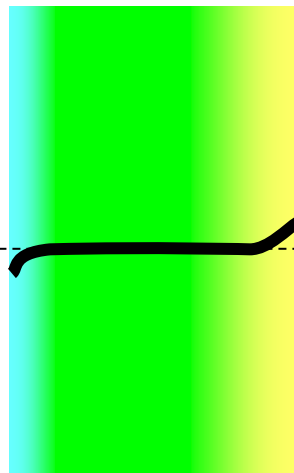
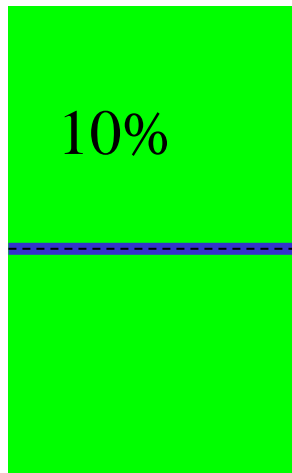
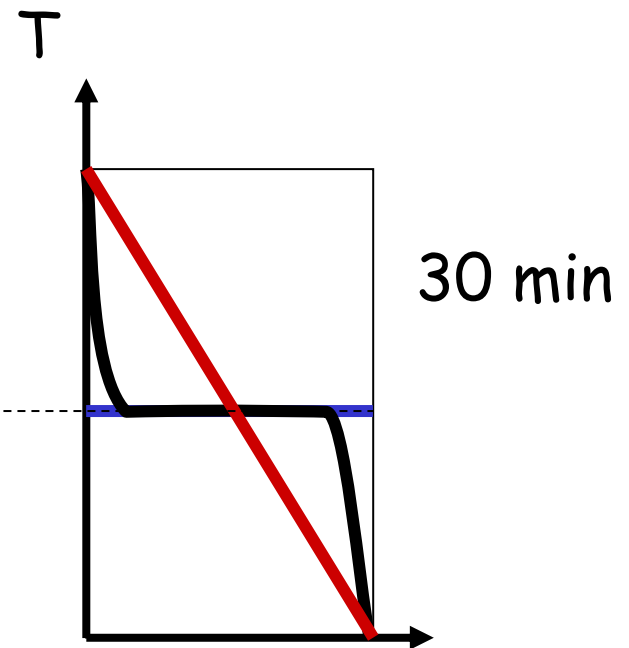
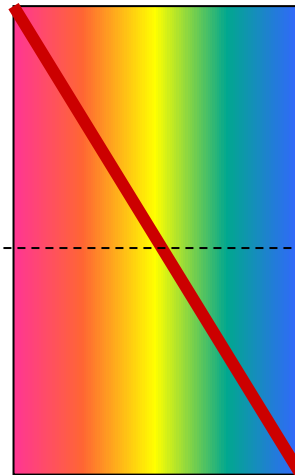
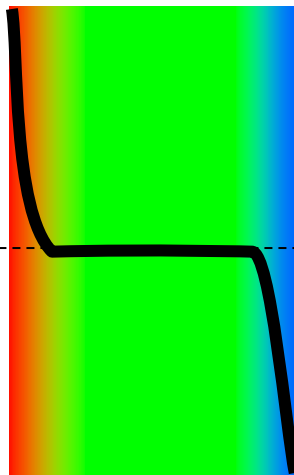
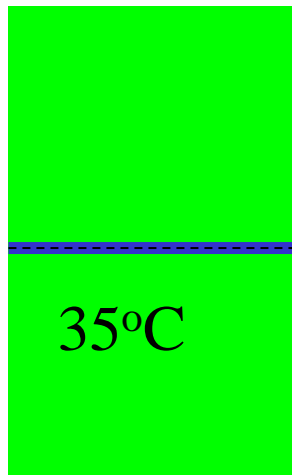


$$T_1 > T_2$$

$$\mu_{A1} > \mu_{A2}$$

$$x_{A1} < x_{A2}$$

termo-difusão

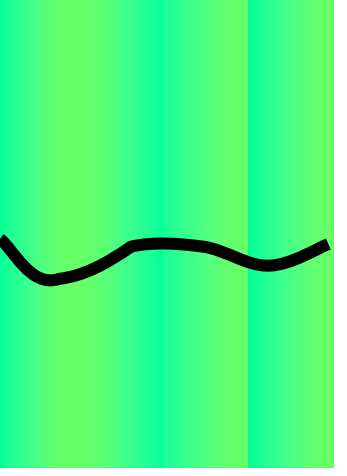
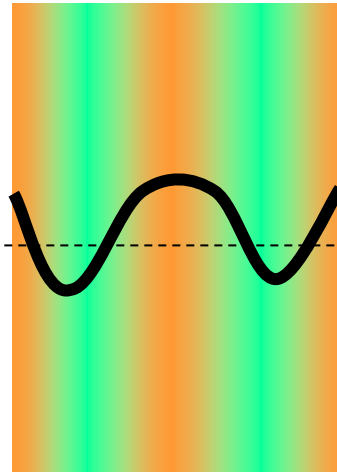
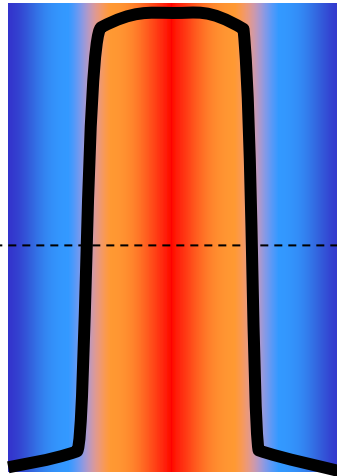
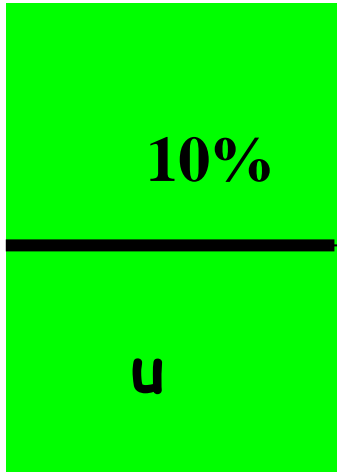
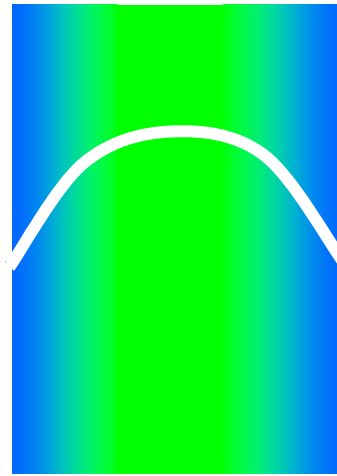
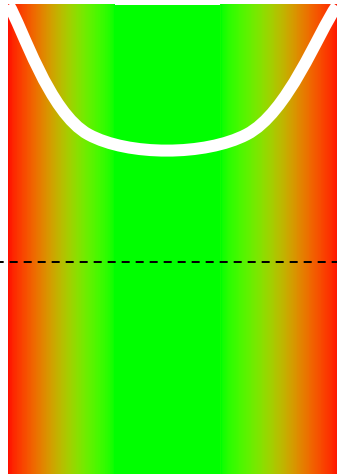
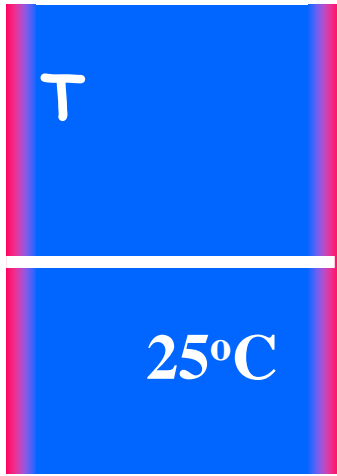


$$\vec{j}_u = -D_u \text{gr} \vec{a} du$$

$$\vec{j}_u = -D_{uu} \text{gr} \vec{a} du + D_{Tu} \text{gr} \vec{a} dT$$

picanha

$$\vec{j}_u = -D(\text{grādu} - \delta \text{grād}T)$$



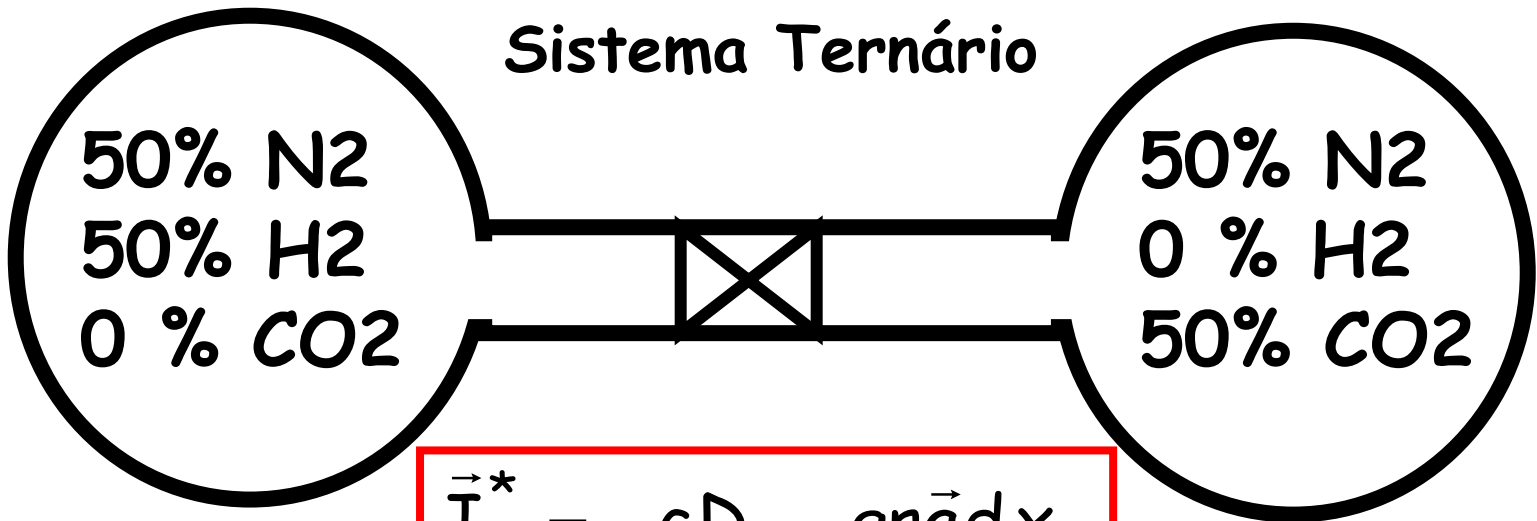
início

fogo

frio

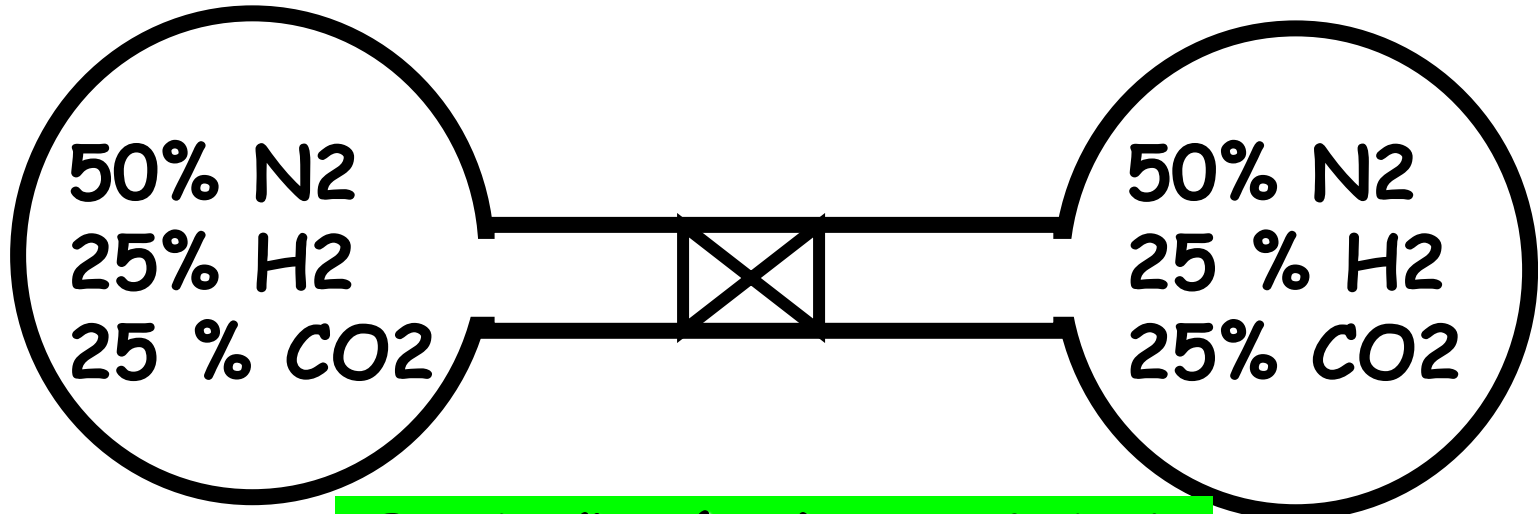
fogo

Sistema Ternário



$$\vec{J}_A^* = -cD_{AB} \text{grad} x_A$$

$$\rho \frac{Dw_A}{Dt} = \frac{\partial \rho_A}{\partial t} + \text{div} \rho_A \vec{v} = \rho D_{AB} \text{lap} w_A + \dot{r}_A$$



BIRD (binário) ex 24.6-1

Difusão Multicomponente

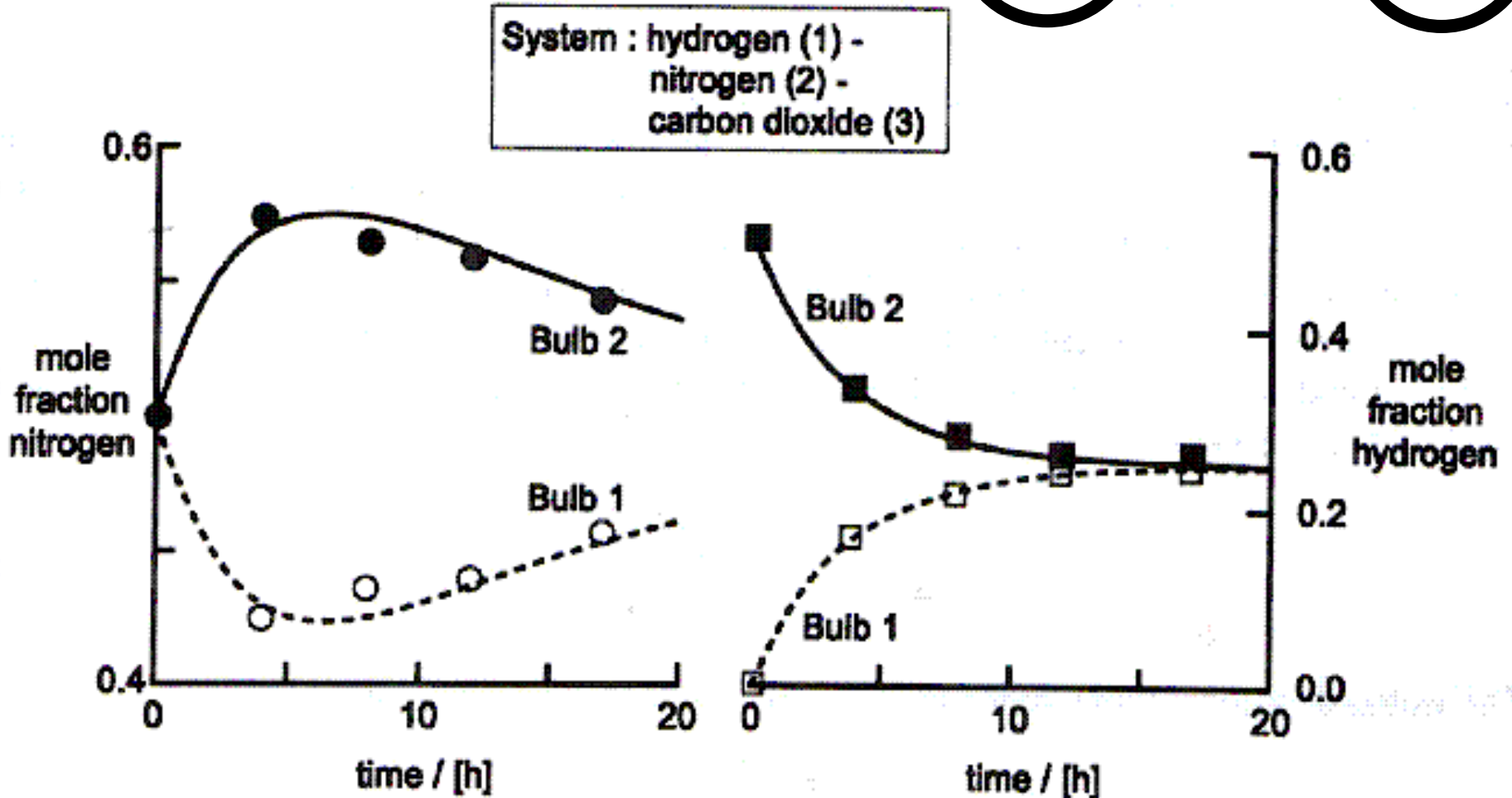
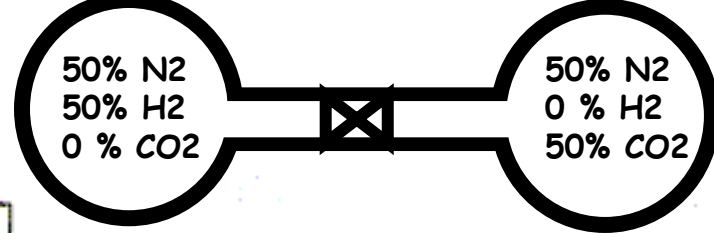


Figure 5.4. Composition-time history in two bulb diffusion cell. Experimental data from Duncan (1960).

~~$$\vec{J}_A^* = -cD_{AB} \text{grad} x_A$$~~

Lei de Fick Generalizada - Sistema Ternário

Dois fluxos e duas forças independentes

$$\vec{J}_1^* = -cD_{11} \text{grad}x_1 - cD_{12} \text{grad}x_2$$

$$\vec{J}_2^* = -cD_{21} \text{grad}x_1 - cD_{22} \text{grad}x_2$$

$D_{12} \neq D_{21}$ e diferentes das difusividades binárias (podem ser negativos)

Lei de Fick Generalizada - Sistema Multicomponente ($i = 1, 2, \dots, n$)

$$\vec{J}_1^* = -cD_{11} \text{grad}x_1 - cD_{12} \text{grad}x_2 \cdots \cdots - cD_{1,n-1} \text{grad}x_{n-1}$$

$$\vec{J}_2^* = -cD_{21} \text{grad}x_1 - cD_{22} \text{grad}x_2 \cdots \cdots - cD_{2,n-1} \text{grad}x_{n-1}$$

.

$$\vec{J}_i^* = -cD_{i1} \text{grad}x_1 - cD_{i2} \text{grad}x_2 \cdots \cdots - cD_{i,n-1} \text{grad}x_{n-1}$$

.

$$\vec{J}_{n-1}^* = -cD_{n-1,1} \text{grad}x_1 - cD_{n-1,2} \text{grad}x_2 \cdots \cdots - cD_{n-1,n-1} \text{grad}x_{n-1}$$

$$\vec{J}_i^* = \begin{pmatrix} \vec{J}_1^* \\ \vec{J}_2^* \\ \vdots \\ \vec{J}_{n-1}^* \end{pmatrix} \quad \sum_{i=1}^N \vec{J}_i^* = 0 \quad \text{grad } x = \begin{pmatrix} \text{grad } x_1 \\ \text{grad } x_2 \\ \vdots \\ \text{grad } x_{n-1} \end{pmatrix}$$

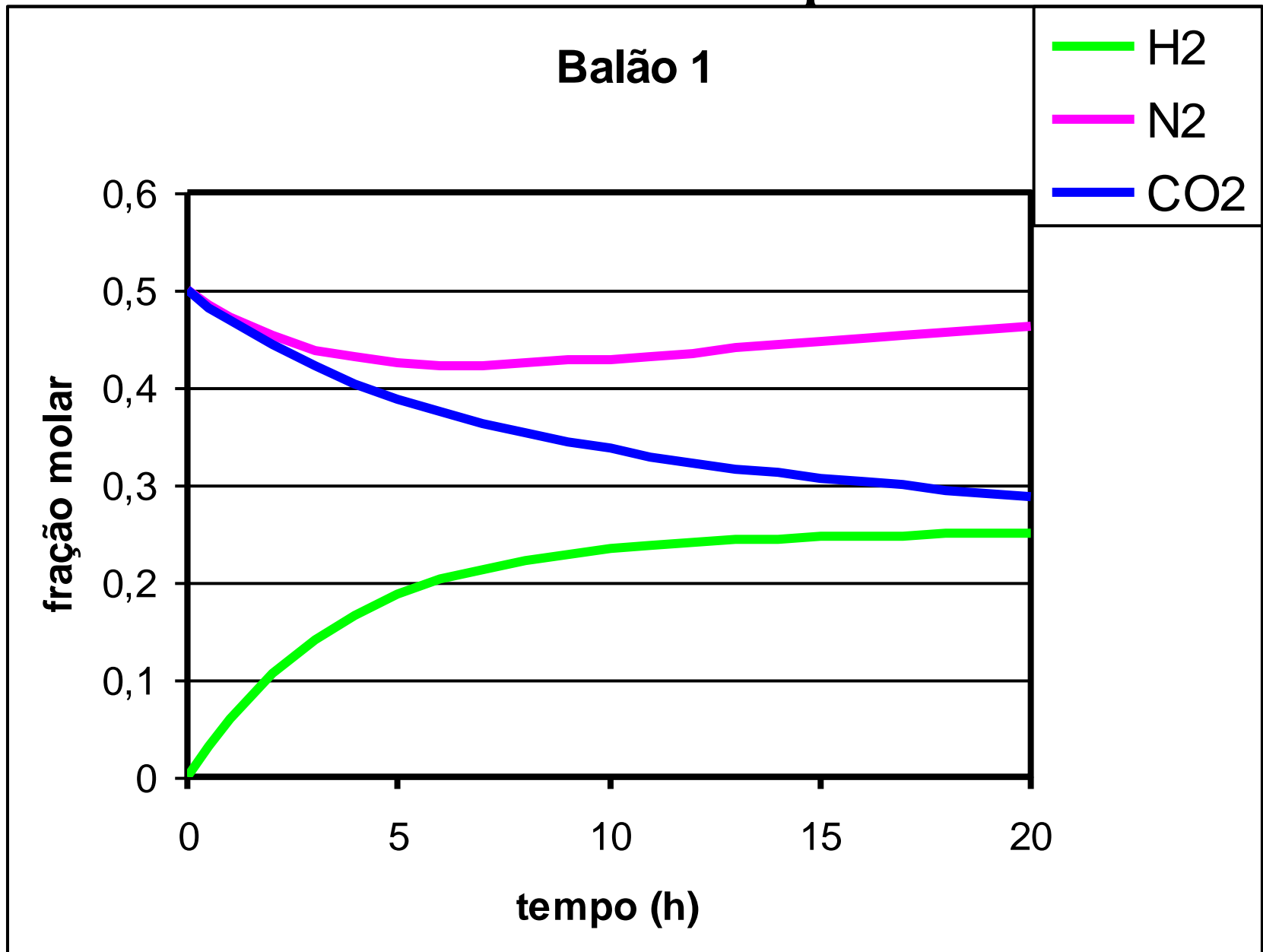
$$[D] = \begin{pmatrix} D_{11} & D_{12} & \cdots & D_{1,n-1} \\ D_{21} & D_{22} & \cdots & D_{2,n-1} \\ \vdots & & & \vdots \\ D_{n-1,1} & D_{n-1,2} & \cdots & D_{n-1,n-1} \end{pmatrix}$$

$$[\vec{J}^*] = -c [D] [\text{grad } x]$$

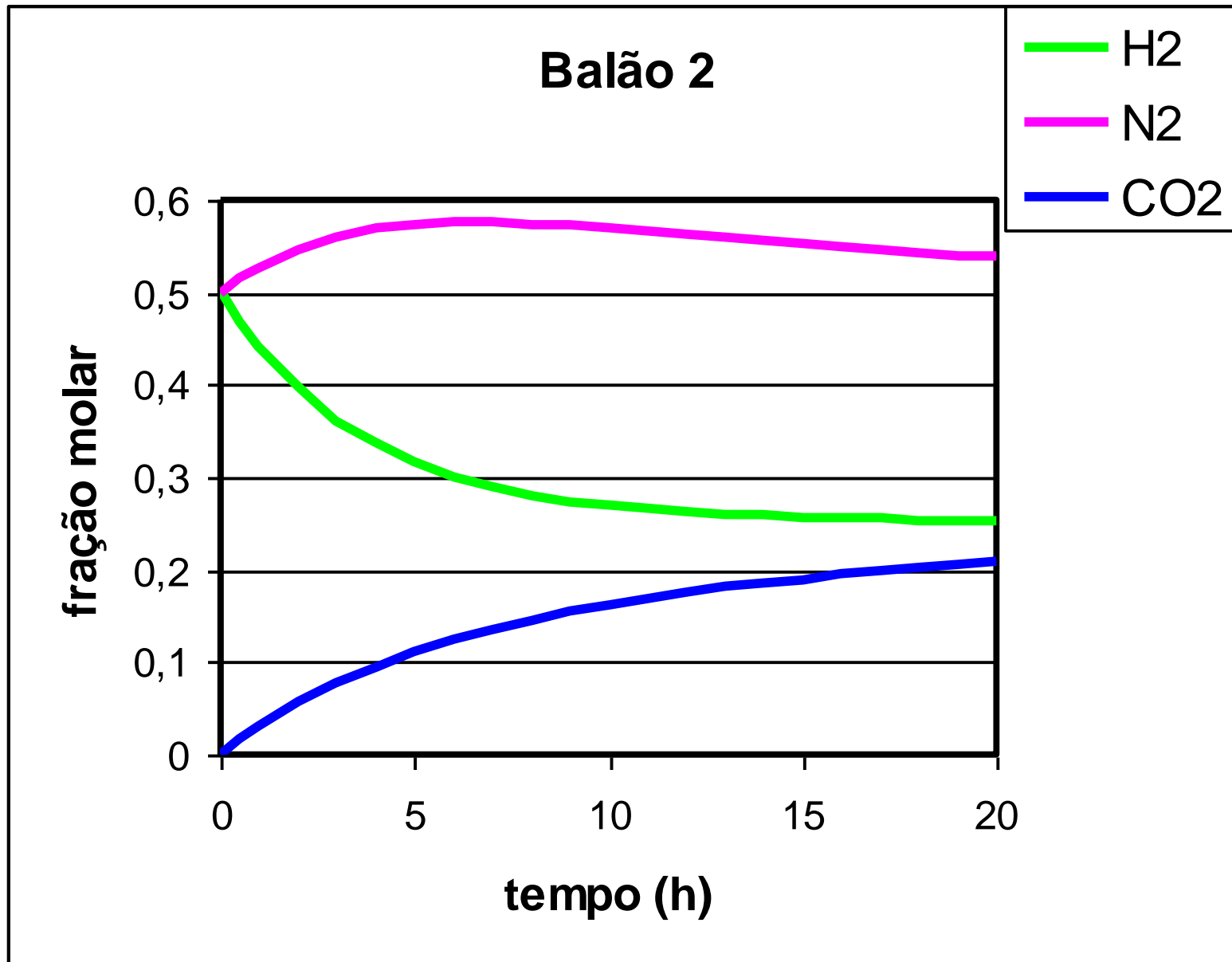
Difusão Multicomponente

tempo (h)	X10	X20	X30	X1L	X2L	X3L
0,0	0,000	0,501	0,499	0,501	0,499	0,000
0,5	0,032	0,485	0,483	0,469	0,514	0,016
1,0	0,060	0,472	0,468	0,442	0,527	0,031
2,0	0,106	0,452	0,442	0,396	0,547	0,056
3,0	0,141	0,438	0,421	0,362	0,561	0,078
4,0	0,167	0,430	0,403	0,336	0,569	0,095
5,0	0,187	0,425	0,388	0,316	0,574	0,111
6,0	0,202	0,423	0,374	0,301	0,576	0,124
7,0	0,214	0,423	0,363	0,289	0,576	0,135
8,0	0,223	0,424	0,353	0,280	0,575	0,145
9,0	0,229	0,427	0,344	0,274	0,572	0,154
10,0	0,234	0,429	0,336	0,269	0,570	0,162
11,0	0,238	0,433	0,329	0,265	0,567	0,169
12,0	0,241	0,436	0,323	0,262	0,563	0,175
13,0	0,244	0,439	0,317	0,260	0,560	0,181
14,0	0,245	0,443	0,312	0,258	0,556	0,186
15,0	0,247	0,446	0,307	0,257	0,553	0,190
16,0	0,248	0,450	0,303	0,256	0,550	0,195
17,0	0,248	0,453	0,299	0,255	0,547	0,199
18,0	0,249	0,456	0,295	0,254	0,543	0,202
19,0	0,249	0,459	0,292	0,254	0,541	0,206
20,0	0,250	0,461	0,289	0,253	0,538	0,209

Difusão Multicomponente



Difusão Multicomponente



Difusão Multicomponente

N2 - Balão 1 e 2

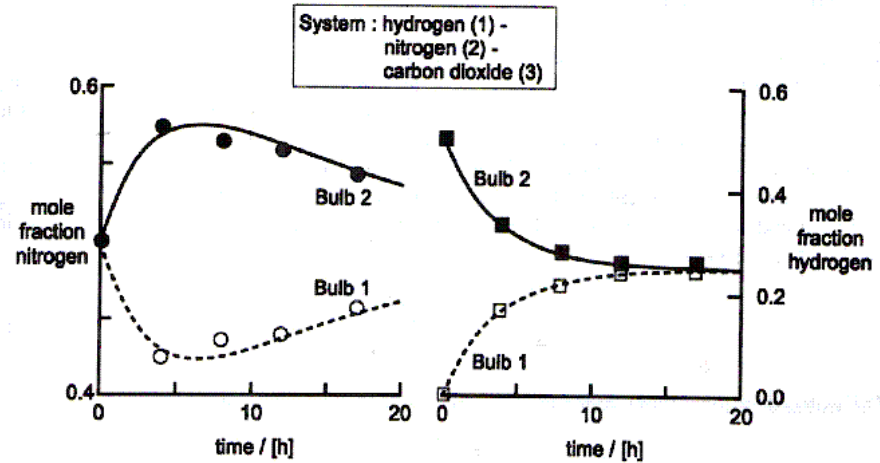
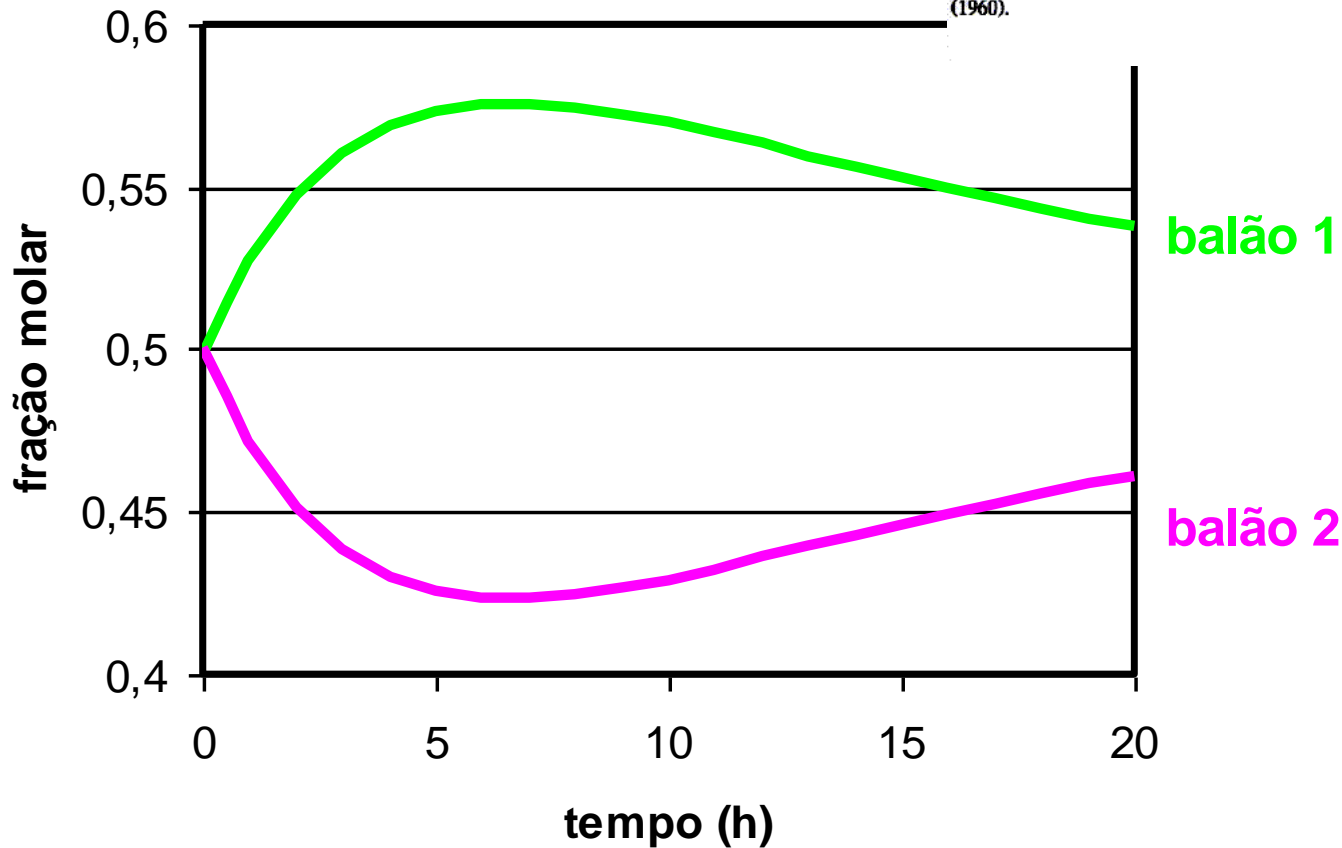


Figure 5.4. Composition-time history in two bulb diffusion cell. Experimental data from Duncan (1960).