Escola Politécnica da Universidade de São Paulo
Departamento de Engenharia Mecatrônica e de Sistemas Mecânicos - PMR

Aula 12

Projeto de Controle – Exemplos

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PMR 3409 – Controle II

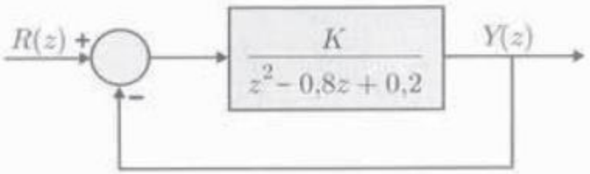
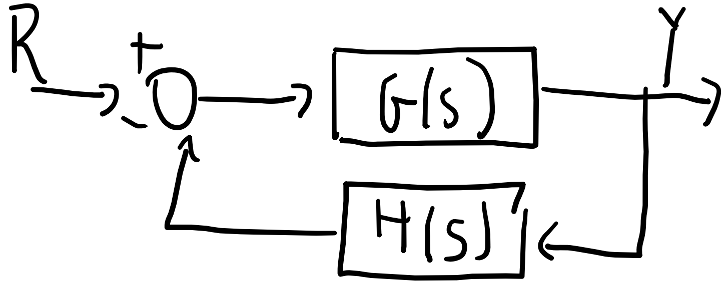


Figura 12.49 Sistema discreto.

Desenhe o lugar das raízes e determine a faixa de valores do ganho K de modo que o sistema seja estável em malha fechada.

RECORDAÇÃO



$$\frac{Y}{R} = \frac{G(s)}{1+G(s)H(s)} \rightsquigarrow$$ POLOS EM MF

$$1 + G(s)H(s) = 0$$

$$G(s)H(s) = -1 + 0j$$

⇒ COND. MÓDULO

$$|G(s) \cdot H(s)| = 1$$

⇒ COND. FASE

$$\underline{|G(s)H(s)} = 180^{0} + K \cdot 360^{0}$$

⇓

REGRAS PARA TRAÇAR LUGAR DAS RAÍZES

↓

VALE A MESMA COISA SE SUBSTITUIR $S \to Z$

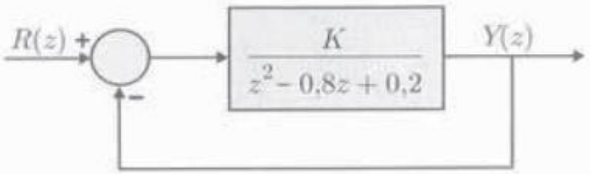
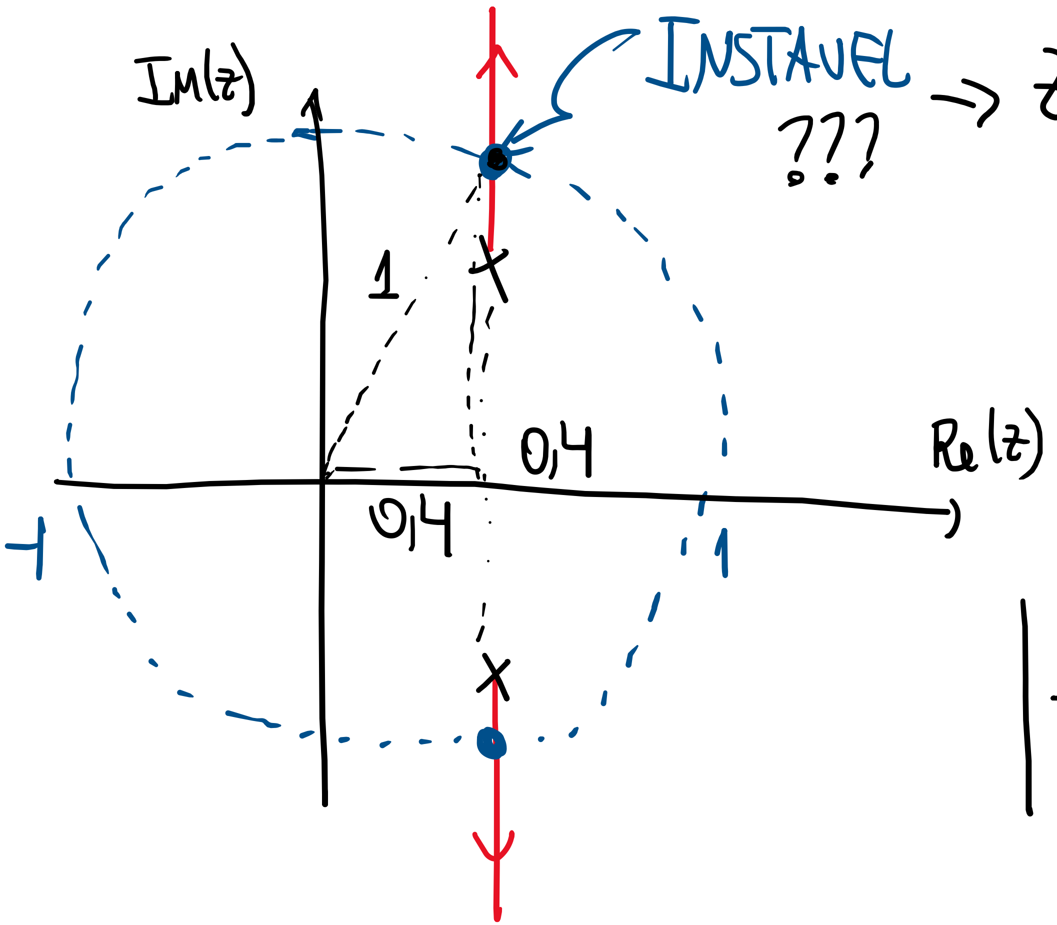


Figura 12.49 Sistema discreto.

Desenhe o lugar das raízes e determine a faixa de valores do ganho K de modo que o sistema seja estável em malha fechada.

$$G(z) = \frac{K}{z^2 - 0,8z + 0,2}$$

$$z_{1,2} = 0,4 \pm 0,2j$$



INSTÁVEL ???

$\rightarrow z_{inst} = 0,4 + 0,9165j$

E O K ASSOCIADO?

COND. MÓDULO

$$\left| \frac{K}{z^2 - 0,8z + 0,2} \right|_{z = 0,4 + 0,91j} = 1$$

$\rightarrow z_{inst} = 0{,}4 + 0{,}9165j$

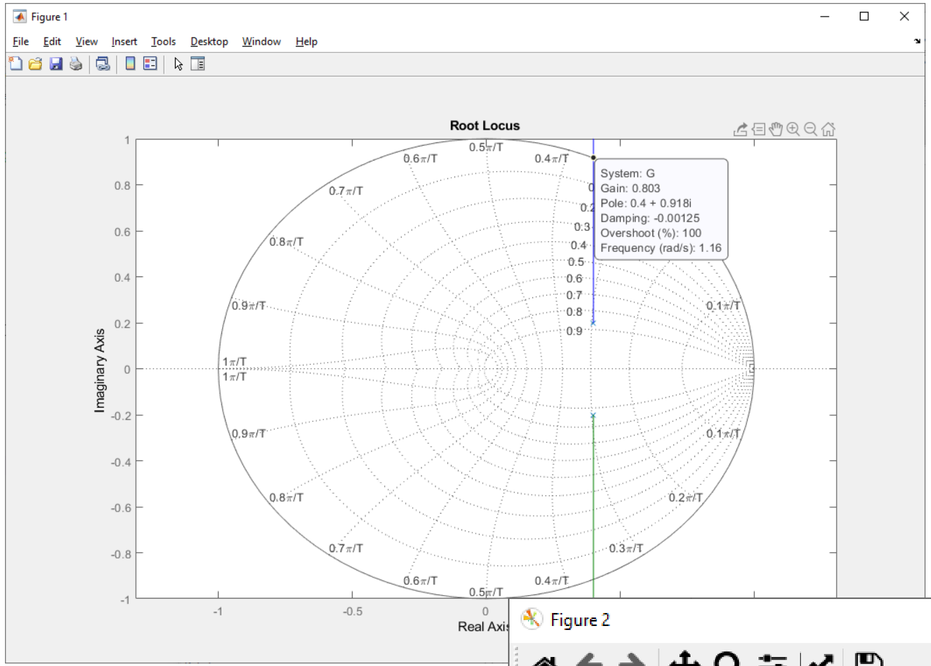
(z) e o K ASSOCIADO?

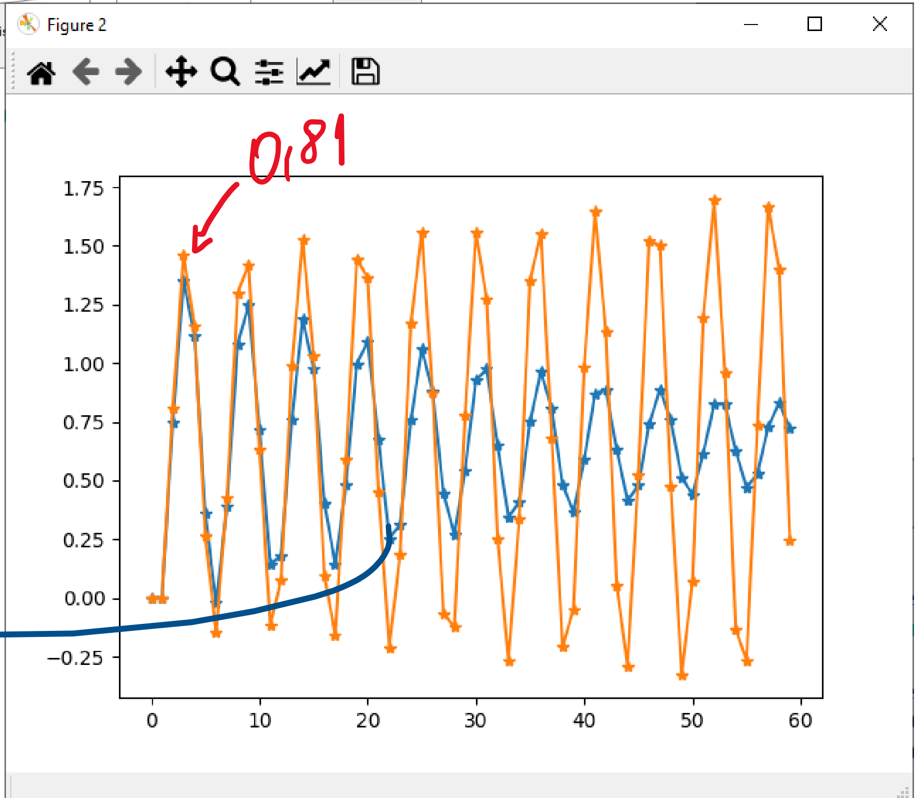
COND. MÓDULO

$$\left| \frac{K}{z^2 - 0{,}8z + 0{,}2} \right|_{z = 0{,}4 + 0{,}91j} = 1$$

$$\Downarrow$$

$$\boxed{K = 0{,}8}$$





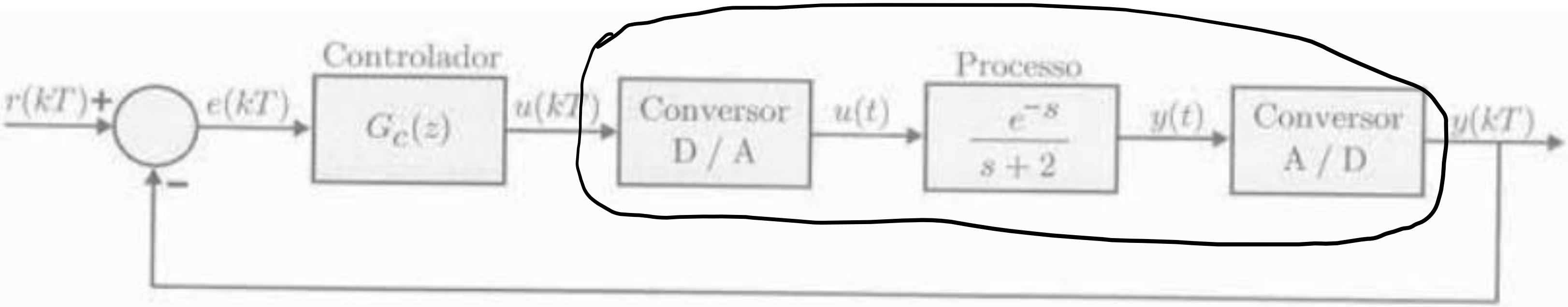


Figura 12.51 Diagrama de blocos de um sistema em malha fechada.

Projete um controlador $G_c(z)$ de modo que as seguintes especificações sejam satisfeitas:

- erro estacionário nulo para entrada $r(kT)$ do tipo degrau unitário e
- polos de malha fechada dominantes com coeficiente de amortecimento $\xi = 0.6$ e frequência natural $\omega_n = 1$ (rad/s).

Suponha que o período de amostragem seja $T = 1$ s.

1) POLOS DOMINANTES EM s $\xrightarrow{\text{EM } z}$ $z = e^{Ts}$

$$s = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

$$z = e^{-0{,}6T \pm 0{,}8Tj}$$

$$s = -0{,}6 \pm 0{,}8j$$

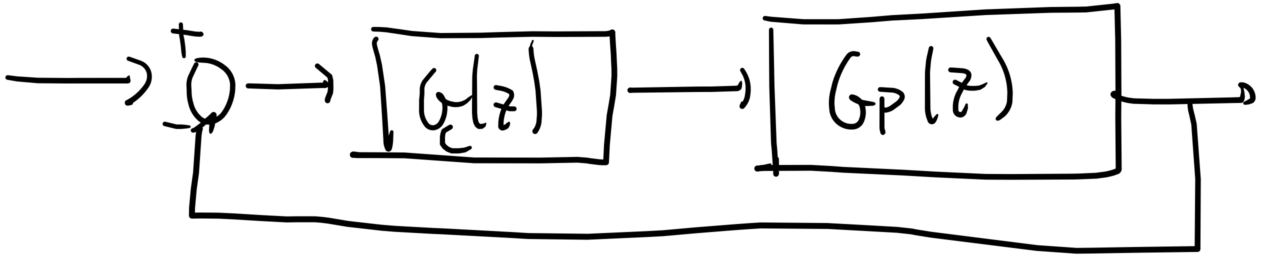
$$\boxed{z = 0{,}38 \pm 0{,}39j}$$

$\rightarrow M_P = 9{,}5\%$ ∴ $\exp\left(\frac{-\pi\xi}{\sqrt{1-\xi^2}}\right)$

VAMOS RESOLVER PELO MÉTODO DIRETO (TUDO EM z)

```
Ts = 1
G = tf(1,[1,2])
Gp = tf(1,[1,0],1) * c2d(G,1,'zoh')
```

$$Gp = \frac{0.4323}{z^2 - 0.1353z} \quad dt = 1$$



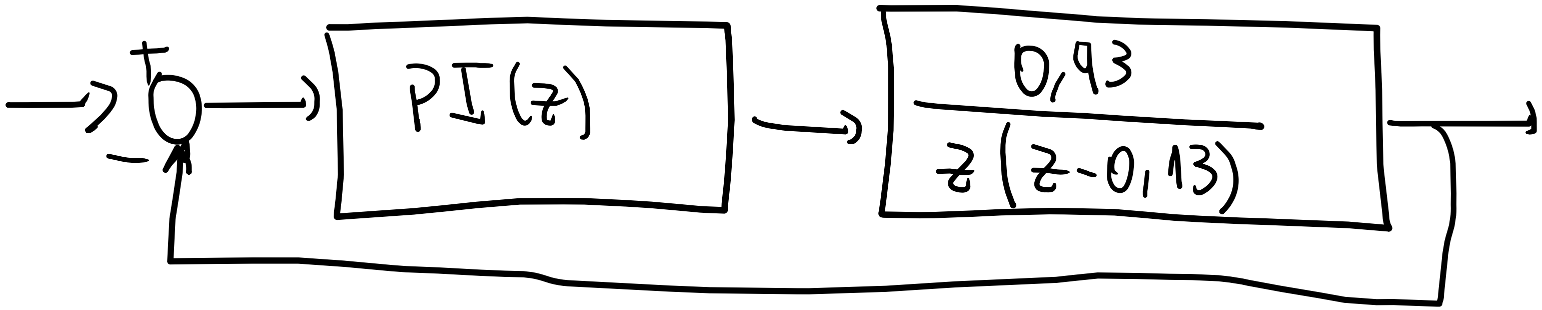
$Gp(z)$ = EQUIVALENTE DISCRETO PLANTA

$$Gp(z) = (1-z^{-1})\,\mathcal{Z}\left\{\mathcal{L}^{-1}\left(\frac{G(s)}{s}\right)\right\} = (1-z^{-1})\,\mathcal{Z}\left\{\mathcal{L}^{-1}\left(\frac{e^{-s}}{(s+2)s}\right)\right\}$$

z^{-1}

$$Gp(z) = z^{-1}\cdot(1-z^{-1})\,\mathcal{Z}\left\{\mathcal{L}^{-1}\left(\frac{1}{s(s+2)}\right)\right\}$$

$c2d\left(\frac{1}{s+2}, \text{'zoh'}\right)$



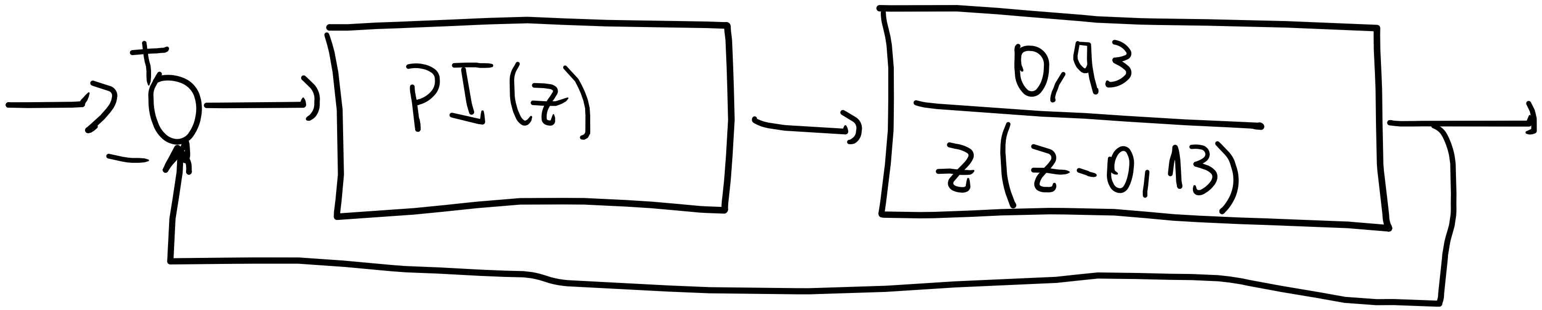
$$PI(z) = K_c + \frac{K_I}{1-z^{-1}} \quad \text{OU} \quad PI(z) = \frac{K(z+C)}{z-1}$$

$$\Rightarrow M.A. = \frac{K(z+C)}{z-1} \cdot \frac{0,43}{z(z-0,13)}$$

$z_{1,2} = 0,38 \pm 0,39j$ (POLOS DESEJADOS)

$\Rightarrow$ COND. FASE $\rightarrow$ $\angle z+C - \angle z-1 - \angle z - \angle z+0,13 = -180 + k \cdot 360°$

$$\arctan\left(\frac{0,39}{0,38+C}\right) = 71° \rightarrow \boxed{C \cong -0,24}$$

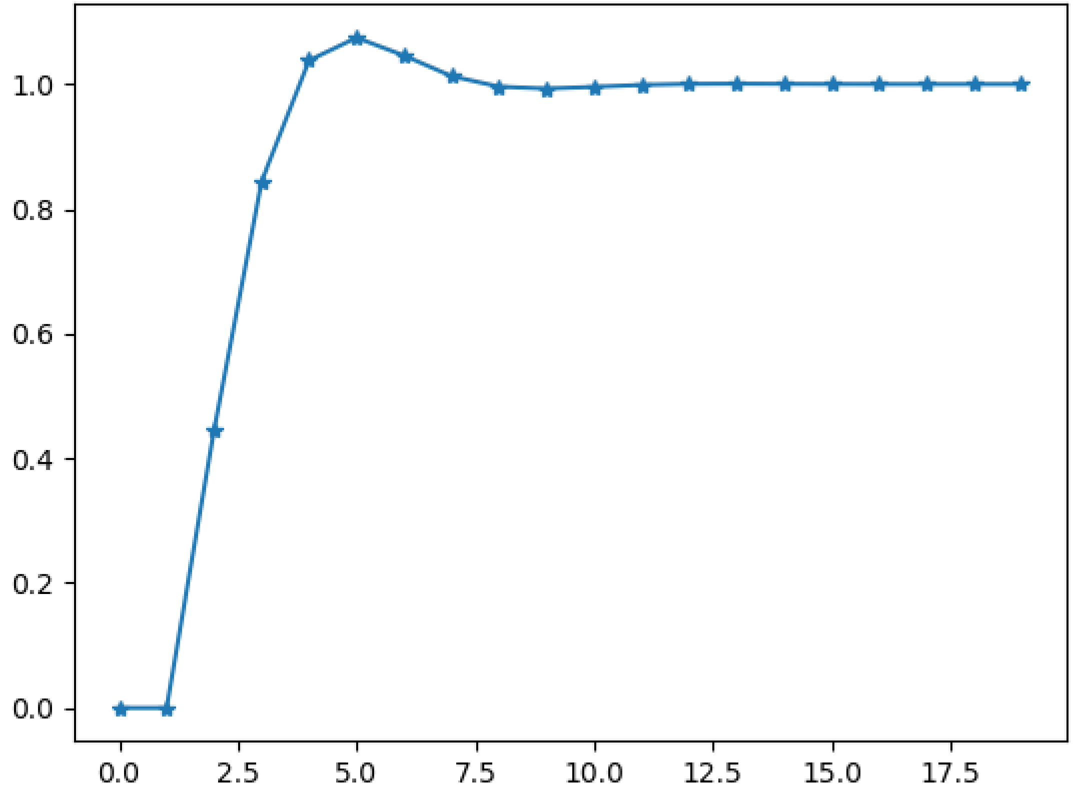


COND. MÓDULO → P/ GANHO $\underline{K}$

$$|MA(z)|_{z=0,38\pm0,39j} = 1 \Rightarrow \left| \frac{K(z-0,24)}{z-1} \cdot \frac{0,43}{z(z-0,13)} \right|_{z_{1,2}} = 1$$

$$\Rightarrow K = 1,03$$

$$\Rightarrow \boxed{PI(z) = \frac{1,03\,(z-0,24)}{z-1}}$$




```
In [11]: Ts = 1
    ...: G = tf(1,[1,2])
    ...: Gp = tf(1,[1,0],1) * c2d(G,1,'zoh')
    ...: Gc = tf([1.03,-1.03*0.24],[1,-1],Ts)
    ...:
    ...: Gmf = feedback(Gc*Gp,1)
    ...: damp(Gmf)
_____Eigenvalue______ Damping___ Frequency_
    0.3931   +0.3894j     0.6041     0.9796
    0.3931   -0.3894j     0.6041     0.9796
     0.349                    1     -0.349
```

4. Design a discrete controller for a DC-motor that is preceeded by a zero-order hold (see 1), so that the closed-loop system has an overshoot of no more than 20%, a rise time $T_r < 0.3$ s, and a settling time of not more than 2s. Use the discrete root locus method to evaluate different controller types and to tune the parameters of the appropriate controller. The sampling time is $T_P = 0.1$ ms .
The DC-motor can be approximately described in continuous-time by

$$G(s) = \frac{1}{s(s+1)} \quad .$$

$t_r = \frac{\pi - \beta}{\omega_d} = 0.3$ $t_s = \frac{4}{\zeta\omega_n} = 2$

$M_p = 20\%$

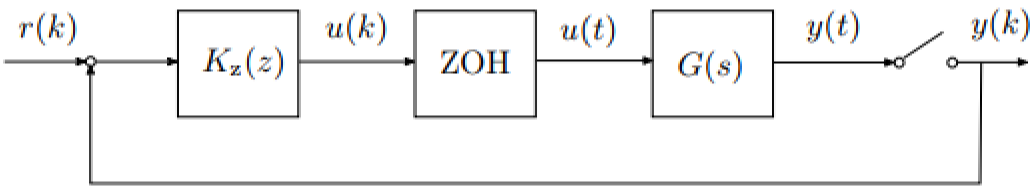


Figure 1: Diagram of a discrete time system

(a) Assume that the closed-loop continuous system can be approximated by a dominant pole pair. Translate the specifications of the continuous closed-loop system into corresponding requirements on the step response, using the dynamic behavior heuristics in Table 1.

(b) Discretize the DC-motor preceeded by the ZOH.

(c) (MATLAB) Consider a proportional controller $K_z(z) = k_p$. Plot the root locus of the closed-loop system with respect to k_p. Which value of k_p allows us to meet the design objectives?

(d) (MATLAB) Consider a lead compensator $K_z(z) = k_p \frac{z - z_{01}}{z - z_1}$. Choose the parameters z_{01}, z_1, and k_p so that the design objectives are met.
Hint: Use the MATLAB-function `rltool` and activate the grid over the context menu.

(e) What is the steady state error for a step input of the closed-loop system using the lead-compensator from d).

$$\frac{\pi-\beta}{\omega_d} = 0{,}3 \qquad \frac{4}{\zeta\omega_N} = 2$$

$$\Rightarrow S_{1,2} \rightarrow \begin{cases} \zeta = 0{,}45 \\ \omega_N = 7{,}6\ rad/s \end{cases}$$

$M_p = 20\%$

1) $\exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = 0{,}2$

$$\boxed{\zeta = 0{,}45}$$

$\beta = \arccos(\zeta) \Rightarrow \beta = 63^\circ$

2) $\frac{4}{0{,}45\,\omega_N} = 2 \Rightarrow \boxed{\omega_N = 4{,}4\ rad/s}$

3) $\frac{\pi-\beta}{\omega_N\sqrt{1-\zeta^2}} = 0{,}3 \Rightarrow \boxed{\omega_N = 7{,}6\ rad/s}$

b) $G_p(z) = c2d\left(\frac{1}{S(S+1)}, 0{,}1seg, 'zoh'\right)$

$G_p(z)$

```
   ....
   ...: Ts = 0.1
   ...:
   ...: G = tf(1,[1,1,0])
   ...:
   ...: Gp = c2d(G,Ts,'zoh')

In [16]: Gp
Out[16]:
```

$\frac{0.004837z + 0.004679}{z^2 - 1.905z + 0.9048}$ $dt = 0.1$ → Polos $+1$; $+0{,}9048$

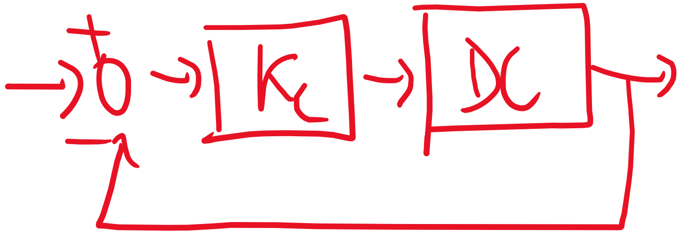
c) $S_{1,2} = -\zeta\omega_N \pm \omega_N\sqrt{1-\zeta^2}\,j$ → $\zeta = 0{,}45$

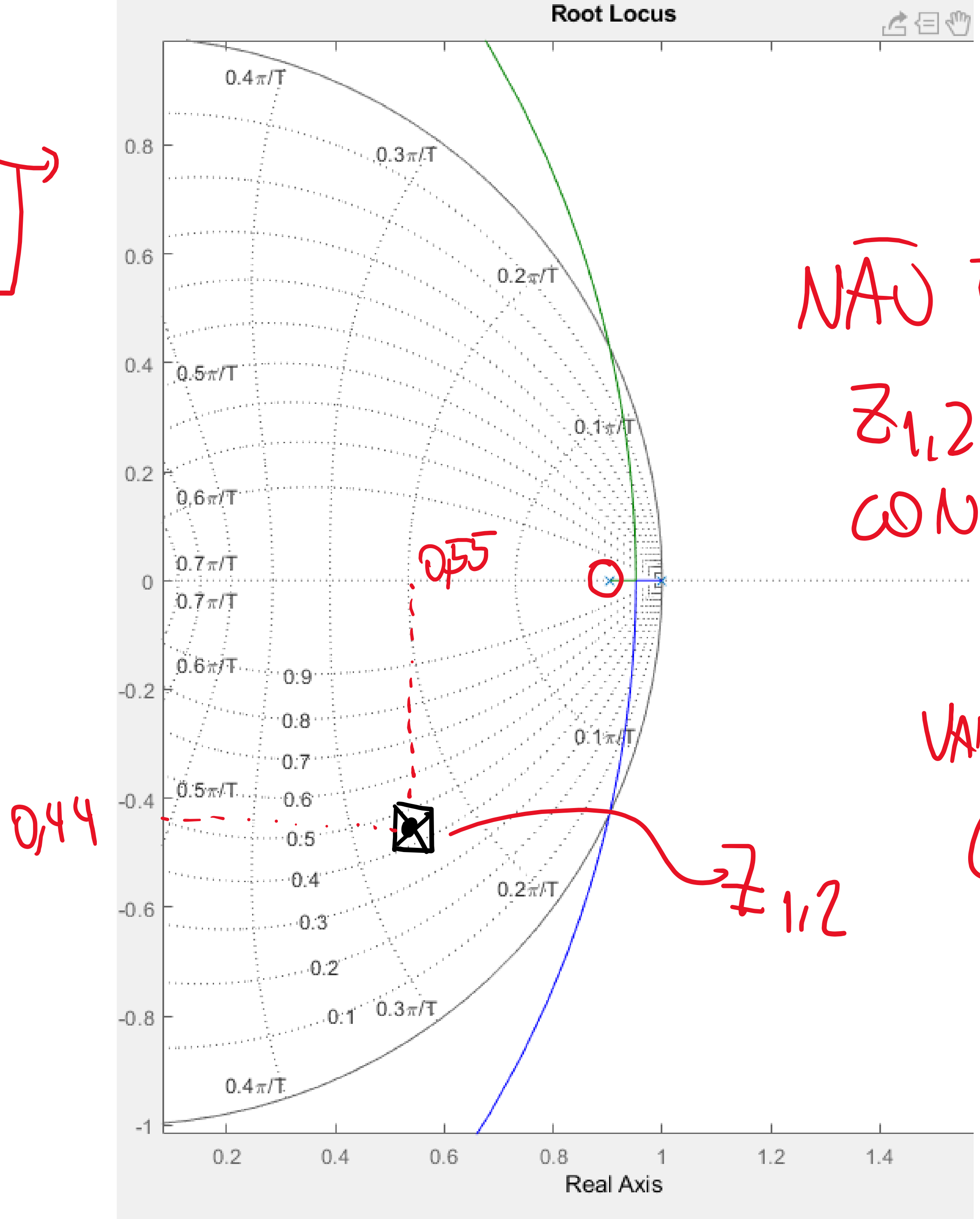
$\omega_N = 7{,}6\ rad/s$

$S_{1,2} = -3{,}42 \pm 6{,}78j$

$\frac{n.\pi}{T} = 7{,}6 \to n = \frac{7{,}6 . 0{,}1}{\pi} = 0{,}24$

$Z_{1,2} = e^{Ts} = \underline{0{,}55 \pm 0{,}44j}$





NÃO PASSA EM $Z_{1,2}$ APENAS COM CONTROLE PROPORCIONAL

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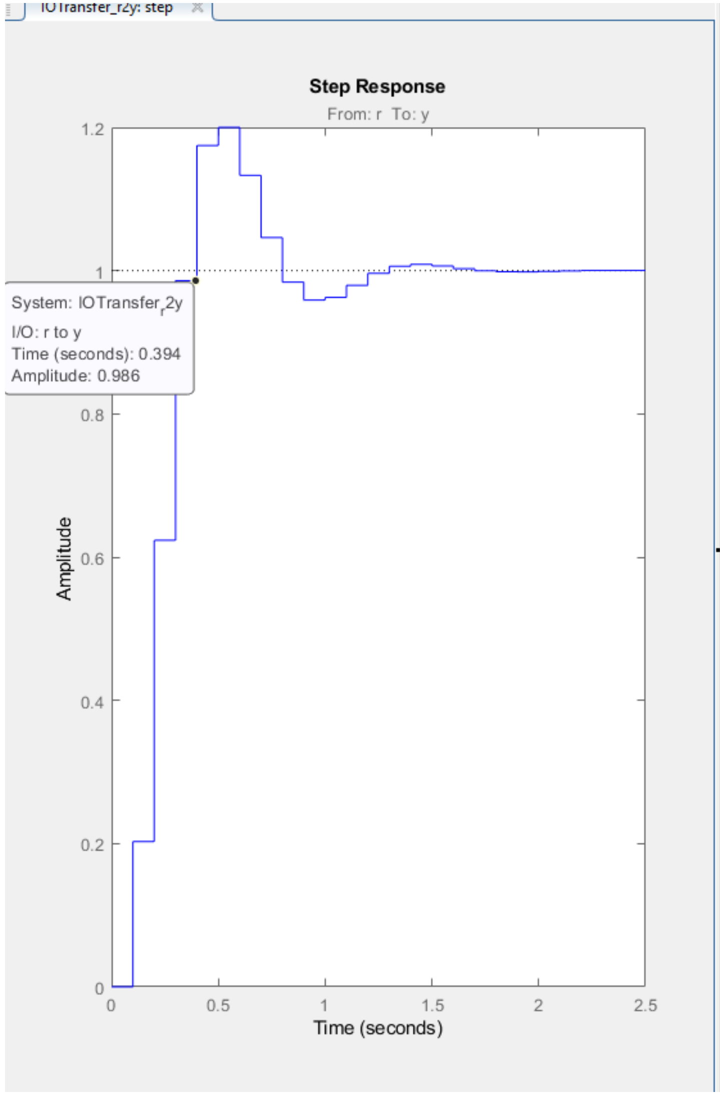
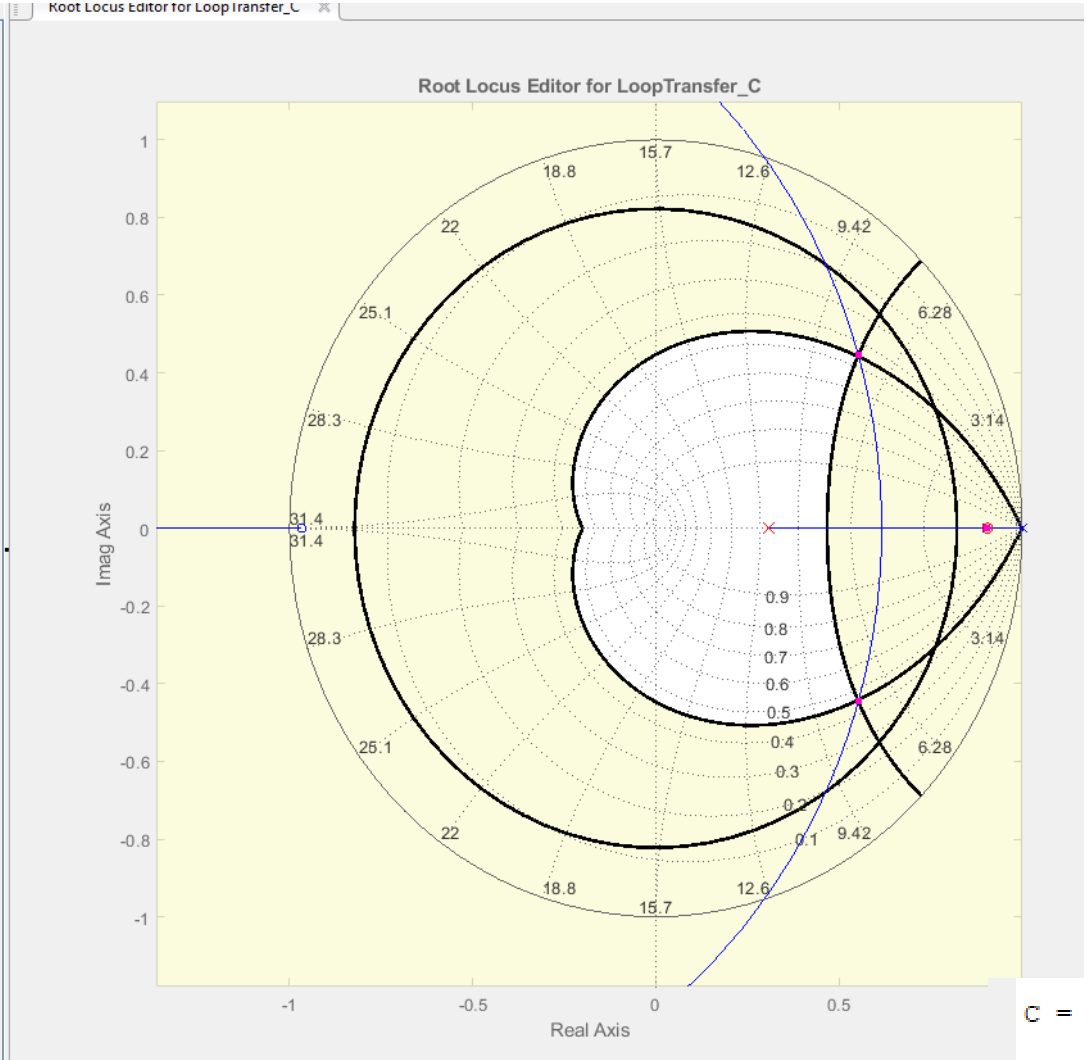
VAMOS TENTAR

$$G_c = K_c \cdot \frac{z - 0{,}9048}{z - p}$$






```
C =

  41.948 (z-0.9048)
  -----------------
     (z-0.3084)

Name: C
Sample time: 0.1 seconds
Discrete-time zero/pole/gain model.
```