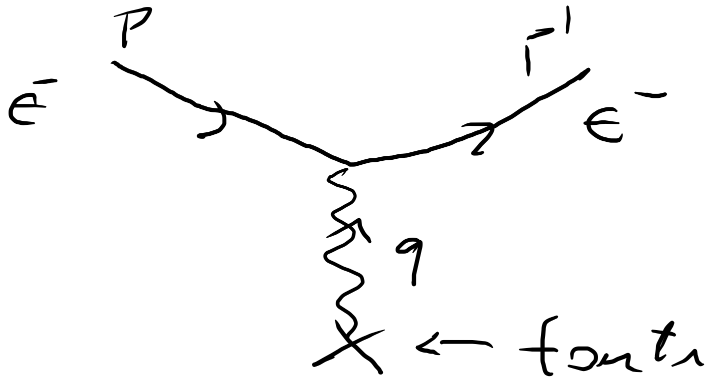
Campo Externo A^μ_e

$\gamma^\mu \partial_\mu \psi \to (\gamma^\mu \partial_\mu + i e A_\mu)\psi$

$$S = \sum_{n=0}^{\infty} \frac{(ie)^n}{n!} \int \cdots \int d^4x_1 \cdots d^4x_n \, T\left(:\bar{\psi}(\cancel{A} + \cancel{A}_e)\psi:_{x_1} \cdots \cdots :\bar{\psi}(\cancel{A} + \cancel{A}_e)\psi:_{x_n} \right)$$

$$A^\mu_e(\vec{x}) = \frac{1}{(2\pi)^3} \int d^3\vec{q} \; e^{i\vec{q}\cdot\vec{x}} \, A^\mu_e(\vec{q})$$

$$S^{(1)}_e = ie \int d^4x :\bar{\psi}^-(x) \cancel{A}_e(x) \psi^+(x):$$



inicial $p = (E, \vec{p})$ $u_r(p)$

final $p' = (E', \vec{p}')$ $u_s(p')$

$\langle f | S_e^{(1)} | i \rangle$

$c^\dagger(p)|0\rangle$

$$\psi^+(x)\,|e^- p\rangle = |0\rangle \left(\frac{m}{V E_p}\right)^{1/2} u_r(p)\, e^{-ipx}$$

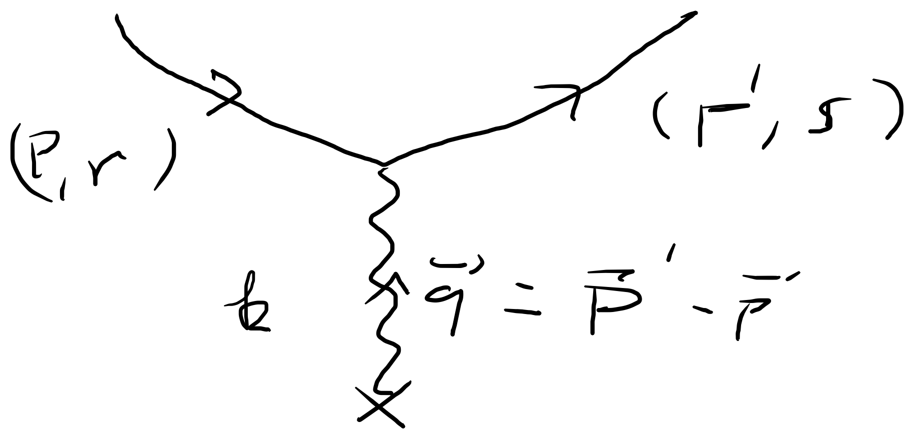
$$\bar{\psi}^- |0\rangle = \sum_{\bar{p}} |e^- \bar{p}\rangle \left(\frac{m}{V E_{\bar{p}}}\right)^{1/2} \bar{u}_s(\bar{p})\, e^{i\bar{p}x}$$

$$\langle f | S_e^{(1)} | i \rangle = \int d^3x\, e^{i(-\vec{p} + \vec{\bar{p}}' + \vec{q})\vec{x}} \sim \boxed{\delta^{(3)}(\vec{p} - \vec{p}' + \vec{q})}$$

$$\langle f|S_0^{(1)}|i\rangle = \underline{2\pi\,\delta(E-E')}\left(\frac{m}{VE}\right)^{1/2}\left(\frac{m}{VE'}\right)^{1/2} M$$

$|\vec{p}\,'| = |\vec{p}\,|$

$$M = ie\,\bar{u}_s(p')\,\cancel{A}_e(\vec{q} = \vec{p}\,' - \vec{p}\,)\,u_r(p)$$



$$\left(\frac{1}{2V\omega_k}\right)^{1/2}\varepsilon_\alpha|k)$$

$$(2\pi)^4\,\delta^{(4)}(p' + k' - p)$$

1) $\langle f|S|i\rangle = \delta_{fi} + \left[(2\pi)^4 \delta^{(4)}(p_f - p_i) \prod_{ext}\left(\frac{m}{VE}\right)^{1/2} \prod_{ext}\left(\frac{1}{2V\omega}\right)^{1/2}\right] M$

$2\pi\delta(E_f - E_i)$

2) Para cada interação de uma part. carregada com um campo externo A_e escreve o fator

$$\underline{A^\mu_e(\vec{q})} = \int d^3x\, e^{-i\vec{q}\cdot\vec{x}} A^\mu_e(x)$$

$$\boxed{w = \frac{1}{T}\left|\langle f | S_\epsilon^{(1)} | i\rangle\right|^2 = 2\pi\,\delta(E-E')\left(\frac{m}{VE}\right)^2 |M|^2}$$

$$\underline{\delta(E-E')}\int_{-\frac{T}{2}}^{\frac{T}{2}} dt\; \underbrace{e^{it\overbrace{(E-E')}^{0}}}_{1}$$

$$T$$

$$2\pi\,\delta(E-E')\,T$$

$$E'^2 = |\vec{p}'|^2 + m^2$$

$$dE'^2 = d|\vec{p}'|^2$$

$$\frac{V d^3\vec{p}'}{(2\pi)^3} = \frac{V|\vec{p}'|^2 d|\vec{p}'| d\Omega'}{(2\pi)^3}$$

$$= \frac{V|\vec{p}'|\frac{1}{2}d|\vec{p}'|^2 d\Omega'}{(2\pi)^3}$$

$$= \frac{V|\vec{p}'|E' dE' d\Omega'}{(2\pi)^3}$$

$$\frac{v}{V} = \frac{|\vec{p}|}{VE}$$

$$E = \gamma m \qquad |\vec{p}| = \gamma m v$$

$$d\sigma \frac{v}{V} = w \frac{V d^3\vec{p}\,'}{(2\pi)^3}$$

$$d\sigma = 2\pi\,\delta(E-E')\left(\frac{m}{VE}\right)^2 |M|^2 \frac{VE}{\cancel{|\vec{p}\,'|}} \frac{V\cancel{|\vec{p}\,'|}E'\,dE'\,d\Omega'}{(2\pi)^3}$$

$$A^\mu_c \gamma^\mu$$

$$\boxed{\frac{d\sigma}{d\Omega'} = \frac{m^2}{(2\pi)^2}|M|^2 = \left(\frac{me}{2\pi}\right)^2 \left|\bar{u}_s(\vec{p}\,') \not{A}_c(\vec{q}=\vec{p}\,'-\vec{p})\, u_r(\vec{p})\right|^2}$$

$\vec{q} = \vec{p}\,' - \vec{p}$ $\quad |\vec{p}\,'| = |\vec{p}|$

$$A^{\mu}_{e}(x) = \left(\frac{Ze}{4\pi|\vec{x}|}, 0, 0, 0 \right)$$

$$A^{\mu}_{e}(\vec{q}) = \left(\frac{Ze}{|\vec{q}|^2}, 0, 0, 0 \right)$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{\cancel{M.P.}} = \left(\frac{m\,e}{2\pi}\right)^2 (Z e)^2 \frac{1}{2}\sum_{r,s} \left| \bar{u}_s(\vec{p}\,') \frac{\gamma^0}{|\vec{q}|^2} u_r(\vec{p}) \right|^2$$

$$\alpha = \frac{e^2}{4\pi}$$

$$\gamma^{\mu +} = \gamma^0 \gamma^\mu \gamma^0$$

$$= (2 m Z \alpha)^2 \frac{1}{2} \sum_{r,s} \bar{u}_s(p')_\alpha \, \gamma^0_{\alpha\beta} \, \underline{u_r(p)_\beta \, \bar{u}_r(p)_\gamma} \, \gamma^0_{\gamma\delta} \, u_s(p')_\delta$$

$$\sum_r u_r(p)\bar{u}_r(p) = \frac{\cancel{p} + m}{2m}$$

$$= \frac{(2 m Z \alpha)^2}{|\vec{q}|^4} \frac{1}{2} \mathrm{Tr}\left(\frac{\cancel{p}' + m}{2m} \gamma^0 \frac{\cancel{p} + m}{2m} \gamma^0 \right)$$

$$= \frac{(\alpha Z)^2}{2|\vec{q}|^4} \left[\mathrm{Tr}\left(\cancel{p}' \gamma^0 \cancel{p} \gamma^0\right) + m^2 \, \underbrace{\mathrm{Tr}(\gamma^0\gamma^0)}_{4} \right]$$

$$-(\cancel{E'E} - \vec{p}\,' \cdot \vec{p})$$

$$= \frac{2(\alpha Z)^2}{|\vec{q}|^4} \left[E'E - \vec{p}\,' \cdot \vec{p} + \cancel{E'E} + m^2 \right]$$

$$\left(\frac{d\sigma}{d\Omega'}\right)_{n.p} = \frac{2(\alpha Z)^2}{|\vec q|^4}\left\{E^2 + \vec p'\cdot\vec p + m^2\right\}$$

$$\vec p'\cdot\vec p = |\vec p|^2\cos\theta$$

$$|\vec q|^2 = |\vec p' - \vec p|^2 = 2|\vec p|^2 - 2|\vec p|^2\cos\theta$$

$$= 2|\vec p|^2(1-\underbrace{\cos\theta}_{\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}}) = 4|\vec p|^2\sin^2\frac{\theta}{2}$$

$$E = \gamma m \qquad |\vec p| = E v$$

$$|\vec p| = \gamma m v$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{M.P} = \frac{2(\alpha Z)^2}{16E^4 v^4 \sin^4\frac{\theta}{2}} \left[E^2 + |\vec{p}|^2\cos\theta + m^2\right]$$

$$|\vec{p}|^2 = E^2 v^2 \qquad m^2 = E^2 - |\vec{p}|^2$$

$$= \frac{(\alpha Z)^2}{8E^4 v^4 \sin^4\frac{\theta}{2}} \left[\underline{E^2} + E^2 v^2 \cos\theta + \underline{E^2} - E^2 v^2\right]$$

$$E^2 v^2 (1-\cos\theta) = 2\sin^2\frac{\theta}{2}$$

$$\boxed{\left(\frac{d\sigma}{d\Omega}\right)_{M.P} = \frac{(\alpha Z)^2}{4E^2 v^4 \sin^4\frac{\theta}{2}} \left[1 - \frac{v^2}{c^2}\sin^2\frac{\theta}{2}\right]}$$

fórmula de Mott

$$\frac{v}{c} << 1$$

$$E \sim m$$

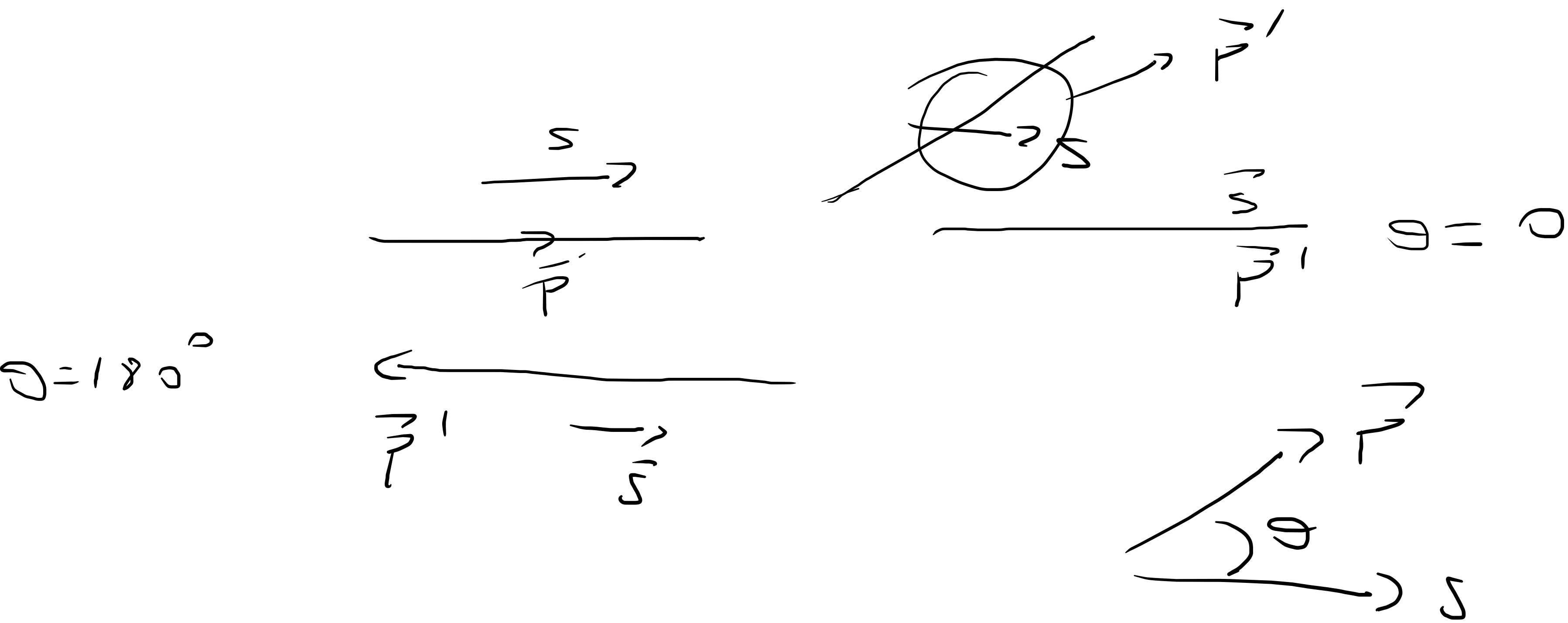
$$\left(\frac{d\sigma}{d\Omega}\right)_R = \frac{(\alpha Z)^2}{4 m^2 v^2 \sin^4\frac{\theta}{2}}$$ Rutherford (1911)

$\vec{E}$ $\vec{B}$

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ \vdots & 0 & B & B \\ \vdots & & 0 & B \\ \vdots & & & 0 \end{pmatrix}$$

Não Relativístico

Spin é conservado   helicidade não é conservada



$P_{h=+1}$ $\cos^2\theta$

$P_{h=-1}$ $\sin^2\theta$

$$|\bar{u}_s(p')\gamma^0 u_r(p)|^2 = |u_s^\dagger(p')\, u_r(p)|^2 = \delta_{rs}$$

$$u_1 = A\begin{pmatrix} 1 \\ 0 \\ B\,p_3 \\ B(p_1 + i p_2) \end{pmatrix} \qquad u_2 = A\begin{pmatrix} 0 \\ 1 \\ B(p_1 - i p_2) \\ B\,p_3 \end{pmatrix}$$

$$A = \left(\frac{E_p + m}{2m}\right)^2$$

$$B = \frac{1}{E_p + m}$$

N.R.L.

$\underline{p \ll m}$ $\quad E \sim m$

$A \sim 1 \qquad B \sim \frac{1}{2m}$

$$u_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad u_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

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e^- inicial $\quad u_1(p) \quad \underline{r = 1}$

$$X_s = \left| \bar{u}_s(p')\, \gamma^0\, \underline{u_1(p)} \right|^2$$

$\color{red}{\Pi^+(p)\, u_r(\vec{p})}$

$s = 1$ hel pos

$s = 2$ " neg

$$\Pi^\pm(p) = \frac{1}{2}\left(1 + \underline{\sigma_p}\right) \qquad \frac{\vec{p}\cdot\vec{\sigma}}{|\vec{p}|}$$

$$\Pi^{\color{red}{\pm}}(p)\, u_r(p) = \delta_{1,r \atop \color{red}{2}}\, u_r(\vec{p})$$

$$X_s = \bar{u}_s(p')\gamma^0 u_1(p)\ \bar{u}_1(p)\gamma^0 u_s(p') \qquad r=1$$

$$= \bar{u}_s(p')\gamma^0 \pi^{+}(p) u_1(p)\ \bar{u}_1(p)\gamma^0 \pi^{\pm} u_s(p') \qquad \begin{array}{l} + \quad s=1 \\ - \quad s=2 \end{array}$$

$$= \sum_{r,q} \bar{u}_q(p')\gamma^0 \pi^{+}(p) u_r(p)\ \bar{u}_r(p)\gamma^0 \pi^{\pm}(p') u_q(p')$$

$$= \mathrm{Tr}\left(\frac{\not{p}' + m}{2m}\gamma^0 \pi^{+}(p) \frac{\not{p}+m}{2m}\gamma^0 \pi^{\pm}(p') \right) \qquad r=1$$

Ultra relativistico

$$E = E' \gg m \qquad \pi^{\pm} = \frac{1}{2}(1 \pm \gamma_5)$$

$$X_5 = \frac{1}{16m^2} \mathrm{Tr}\left((\cancel{p}' + m) \gamma^0 (1+\gamma_5) \cancel{\gamma^0} \gamma^0 (\cancel{p} + m)(1 \pm \gamma_5) \right)$$

$$\gamma^{5\,2} = 1 \qquad \mathrm{Tr}(\gamma^5) = \mathrm{Tr}(\gamma^5\gamma^\mu) = \mathrm{Tr}(\gamma^5\gamma^\mu\gamma^\nu)$$

$$= \mathrm{Tr}(\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho) = 0$$

$$\gamma^0(1+\gamma^5)\cancel{p}\,\gamma^0(1\pm\gamma^5)$$

$$\underbrace{\gamma^0\cancel{p}\gamma^0}_{=0} \qquad \underbrace{\gamma^0\gamma^5\cancel{p}\gamma^0}_{=0} \pm \left(\underbrace{\gamma^0\cancel{p}\gamma^0\gamma^5}_{=0} + \underbrace{\gamma^0\gamma^5\cancel{p}\gamma^0\gamma^5}_{=0} \right)$$

$\underline{\underline{m^2}}$

$$\gamma^0(1+\gamma^5)\gamma^0(1\pm\gamma^5)$$

$$\mathrm{Tr}\left(\gamma^0\gamma^0 + \underbrace{\gamma^0\gamma^5\gamma^0}_{=0} \pm \left(\underbrace{\gamma^0\gamma^0\gamma^5}_{=0} + \gamma^0\gamma^5\gamma^0\gamma^5\right)\right)$$

$$\mathrm{Tr}\left(1 \mp \underbrace{\gamma^0\gamma^0\gamma^5\gamma^5}_{=1}\right) = \begin{cases} 0 & \text{no flip} \\ \neq 0 & \text{flip} \end{cases}$$

$$\not{p}'\gamma^0(1+\gamma^5)\not{p}\gamma^0(1\pm\gamma^5)$$

$$\not{p}'\gamma^0\not{p}\gamma^0 + \underbrace{\not{p}'\gamma^0\gamma^5\not{p}\gamma^0}_{\to 0}$$

$$\pm\left(\underbrace{\not{p}'\gamma^0\not{p}\gamma^0\gamma^5}_{\to 0} + \not{p}'\gamma^0\gamma^5\overbrace{\not{p}\gamma^0\gamma^5}\right)$$

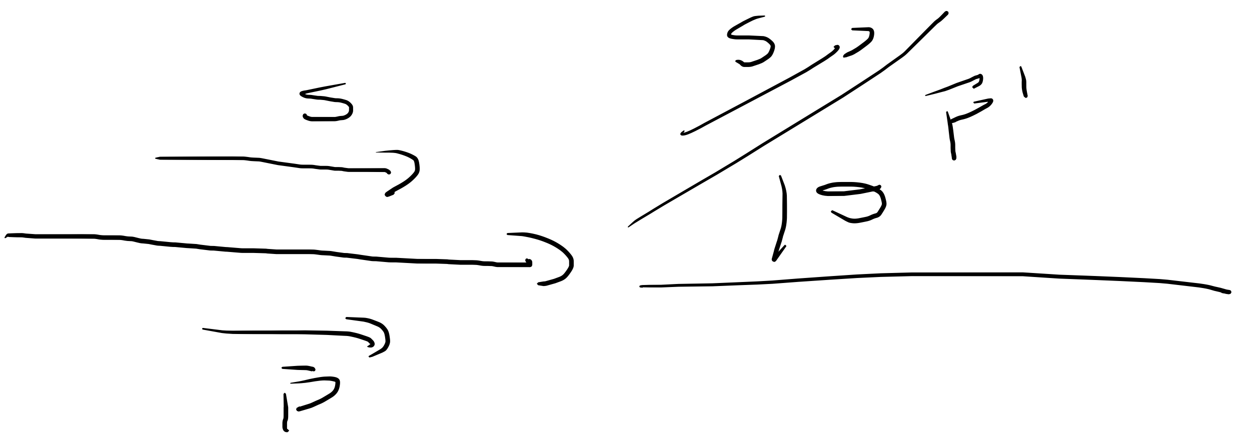
$$\mathrm{Tr}(\not{p}'\gamma^0\not{p}\gamma^0) \pm \mathrm{Tr}(\not{p}'\gamma^0\not{p}\gamma^0)$$

$$= \begin{cases} \neq 0 & \underline{\text{no flip}} \\ = 0 & \text{flip} \end{cases}$$

$$X_s \approx p p' \delta_{s,1} + m^2 \delta_{s,2}$$

$$E E' \gg m^2$$

no flip (ultra relativistico)



helicidad

conservada

$P \quad P'$