Chapter Seven

Power Dividers and Directional Couplers

Power dividers and directional couplers are passive microwave components used for power division or power combining, as illustrated in Figure 7.1. In power division, an input signal is divided into two (or more) output signals of lesser power, while a power combiner accepts two or more input signals and combines them at an output port. The coupler or divider may have three ports, four ports, or more, and may be (ideally) lossless. Three-port networks take the form of T-junctions and other power dividers, while four-port networks take the form of directional couplers and hybrids. Power dividers usually provide in-phase output signals with an equal power division ratio (3 dB), but unequal power division ratios are also possible. Directional couplers can be designed for arbitrary power division, while hybrid junctions usually have equal power division. Hybrid junctions have either a 90° or a 180° phase shift between the output ports.

A wide variety of waveguide couplers and power dividers were invented and characterized at the MIT Radiation Laboratory in the 1940s. These included E- and H-plane waveguide T-junctions, the Bethe hole coupler, multihole directional couplers, the Schwinger coupler, the waveguide magic-T, and various types of couplers using coaxial probes. In the mid-1950s through the 1960s, many of these couplers were reinvented to use stripline or microstrip technology. The increasing use of planar lines also led to the development of new types of couplers and dividers, such as the Wilkinson divider, the branch line hybrid, and the coupled line directional coupler.

We will first discuss some of the general properties of three- and four-port networks, and then treat the analysis and design of several of the most common types of power dividers, couplers, and hybrids.

7.1 BASIC PROPERTIES OF DIVIDERS AND DOUPLERS

In this section we will use properties of the scattering matrix developed in Section 4.3 to derive some of the basic characteristics of three- and four-port networks. We will also define

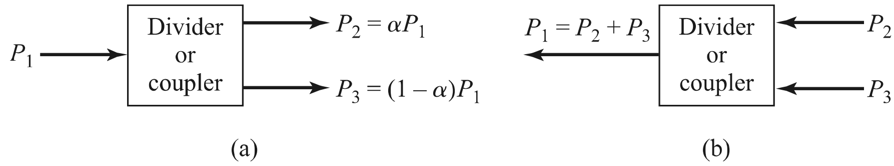


FIGURE 7.1 Power division and combining. (a) Power division. (b) Power combining.

isolation, coupling, and directivity, which are important quantities for the characterization of couplers and hybrids.

Three-Port Networks (T-Junctions)

The simplest type of power divider is a *T-junction*, which is a three-port network with two inputs and one output. The scattering matrix of an arbitrary three-port network has nine independent elements:

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}. \tag{7.1}$$

If the device is passive and contains no anisotropic materials, then it must be reciprocal and its scattering matrix will be symmetric ($S_{ij} = S_{ji}$). Usually, to avoid power loss, we would like to have a junction that is lossless and matched at all ports. We can easily show, however, that it is impossible to construct such a three-port lossless reciprocal network that is matched at all ports.

If all ports are matched, then $S_{ii} = 0$, and if the network is reciprocal, the scattering matrix of (7.1) reduces to

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}. \tag{7.2}$$

If the network is also lossless, then energy conservation requires that the scattering matrix satisfy the unitary properties of (4.53), which leads to the following conditions [1, 2]:

$$|S_{12}|^2 + |S_{13}|^2 = 1, \tag{7.3a}$$

$$|S_{12}|^2 + |S_{23}|^2 = 1, \tag{7.3b}$$

$$|S_{13}|^2 + |S_{23}|^2 = 1, \tag{7.3c}$$

$$S_{13}^* S_{23} = 0, \tag{7.3d}$$

$$S_{23}^* S_{12} = 0, \tag{7.3e}$$

$$S_{12}^* S_{13} = 0. \tag{7.3f}$$

Equations (7.3d)–(7.3f) show that at least two of the three parameters (S_{12}, S_{13}, S_{23}) must be zero. However, this condition will always be inconsistent with one of equations (7.3a)–(7.3c), implying that a three-port network cannot be simultaneously lossless, reciprocal, and matched at all ports. If any one of these three conditions is relaxed, then a physically realizable device is possible.

If the three-port network is nonreciprocal, then $S_{ij} \neq S_{ji}$, and the conditions of input matching at all ports and energy conservation can be satisfied. Such a device is known as a *circulator*, and generally relies on an anisotropic material, such as ferrite, to achieve nonreciprocal behavior. Ferrite circulators will be discussed in more detail in Chapter 9, but we can demonstrate here that any matched lossless three-port network must be nonreciprocal and, thus, a circulator. The scattering matrix of a matched three-port network has the following form:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}. \tag{7.4}$$

If the network is lossless, $[S]$ must be unitary, which implies the following conditions:

$$S_{31}^* S_{32} = 0, \tag{7.5a}$$

$$S_{21}^* S_{23} = 0, \tag{7.5b}$$

$$S_{12}^* S_{13} = 0, \tag{7.5c}$$

$$|S_{12}|^2 + |S_{13}|^2 = 1, \tag{7.5d}$$

$$|S_{21}|^2 + |S_{23}|^2 = 1, \tag{7.5e}$$

$$|S_{31}|^2 + |S_{32}|^2 = 1. \tag{7.5f}$$

These equations can be satisfied in one of two ways. Either

$$S_{12} = S_{23} = S_{31} = 0, \quad |S_{21}| = |S_{32}| = |S_{13}| = 1, \tag{7.6a}$$

or

$$S_{21} = S_{32} = S_{13} = 0, \quad |S_{12}| = |S_{23}| = |S_{31}| = 1. \tag{7.6b}$$

These results shows that $S_{ij} \neq S_{ji}$ for $i \neq j$, which implies that the device must be nonreciprocal. The scattering matrices for the two solutions of (7.6) are shown in Figure 7.2, together with the symbols for the two possible types of circulators. The only difference between the two cases is in the direction of power flow between the ports: solution (7.6a) corresponds to a circulator that allows power flow only from port 1 to 2, or port 2 to 3, or port 3 to 1, while solution (7.6b) corresponds to a circulator with the opposite direction of power flow.

Alternatively, a lossless and reciprocal three-port network can be physically realized if only two of its ports are matched [1]. If ports 1 and 2 are the matched ports, then the scattering matrix can be written as

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}. \tag{7.7}$$

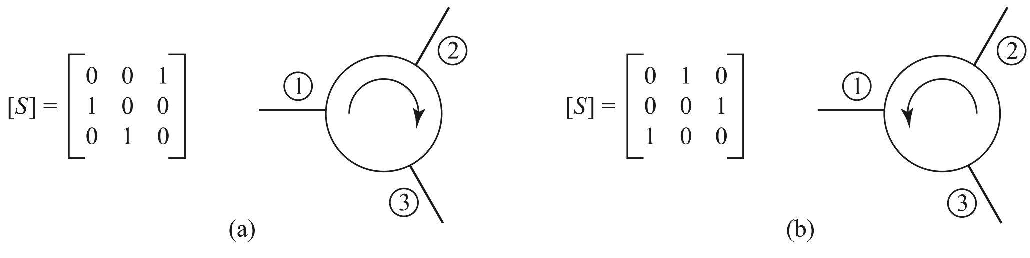


FIGURE 7.2 Two types of circulators and their scattering matrices. (a) Clockwise circulation. (b) Counterclockwise circulation. The phase references for the ports are arbitrary.

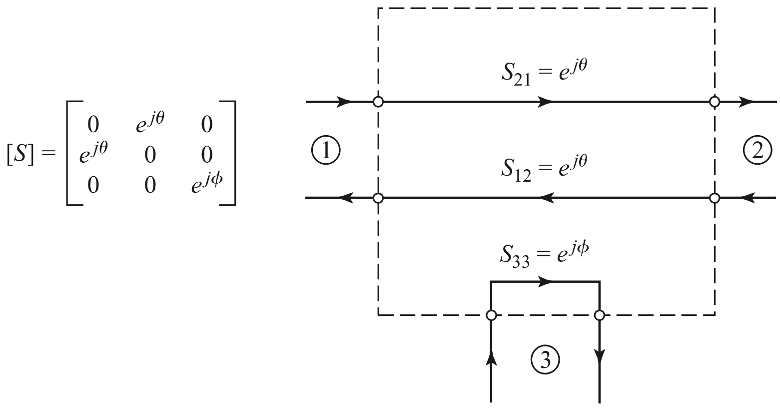


FIGURE 7.3 A reciprocal lossless three-port network matched at ports 1 and 2.

To be lossless, the following unitarity conditions must be satisfied:

$$S_{13}^* S_{23} = 0, \tag{7.8a}$$

$$S_{12}^* S_{13} + S_{23}^* S_{33} = 0, \tag{7.8b}$$

$$S_{23}^* S_{12} + S_{33}^* S_{13} = 0, \tag{7.8c}$$

$$|S_{12}|^2 + |S_{13}|^2 = 1, \tag{7.8d}$$

$$|S_{12}|^2 + |S_{23}|^2 = 1, \tag{7.8e}$$

$$|S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 = 1. \tag{7.8f}$$

Equations (7.8d) and (7.8e) show that $|S_{13}| = |S_{23}|$, so (7.8a) leads to the result that $S_{13} = S_{23} = 0$. Then, $|S_{12}| = |S_{33}| = 1$. The scattering matrix and corresponding signal flow graph for this network are shown in Figure 7.3, where it is seen that the network actually degenerates into two separate components—one a matched two-port line and the other a totally mismatched one-port.

Finally, if the three-port network is allowed to be lossy, it can be reciprocal and matched at all ports; this is the case of the *resistive divider*, which will be discussed in Section 7.2. In addition, a lossy three-port network can be made to have isolation between its output ports (e.g., $S_{23} = S_{32} = 0$).

Four-Port Networks (Directional Couplers)

The scattering matrix of a reciprocal four-port network matched at all ports has the following form:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}. \tag{7.9}$$

If the network is lossless, 10 equations result from the unitarity, or energy conservation, condition [1, 2]. Consider the multiplication of row 1 and row 2, and the multiplication of row 4 and row 3:

$$S_{13}^* S_{23} + S_{14}^* S_{24} = 0, \tag{7.10a}$$

$$S_{14}^* S_{13} + S_{24}^* S_{23} = 0. \tag{7.10b}$$

Multiply (7.10a) by S^*_{24}, and (7.10b) by S^*_{13}, and subtract to obtain

$$S^*_{14}(|S_{13}|^2 - |S_{24}|^2) = 0. \tag{7.11}$$

Similarly, the multiplication of row 1 and row 3, and the multiplication of row 4 and row 2, gives

$$S^*_{12}S_{23} + S^*_{14}S_{34} = 0, \tag{7.12a}$$

$$S^*_{14}S_{12} + S^*_{34}S_{23} = 0. \tag{7.12b}$$

Multiply (7.12a) by S_{12}, and (7.12b) by S_{34}, and subtract to obtain

$$S_{23}(|S_{12}|^2 - |S_{34}|^2) = 0. \tag{7.13}$$

One way for (7.11) and (7.13) to be satisfied is if $S_{14} = S_{23} = 0$, which results in a directional coupler. Then the self-products of the rows of the unitary scattering matrix of (7.9) yield the following equations:

$$|S_{12}|^2 + |S_{13}|^2 = 1, \tag{7.14a}$$

$$|S_{12}|^2 + |S_{24}|^2 = 1, \tag{7.14b}$$

$$|S_{13}|^2 + |S_{34}|^2 = 1, \tag{7.14c}$$

$$|S_{24}|^2 + |S_{34}|^2 = 1, \tag{7.14d}$$

which imply that $|S_{13}| = |S_{24}|$ [using (7.14a) and (7.14b)], and that $|S_{12}| = |S_{34}|$ [using (7.14b) and (7.14d)].

Further simplification can be made by choosing the phase references on three of the four ports. Thus, we choose $S_{12} = S_{34} = \alpha$, $S_{13} = \beta e^{j\theta}$, and $S_{24} = \beta e^{j\phi}$, where α and β are real, and θ and ϕ are phase constants to be determined (one of which we are still free to choose). The dot product of rows 2 and 3 gives

$$S^*_{12}S_{13} + S^*_{24}S_{34} = 0, \tag{7.15}$$

which yields a relation between the remaining phase constants as

$$\theta + \phi = \pi \pm 2n\pi. \tag{7.16}$$

If we ignore integer multiples of 2π, there are two particular choices that commonly occur in practice:

1. *A Symmetric Coupler*: $\theta = \phi = \pi/2$. The phases of the terms having amplitude β are chosen equal. Then the scattering matrix has the following form:

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}. \tag{7.17}$$

2. *An Antisymmetric Coupler*: $\theta = 0$, $\phi = \pi$. The phases of the terms having amplitude β are chosen to be 180° apart. Then the scattering matrix has the following form:

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}. \tag{7.18}$$

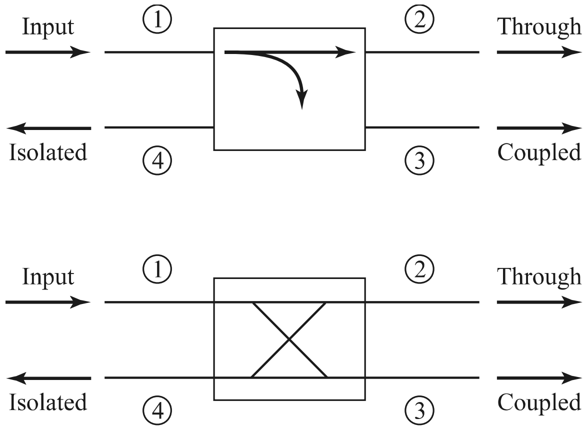


FIGURE 7.4 Two commonly used symbols for directional couplers, and power flow conventions.

Note that these two couplers differ only in the choice of reference planes. In addition, the amplitudes α and β are not independent, as (7.14a) requires that

$$\alpha^2 + \beta^2 = 1. \tag{7.19}$$

Thus, apart from phase references, an ideal four-port directional coupler has only one degree of freedom, leading to two possible configurations.

Another way for (7.11) and (7.13) to be satisfied is if $|S_{13}| = |S_{24}|$ and $|S_{12}| = |S_{34}|$. If we choose phase references, however, such that $S_{13} = S_{24} = \alpha$ and $S_{12} = S_{34} = j\beta$ [which satisfies (7.16)], then (7.10a) yields $\alpha(S_{23} + S_{14}^*) = 0$, and (7.12a) yields $\beta(S_{14}^* - S_{23}) = 0$. These two equations have two possible solutions. First, $S_{14} = S_{23} = 0$, which is the same as the above solution for the directional coupler. The other solution occurs for $\alpha = \beta = 0$, which implies that $S_{12} = S_{13} = S_{24} = S_{34} = 0$. This is the degenerate case of two decoupled two-port networks (between ports 1 and 4, and ports 2 and 3), which is of trivial interest and will not be considered further. We are thus left with the conclusion that any reciprocal, lossless, matched four-port network is a directional coupler.

The basic operation of a directional coupler can be illustrated with the aid of Figure 7.4, which shows two commonly used symbols for a directional coupler and the port definitions. Power supplied to port 1 is coupled to port 3 (the *coupled* port) with the coupling factor $|S_{13}|^2 = \beta^2$, while the remainder of the input power is delivered to port 2 (the *through* port) with the coefficient $|S_{12}|^2 = \alpha^2 = 1 - \beta^2$. In an ideal directional coupler, no power is delivered to port 4 (the *isolated* port).

The following quantities are commonly used to characterize a directional coupler:

$$\text{Coupling} = C = 10 \log \frac{P_1}{P_3} = -20 \log \beta \text{ dB}, \tag{7.20a}$$

$$\text{Directivity} = D = 10 \log \frac{P_3}{P_4} = 20 \log \frac{\beta}{|S_{14}|} \text{ dB}, \tag{7.20b}$$

$$\text{Isolation} = I = 10 \log \frac{P_1}{P_4} = -20 \log |S_{14}| \text{ dB}, \tag{7.20c}$$

$$\text{Insertion loss} = L = 10 \log \frac{P_1}{P_2} = -20 \log |S_{12}| \text{ dB}. \tag{7.20d}$$

The *coupling factor* indicates the fraction of the input power that is coupled to the output port. The *directivity* is a measure of the coupler's ability to isolate forward and backward waves (or the coupled and uncoupled ports). The *isolation* is a measure of the power delivered to the uncoupled port. These quantities are related as

$$I = D + C \text{ dB}. \tag{7.21}$$

The *insertion loss* accounts for the input power delivered to the through port, diminished by power delivered to the coupled and isolated ports. The ideal coupler has infinite directivity and isolation ($S_{14} = 0$). Then both α and β can be determined from the coupling factor, C.

Hybrid couplers are special cases of directional couplers, where the coupling factor is 3 dB, which implies that $\alpha = \beta = 1/\sqrt{2}$. There are two types of hybrids. The *quadrature hybrid* has a 90° phase shift between ports 2 and 3 ($\theta = \phi = \pi/2$) when fed at port 1, and is an example of a symmetric coupler. Its scattering matrix has the following form:

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}. \tag{7.22}$$

The *magic-T hybrid* and the *rat-race hybrid* have a 180° phase difference between ports 2 and 3 when fed at port 4, and are examples of an antisymmetric coupler. Its scattering matrix has the following form:

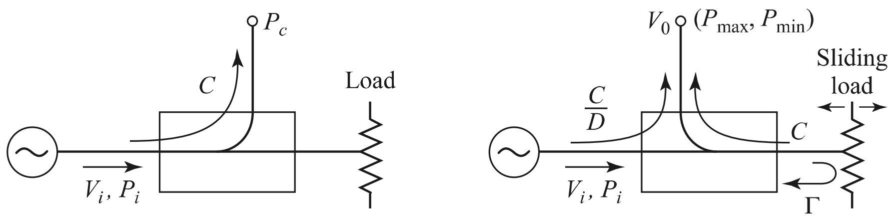
$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}. \tag{7.23}$$

POINT OF INTEREST: Measuring Coupler Directivity

The directivity of a directional coupler is a measure of the coupler's ability to separate forward and reverse wave components, and applications of directional couplers often require high (35 dB or greater) directivity. Poor directivity will limit the accuracy of a reflectometer, and can cause variations in the coupled power level from a coupler when there is even a small mismatch on the through line.

The directivity of a coupler generally cannot be measured directly because it involves a low-level signal that can be masked by coupled power from a reflected wave on the through arm. For example, if a coupler has $C = 20$ dB and $D = 35$ dB, with a load having a return loss RL $= 30$ dB, the signal level through the directivity path will be $D + C = 55$ dB below the input power, but the reflected power through the coupled arm will only be RL $+ C = 50$ dB below the input power.

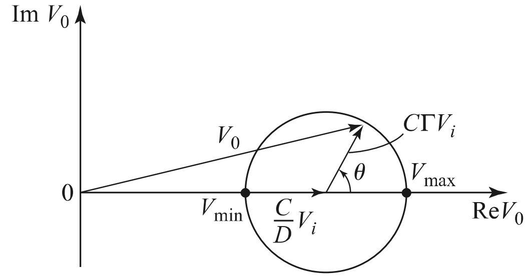
One way to measure coupler directivity uses a sliding matched load, as follows. First, the coupler is connected to a source and a matched load, as shown in the accompanying left-hand figure, and the coupled output power is measured. If we assume an input power P_i, this power will be $P_c = C^2 P_i$, where $C = 10^{(-C\text{dB})/20}$ is the numerical voltage coupling factor of the coupler. Next, the position of the coupler is reversed, and the through line is terminated with a sliding load, as shown in the right-hand figure.



Changing the position of the sliding load introduces a variable phase shift in the signal reflected from the load and coupled to the output port. The voltage at the output port can be written as

$$V_0 = V_i\left(\frac{C}{D} + C|\Gamma|e^{-j\theta}\right),$$

where V_i is the input voltage, $D = 10^{(D\ \mathrm{dB})/20} \geq 1$ is the numerical value of the directivity, $|\Gamma|$ is the reflection coefficient magnitude of the load, and θ is the path length difference between the directivity and reflected signals. Moving the sliding load changes θ, so the two signals will combine to trace out a circular locus, as shown in the following figure.



The minimum and maximum output powers are given by

$$P_{\min} = P_i\left(\frac{C}{D} - C|\Gamma|\right)^2, \quad P_{\max} = P_i\left(\frac{C}{D} + C|\Gamma|\right)^2.$$

Let M and m be defined in terms of these powers as follows:

$$M = \frac{P_c}{P_{\max}} = \left(\frac{D}{1+|\Gamma|D}\right)^2, \quad m = \frac{P_{\max}}{P_{\min}} = \left(\frac{1+|\Gamma|D}{1-|\Gamma|D}\right)^2.$$

These ratios can be accurately measured directly by using a variable attenuator between the source and coupler. The coupler directivity (numerical) can then be found as

$$D = M\left(\frac{2m}{m+1}\right).$$

This method requires that $|\Gamma| < 1/D$ or, in dB, RL $> D$.

Reference: M. Sucher and J. Fox, eds., *Handbook of Microwave Measurements*, 3rd edition, Volume II, Polytechnic Press, New York, 1963.

7.2 THE T-JUNCTION POWER DIVIDER

The T-junction power divider is a simple three-port network that can be used for power division or power combining, and it can be implemented in virtually any type of transmission line medium. Figure 7.5 shows some commonly used T-junctions in waveguide and microstrip line or stripline form. The junctions shown here are, in the absence of transmission line loss, lossless junctions. Thus, as discussed in the preceding section, such junctions cannot be matched simultaneously at all ports. We will analyze the T-junction divider below, followed by a discussion of the resistive power divider, which can be matched at all ports but is not lossless.

Lossless Divider

The lossless T-junction dividers of Figure 7.5 can all be modeled as a junction of three transmission lines, as shown in Figure 7.6 [3]. In general, there may be fringing fields and

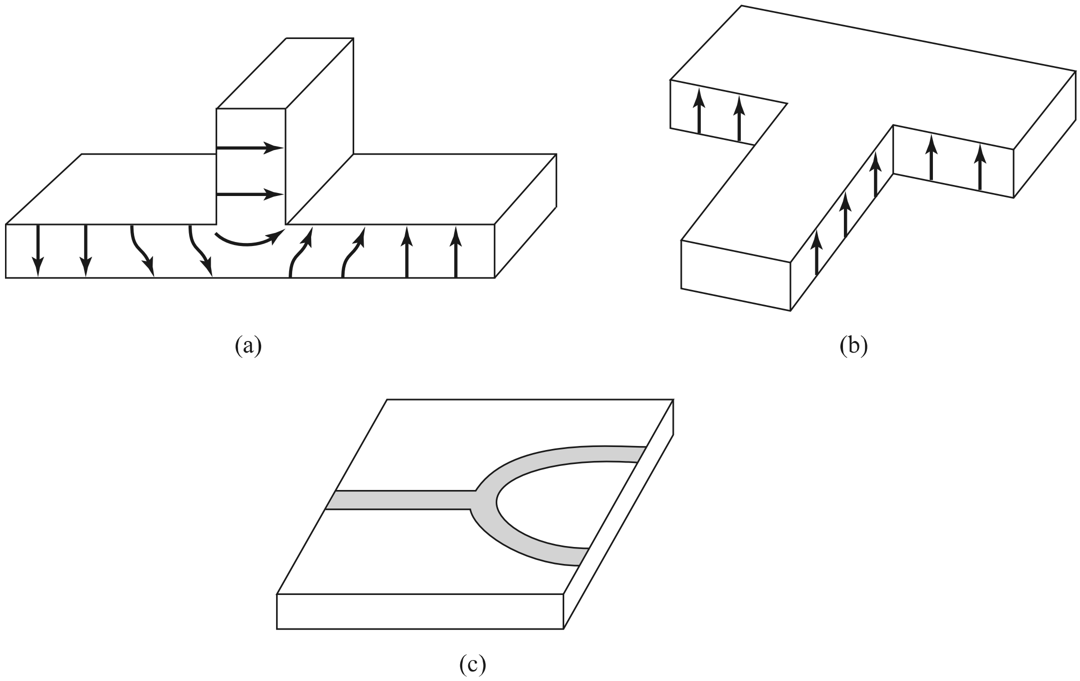


FIGURE 7.5 Various T-junction power dividers. (a) E-plane waveguide T. (b) H-plane waveguide T. (c) Microstrip line T-junction divider.

higher order modes associated with the discontinuity at such a junction, leading to stored energy that can be accounted for by a lumped susceptance, B. In order for the divider to be matched to the input line of characteristic impedance Z_0, we must have

$$Y_{\text{in}} = jB + \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0}. \tag{7.24}$$

If the transmission lines are assumed to be lossless (or of low loss), then the characteristic impedances are real. If we also assume $B = 0$, then (7.24) reduces to

$$\frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_0}. \tag{7.25}$$

In practice, if B is not negligible, some type of discontinuity compensation or a reactive tuning element can usually be used to cancel this susceptance, at least over a narrow frequency range.

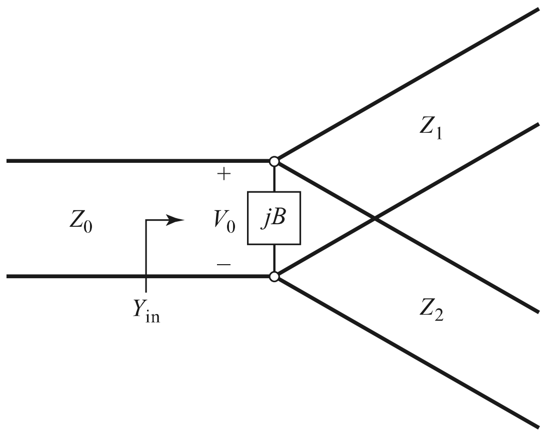


FIGURE 7.6 Transmission line model of a lossless T-junction divider.

The output line impedances, Z_1 and Z_2, can be selected to provide various power division ratios. Thus, for a 50 Ω input line, a 3 dB (equal split) power divider can be made by using two 100 Ω output lines. If necessary, quarter-wave transformers can be used to bring the output line impedances back to the desired levels. If the output lines are matched, then the input line will be matched. There will be no isolation between the two output ports, however, and there will be a mismatch looking into the output ports.

EXAMPLE 7.1 THE T-JUNCTION POWER DIVIDER

A lossless T-junction power divider has a source impedance of 50 Ω. Find the output characteristic impedances so that the output powers are in a 2:1 ratio. Compute the reflection coefficients seen looking into the output ports.

Solution

If the voltage at the junction is V_0, as shown in Figure 7.6, the input power to the matched divider is

$$P_{\text{in}} = \frac{1}{2}\frac{V_0^2}{Z_0},$$

while the output powers are

$$P_1 = \frac{1}{2}\frac{V_0^2}{Z_1} = \frac{1}{3}P_{\text{in}},$$

$$P_2 = \frac{1}{2}\frac{V_0^2}{Z_2} = \frac{2}{3}P_{\text{in}}.$$

These results yield the characteristic impedances as

$$Z_1 = 3Z_0 = 150\ \Omega,$$

$$Z_2 = \frac{3Z_0}{2} = 75\ \Omega.$$

The input impedance to the junction is

$$Z_{\text{in}} = 75||150 = 50\ \Omega,$$

so that the input is matched to the 50 Ω source.

Looking into the 150 Ω output line, we see an impedance of $50\,||\,75 = 30\ \Omega$, while at the 75 Ω output line we see an impedance of $50\,||\,150 = 37.5\ \Omega$. The reflection coefficients seen looking into these ports are

$$\Gamma_1 = \frac{30 - 150}{30 + 150} = -0.666,$$

$$\Gamma_2 = \frac{37.5 - 75}{37.5 + 75} = -0.333.$$

■

Resistive Divider

If a three-port divider contains lossy components, it can be made to be matched at all ports, although the two output ports may not be isolated [3]. The circuit for such a divider is illustrated in Figure 7.7, using lumped-element resistors. An equal-split (−3 dB) divider is shown, but unequal power division ratios are also possible.

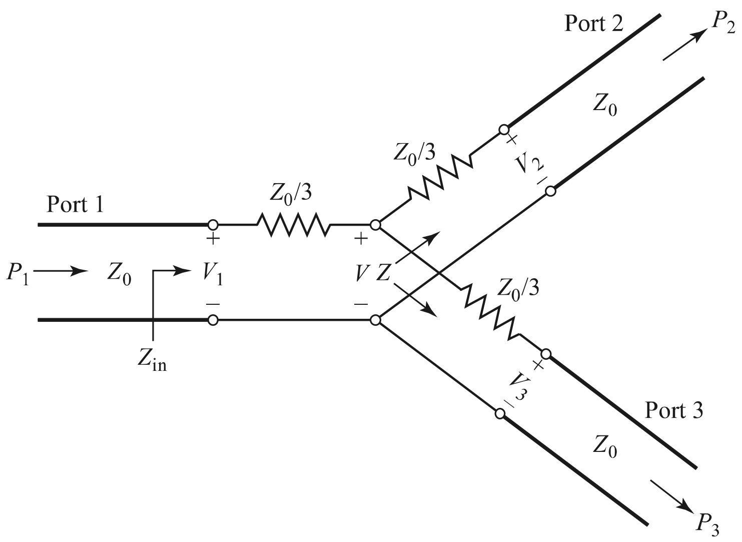


FIGURE 7.7 An equal-split three-port resistive power divider.

The resistive divider of Figure 7.7 can easily be analyzed using circuit theory. Assuming that all ports are terminated in the characteristic impedance Z_0, the impedance Z, seen looking into the $Z_0/3$ resistor followed by a terminated output line, is

$$Z = \frac{Z_0}{3} + Z_0 = \frac{4Z_0}{3}. \tag{7.26}$$

Then the input impedance of the divider is

$$Z_{in} = \frac{Z_0}{3} + \frac{2Z_0}{3} = Z_0, \tag{7.27}$$

which shows that the input is matched to the feed line. Because the network is symmetric from all three ports, the output ports are also matched. Thus, $S_{11} = S_{22} = S_{33} = 0$.

If the voltage at port 1 is V_1, then by voltage division the voltage V at the center of the junction is

$$V = V_1 \frac{2Z_0/3}{Z_0/3 + 2Z_0/3} = \frac{2}{3}V_1, \tag{7.28}$$

and the output voltages are, again by voltage division,

$$V_2 = V_3 = V \frac{Z_0}{Z_0 + Z_0/3} = \frac{3}{4}V = \frac{1}{2}V_1. \tag{7.29}$$

Thus, $S_{21} = S_{31} = S_{23} = 1/2$, so the output powers are 6 dB below the input power level. The network is reciprocal, so the scattering matrix is symmetric, and it can be written as

$$[S] = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}. \tag{7.30}$$

The reader may verify that this is not a unitary matrix.

The power delivered to the input of the divider is

$$P_{in} = \frac{1}{2}\frac{V_1^2}{Z_0}, \tag{7.31}$$

while the output powers are

$$P_2 = P_3 = \frac{1}{2}\frac{(1/2V_1)^2}{Z_0} = \frac{1}{8}\frac{V_1^2}{Z_0} = \frac{1}{4}P_{\text{in}}, \tag{7.32}$$

which shows that half of the supplied power is dissipated in the resistors.

7.3 THE WILKINSON POWER DIVIDER

The lossless T-junction divider suffers from the disadvantage of not being matched at all ports, and it does not have isolation between output ports. The resistive divider can be matched at all ports, but even though it is not lossless, isolation is still not achieved. From the discussion in Section 7.1, however, we know that a lossy three-port network can be made having all ports matched, with isolation between output ports. The *Wilkinson power divider* [4] is such a network, with the useful property of appearing lossless when the output ports are matched; that is, only reflected power from the output ports is dissipated.

The Wilkinson power divider can be made with arbitrary power division, but we will first consider the equal-split (3 dB) case. This divider is often made in microstrip line or stripline form, as depicted in Figure 7.8a; the corresponding transmission line circuit is given in Figure 7.8b. We will analyze this circuit by reducing it to two simpler circuits driven by symmetric and antisymmetric sources at the output ports. This "even-odd" mode analysis technique [5] will also be useful for other networks that we will study in later sections.

Even-Odd Mode Analysis

For simplicity, we can normalize all impedances to the characteristic impedance Z_0, and redraw the circuit of Figure 7.8b with voltage generators at the output ports as shown in Figure 7.9. This network has been drawn in a form that is symmetric across the midplane; the two source resistors of normalized value 2 combine in parallel to give a resistor of normalized value 1, representing the impedance of a matched source. The quarter-wave lines have a normalized characteristic impedance Z, and the shunt resistor has a normalized value of r; we shall show that, for the equal-split power divider, these values should be $Z = \sqrt{2}$ and $r = 2$, as given in Figure 7.8.

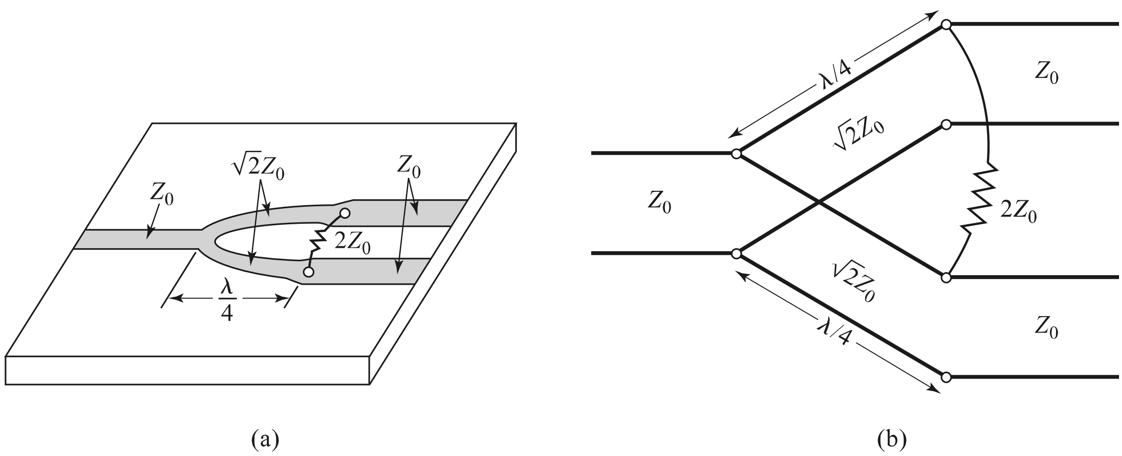


FIGURE 7.8 The Wilkinson power divider. (a) An equal-split Wilkinson power divider in microstrip line form. (b) Equivalent transmission line circuit.

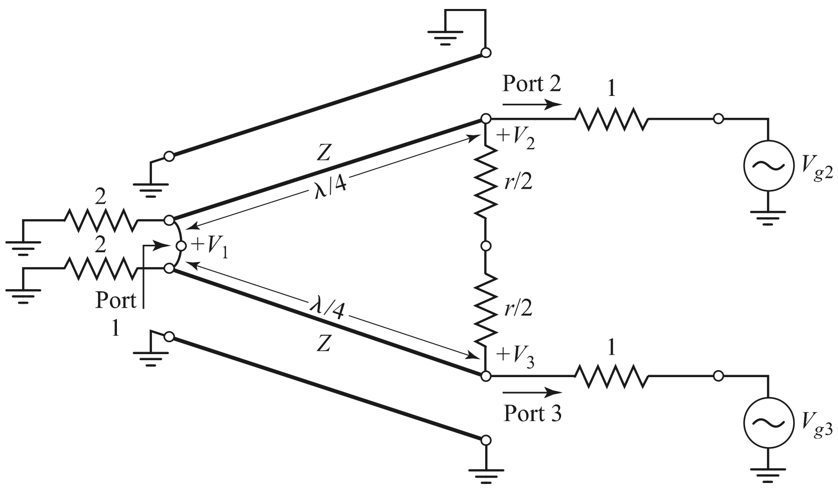


FIGURE 7.9 The Wilkinson power divider circuit in normalized and symmetric form.

Now define two separate modes of excitation for the circuit of Figure 7.9: the *even mode*, where $V_{g2} = V_{g3} = 2V_0$, and the *odd mode*, where $V_{g2} = -V_{g3} = 2V_0$. Superposition of these two modes effectively produces an excitation of $V_{g2} = 4V_0$ and $V_{g3} = 0$, from which we can find the scattering parameters of the network. We now treat these two modes separately.

Even mode: For even-mode excitation, $V_{g2} = V_{g3} = 2V_0$, so $V_2^e = V_3^e$, and therefore no current flows through the $r/2$ resistors or the short circuit between the inputs of the two transmission lines at port 1. We can then bisect the network of Figure 7.9 with open circuits at these points to obtain the network of Figure 7.10a (the grounded side of the $\lambda/4$ line is not shown). Then, looking into port 2, we see an impedance

$$Z_{\text{in}}^e = \frac{Z^2}{2}, \tag{7.33}$$

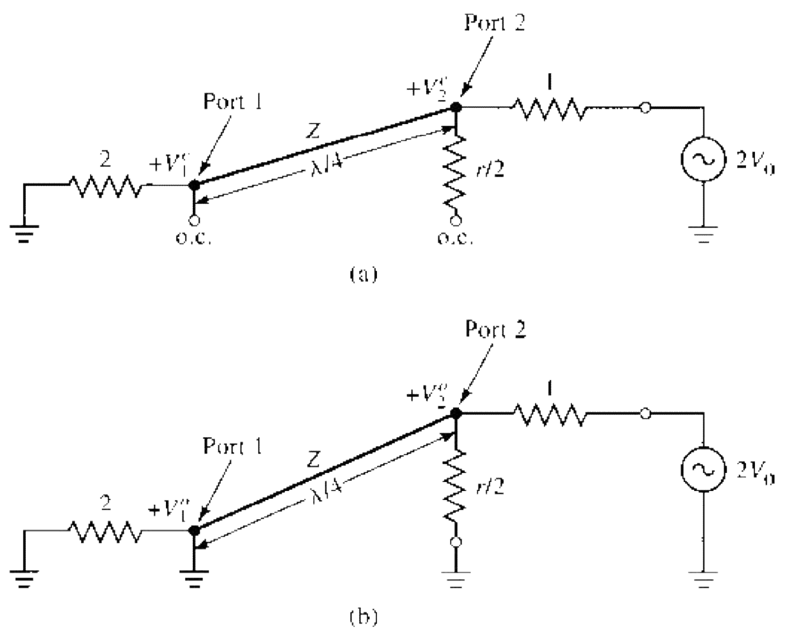


FIGURE 7.10 Bisection of the circuit of Figure 7.9. (a) Even-mode excitation. (b) Odd-mode excitation.

since the transmission line looks like a quarter-wave transformer. Thus, if $Z = \sqrt{2}$, port 2 will be matched for even-mode excitation; then $V_2^e = V_0$ since $Z_{in}^e = 1$. The $r/2$ resistor is superfluous in this case since one end is open-circuited. Next, we find V_1^e from the transmission line equations. If we let $x = 0$ at port 1 and $x = -\lambda/4$ at port 2, we can write the voltage on the transmission line section as

$$V(x) = V^+(e^{-j\beta x} + \Gamma e^{j\beta x}).$$

Then

$$V_2^e = V(-\lambda/4) = jV^+(1 - \Gamma) = V_0, \tag{7.34a}$$

$$V_1^e = V(0) = V^+(1 + \Gamma) = jV_0 \frac{\Gamma + 1}{\Gamma - 1}. \tag{7.34b}$$

The reflection coefficient Γ is that seen at port 1 looking toward the resistor of normalized value 2, so

$$\Gamma = \frac{2 - \sqrt{2}}{2 + \sqrt{2}},$$

and

$$V_1^e = -jV_0\sqrt{2}. \tag{7.35}$$

Odd mode: For odd-mode excitation, $V_{g2} = -V_{g3} = 2V_0$, and so $V_2^o = -V_3^o$, and there is a voltage null along the middle of the circuit in Figure 7.9. We can then bisect this circuit by grounding it at two points on its midplane to give the network of Figure 7.10b. Looking into port 2, we see an impedance of $r/2$ since the parallel-connected transmission line is $\lambda/4$ long and shorted at port 1, and so looks like an open circuit at port 2. Thus, port 2 will be matched for odd-mode excitation if we select $r = 2$. Then $V_2^o = V_0$ and $V_1^o = 0$; for this mode of excitation all power is delivered to the $r/2$ resistors, with none going to port 1.

Finally, we must find the input impedance at port 1 of the Wilkinson divider when ports 2 and 3 are terminated in matched loads. The resulting circuit is shown in Figure 7.11a, where it is seen that this is similar to an even mode of excitation since $V_2 = V_3$. No current flows through the resistor of normalized value 2, so it can be removed, leaving the circuit of Figure 7.11b. We then have the parallel connection of two quarter-wave transformers terminated in loads of unity (normalized). The input impedance is

$$Z_{in} = \frac{1}{2}(\sqrt{2})^2 = 1. \tag{7.36}$$

In summary, we can establish the following scattering parameters for the Wilkinson divider:

$S_{11} = 0$	($Z_{in} = 1$ at port 1)
$S_{22} = S_{33} = 0$	(ports 2 and 3 matched for even and odd modes)
$S_{12} = S_{21} = \dfrac{V_1^e + V_1^o}{V_2^e + V_2^o} = -j/\sqrt{2}$	(symmetry due to reciprocity)
$S_{13} = S_{31} = -j/\sqrt{2}$	(symmetry of ports 2 and 3)
$S_{23} = S_{32} = 0$	(due to short or open at bisection)

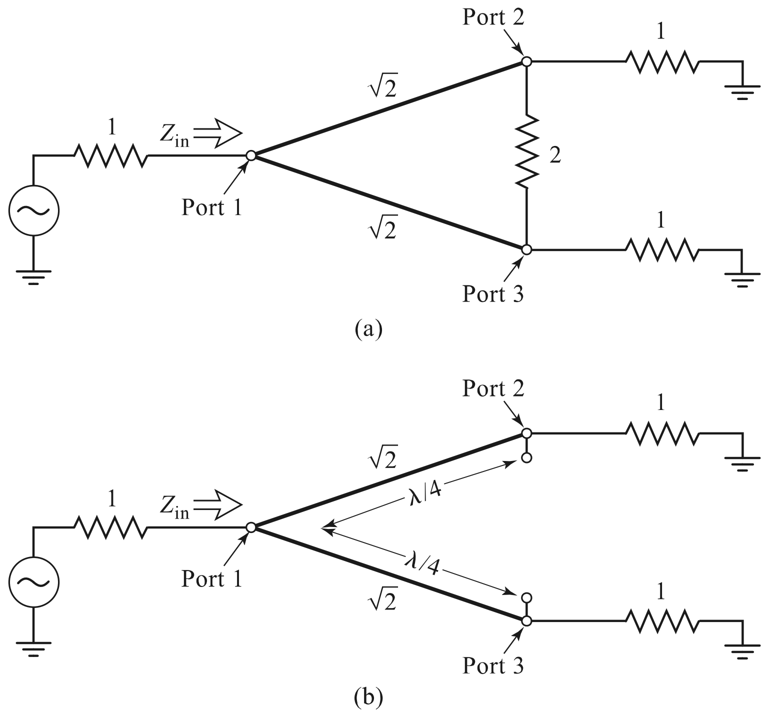


FIGURE 7.11 Analysis of the Wilkinson divider to find S_{11}. (a) The terminated Wilkinson divider. (b) Bisection of the circuit in (a).

The preceding formula for S_{12} applies because all ports are matched when terminated with matched loads. Note that when the divider is driven at port 1 and the outputs are matched, no power is dissipated in the resistor. Thus the divider is lossless when the outputs are matched; only reflected power from ports 2 or 3 is dissipated in the resistor. Because $S_{23} = S_{32} = 0$, ports 2 and 3 are isolated.

EXAMPLE 7.2 DESIGN AND PERFORMANCE OF A WILKINSON DIVIDER

Design an equal-split Wilkinson power divider for a 50 Ω system impedance at frequency f_0, and plot the return loss (S_{11}), insertion loss ($S_{21} = S_{31}$), and isolation ($S_{23} = S_{32}$) versus frequency from $0.5 f_0$ to $1.5 f_0$.

Solution

From Figure 7.8 and the above derivation, we have that the quarter-wave transmission lines in the divider should have a characteristic impedance of

$$Z = \sqrt{2} Z_0 = 70.7\ \Omega,$$

and the shunt resistor a value of

$$R = 2Z_0 = 100\ \Omega.$$

The transmission lines are $\lambda/4$ long at the frequency f_0. Using a computer-aided design tool for the analysis of microwave circuits, the scattering parameter magnitudes were calculated and plotted in Figure 7.12. ■

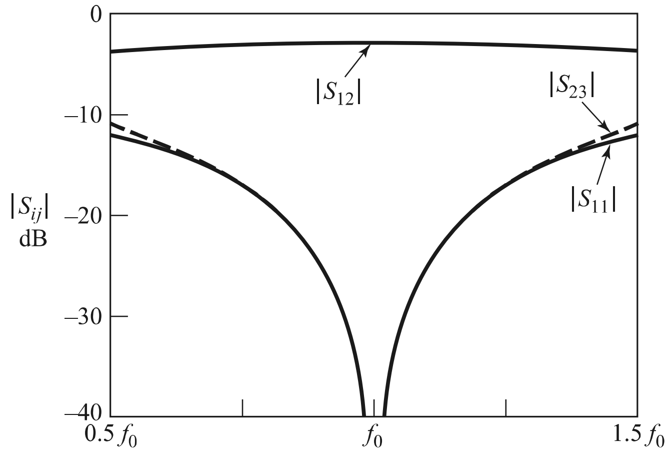


FIGURE 7.12 Frequency response of an equal-split Wilkinson power divider. Port 1 is the input port; ports 2 and 3 are the output ports.

Unequal Power Division and *N*-Way Wilkinson Dividers

Wilkinson-type power dividers can also be made with unequal power splits; a microstrip line version is shown in Figure 7.13. If the power ratio between ports 2 and 3 is $K^2 = P_3/P_2$, then the following design equations apply:

$$Z_{03} = Z_0\sqrt{\frac{1+K^2}{K^3}}, \tag{7.37a}$$

$$Z_{02} = K^2 Z_{03} = Z_0\sqrt{K(1+K^2)}, \tag{7.37b}$$

$$R = Z_0\left(K + \frac{1}{K}\right). \tag{7.37c}$$

Note that the above results reduce to the equal-split case for $K=1$. Also observe that the output lines are matched to the impedances $R_2 = Z_0 K$ and $R_3 = Z_0/K$, as opposed to the impedance Z_0; matching transformers can be used to transform these output impedances.

The Wilkinson divider can also be generalized to an N-way divider or combiner [4], as shown in Figure 7.14. This circuit can be matched at all ports, with isolation between all ports. A disadvantage, however, is the fact that the divider requires crossovers for the resistors for $N \geq 3$, which makes fabrication difficult in planar form. The Wilkinson divider can also be made with stepped multiple sections, for increased bandwidth. A photograph of a four-way Wilkinson divider network is shown in Figure 7.15.

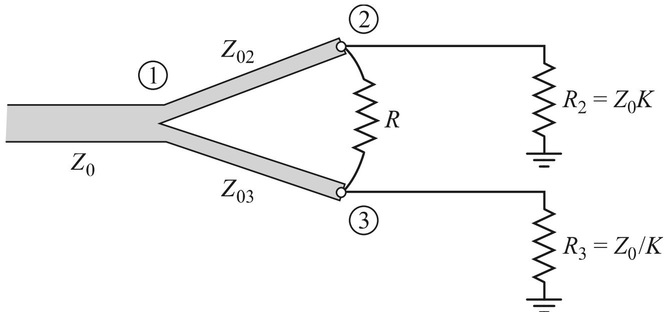


FIGURE 7.13 A Wilkinson power divider in microstrip form having unequal power division.

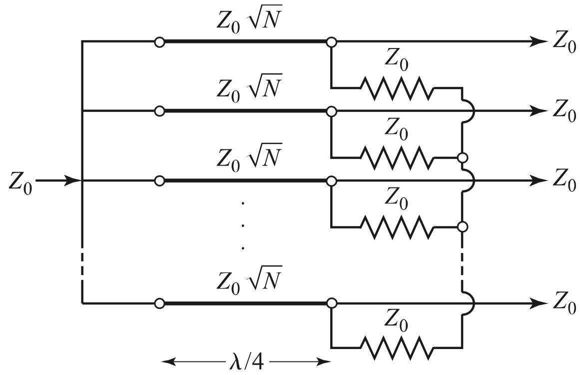


FIGURE 7.14 An N-way, equal-split Wilkinson power divider.

7.4 WAVEGUIDE DIRECTIONAL COUPLERS

We now turn our attention to directional couplers, which are four-port devices with the characteristics discussed in Section 7.1. To review the basic operation, consider the directional coupler schematic symbols shown in Figure 7.4. Power incident at port 1 will couple to port 2 (the through port) and to port 3 (the coupled port), but not to port 4 (the isolated port). Similarly, power incident in port 2 will couple to ports 1 and 4, but not 3. Thus, ports 1 and 4 are decoupled, as are ports 2 and 3. The fraction of power coupled from port 1 to port 3 is given by C, the coupling coefficient, as defined in (7.20a), and the leakage of power from port 1 to port 4 is given by I, the isolation, as defined in (7.20c). Another quantity that characterizes a coupler is the directivity, $D = I - C$ (dB), which is the ratio

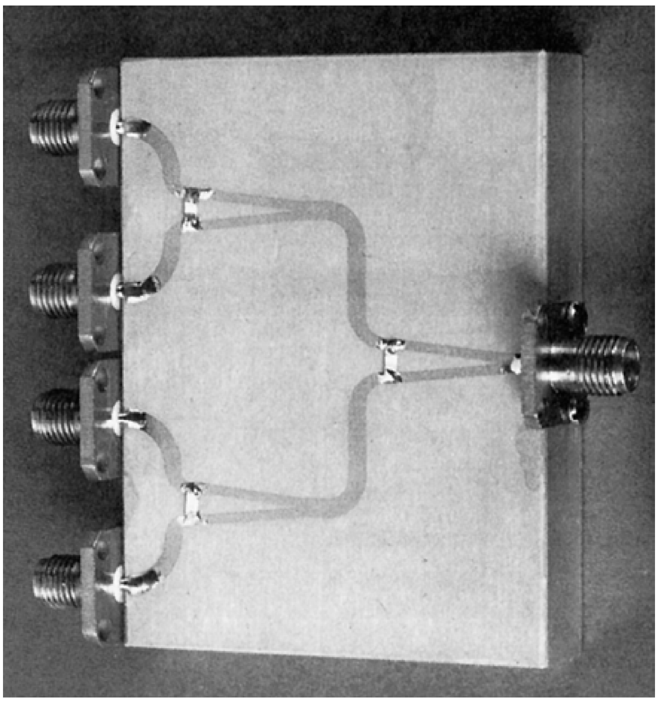


FIGURE 7.15 Photograph of a four-way corporate power divider network using three microstrip Wilkinson power dividers. Note the isolation chip resistors.

Courtesy of M. D. Abouzahra, MIT Lincoln Laboratory, Lexington, Mass.