Galloping, flutter and vibrations induced by the internal flow in pipes discharging fluid

PEF 6000 - Special topics on dynamics of structures

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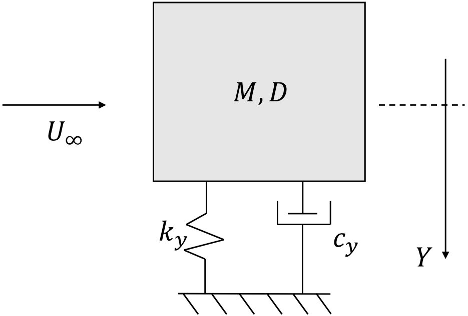
- The graduate course PNV5203 - Fluid-Structure Interaction 1 brings deeplier concepts on the theme.

- Contrary to VIV, galloping is not resonant;
- Example of application: In some countries with severe winters, the effective cross-section of cables is changed due to ice deposition;
- Galloping may cause very large oscillation amplitudes;
- Focus of the class: prismatic rigid bodies of mass *per* unit length M, assembled onto an elastic support of stiffness k_y and damping constant c_y, both *per* unit length. The characteristic dimension of the cross-section is D.

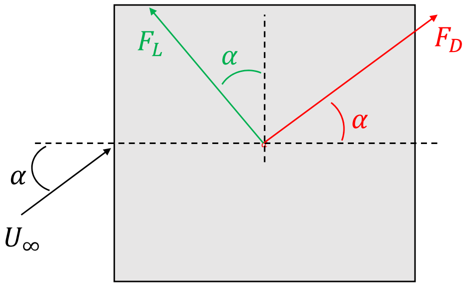


Extracted from Franzini (2019).

Equation of motion

$$M\frac{d^2Y}{dt^2} + c_y\frac{dY}{dt} + k_yY = F_y = \frac{1}{2}\rho U_\infty^2 DC_y \qquad (1)$$

- The force coefficient C_y is obtained by using the quasi-steady hypothesis;
- Consider an experiment in which the cylinder is fixed, with a certain angle of attack α and immersed on a free-stream of intensity U_∞;
- $C_{D,L}(\alpha) = F_{D,L}/0.5\rho U_\infty^2 D$



Extracted from Franzini (2019).

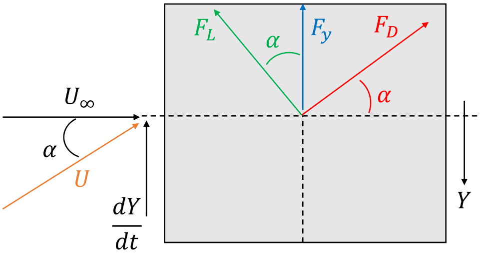
- Two time-scales can be defined, namely the one associated with the prism oscillation (its natural period) and that characteristic of the flow $T_f = D/U_\infty$;
- For the validity of the quasi-steady approach, it is necessary:

$$\frac{T_{n,y}}{T_f} >> 1 \leftrightarrow \frac{\frac{1}{f_{n,y}}}{\frac{D}{U_\infty}} >> 1 \leftrightarrow \frac{U_\infty}{f_{n,y} D} >> 1 \qquad (2)$$

- Blevins (2001) suggests:

$$\frac{U_\infty}{f_{n,y} D} > 20 \qquad (3)$$

- Physically, the validity of the quasi-steady hypothesis means that the vortex-shedding frequency is much higher than the structural natural frequency;
- Concomitant VIV and galloping may occur. However, this is out of the scope of this class. The interested reader should read Paidoussis et al. (2011).



Extracted from Franzini (2019).

$$\tan\alpha = \frac{\frac{dY}{dy}}{U_\infty} \tag{4}$$

$$U = \sqrt{\left(\frac{dY}{dt}\right)^2 + U_\infty^2} \tag{5}$$

- From the above figure, we have:

$$F_L = \frac{1}{2}\rho U^2 D C_L \tag{6}$$

$$F_D = \frac{1}{2}\rho U^2 D C_D \tag{7}$$

$$F_y = \frac{1}{2}\rho U_\infty^2 D C_y = -F_D \sin\alpha - F_L \cos\alpha \tag{8}$$

$$C_y = \left(\frac{U}{U_\infty}\right)^2 (-C_D \sin\alpha - C_L \cos\alpha) \tag{9}$$

- Assuming that the prism velocity is much smaller than the free-stream velocity

$$\tan\alpha \approx \alpha = \frac{\frac{dY}{dt}}{U_\infty} \tag{10}$$

$$U \approx U_\infty \tag{11}$$

$$C_y \approx -C_L(0) - \left(\frac{\partial C_L}{\partial \alpha} + C_D\right)_0 \alpha \tag{12}$$

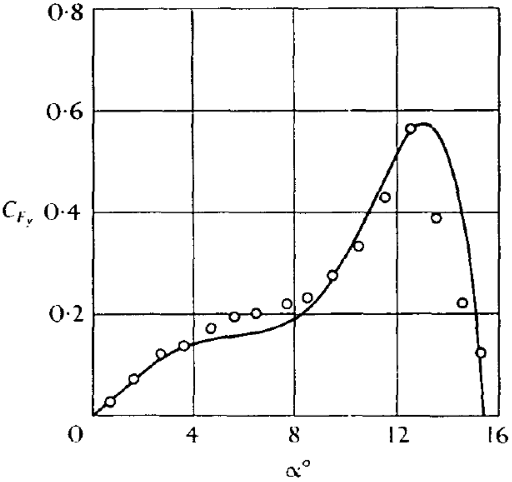
- Since $\alpha = \frac{\frac{dY}{dt}}{U_\infty}$, if $\left(\frac{\partial C_L}{\partial \alpha} + C_D\right)_0 < 0$, the trivial solution may become unstable, depending on $U\infty$;
- Usually, C_y is written in the form of a polynomial function of α in the form

$$C_y = \sum_{k=1}^{N} a_k \alpha^k = \sum_{k=1}^{N} a_k \left(\frac{\frac{dY}{dt}}{U_\infty}\right)^k \tag{13}$$

a_k being coefficients obtained from experiments.

- Using Eq. 13, the equation of motion for the prism under galloping reads

$$M\frac{d^2Y}{dt^2} + c_y\frac{dY}{dt} + k_y Y = \frac{1}{2}\rho U_\infty^2 D \sum_{k=1}^{N} a_k \left(\frac{\frac{dY}{dt}}{U_\infty}\right)^k \tag{14}$$



Extracted from Parksinson & Smith (1964).

- We consider the linearized version of the equation of motion. Notice that the term of the fluid force proportional to $\frac{dY}{dt}$ is included on the LHS:

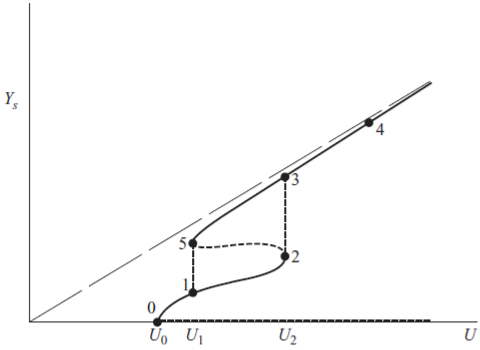
$$M\frac{d^2Y}{dt^2} + \left(c_y - \frac{1}{2}\rho D U_\infty a_1\right)\frac{dY}{dt} + k_y Y = 0 \tag{15}$$

- The equivalent damping is composed of two terms, one associated with the structure (c_y) and the second one from the fluid force.
- If $a_1 > 0$, the equivalent damping may be negative, depending on the free-stream velocity U_∞. In this case, the equivalent damping is null if

$$U_\infty^c = \frac{2c_y}{\rho a_1 D} \tag{16}$$

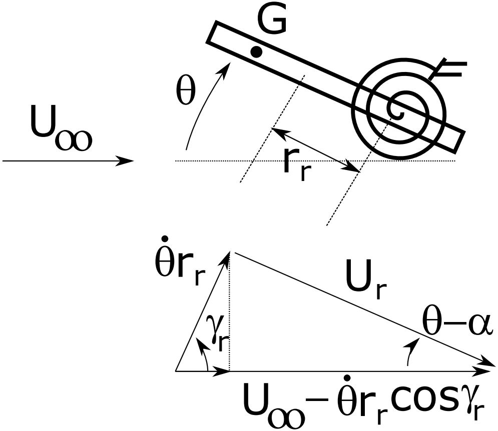
- If $U_\infty < U_\infty^c$ (sub-critical velocity), the structural response to small disturbances on the initial conditions are damped.
- On the other hand, the structural response to small disturbances grows as $t \to \infty$ if $U_\infty > U_\infty^c$;
- U_c^∞ is a critical velocity, above which the equilibrium position losses stability through a Hopf bifurcation.

- In the linear model, the structural response infinitely grows;
- Notice, however, that unbounded responses are not physically observed. As the responses grow, the neglected non-linear terms play the important role of limiting the structural responses, leading to a post-critical behavior;



Extracted from Paidoussis et al. (2011).

- The angle of attack varies from point to point due to the angular velocity. Hence, we can not define an equivalent static configuration for the proper application of the quasi-steady approach;
- A reference distance r_r is adopted for the quasi-steady model;



- From the above figure:

$$\theta - \alpha = atan\left(\frac{\dot{\theta} r_r \sin\gamma_r}{U_\infty - \dot{\theta} r_r \cos\gamma_r}\right) \tag{17}$$

- For small values of $\dot{\theta}$, $U_r \approx U_\infty$ and

$$\theta - \alpha = atan\left(\frac{\dot{\theta} r_r \sin\gamma_r}{U_\infty}\right) \approx \frac{\dot{\theta} r_r \sin\gamma_r}{U_\infty} \rightarrow \alpha = \theta - \frac{\dot{\theta} R}{U_\infty}; R = r_r \sin\gamma_r \tag{18}$$

- Equation of motion:

$$J\ddot{\theta} + c_\theta \dot{\theta} + k_\theta \theta = \frac{1}{2}\rho U_\infty^2 h^2 C_M(\alpha) \tag{19}$$

where h is the characteristic dimension of the body and C_M is the aerodynamic moment coefficient.

- We expand C_M is Taylor series around $\alpha = 0$ as:

$$C_M = C_M(0) + \left(\frac{\partial C_M}{\partial \alpha}\right)_0 \alpha + \mathcal{O}(\alpha^2) \tag{20}$$

- Substituting the above expression into the equation of motion:

$$J\ddot{\theta} + \left(c_\theta + \frac{1}{2}\rho U_\infty^2 h^2 \left(\frac{\partial C_M}{\partial \alpha}\right)_0 \frac{R}{U_\infty}\right)\dot{\theta} + \left(k_\theta - \frac{1}{2}\rho U_\infty^2 h^2 \left(\frac{\partial C_M}{\partial \alpha}\right)_0\right)\theta = \\ = \frac{1}{2}\rho U_\infty^2 h^2 C_M(0) \tag{21}$$

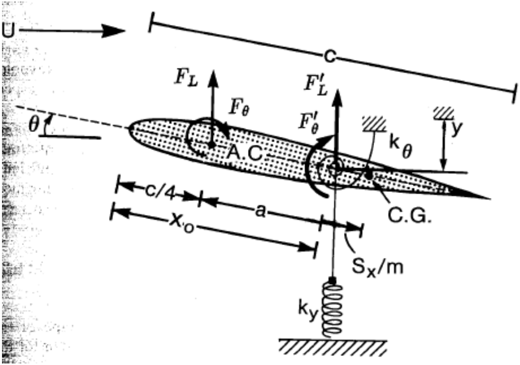
- Static instability (divergence) occurs if

$$k_\theta - \frac{1}{2}\rho U_\infty^2 h^2 \left(\frac{\partial C_M}{\partial \alpha}\right)_0 < 0 \tag{22}$$

- Dynamic instability occurs if

$$c_\theta + \frac{1}{2}\rho U_\infty h^2 \left(\frac{\partial C_M}{\partial \alpha}\right)_0 R < 0 \tag{23}$$

Sketch



Extracted from Blevins (2001).

- Equations of motion

$$m\ddot{y} + c_y\dot{y} + k_y y + S_x\ddot{\theta} = F_y' \tag{24}$$

$$J_\theta\ddot{\theta} + c_\theta\dot{\theta} + k_\theta\theta + S_x\ddot{y} = F_\theta' \tag{25}$$

- $F_{L,D} = \frac{1}{2}\rho U^2 c C_{L,D}$ and $F_\theta = \frac{1}{2}\rho U^2 c^2 C_M$
- For small angles of attack α (without stall), $C_L >> C_D$ and, consequently $F_y \approx -F_L$ (pay attention to the signal convention);
- Since the moment at the aerodynamic center (point associated with minimum aerodynamic moment) is small, $F_\theta' = F_\theta + F_L a = F_L a$;
- Using quasi-steady hypothesis, the relative velocity is similar to the one obtained for the torsional galloping. Notice, however, that the structural velocity $\dot{y}$ must be also taken into account. Then, we have

$$\alpha = \theta - \frac{R}{U}\dot{\theta} + \frac{\dot{y}}{U_\infty} \tag{26}$$

- In a steady hypothesis, $\alpha = \theta$.

- Notice that

$$F_L = \frac{1}{2}\rho U^2 c C_L(\alpha) = \frac{1}{2}\rho U^2 c \left(C_L(0) + \left(\frac{\partial C_L}{\partial \alpha}\right)_0 \alpha + \mathcal{O}(\alpha^2) \right) \tag{27}$$

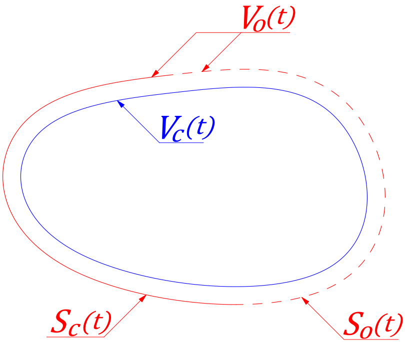
- Substituting this linearized equation in α into the equations of motion, we have:

$$m\ddot{y} + c_y \dot{y} + k_y y + S_x \ddot{\theta} = -\frac{1}{2}\rho U^2 c C_L(\alpha) = \frac{1}{2}\rho U^2 c \left(C_L(0) + \left(\frac{\partial C_L}{\partial \alpha}\right)_0 \theta \right) \tag{28}$$

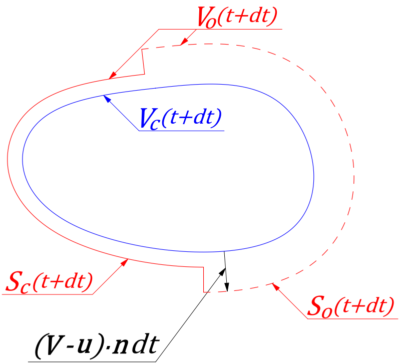
$$J_\theta \ddot{\theta} + c_\theta \dot{\theta} + k_\theta \theta + S_x \ddot{y} = \frac{1}{2}\rho U^2 a c C_L(\alpha) = \frac{1}{2}\rho U^2 c \left(C_L(0) + \left(\frac{\partial C_L}{\partial \alpha}\right)_0 \theta \right) \tag{29}$$

- The above equations define an autonomous system. The stability of the equilibrium points can be easily studied using the Lyapunov indirect's method with U as the control parameter.

- Classical and important dynamic problem, found in a series of engineering applications such as heat exchangers and risers. A survey on the theme is found in Païdoussis (1998). This class follows the derivation presented in Païdoussis (1998) and also detailed in the qualifying report Maciel (2020),;
- Open-system problem: The total mass is not constant or the system is not composed by the same particles $\rightarrow$ There is flux of momentum and/or kinetic energy across the boundary;
- Focus of the class: Linear and planar dynamics of a cantilevered pipe discharging fluid.
- Hypothesis:
  1. Small displacements, rotations and strains (linear elasticity);
  2. Bernoulli-Euler beam model;
  3. Horizontal pipe (no gravitational effect is considered) of mass *per* unit length m, length L and bending stiffness EI;
  4. plug-flow model: Fluid has mass *per* unit length M, incompressible flow, the velocity profile is uniform (there is no internal boundary layer) and is equal to U (in relation to the pipe).



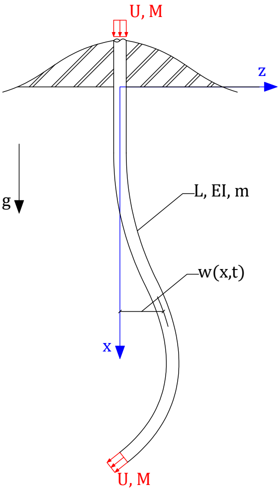
(a) System at t.



(b) System at $t + dt$.

Extracted from Maciel (2020).

- Consider the closed system, defined by a volumn $V_c(t)$ and bounded by $S_c(t)$. In addition, consider an open system $V_o(t)$ bounded by $S_c(t) \cup S_o(t)$.
- $\boldsymbol{v_p}$ is the instantaneous velocities of the particles pertaining to the pipe and $\boldsymbol{u}$ refers to the velocities of the fluid particles;
- We interpret $S_c(t)$ as the surface associated with the pipe wall and $S_o(t)$ the inlet/outlet. S_o moves with velocity $(\boldsymbol{V}.\boldsymbol{n}).boldsymboln$, $\boldsymbol{n}$ is the outward normal.

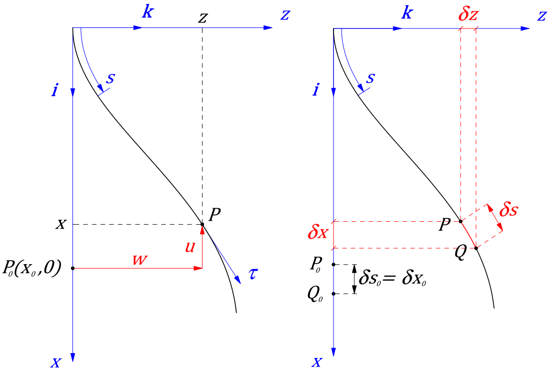


Extracted from Maciel (2020).

- From McIver (1973), the formulation for an open system reads:

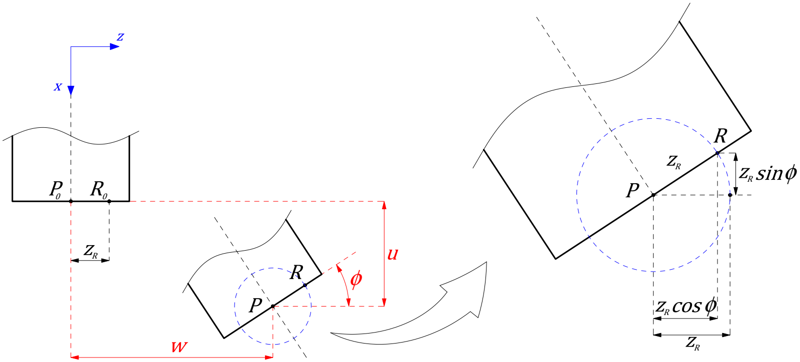
$$\int_{t_1}^{t_2} \left\{ \delta L_o + \delta W + \iint_{B_o(t)} \rho(\mathbf{u} \cdot \delta \mathbf{r})(\mathbf{V} - \mathbf{u}) \cdot \mathbf{n} dS \right\} dt = 0 \quad (30)$$

- L_o is the Lagrangean of the open system, δW is the virtual work of the non-conservative forces and the third term is the flux of momentum across the surface of the open system;
- Casetta & Pesce (2013) obtained a more general formulation for open systems which includes the flux of kinetic energy accross the surface. Notice, however, that for compressible internal flows, the latter term is important.



(c) Definitions. (d) Inextensibility.

Extracted from Maciel (2020).



Extracted from Maciel (2020).

- It trivial to observe that

$$
\begin{aligned}
u &= x - x_0 \\
w &= z - z_0 = z
\end{aligned}
\tag{31}
$$

- For small rotations

$$
\begin{aligned}
&\sin\phi \approx \tan\phi \approx \phi \\
&\cos\phi \approx 1
\end{aligned}
\tag{32}
$$

- From the above approximation:

$$
\begin{aligned}
u_R &= u - z_R \frac{\partial w}{\partial x} \\
w_R &= w, \, w/L \sim \mathcal{O}(\epsilon)
\end{aligned}
\tag{33}
$$

- The inextensibility condition is written as $\delta s = \delta s_0$, with

$$
\begin{aligned}
(\delta s)^2 &= (\delta x)^2 + (\delta z)^2 \\
(\delta s_0)^2 &= (\delta x_0)^2 + (\delta z_0)^2 = (\delta x_0)^2
\end{aligned}
\tag{34}
$$

- The inextensibility condition can be written as function of the displacements as:

$$\left(1+\frac{\partial u}{\partial s}\right)^2+\left(\frac{\partial w}{\partial s}\right)^2=1 \tag{35}$$

- Expanding the inextensibility condition

$$1+2\frac{\partial u}{\partial s}+\left(\frac{\partial u}{\partial s}\right)^2+\left(\frac{\partial w}{\partial s}\right)^2=1 \tag{36}$$

- Neglecting the term $\left(\frac{\partial u}{\partial s}\right)^2$, we have

$$u \cong -\int_0^s \frac{1}{2}\left(\frac{\partial w}{\partial s}\right)^2 ds,\, u/L \sim \mathcal{O}(\epsilon^2) \tag{37}$$

- The velocity of a point pertaining to the centreline is

$$\mathbf{v}_p = \dot{x}\mathbf{i} + \dot{z}\mathbf{k} \tag{38}$$

- The velocity of a particle of fluid reads ($\boldsymbol{\tau} = \frac{\partial x}{\partial s}\boldsymbol{i} + \frac{\partial z}{\partial s}\boldsymbol{k} = x'\boldsymbol{i} + z'\boldsymbol{k}$):

$$\mathbf{u} = \mathbf{v}_p + U\boldsymbol{\tau} = \dot{x}\mathbf{i} + \dot{z}\mathbf{k} + Ux'\mathbf{i} + Uz'\mathbf{k} = (\dot{x} + Ux')\mathbf{i} + (\dot{z} + Uz')\mathbf{k} \tag{39}$$

- Term of the kinetic energy associated with the pipe:

$$\mathcal{T}_p = \frac{1}{2}m\int_0^L (\mathbf{v}_p \cdot \mathbf{v}_p)ds = \frac{1}{2}m\int_0^L (\dot{x}^2 + \dot{z}^2)ds \tag{40}$$

- Term of the kinetic energy associated with the internal fluid:

$$\mathcal{T}_f = \frac{1}{2}M\int_0^L (\mathbf{u} \cdot \mathbf{u})ds = \frac{1}{2}M\int_0^L [(\dot{x} + Ux')^2 + (\dot{z} + Uz')^2]ds \tag{41}$$

- We are interested in obtaining a linear mathematical model. Hence, the kinetic energy must contain up to quadratic terms. With this in mind and recalling that $\dot{z} = \dot{w}$, we have the following quantities associated with the pipe

$$\mathcal{T}_p = \frac{1}{2} m \int_0^L (\dot{x}^2 + \dot{z}^2) ds = \frac{1}{2} m \int_0^L \dot{z}^2 ds = \frac{1}{2} m \int_0^L \dot{w}^2 ds \tag{42}$$

$$\int_{t_1}^{t_2} \delta \mathcal{T}_p dt = \int_{t_1}^{t_2} \left(\frac{1}{2} m \int_0^L 2\dot{w}\delta\dot{w} ds \right) dt = \frac{1}{2} m \int_0^L \int_{t_1}^{t_2} 2\dot{w}\delta\dot{w} dt ds =$$

$$= \frac{1}{2} m \int_0^L \left[[2\dot{w}\delta w]_{t_1}^{t_2} - \int_{t_1}^{t_2} 2\ddot{w}\delta w dt \right] ds = - \int_{t_1}^{t_2} \int_0^L m\ddot{w}\delta w ds dt \tag{43}$$

- We have the expressions below for the fluid

$$\mathcal{T}_f = \frac{1}{2} M \int_0^L (U^2 + \dot{w}^2 + 2U\dot{w}w' + 2U\dot{u}) ds \tag{44}$$

$$\int_{t_1}^{t_2} \delta \mathcal{T}_f dt = - \int_{t_1}^{t_2} \int_0^L (M\ddot{w} + 2MU\dot{w}') \delta w ds dt + \int_{t_1}^{t_2} (MU\dot{w}_L \delta w_L) dt \tag{45}$$

- Since we are considering a horizontal pipe, the gravitational acceleration plays no role. However, it can be easily included in the formulation.
- The relevant strain is

$$\varepsilon_R = \frac{\partial(u_R)}{\partial s} = \frac{\partial(u - z_R w')}{\partial s} = u' - z_R w'' \tag{46}$$

- From the constitutive equation (Hooke's law), $\sigma_R = E\varepsilon_R$. The potential energy due to the solid deformation and its first variation are:

$$\mathcal{U}_{def} = \iiint_V \frac{\sigma_R \varepsilon_R}{2} dV = \int_0^L \iint_A \frac{\sigma_R \varepsilon_R}{2} dAds \quad = \int_0^L \iint_A \frac{E\varepsilon_R^2}{2} dAds \tag{47}$$

$$\delta\mathcal{U}_{def} = \int_0^L \iint_A E\varepsilon_R \delta\varepsilon_R dAds \tag{48}$$

- Neglecting higher-order terms and after some manipulations already discussed in the course, we have

$$\delta\mathcal{U}_{def} = \int_0^L EIw'' \delta w'' ds \tag{49}$$

- Integrating by parts twice, recalling that virtual displacements vanish at t_1 and t_2 and using the essential boundary conditions $\delta w_0 = \delta w_L' = 0$, we write

$$\int_{t_1}^{t_2} \delta \mathcal{U}_{def}\, dt = \int_{t_1}^{t_2} \left(EIw_L'' \delta w_L' - EIw_L''' \delta w_L + \int_0^L EIw'''' \delta w ds \right) dt \qquad (50)$$

- Now, we consider the term associated with the non-conservative force due to pressure. The control surface is $\partial V_o = S_i \cup S_e \cup S_c$, where S_i is the inlet, S_e the outlet (both open) and S_c is the pipe wall (closed surface).

$$\iint_{\partial V_o} p(\delta \mathbf{r} \cdot \mathbf{n}) d\partial V_o \qquad (51)$$

- In the above equation, $\delta \boldsymbol{r} = 0$ at the inlet (clamp). We assume that the pipe (excepted the inlet) is at atmospheric pressure ($p = 0$), this integral vanishes.

- The term associated with the flux of momentum is

$$\iint_{\partial V_o} \rho(\mathbf{u} \cdot \delta \mathbf{r})(\mathbf{u} - \mathbf{V}) \cdot \mathbf{n} d\partial V_o \tag{52}$$

- At the inlet, $\delta \boldsymbol{r} = 0$. On the pipe wall, there is no flux of momentum. Hence, only the outlet contributes with this term.
- At the outlet, $\boldsymbol{u_L} = \dot{\boldsymbol{r}}_{\boldsymbol{L}} + U\boldsymbol{\tau_L}$ and $(\boldsymbol{u} - \boldsymbol{V}) \cdot \boldsymbol{n} = U$ and the above integral is

$$MU(\dot{\boldsymbol{r}}_{\boldsymbol{L}} + U\boldsymbol{\tau_L}) \cdot \delta \boldsymbol{r_L} =$$

$$MU\dot{u}_L\delta u_L + MU^2\delta u_L - \frac{1}{2}MU^2 {w_L'}^2 \delta u_L + MU\dot{w}_L\delta w_L + MU^2 w_L' \delta w_L \tag{53}$$

- Neglecting higher-order terms, the above equation is given by

$$MU^2\delta u_L + MU(\dot{w}_L + Uw_L')\delta w_L \tag{54}$$

- Notice that

$$-MU^2 \int_{t_1}^{t_2} \delta u_L dt = -MU^2 \int_{t_1}^{t_2} \delta \left[\int_0^L \left(-\frac{1}{2} {w'}^2 \right) ds \right] dt = +MU^2 \int_{t_1}^{t_2} \int_0^L w' \delta w' ds dt$$

Final equation of (transverse) motion

- Integrating from t_1 to t_2 of the classical variational problem:

$$\int_{t_1}^{t_2}\left[\delta W - \iint_{\partial V_o} \rho(\mathbf{u}\cdot\delta\mathbf{r})(\mathbf{u}-\mathbf{V})\cdot\mathbf{n}d\partial V_o\right]dt = \tag{55}$$

$$= -MU^2\int_{t_1}^{t_2}\int_0^L w''\delta w ds dt - MU\int_{t_1}^{t_2}\dot{w}_L\delta w_L dt$$

- Using the discussed quantities into the extended version of the Hamilton's principle leads to the equation of motion given by:

$$EIw'''' + MU^2w'' + 2MU\dot{w}' + (m+M)\ddot{w} = 0 \tag{56}$$

- Notice that the second and the third terms are associated with the centrifugal and the Corioli's forces, respectively.

- The use of the extended version of the Hamilton's principle leads to the following natural boundary conditions:

$$EIw_L''' = 0$$
$$EIw_L'' = 0$$

  corresponding to null shear force and bending moment at the tip of the cantilevered pipe discharging fluid.
- The dynamics of the pipe discharging fluid can be investigated using the Galerkin's method. For this, the natural modes of the cantilevered beam not conveying fluid can be used as projection functions.
- The application of the Galerkin's method leads, after some algebraic work, to a system of first-order autonomous equations.
- The stability of the vertical configuration can then be investigated using the Lyapunov's indirect method and the internal flow velocity as the control parameter.
- Usually, the minimum value of U that leads to instability of the vertical equilibrium configuration is know as critical velocity.
- Investigations concerning the effects of lumped-masses placed along the pipe can be easily made using the Dirac delta function (see the qualification report by Maciel (2020)).