PMR3523 Controle Moderno

Apoio à Aula

Controle Ótimo

CONTROLE ÓTIMO

$$\dot{x} = Ax + Bu$$

$$u = -G \cdot x$$

$$V = \int_t^T \overbrace{x'(\tau) Q(\tau) x(\tau)}^{\text{ERRO}} + \underbrace{u'(\tau) R(\tau) u(\tau)}_{\text{ENERGIA}} \, d\tau$$

$Q, R \rightarrow$ MATRIZES DE PONDERAÇÃO, PARÂMETROS CONTROLE

Ex)

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \text{OTIMIZA} \quad x_1^2 + c^2 x_2^2$$

Ex) $y = C \cdot x \rightarrow$ SAÍDA

$y \hookrightarrow$ ESCALAR

$$y^2 = x' \cdot C' \cdot C \cdot x \rightarrow \underline{Q = C' \cdot C}$$

$\hookrightarrow$ ERRO NA SAÍDA

COMO OBTER $u = -\underline{\underline{G}} \cdot x$ QUE GARANTA $\min V$?

$$\dot{x} = Ax - BGx = A_c \cdot x$$

$$A_c = A - B \cdot G$$

$$\dot{x} = Ax - BGx = A_c \cdot x$$

$$A_c = A - B.G$$

$$x(\tau) = \phi_c(\tau,t).x(t)$$

↳ MATRIZ TRANSIÇÃO ESTADOS

$$V = \int_t^T \underline{x'(\tau)}\, Q(\tau)\, \underline{x(\tau)} + \underbrace{u'(\tau)}_{-G.x(\tau)} R(\tau)\, u(\tau)\, d\tau$$

$$V = \int_t^T x'(t).\phi_c'(\tau,t)\{Q + G'RG\}\phi_c(\tau,t)\, x(t)\, d\tau$$

$x(t)$ PODE SER COLOCADO P/ FORA DA INTEGRAL

$$V = x'(t).M(t,T)\,x(t)$$

c/ $M(t,T) = \int_t^T \phi_c'(t,\tau)\{Q + G'RG\}\phi(t,\tau)\, d\tau$

↳ É SIMÉTRICA

MAS

$$V = \int_t^T x'(\tau).Q.x(\tau) + x'(\tau).G'.R.G.x(\tau)\, d\tau$$

$$V = \int_t^\tau x'(\tau).L(\tau)\,x(\tau)\, d\tau$$

COM $L(\tau) = Q + G'RG$

$$V=\int_t^T x'(\tau).Q.x(\tau)+x'(\tau).G'.R.G.x(\tau)\,d\tau$$

$$V=\int_t^\tau x'(\tau).L(\tau)x(\tau)d\tau$$

com $L(\tau)=Q+G'RG$

(A) $$\frac{dV}{dt}=-x'(\tau)L(\tau)x(\tau)\Big|_{\tau=t}=$$

$$=-x'(t)L(t)x(t)$$

(B) $$V=x'(t).M(t,T).x(t)$$

$$\frac{dV}{dt}=\dot{x}'Mx+x'\dot{M}x+x'M.\dot{x}$$

$\dot{M} \to \frac{\partial M(t,t)}{\partial t}$

$$\dot{x}=A_c.x \Rightarrow$$

$$\frac{dV}{dt}=x'(t)\left[A_c'M+\dot{M}+MA_c\right]x(t)$$

DUAS FORMAS P/ $\frac{dV}{dt}$

$$\Rightarrow -L=A_c'M+\dot{M}+MA_c$$

ou $$\boxed{-\dot{M}=MA_c+A_c'M+L}$$ (C)

$M=M(t,T)$ COND. FINAL $M(T,T)=0$

$A_c=A_c(t)$ POIS $M(t,T)=\int_t^T \phi_c'(\tau,t)L(t)\phi_c(\tau,t)d\tau$

$L=L(t)$

$$\boxed{-\dot{M} = MA_c + A_c' M + L}$$

PARA OBTER G

$$-\dot{M} = M(A-BG) + (A'-G'B')M + Q + G'RG$$

LEMBRAMOS QUE V QUE QUEREMOS MINIMIZAR É:

$$V = x'(t)M(t,\tau)x(t)$$

LOGO MINIMIZAR V REPRESENTA MINIMIZAR M E BUSCAMOS A MATRIZ $\hat{M}$ QUE:

$$\hat{V} = x'\hat{M}x < x'M.x \quad p/ \forall x(t) \text{ e } \hat{M} \neq M$$

ASSUMINDO QUE O GANHO $\hat{G}$ CORRESPONDE À $\hat{M}$

$$-\dot{\hat{M}} = \hat{M}(A-B\hat{G}) + (A'-\hat{G}'B')M + \hat{G}'R\hat{G} + Q$$

$\Rightarrow \forall$ MATRIZ DE GANHO G NÃO ÓTIMO E SUA CORRESPONDENTE MATRIZ M SÃO EXPRESSAS POR:

$$M = \hat{M} + N$$

$$G = \hat{G} + Z$$

(i)

$$-\dot{\hat{M}} = \hat{M}(A - B\hat{G}) + (A' - \hat{G}'B')M + \hat{G}'R\hat{G} + Q$$

$$M = \hat{M} + N$$

$$G = \hat{G} + Z$$

(ii) $-(\dot{\hat{M}} + \dot{N}) = (\hat{M} + N) \dots$

(ii)-(i) ⇒

$$-\dot{N} = NA_c + A_c'N + (\hat{G}'R - \hat{M}B)Z + Z'(R\hat{G} - B'\hat{M}) + Z'RZ$$

MESMA FORMA DA EQ.(C) COM L =

SOLUÇÃO É

$$N(t,\tau) = \int_t^T \phi_c'(\tau,t) . L . \phi(\tau,t)\, d\tau$$

se $\hat{V}$ É MÍNIMO ⇒

$$\hat{V} = x'.\hat{M}.x \leq x'.(\hat{M} + N).x =$$

$$= x\hat{M}x + x'N.x$$

⇒ $x'Nx \geq 0$ ⇒ N É POSITIVA DEFINIDA

ORA $N = \int_t^\tau \phi_c' . L . \phi_c \, d\tau$

E $L = (\hat{G}'R - \hat{M}B)Z + Z'(R\hat{G} - B'\hat{M}) + Z'RZ$

N SERÁ P.D. $\forall z$ SE

$$G'R - \hat{M}B = 0 \quad \text{OU}$$

$$R.G - B'.\hat{M} = 0$$

$$\boxed{G = R^{-1}B'.\hat{M}} \rightarrow \text{GANHO ÓTIMO}$$

SUBST. O GANHO ÓTIMO EM:

$$-\dot{M} = M(A-BG) + (A' - G'B')M + Q + G'RG$$

$$\boxed{-\dot{\hat{M}} = \hat{M}A + A'\hat{M} - \hat{M}BR^{-1}B'\hat{M} + Q}$$

EQ. RICCATI

EQ. ALGÉBRICA

$$T = \infty \Rightarrow V_{\infty} = \int_t^{\infty} x'Q.x + u'.R.u \, d\tau$$

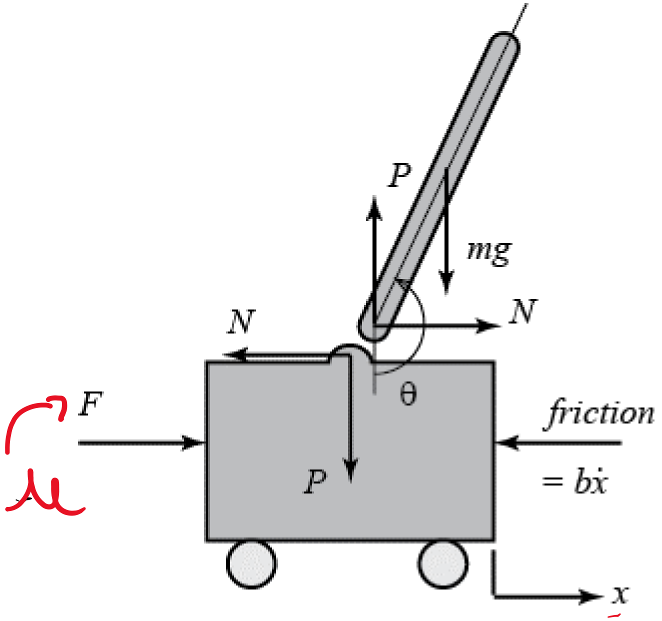
NESTE CASO M CONVERGE P/ VALOR CONSTANTE $\dot{\hat{M}} = 0$

$$\boxed{\begin{array}{l} \bar{M}A + A'\bar{M} - \bar{M}BR^{-1}B'\bar{M} + Q = 0 \\ G = R^{-1}B'\bar{M} \end{array}}$$

lqr MATLAB

$$x^T \cdot Q \cdot x$$

$$\underset{1\times4}{} \quad \underset{4\times4}{} \quad \underset{4\times1}{} \rightarrow \underline{1\times1}$$

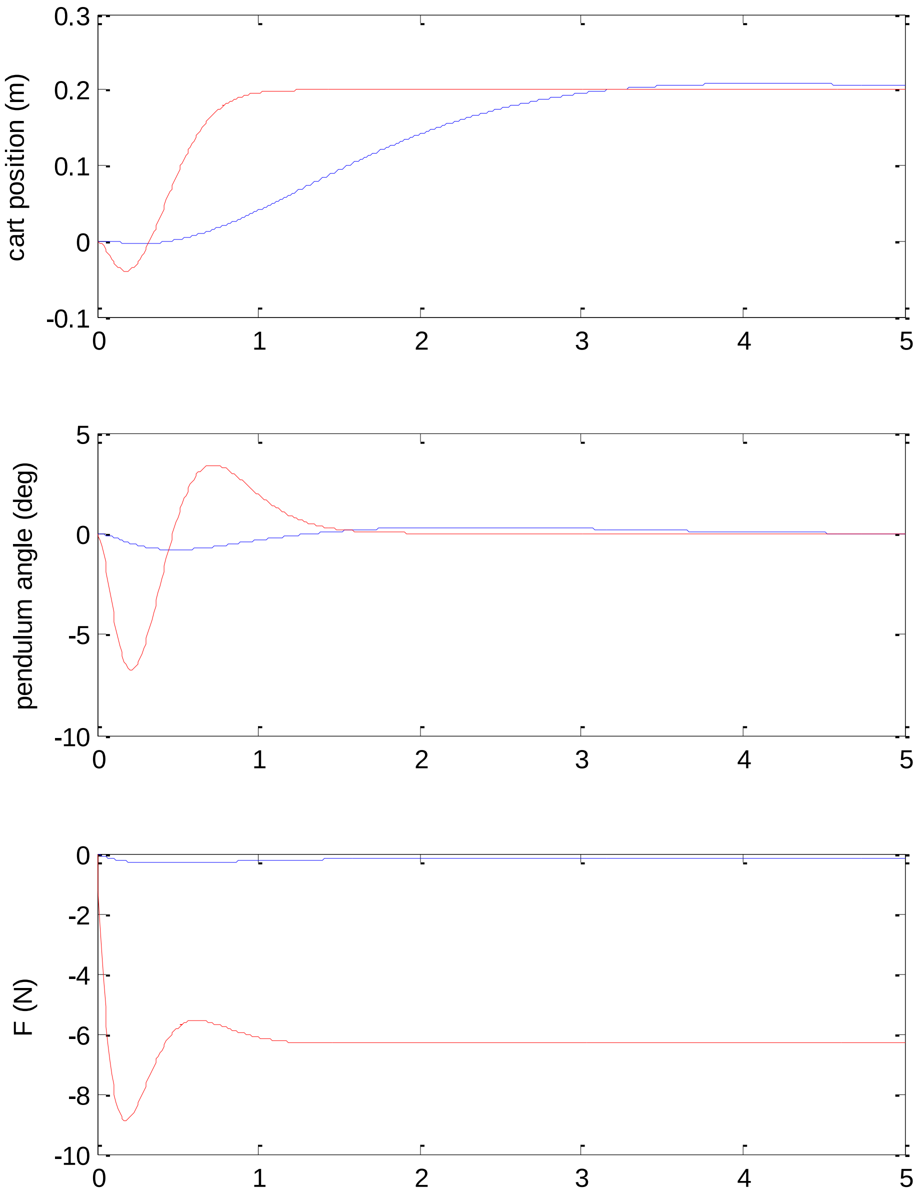


ESTADOS

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\phi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.1818 & 2.6727 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.4545 & 31.1818 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 1.8182 \\ 0 \\ 4.5455 \end{bmatrix} u$$

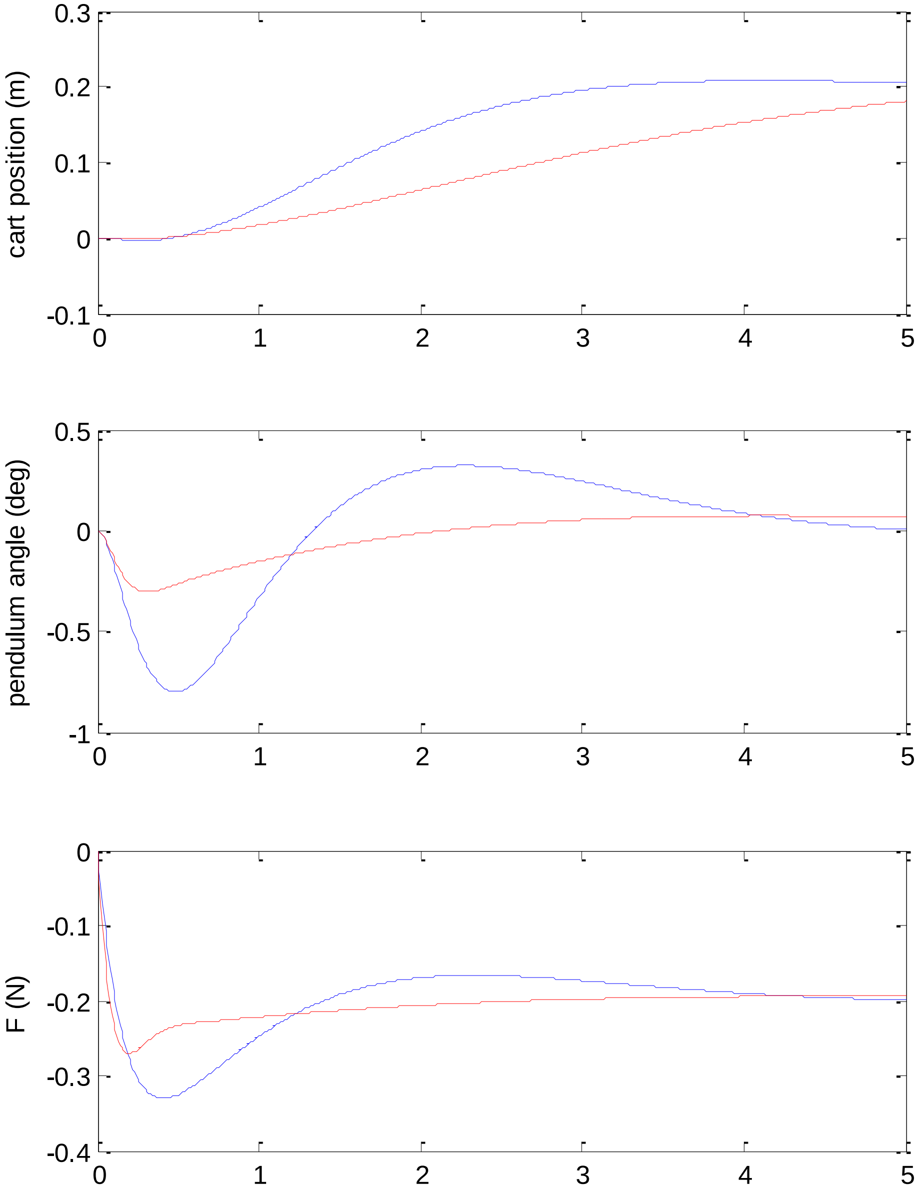
$$Q = \begin{pmatrix} c_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Bigg\} \quad \min \underline{c_1 x^2 + c_2 \phi^2 + c_3 u^2}$$

$$R = c_3$$



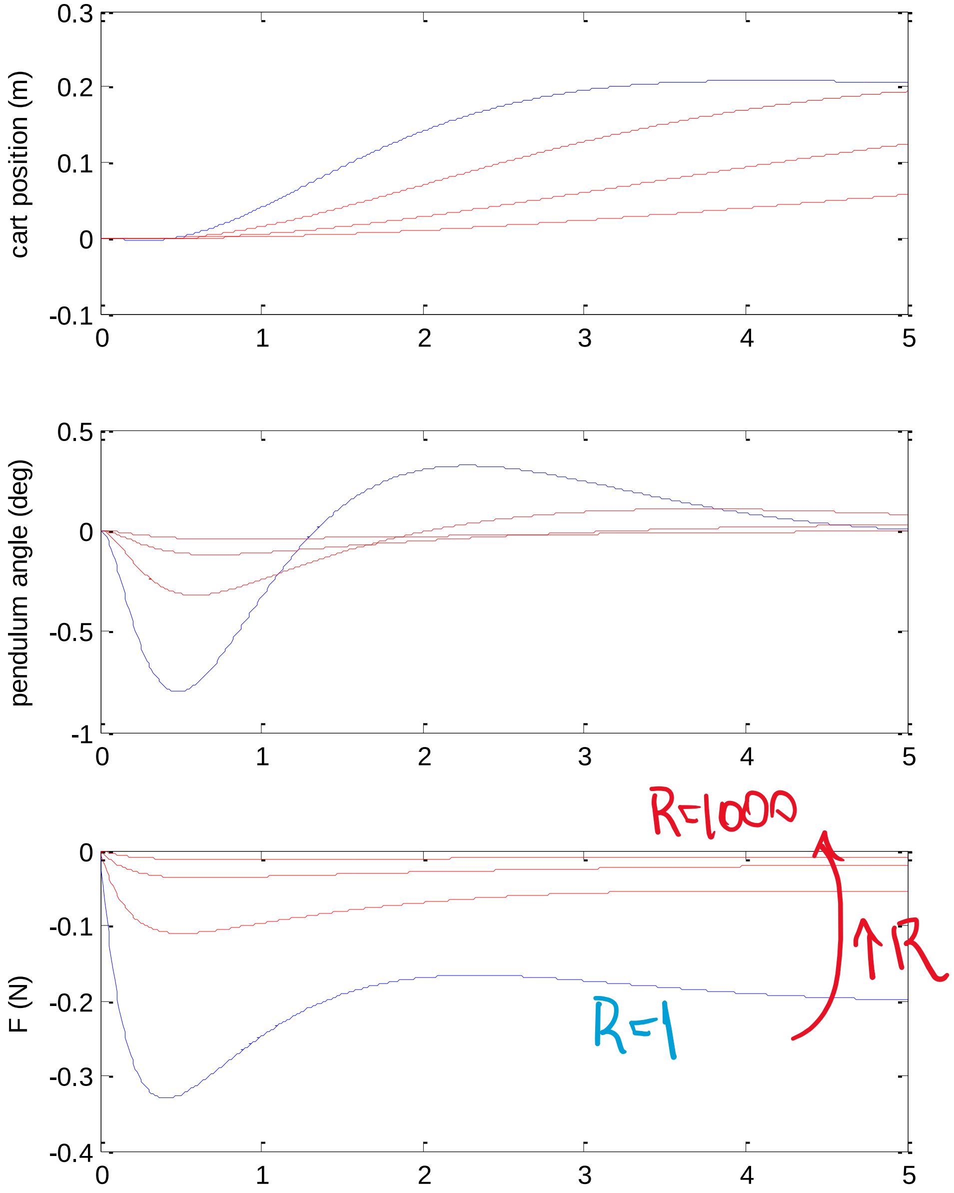
$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$Q = \begin{pmatrix} 100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



$$- \; Q = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$- \; Q = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



$$Q = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$R = 1, 10, 100, 1000$$