$$\mathcal{M} = \varepsilon^{\mu}_{r_1}(k_1)\,\varepsilon^{\nu}_{r_2}(k_2)\,\cdots\,\mathcal{M}_{\mu\nu\cdots}(k_1, k_2 \cdots)$$

$$A^{\mu}(x) = c\,\varepsilon^{\mu}_{r}(k)\,e^{\pm ikx}$$

$$\partial_{\mu} A^{\mu} = 0 \qquad k_{\mu}\varepsilon^{\mu}_{r}(k) = 0$$

$$A_{\mu} \to A_{\mu} + \partial_{\mu} f \qquad f(x) = \tilde{f}(k)\,e^{\pm ikx}$$

$$\varepsilon^{\mu}_{r}\,e^{\pm ikx} \to \left(\varepsilon^{\mu}_{r} \pm i k^{\mu}\tilde{f}\right]e^{\pm ikx}$$

$$k_1^{\mu} M_{\mu\nu\ldots} = 0 = k_2^{\nu} M_{\mu\nu\ldots} = 0$$

$$k_1^{\mu} k_2^{\nu} M_{\mu\nu\ldots} = 0$$

$$M_r(k) = \varepsilon_r^{\mu}(k)\, \underline{M_{\mu}(k)}$$

$$\underline{k^{\mu} M_{\mu}} = 0$$

$$X = \sum_{r=1}^{2} |M_r(k)|^2 = \underline{M_\mu(k)\, M_\nu(k)^*} \underline{\sum_{r=1}^{2} \varepsilon_r^\mu(k)\, \varepsilon_r^\nu(k)}$$

$$D_F^{\mu\nu}(k) = \frac{-g^{\mu\nu}}{k^2 + i\varepsilon} = \underline{\frac{1}{k^2 + i\varepsilon} \sum_{r=0}^{3} \zeta_r\, \varepsilon_r^\mu(k) \varepsilon_r^\nu(k)}$$

$$D_F^{\mu\nu}(k) = \frac{1}{k^2 + i\varepsilon}\left[\underline{\sum_{r=1}^{2} \varepsilon_r^\mu \varepsilon_r^\nu} + \frac{\left(k^\mu - (k\cdot n)n^\mu\right)\left(k^\nu - (k\cdot n) n^\nu\right)}{(k\cdot n)^2 - k^2} + \underline{(-1)\, n^\mu n^\nu} \right]$$

$$\shortparallel \quad 0$$

$$n^\mu = (1, 0, 0, 0)$$

$$\sum_{r=1}^{2} \epsilon^{\mu}_{r} \epsilon^{\nu}_{r} = -g^{\mu\nu} - \frac{1}{(k\cdot n)^2}\left[k^{\mu}k^{\nu} - (k\cdot n)(k^{\mu}n^{\nu} + k^{\nu}n^{\mu})\right]$$

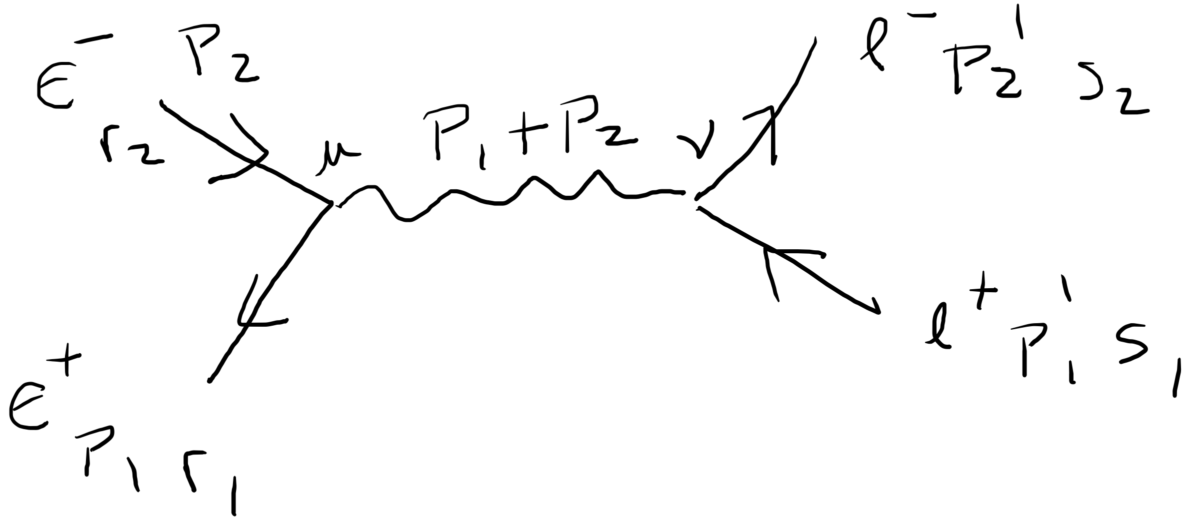
$$\boxed{X = \sum_{r=1}^{2} |M_r(k)|^2 = -M^{\mu}(k)\, M^{*}_{\mu}(k)}$$

Produção de pares de leptons

$e^+ e^-$

$e^+ e^- \to \ell^+ \ell^-$

$\ell = \mu, \tau$



$$M = (i e)^2 \left(\bar{u}_{s_2}(p_2') \gamma^\nu v_{s_1}(p_1') \right) \left(\bar{v}_{r_1}(p_1) \gamma^\mu u_{r_2}(p_2) \right)$$

$$\times \frac{-i\, g_{\mu\nu}}{(p_1+p_2)^2 + \cancel{i\varepsilon}}$$

$(\bar{v} \gamma^\nu u)$

$v^{+} \gamma^{\nu +} \gamma^{0 +} u$

$\gamma^0 \gamma^0$ $\quad$ γ^0

$$(P_1+P_2)^2 = P_1^2 + P_2^2 + 2P_1\cdot P_2 = \underline{2m_e^2} + 2E_1E_2 - 2\vec{P}_1\cdot\vec{P}_2$$

Rest CM $\quad \vec{P}_1 = -\vec{P}_2' \quad E_1 = E_2$

$$2E_1^2 + \underline{2\vec{P}_1^2 + 2\vec{P}_1^2} \rightarrow -\underline{2\vec{P}_1^2}$$

$$= 4\underset{\underset{0.5\ \mathrm{MeV}}{\parallel}}{m_e^2} + 4\underline{\vec{P}_1^2}$$

$$X = \frac{1}{2}\sum_{r_1}\frac{1}{2}\sum_{r_2}\sum_{s_1}\sum_{s_2}|M|^2$$

$$M^* = -i e^2 \frac{g^{\mu\nu}}{(p_1+p_2)^2} \frac{\left[\bar{v}_{s_1}(p_1')\gamma_\mu u_{s_2}(p_2')\right]^*}{\left[\bar{u}_{r_2}(p_2)\gamma_\nu v_{r_1}(p_1)\right]}$$

$$\boxed{\frac{1}{4}\sum_{\substack{s_1,s_2\\ r_1,r_2}} |M|^2 = \frac{e^4}{\left((p_1+p_2)^2\right)^2}\frac{1}{4} A_{(e)\mu\nu} B_{(\epsilon)}^{\mu\nu}}$$

$$A_{e\mu\nu} = \sum_{s_1 s_2} \left(\bar{u}_{s_2}(p_2')_\alpha \gamma_\mu^{\alpha\beta} v_{s_1}(p_1)_\beta\right)\left(\bar{v}_{s_1}(p_1')_\gamma \gamma_\nu^{\gamma\delta} u_{s_2}(p_2')_\delta\right)$$

$$= \mathrm{Tr}\left(\overbrace{\frac{\not{p}_2' + m_e}{2m_e}\gamma_\mu \frac{\not{p}_1' - m_e}{2m_e}}\gamma_\nu\right)$$

$$\boxed{\sum_r \bar{u}_{r\alpha}(p)\, u_{r\beta}(p) = \frac{\not p + m}{2m}}$$

$$B^{\mu\nu}_{(e)} = Tr\left(\frac{\not p_1 - m_e}{2m_e}\gamma^\mu \frac{\not p_2 + m_e}{2m_e}\gamma^\nu\right)$$

$$Tr(\gamma^\mu \gamma^\nu \ldots \gamma^\rho) = 0 \quad \text{impar}$$

$$Tr(\gamma^\mu \gamma^\nu) = 4 g^{\mu\nu}$$

$$Tr(\gamma^\mu \gamma^\nu \gamma^\gamma \gamma^\delta) = 4\left(g^{\mu\nu} g^{\gamma\delta} - g^{\mu\gamma} g^{\nu\delta} + g^{\mu\delta} g^{\nu\gamma}\right)$$

$$A_{(e)}^{\mu\nu} = \frac{1}{4 m_e^2} \mathrm{Tr}\left(\not{p}_2' \gamma^\mu \not{p}_1' \gamma^\nu - m_e^2 \gamma^\mu \gamma^\nu\right)$$

$$= \frac{1}{m_e^2}\left[p_2'^{\mu} p_1'^{\nu} - p_2' \cdot p_1' g^{\mu\nu} + p_2'^{\nu} p_1'^{\mu} - m_e^2 g^{\mu\nu} \right]$$

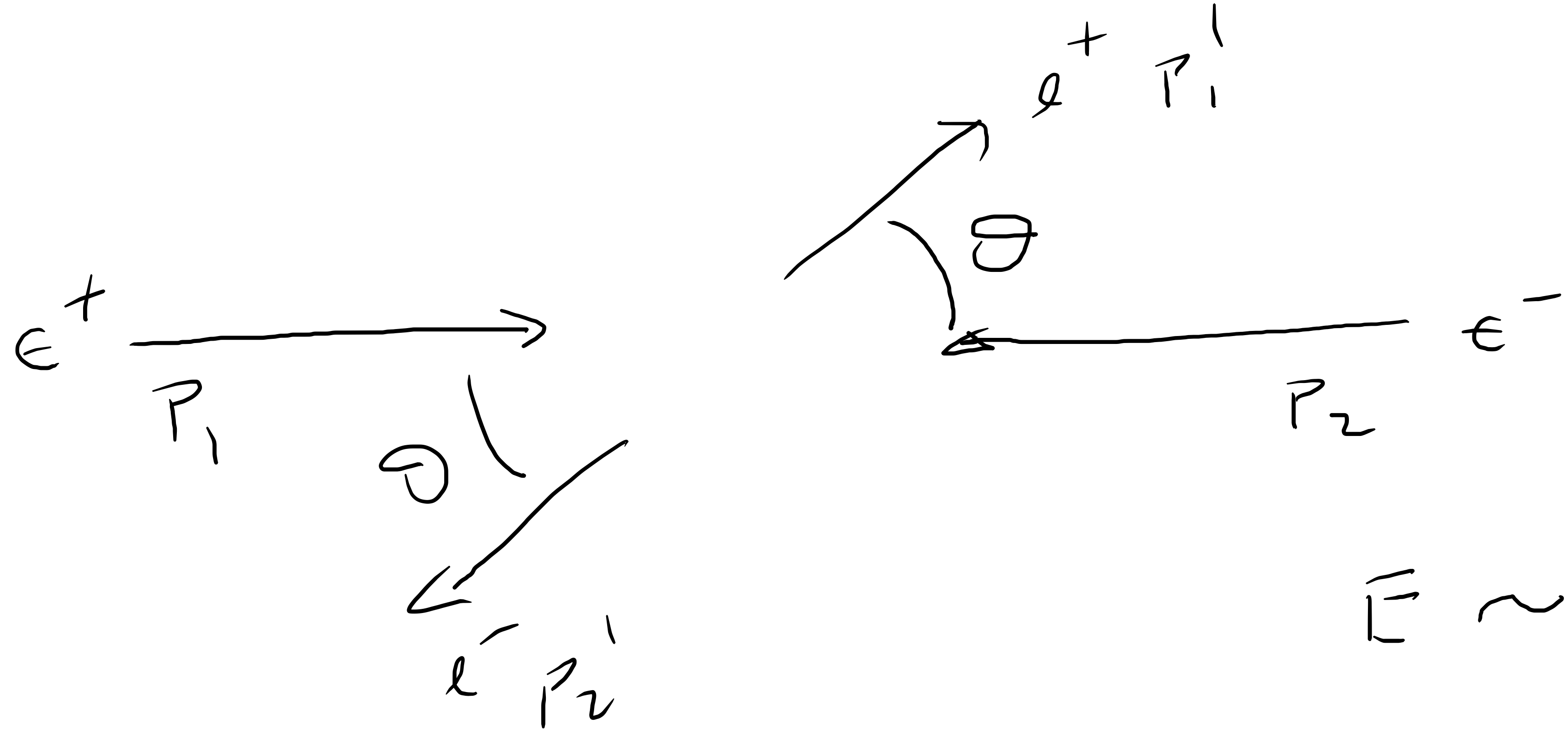
$$B_{\mu\nu} = \frac{1}{m_e^2}\left\{ p_1^{\mu} p_2^{\nu} + p_2^{\mu} p_1^{\nu} - g^{\mu\nu}\left(m_e^2 + p_1 \cdot p_2\right)\right]$$

$$X = \frac{e^4}{4} \frac{1}{\left((p_1+p_2)^2\right)^2} \frac{1}{m_e^2 m_\epsilon^2} \Big[2(p_1 p_1')(p_2 p_2') + 2(p_1 p_2')(p_2 p_1') - 2 p_1' p_2' (m_\epsilon^2 + p_1 p_2) - 2 p_1 p_2 (m_e^2 + p_1' p_2') + 4 (p_1' p_2' + m_e^2)(p_1 p_2 + m_\epsilon^2) \Big]$$

$$\boxed{X = \frac{e^4}{2\left((p_1+p_2)^2\right)^2 m_e^2 m_\epsilon^2} \Big[(p_1 p_1')(p_2 p_2') + (p_1 p_2')(p_2 p_1') + m_e^2 (p_1 p_2) + m_\epsilon^2 (p_1' p_2') + 2 m_e^2 m_\epsilon^2 \Big]}$$

CM

$\vec{P}_1 = -\vec{P}_2 = \vec{P} \qquad \vec{P}_1' = -\vec{P}_2' = \vec{P}'$



$E \sim m_\mu$

6

$E_1 = E_2 = E_1' = E_2' = E$

$$p_1 \cdot p_1' = p_2 p_2' = E^2 - |\vec{p}|\,|\vec{p}'| \cos\theta$$

$$p_1 p_2' = p_2 p_1' = E^2 + |\vec{p}|\,|\vec{p}'| \cos\theta$$

$$p_1 p_2 = E^2 + \vec{p}^{\,2}$$

$$p_1' p_2' = E^2 + \vec{p}'^{\,2}$$

$$(p_1 + p_2)^2 = 4E^2$$

$$E = |\vec{p}|$$

$l^{\pm} = \mu^{\pm}$ or $\tau^{\pm}$

105 MeV   1778 MeV

$E > m_{\mu}, m_{\tau}$

$$E^2 = |\vec{p}|^2 + \cancel{m_e^2}$$

$$X = \frac{e^4}{32 E^4 m_\ell^2 m_e^2}\left[\left(E^2 - p p' \cos\theta\right)^2 + \left(E^2 + p p' \cos\theta\right)^2 + m_\ell^2 \left(E^2 + \underbrace{p^2}_{\simeq E^2}\right)\right]$$

$$\left(\frac{d\sigma}{d\Omega'}\right)_{cm} = \frac{1}{64\pi^2 (E_1 + E_2)^2} \frac{|\vec{p}_1'|}{|\vec{p}_1|} \prod_\ell 2 m_\ell \underbrace{|M|^2}_{\simeq X}$$

$$\left(\frac{d\sigma}{d\Omega'}\right)_{cm} = \frac{1}{64\pi^2 \, 4E^2} \frac{p'}{p} \, 4 m_e^2 \, 4 m_\ell^2 \, \frac{e^4}{32 E^4 m_\ell^2 m_e^2} \left[2E^4 + 2 \underbrace{\vec{p}^2}_{\simeq E^2} \vec{p}'^2 \cos^2\theta + 2 m_\ell^2 E^2 \right]$$

$$\boxed{\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{\alpha^2}{16E^4}\frac{|\vec{p}'|}{E}\left(E^2 + |\vec{p}'|^2\cos^2\theta + m_e^2\right)}$$

$$\alpha = \frac{e^2}{4\pi}$$

$$m_e \ll E$$

$$(\sigma_{TOT})_{CM} = \int_0^{2\pi} d\phi \int_{-1}^{1} d\cos\theta \left(\frac{d\sigma}{d\Omega}\right)_{CM}$$

$$2\pi \left.\frac{\cos^3\theta}{3}\right|_{-1}^{+1} = \frac{4\pi}{3}$$

4π

$$\boxed{(\sigma_{TOT})_{CM} = \frac{\pi\alpha^2}{4E^4}\frac{|\vec{p}'|}{E}\left(E^2 + m_e^2 + \frac{|\vec{p}'|^2}{3}\right)}$$

$$1 + \frac{1}{3} = \frac{4}{3}$$

$$|\vec{p}'|^2 = E^2 - m_e^2$$

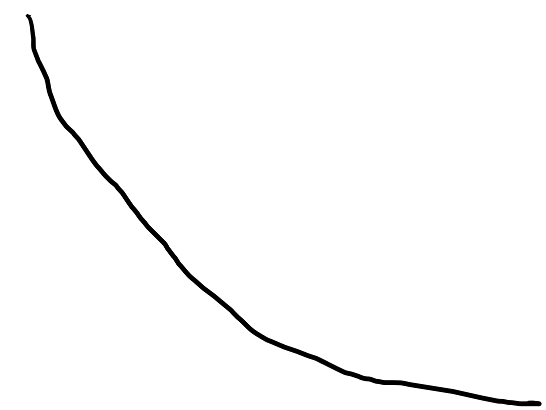
$$E \gg m_e$$

$$E^2 = |\vec{p}|^2 + \cancel{m_e^2}$$

$$E = |\vec{p}|$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{\alpha^2}{16E^2}\left(1+\cos^2\theta\right)$$

$$\left(\sigma_{TOT}\right) = \frac{\pi\alpha^2}{3E^2}$$



τ descoberto $e^+ e^- \to \tau^+ \tau^-$

$(\sigma_{TOT})(E, m_\ell)$

$$\langle 0|\bar{\psi}_\alpha(x_1)\,\psi_\delta(x_2)|0\rangle = -\,\langle 0|\psi_\delta(x_2)\,\bar{\psi}_\alpha(x_1)|0\rangle$$

$$= \langle 0|\bar{\psi}^{+}_\alpha(x_1)\,\psi^{-}_\delta(x_2)|0\rangle$$

$$= \langle 0| -\psi^{-}_\delta(x_2)\,\underline{\bar{\psi}^{+}(x_1)} + \{\bar{\psi}^{+}_\alpha(x_1),\,\psi^{-}_\delta(x_2)\}|0\rangle$$

$$= \langle 0|\{\qquad\}|0\rangle = S_F$$

$$\underbrace{\bar{\psi}_\alpha \psi_\beta} = - \underbrace{\psi_\beta \bar{\psi}_\alpha} = i \left(S_F\right)_{\alpha\beta}(x_1 - x_2)$$