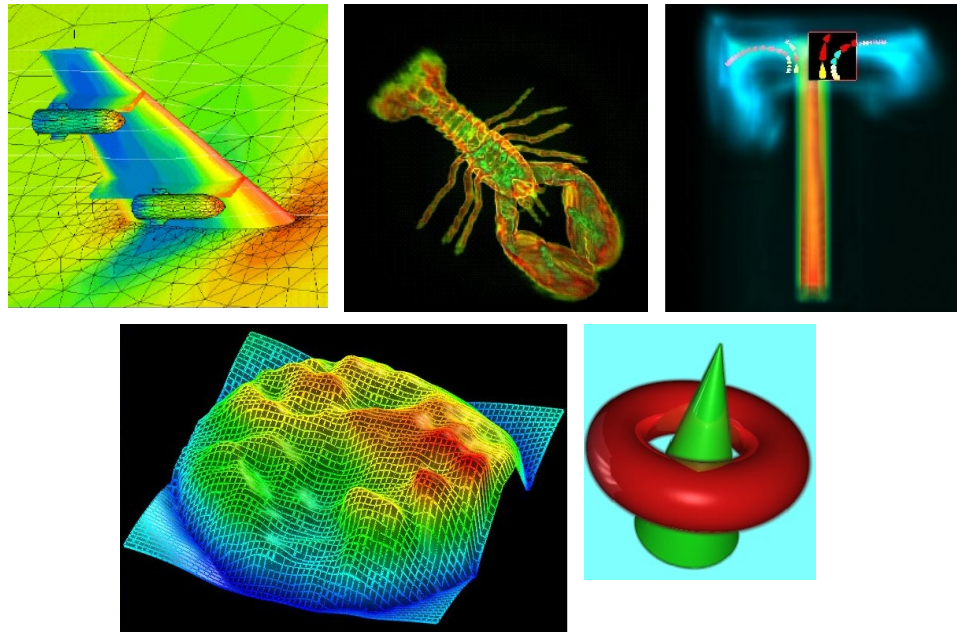# Scientific Visualization Module 3
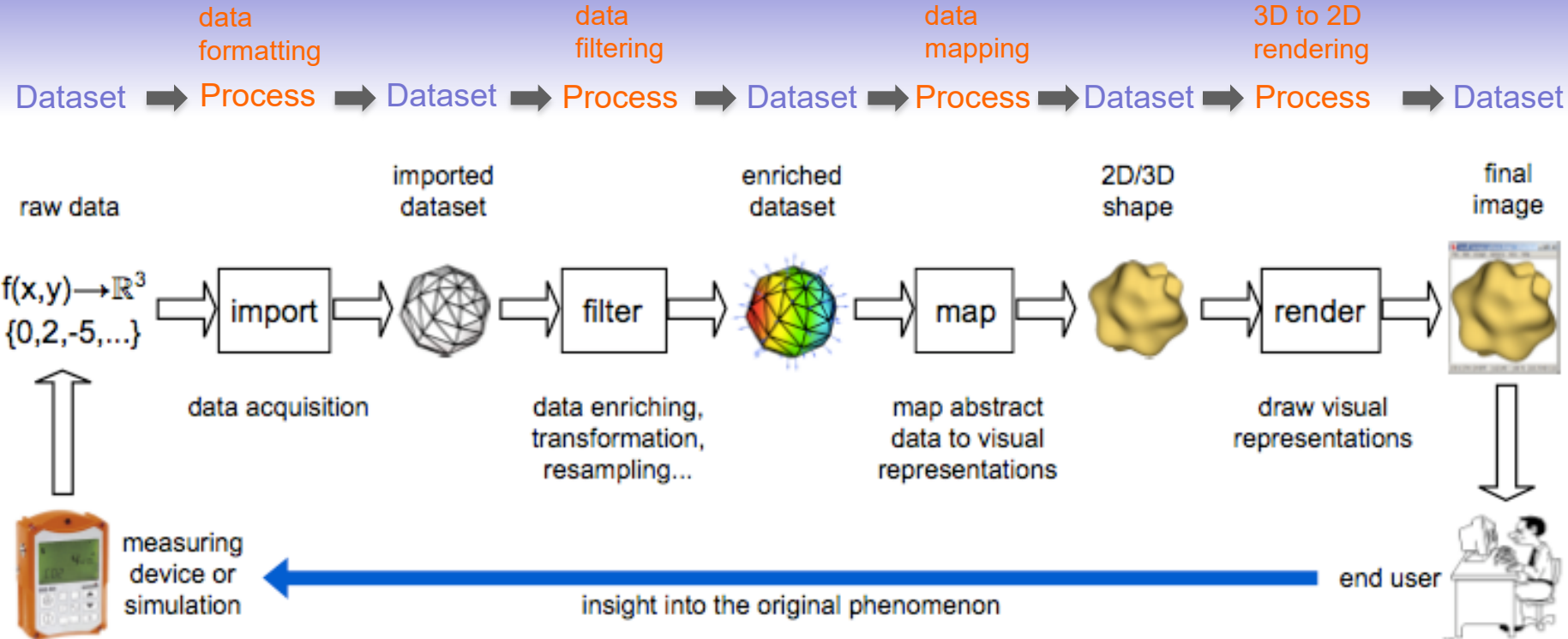## *Scalar Algorithms*

prof. dr. Alexandru (Alex) Telea

Department of Mathematics and Computer Science
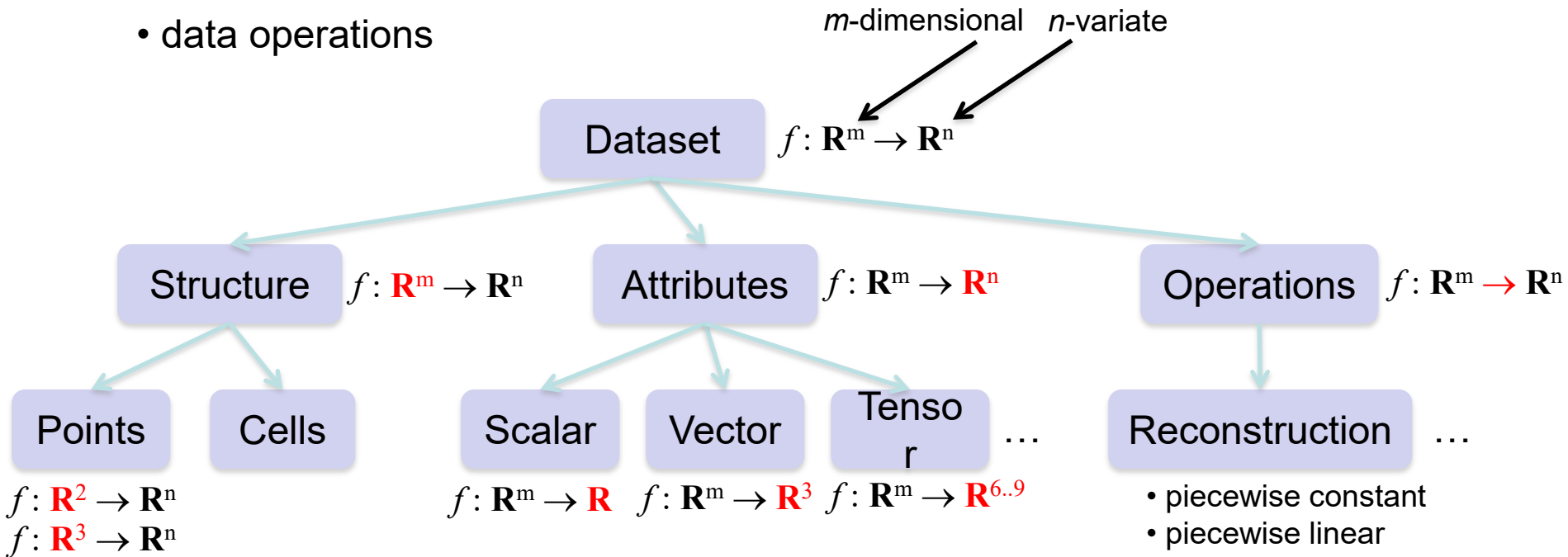University of Groningen, the Netherlands

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The (spatial data) Visualization Pipeline - Recall

data formatting — data filtering — data mapping — 3D to 2D rendering

Dataset ➡ **Process** ➡ Dataset ➡ **Process** ➡ Dataset ➡ **Process** ➡ Dataset ➡ **Process** ➡ Dataset

raw data — imported dataset — enriched dataset — 2D/3D shape — final image

$f(x,y) \rightarrow \mathbb{R}^3$
$\{0,2,-5,...\}$

import → filter → map → render

data acquisition — data enriching, transformation, resampling... — map abstract data to visual representations — draw visual representations

measuring device or simulation ← insight into the original phenomenon — end user

Scientific Visualization – The Dataset

Dataset

- key notion in visualization (SciVis, InfoVis, SoftVis)
- captures all relevant characteristics of a data collection
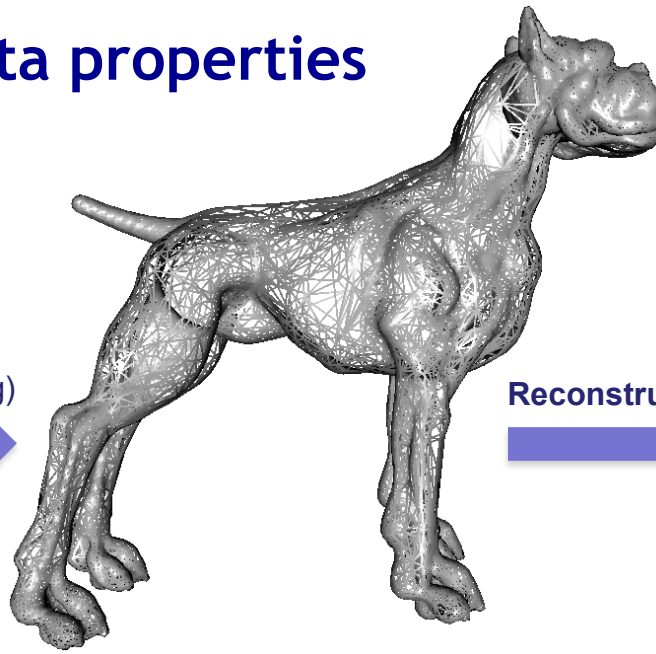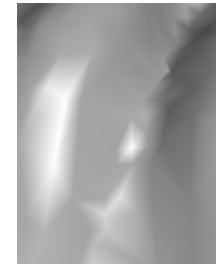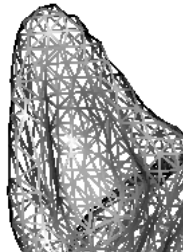 - structure
  - data values
  - data operations

m-dimensional   *n*-variate

Dataset $f : \mathbf{R}^m \to \mathbf{R}^n$

Structure $f : \mathbf{R}^m \to \mathbf{R}^n$ **Attributes** $f : \mathbf{R}^m \to \mathbf{R}^n$ **Operations** $f : \mathbf{R}^m \to \mathbf{R}^n$

Points   **Cells**   **Scalar**   **Vector**   **Tensor** …   **Reconstruction** …

$f : \mathbf{R}^2 \to \mathbf{R}^n$

$f : \mathbf{R}^3 \to \mathbf{R}^n$

$f : \mathbf{R}^m \to \mathbf{R}$ $f : \mathbf{R}^m \to \mathbf{R}^3$ $f : \mathbf{R}^m \to \mathbf{R}^{6..9}$

- piecewise constant
- piecewise linear

We'll detail all these next

Visualization data properties

Sampling
(data importing)

Reconstruction

$$f : \mathrm{D} \rightarrow \mathrm{C}$$
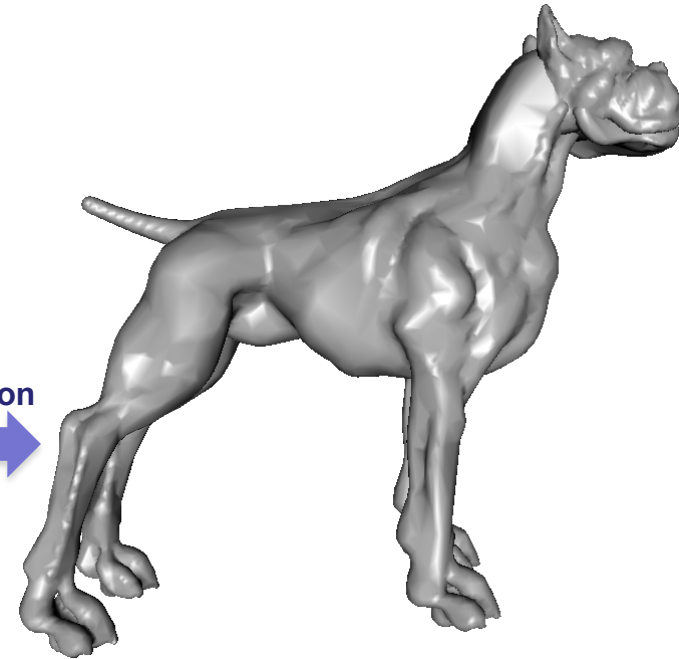
Sampling

$$\{p_i, f_i\} \quad \begin{matrix} p_i \in \mathrm{D} \\ f_i \in \mathrm{C} \end{matrix}$$

Reconstruction

$$\tilde{f} : \mathrm{D} \rightarrow \mathrm{C} \quad \tilde{f}(p_i) = f(p_i) = f_i$$

Continuous data

Measurements (samples)
at discrete set of points

Continuous data, as close as possible to input
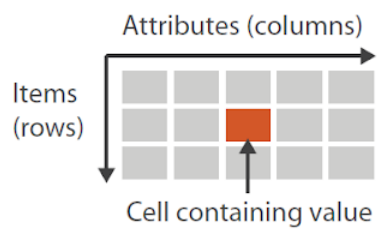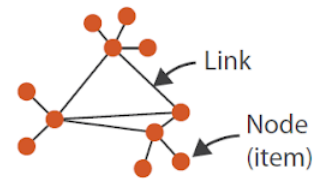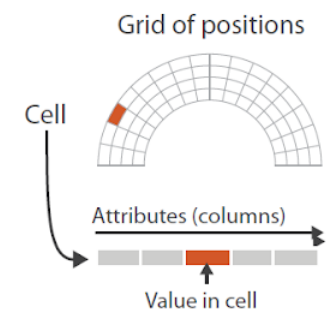
---

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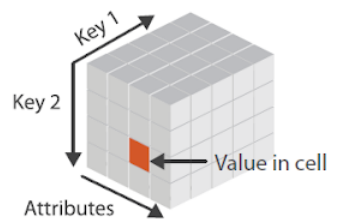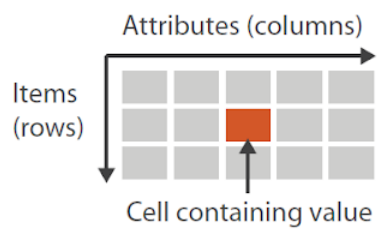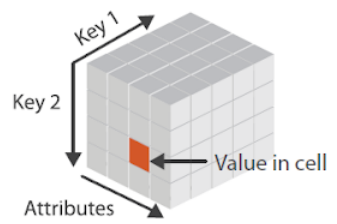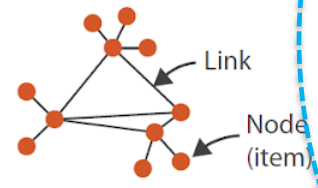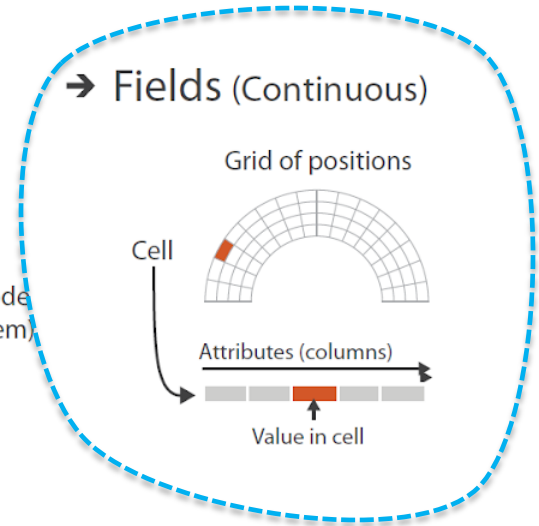
Munzner, Fig. 2.2

Munzner, Fig. 2.2

Algorithm classification

1. **Scalar algorithms**
   - operate on scalar data
   - color mapping, contouring, height plots

2. **Vector algorithms**
   - operate on vector data
   - hedgehogs, glyhps, derived quantities, stream surfaces, image-based methods

3. **Tensor algorithms**
   - operate on symmetric 3x3 tensors
   - tensor glyphs, hyperstreamlines, fiber tracing, principal component analysis

4. **Modeling algorithms**
   - change attributes and/or underlying **grid**
   - implicit functions, distance fields, cutting, selection, grid-less interpolation, grid processing

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Algorithm classification

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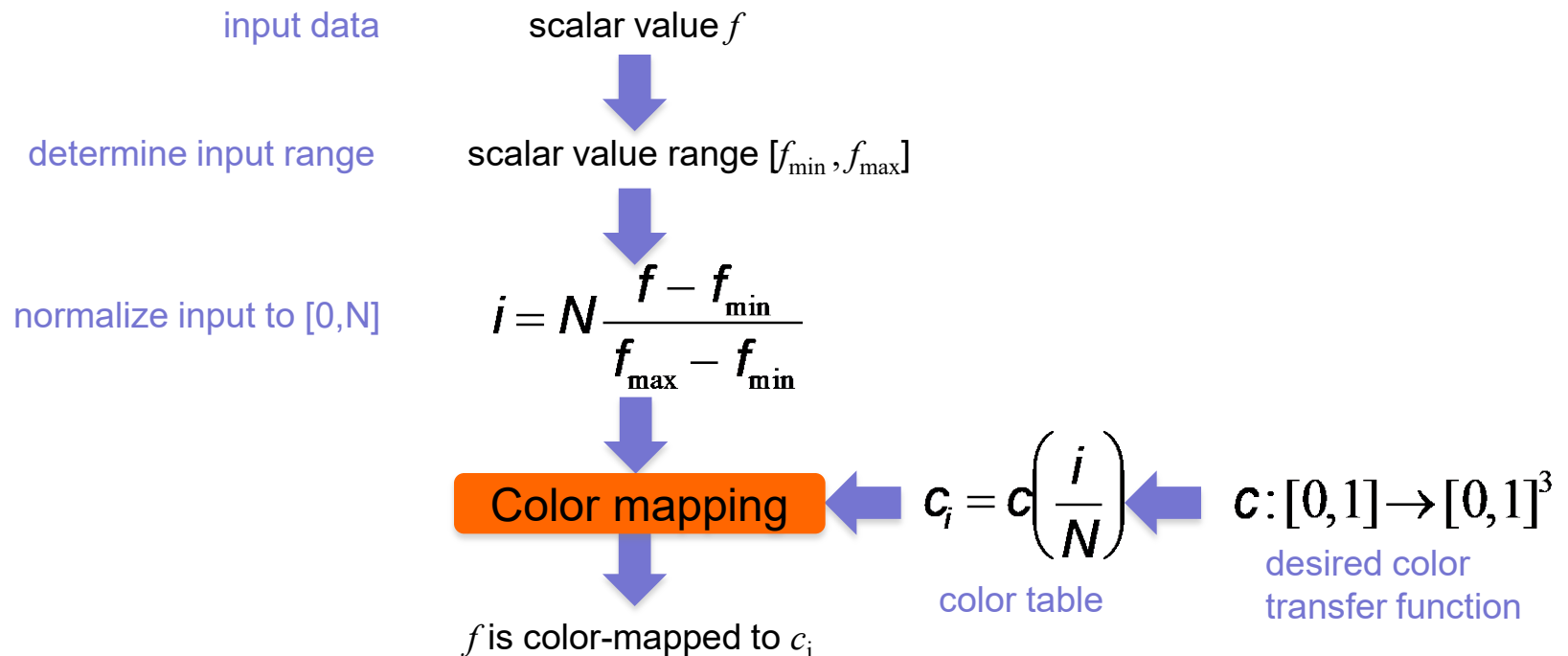   - implicit functions, distance fields, cutting, selection, grid-less interpolation, grid processing

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Color mapping

Basic idea

- Map each scalar value $f \in \mathbf{R}$ at a point to a color via a function $c : [0,1] \rightarrow [0,1]^3$

Color tables

- precompute (sample) c and save results into a table $\{c_i\}_{i=1..N}$
- index table by normalized scalar values

input data   scalar value f

determine input range   scalar value range $[f_{\min}, f_{\max}]$

normalize input to [0,N] $$i = N \frac{f - f_{\min}}{f_{\max} - f_{\min}}$$

Color mapping $\leftarrow$ $c_i = c\left(\dfrac{i}{N}\right)$ $\leftarrow$ $c : [0,1] \rightarrow [0,1]^3$

color table   desired color transfer function
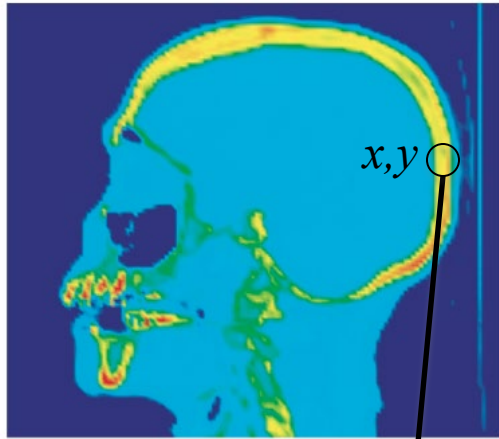
f is color-mapped to c_i

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Colormap design

What makes a good colormap?

- map scalar values to colors *intuitively*…
- …so we can visually *invert* the mapping to tell scalar values from colors

Recall example in Module 1

Data values mapped to RGB colors via a colormap

Invert mapping:
1. look at some point (x,y) in the image $\rightarrow$ color c
2. locate c in colormap at some position p
3. use the colormap legend to derive data value s from p

x,y

blue=0 | red=100

p answer: $s = 90$

Gray-value colormap

- brightness = value
- natural in some domains (X-ray, angiography)

2D slice in 3D CT dataset
Scalar value: tissue density

Gray-value colormap
•white = hard tissues (bone)
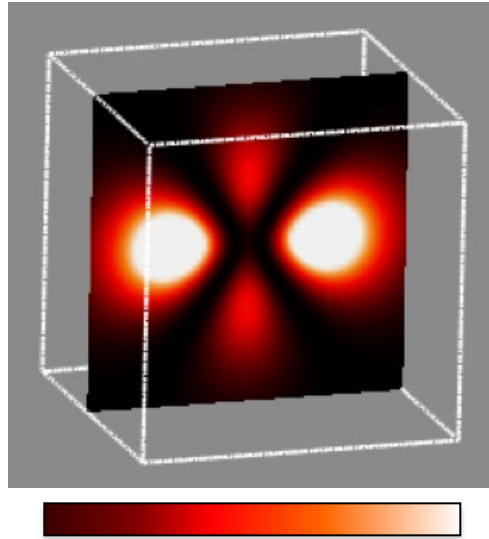•gray = soft tissues (flesh)
•black = air
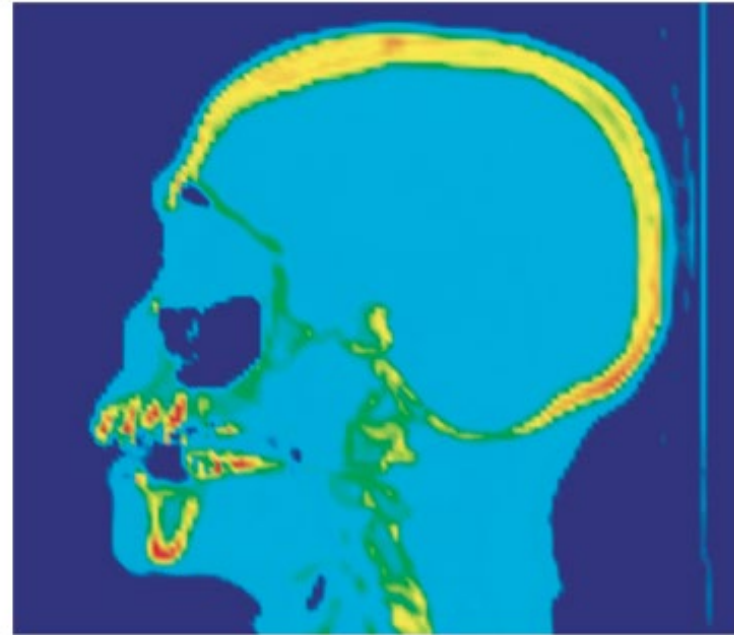
Rainbow colormap
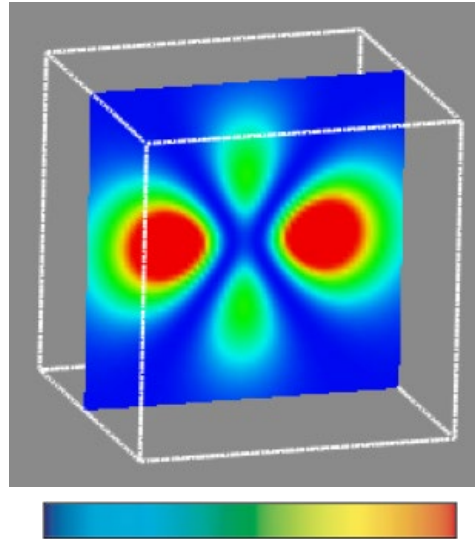•red = hard tissues (bone)
•blue = air
•other colors = soft tissues

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Colormap comparison

2D slice in 3D hydrogen atom potential field

Heat colormap
• maxima highlighted well
• lower values better
separable than with
gray-value colormap

Rainbow colormap
•   maxima not prominent
•   lower values better
•   separable
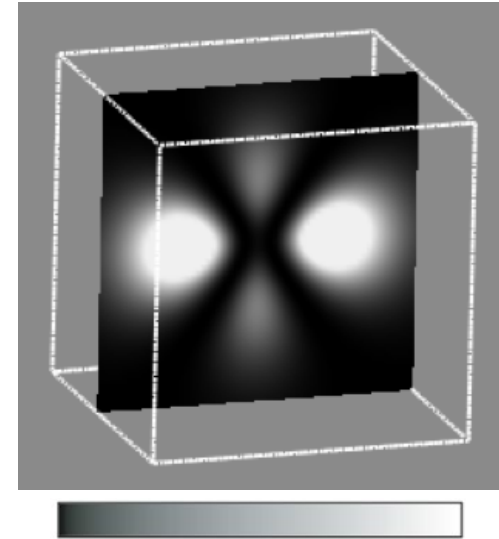
Gray-value colormap
• maxima are highlighted well
• lower values are unclear

Which is the better colormap? Depends on the application context!

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Colormap design techniques

We cannot give universal design rules
- but some technical guidelines/tricks still exist

1. Fully use the perceptual spectrum
- colormap entries should differ in more, rather than less, HSV components

scalar value ~ V; H,S not used

scalar value ~ H; S,V not used

scalar value ~ H,V; S not used

2. Colormap should be easily invertible
- avoid colormap entries with
    - similar HSV entries
    - which are *perceived* as similar (see color blindness issues)
    - which are hard to perceive (e.g. dark or strongly desaturated colors)

Good design guidelines: www.colorbrewer.org

www.cs.rug.nl/svcg

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Colormap design techniques

3. Design based on what you *need* to emphasize

- specific value ranges
- specific values
- value change rate (1st derivative of scalar data)
- …

2D function $f(x,y) = e^{-10(x^4 + y^4)}$
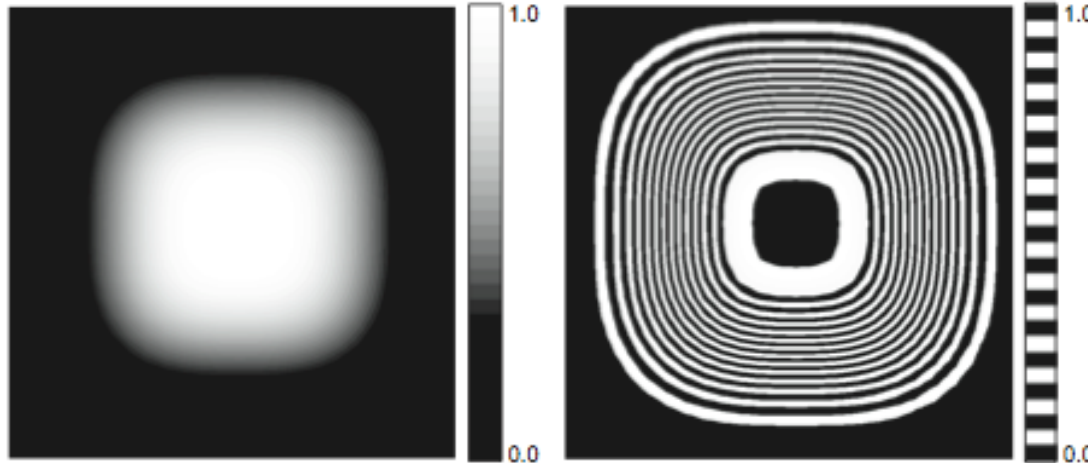
Gray-scale colormap
- highlights plateaus
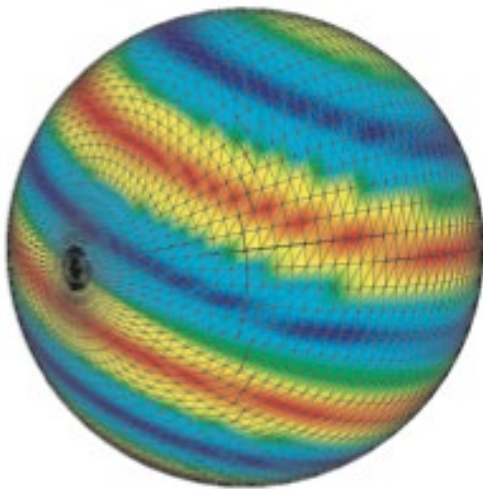- value transitions hard to see

Zebra colormap
- highlights value variations (1st derivative)
- dense, thin bands: fast variation
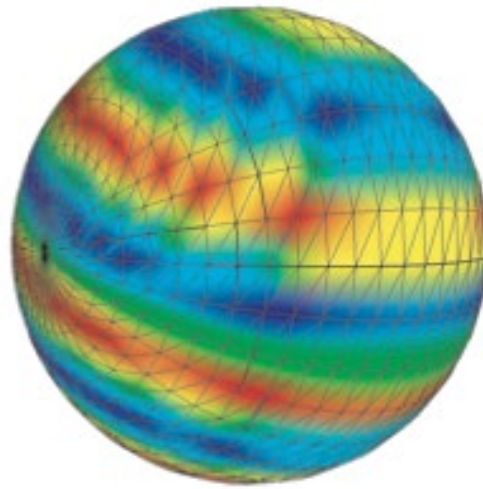- thick bands: slow variation

Colormap implementation details

Where to apply the colormap?

- per grid-cell vertex

2D periodic high-frequency function
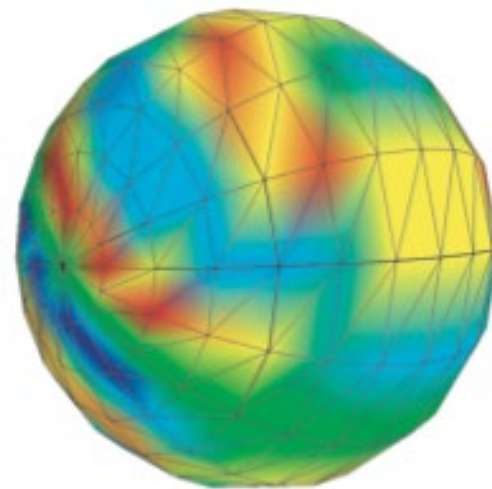
64x64 points   32x32 points   16x16 points

As we decrease the sampling frequency, strong colormapping artifacts appear
Why is this so?

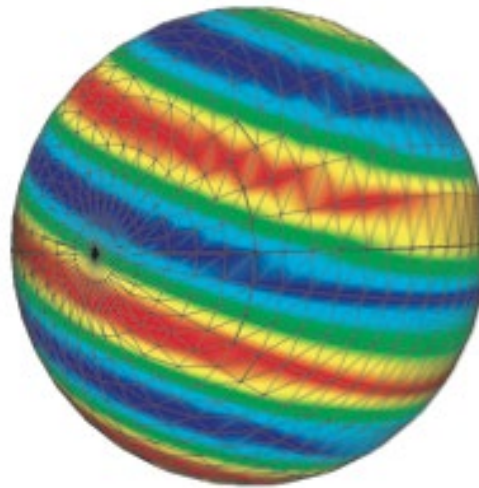www.cs.rug.nl/svcg   university of groningen

Colormap implementation details

Where to apply the colormap?
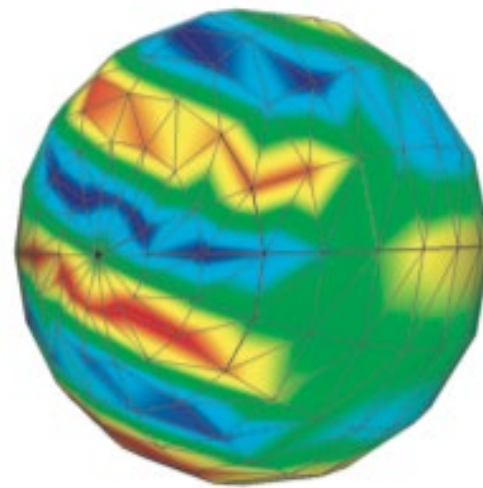- per pixel drawn – better results than per-vertex colormapping
- done using 1D textures

2D periodic high-frequency function

| 64x64 points | 32x32 points | 16x16 points |

Explanation
- per-vertex: $f \rightarrow c(f) \rightarrow \text{interpolation}(c(f))$ color interpolation can fall outside colormap!
- per-pixel: $f \rightarrow \text{interpolation}(f) \rightarrow c(\text{interpolation}(f))$ colors always stay in colormap
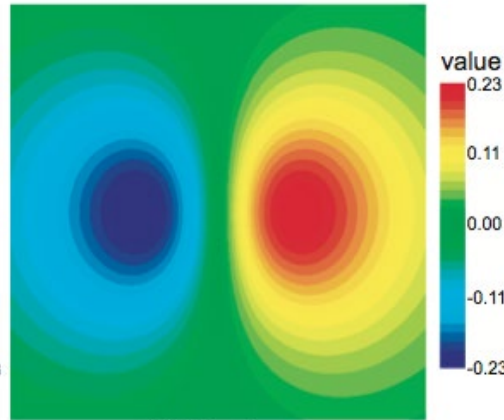
See Sec. 5.2 for details

18

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Color banding

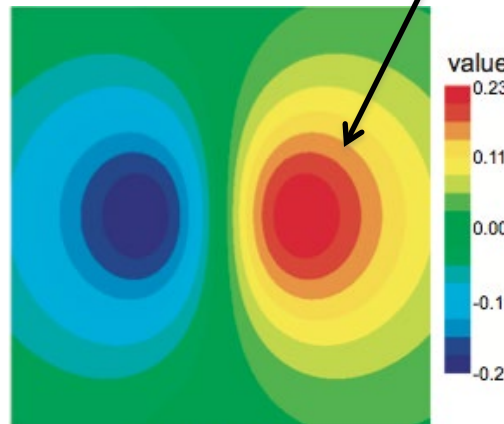**How many distinct colors N to use in a color table?**

- more colors: better sampled c thus smoother results
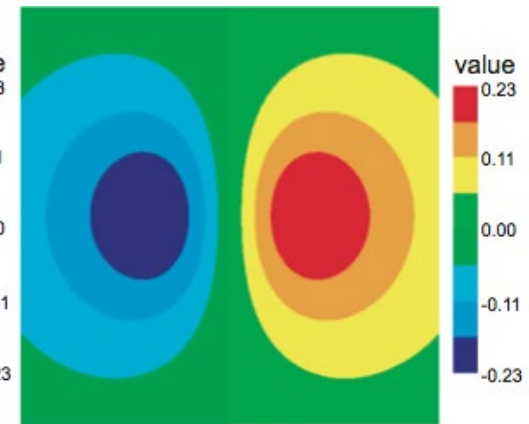- fewer colors: color banding appears
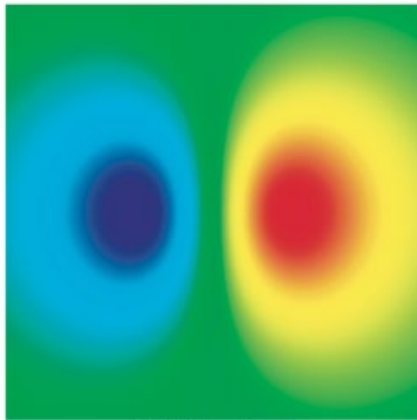
(a) 256 colors

(b) 32 colors

color banding

(c) 16 colors

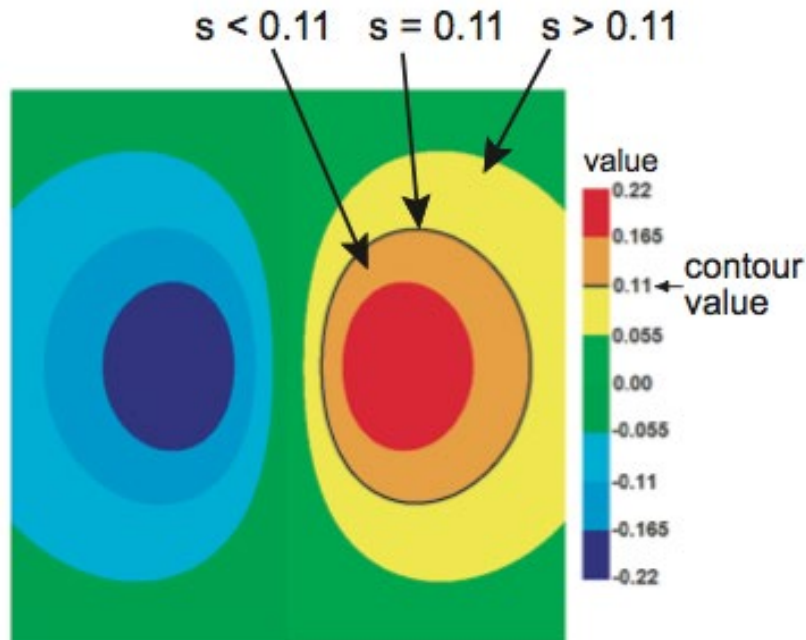(d) 8 colors

Question

- can we see sharp color banding with per-vertex colormapping? Why (not)?

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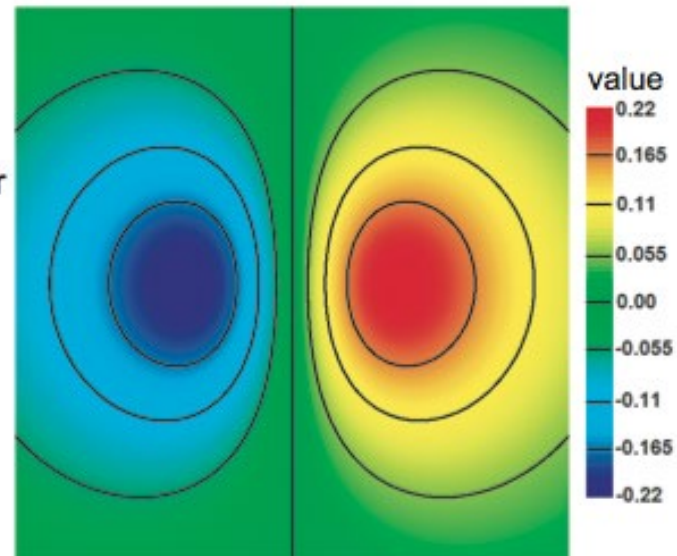
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Contouring

How to see where some given values appear in a dataset?

- recall color banding
- a transition separating two consecutive bands = a contour

contour = all points having
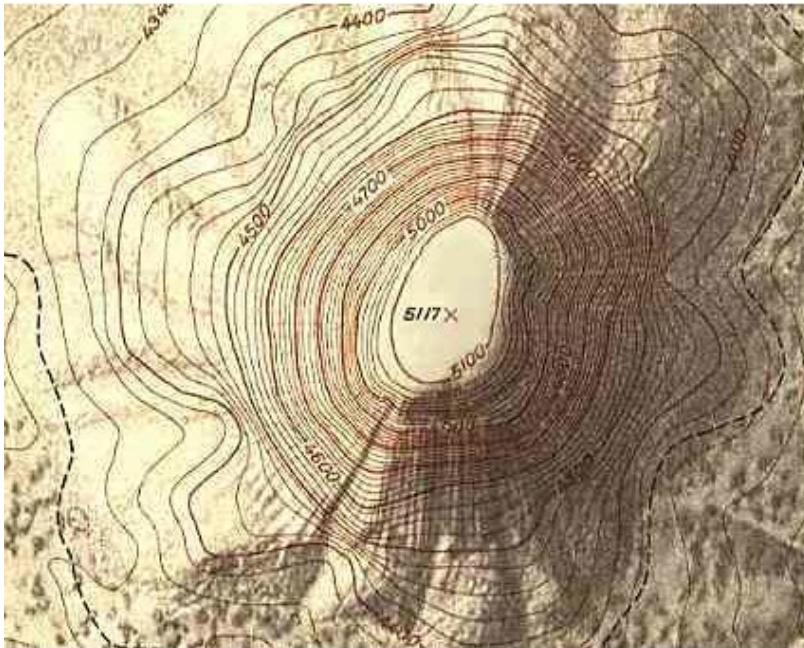the scalar value $s = 0.11$

seven different contours,
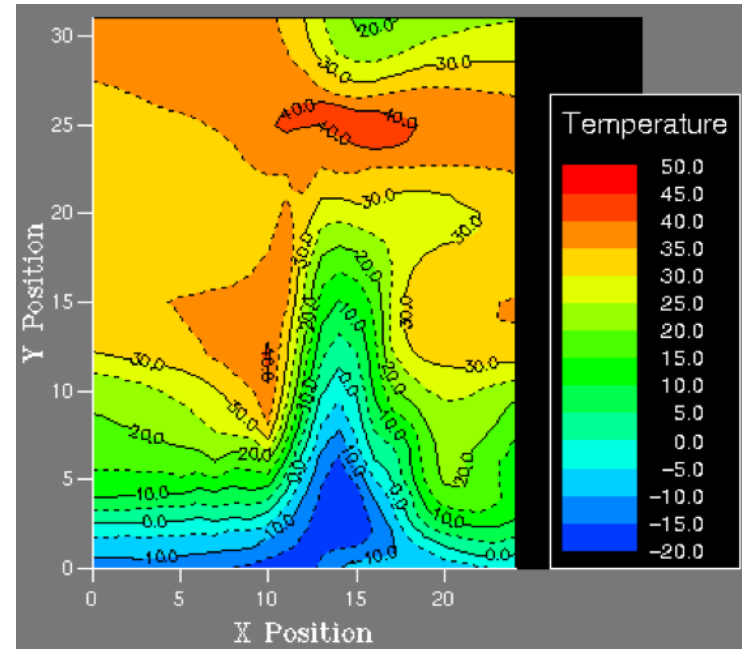equidistant in value space

Contouring

Contours are known for hundreds of years in cartography
• also called *isolines* ('lines of equal value')

hand-drawn contours on
geographical map

computer-generated
contours of temperature map

How to compute contours?

Contour properties

Definition

$$I(f_0) = \left\{ x \in D \,\middle|\, f(x) = f_0 \right\}$$

Contours are always closed curves (except when they exit D)
- why? Recall that f is C^0

Contours never (self-)intersect, thus are nested
- why? Think what would mean if a point belonged to two *different* contours

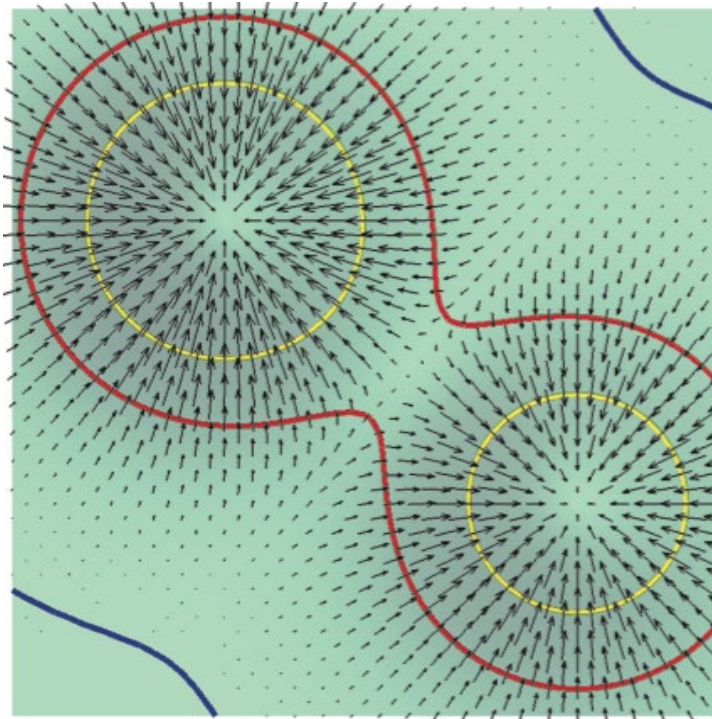**Contours cut D into values smaller resp. larger than the isovalue**
- why? Think of definition

Contour properties

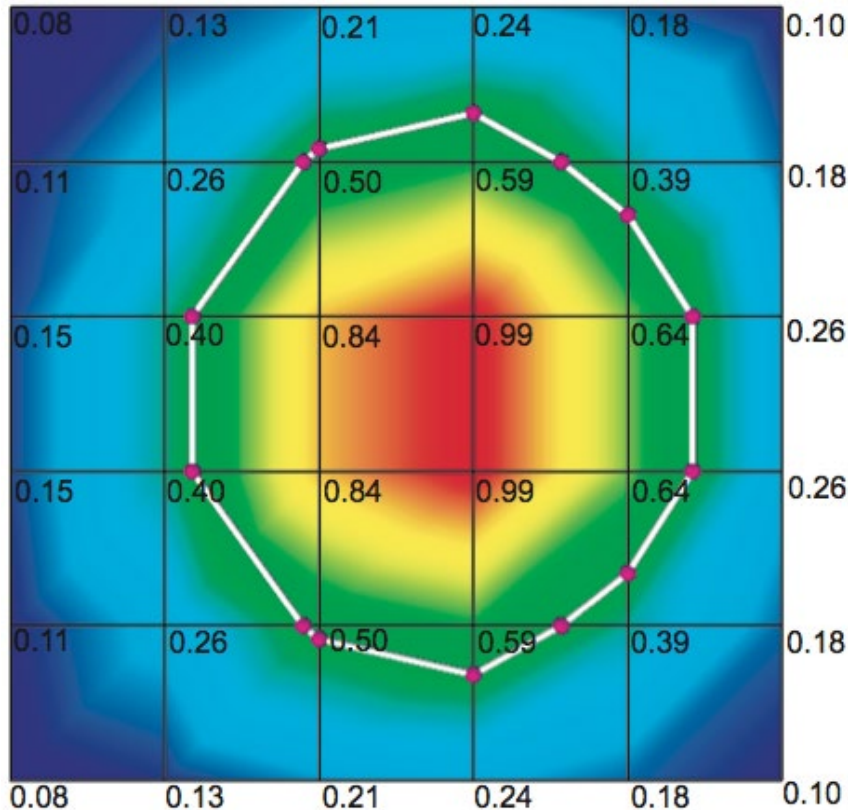**Contours are always orthogonal to the scalar value's gradient**
• why? Recall definitions

$$I(f_0) = \{x \in D \mid f(x) = f_0\}$$

contour: $\dfrac{\partial f}{\partial I} = 0$ since f constant along I

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$$

gradient: $\dfrac{\partial f}{\partial(\nabla f)} = \max$ by definition of gradient

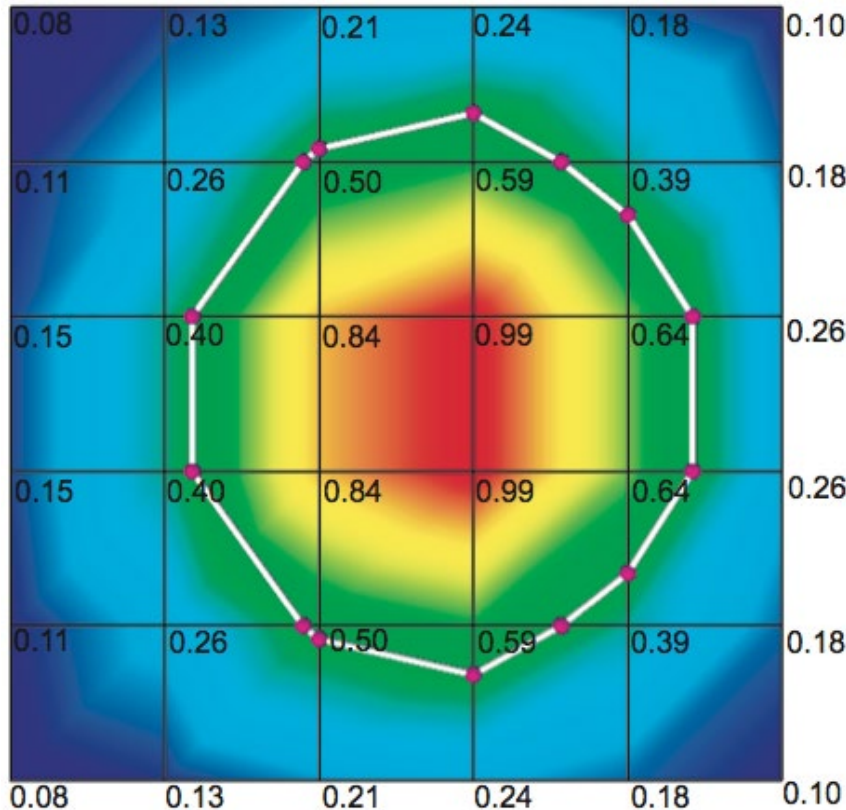gradient of a scalar field
(drawn with arrows) is
orthogonal to contours

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Basic contouring algorithm

c = a cell of the dataset
$e = (p_i, p_j)$ an edge of a cell c
p_i, p_j endpoints of edge e
v_i, v_j attribute values at end points p_i and p_j

Find countour with value $v = 0.48$

Basic contouring algorithm


```
for(each cell c in D)
{
    S = ∅                           //no contour-edge cuts
    for(each edge e=(pi,pj) of c)
    {
        if(vi < v < vj)             //e cuts contour
        {
```

$$q = \frac{p_i(v_j - v) + p_j(v - v_i)}{v_j - v_i}$$

$$S = S \cup q$$

```
        }
    }
    connect points in S with lines to build contour;
}
```
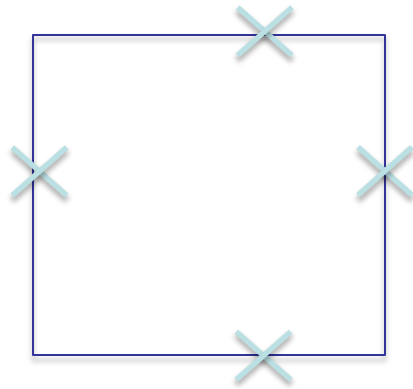
Works OK but it is

- cumbersome: connecting contour-edge cuts into lines is not trivial to program
- slow: edges intersecting contours are processed twice

Question

- Are contours piecewise-linear? Why (not)?
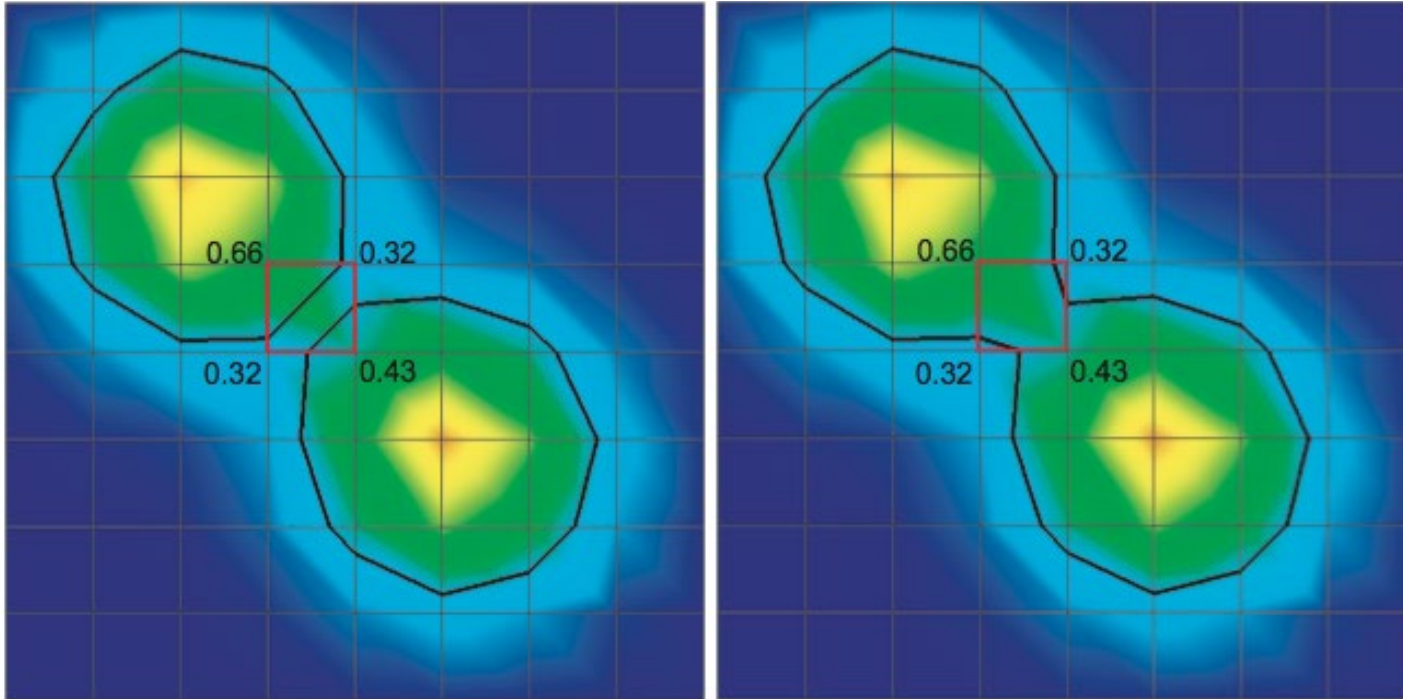
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Contouring amiguity

Consider this situation after identifying the countour intersections in a cell

Contouring ambiguity

Each edge of the red cell intersects the contour
- which is the right contour result?

Both answers are equally correct!
- we could discriminate only if we had higher-level information (e.g. topology)
- at cell level, we cannot determine more
- same would happen if we first split quads into triangles (2 splits possible..)

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Marching squares

Fast implementation of 2D contouring on quad-cell grids

1. Encode inside/outside state of each vertex w.r.t. contour in a 4-bit code

e.g.
inside: $f > f_0$
outside: $f \leq f_0$

4-bit code construction $\boxed{v_3 \, v_2 \, v_1 \, v_0}$
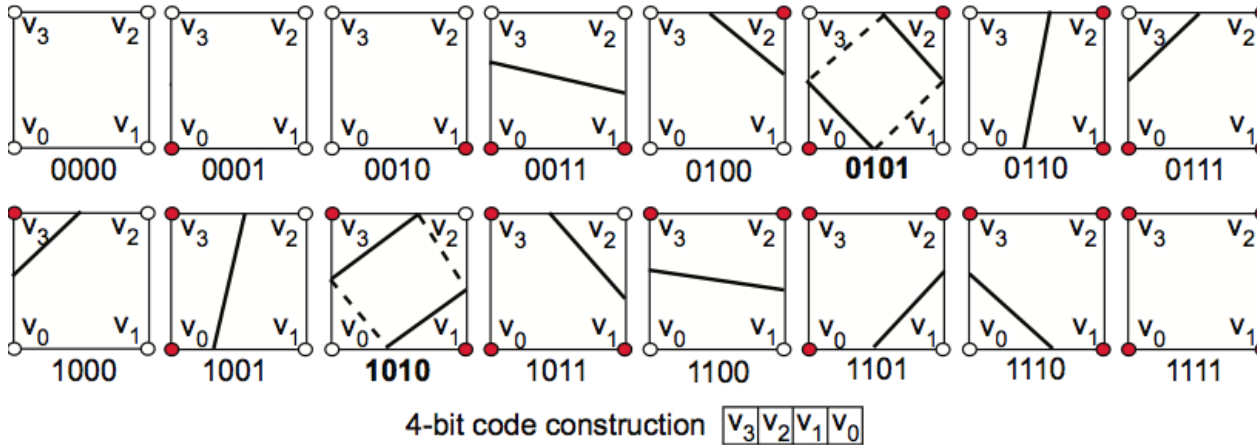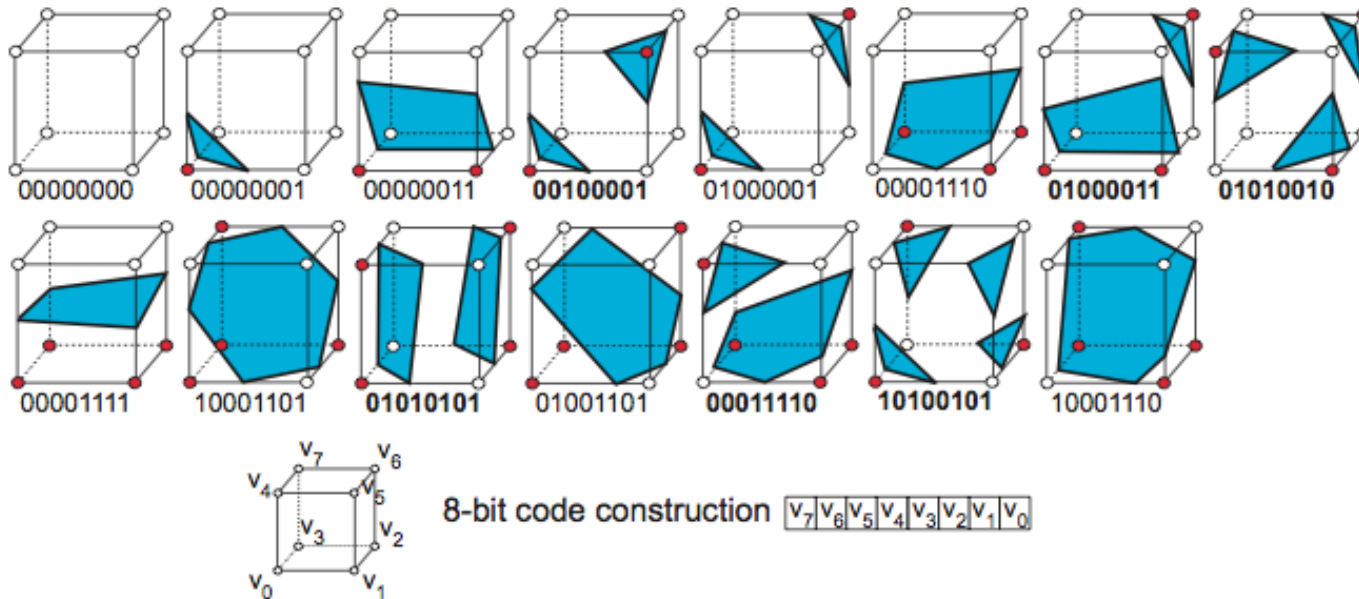
2. Process all dataset cells
- for each cell, use codes as pointers into a jump-table with 16 cases
- each case has hand-optimized code to
  - compute only the existing edge-contour intersections
  - automatically create required contour segments (connect intersections)
  - reuse already-computed contour segment vertices from previous cells

Note: same can be done for triangles ('marching triangles')

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Marching cubes

Fast implementation of 3D contouring (isosurfaces) on parallelepiped-cell grids

1. Encode inside/outside state of each vertex w.r.t. contour in a 8-bit code

8-bit code construction

e.g.
inside: $f > f_0$
outside: $f \leq f_0$

2. Process all dataset cells
* for each cell, use codes as pointers into a jump-table with 15 cases
  (reduce the $2^8 = 256$ cases to 8 by symmetry considerations)

Marching cubes (cont'd)

- For each case
  - compute the cell-contour intersection $\rightarrow$ triangles, quads, pentagons, hexagons
  - triangulate these on-the-fly $\rightarrow$ triangle output only

3. Treat ambiguous cases
- 6 such cases (see **bold**-coded figures on previous slide)
- harder to solve than in 2D (need to prevent false cracks in the surface)
- see Sec. 5.3 for algorithmic details

4. Compute isosurface normals
- by face-to-vertex normal averaging (see Module 2, Data resampling)
- directly from data

$$\forall x \in I,\ n_I(x) = -\frac{\nabla f(x)}{\|\nabla f(x)\|}$$
(gradient is normal to contours, see previous slides)

5. Draw resulting surface as a (shaded) unstructured triangle mesh
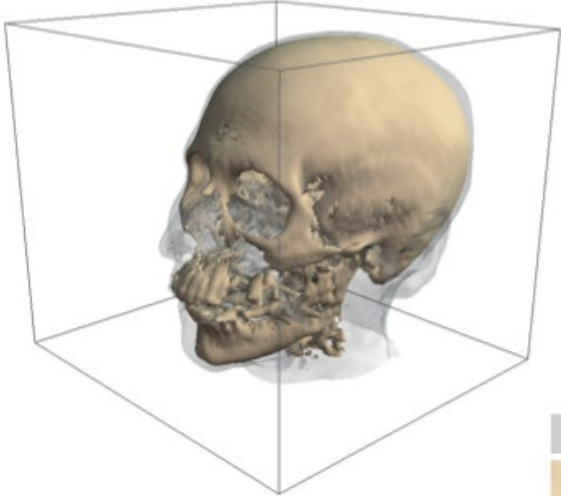
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Marching cubes
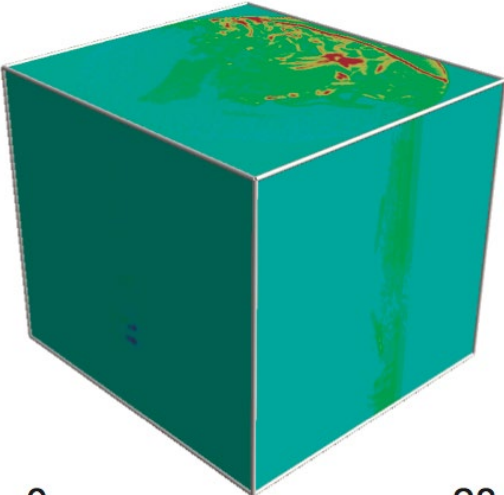
isosurfaces →

scalar CT volume
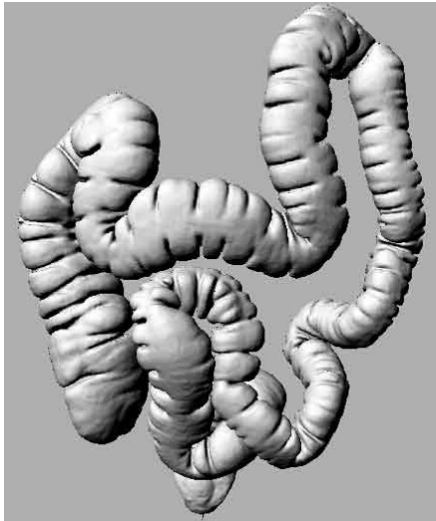(tissue density)
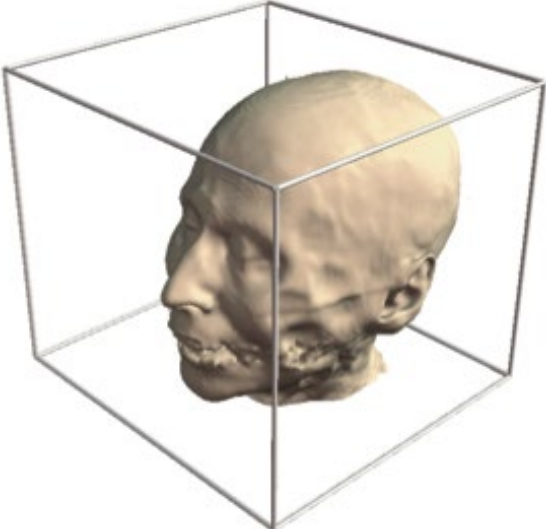
isosurface for scalar value
corresponding to skin

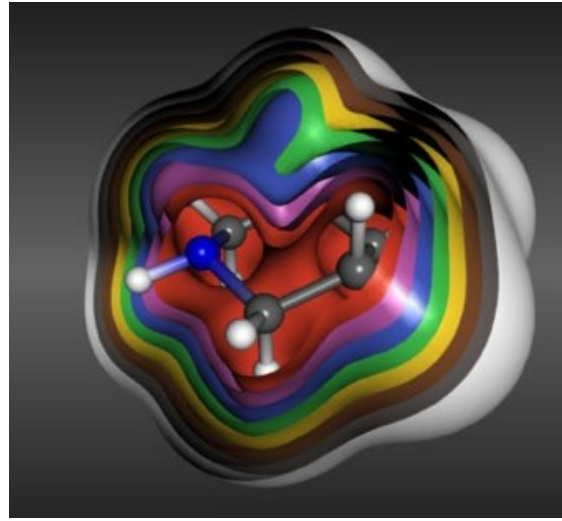isovalue = 65
isovalue = 127

isosurfaces for skin and bone
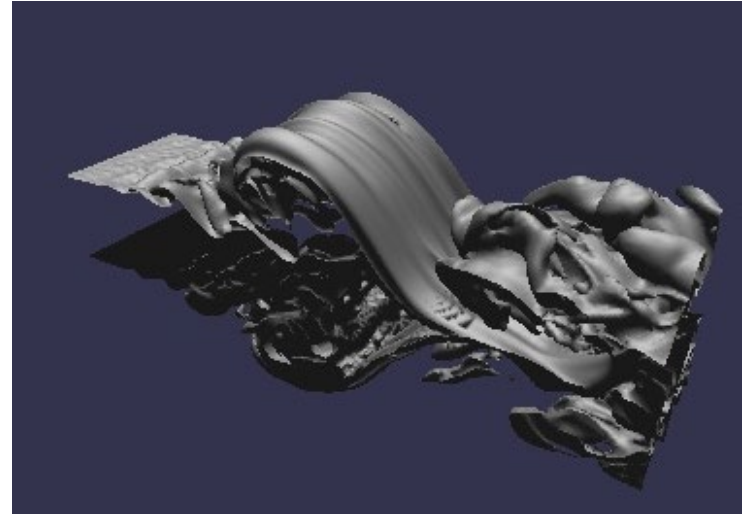
- extremely simple to use tool
- insightful results

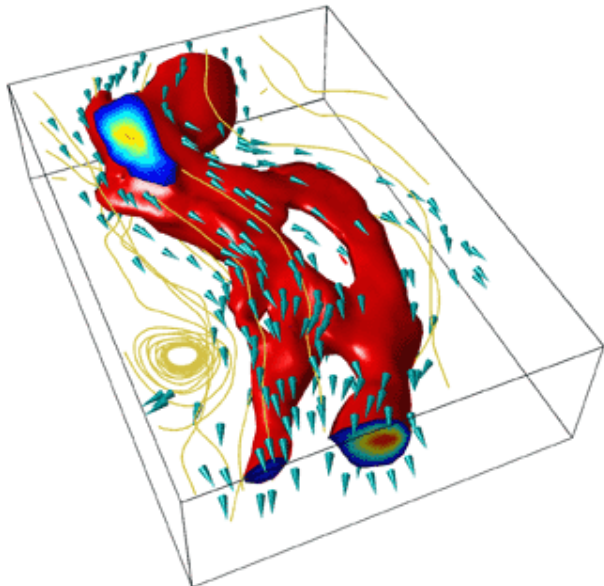university of
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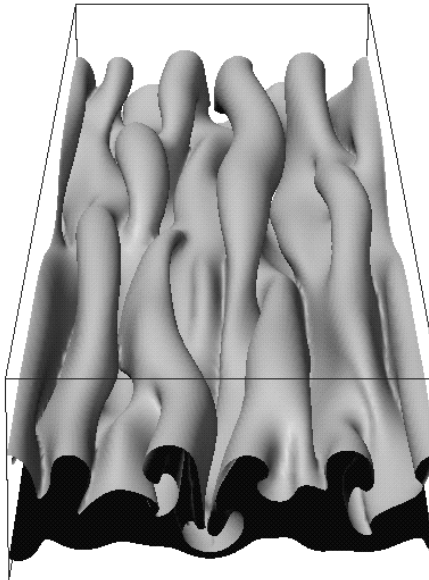# Isosurface examples

colon (CT dataset)

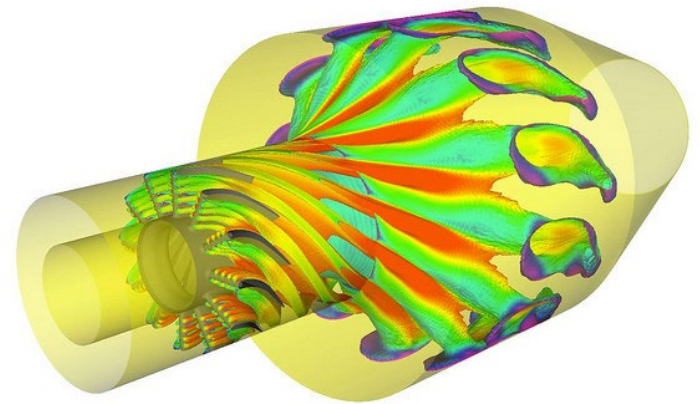electron density in molecule

velocity in 3D fluid flow

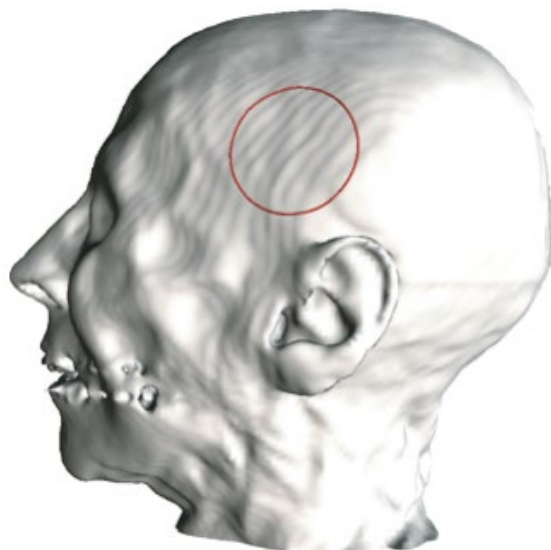velocity in 3D fluid flow
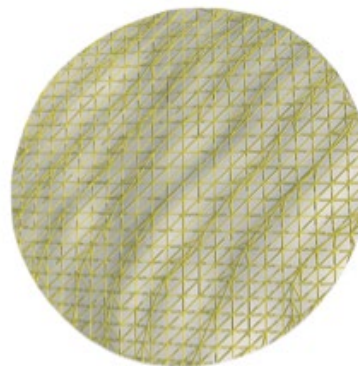

magnetic field in sunspots

fuel concentration, colored
by temperature in jet engine
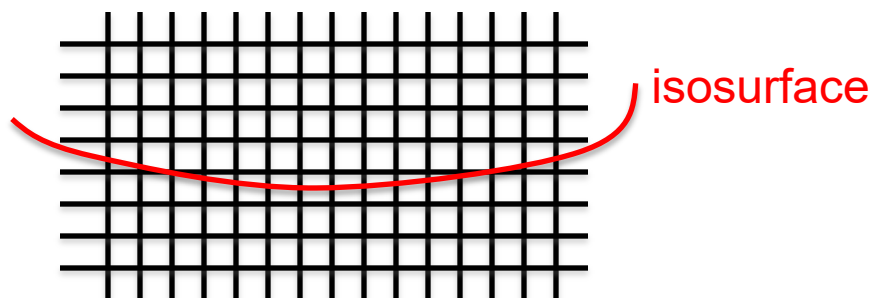
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Marching cubes – technical points

overview

detail

Does this person have wavy wrinkles on his head's skin?
- so it looks from the visualization…
- these are so-called 'ringing artifacts'
    - due to the near-tangent orientation of the isosurface w.r.t. finite-resolution volume grid

isosurface

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Summary

Scalar Algorithms (book Chapter 5)

• colormapping

• contouring (2D and 3D)

• height plots

• displacement plots

• read Ch. 5 *in detail* to understand all the algorithmic issues!

Next module

• volume visualization algorithms

Happy so far?

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