



PMR3523 Controle Moderno

Apoio à Aula

Projeto de controladores por realimentação dos estados
(alocação de polos)
Distúrbio e Referência (Servo)

- CAP 3 - LIVRO TEXTO



- OBJETIVO ANTERIOR $x(t) \rightarrow 0, x(0) \neq 0$ (REGULADOR)

- OBJETIVO AGORA É ACOMPANHAR TRAJETÓRIA (SET-POINT)

$$y = C \cdot x \rightarrow y_d \text{ (set-point)}$$

USAREMOS $u = -G \cdot x + v_d$

$$\Rightarrow \dot{x} = Ax + Bu = Ax + B(-Gx + v_d)$$

$$\dot{x} = (A - BG)x + B \cdot v_d$$

\Rightarrow NO EQUILÍBRIO $\dot{x} = 0$ E $x = x_d$ TAL QUE $y_d = C \cdot x_d$

$$0 = (A - BG) \cdot x_d + B \cdot v_d \Rightarrow x_d = -(A - BG)^{-1} \cdot B \cdot v_d$$

$$y_d = C \cdot x_d = -C \cdot (A - BG)^{-1} \cdot B \cdot v_d$$

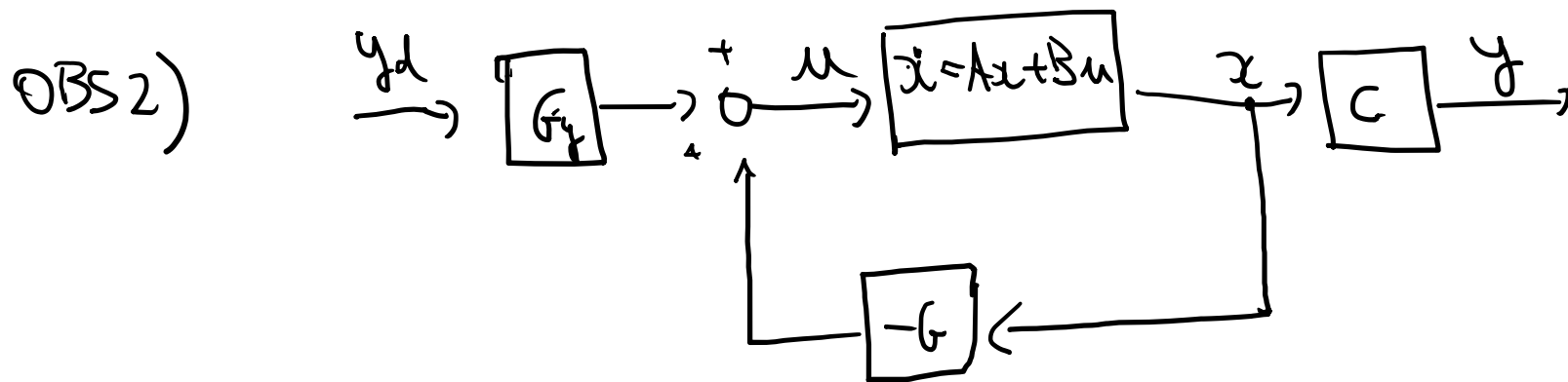


$$y_d = C \cdot x_d = -C \cdot (A - BG)^{-1} \cdot B \cdot v_d$$

$$v_d = - \left[C \cdot (A - BG)^{-1} \cdot B \right]^{-1} \cdot y_d$$

MATRIZ PRÉ COMPENSAÇÃO = G_y
SET-POINT

OBS: $(A - BG)^{-1}$ EXISTE E POSSUI POLOS ESTÁVEIS
POIS É A MATRIZ DO SISTEMA EM MALHA
FECHADA.

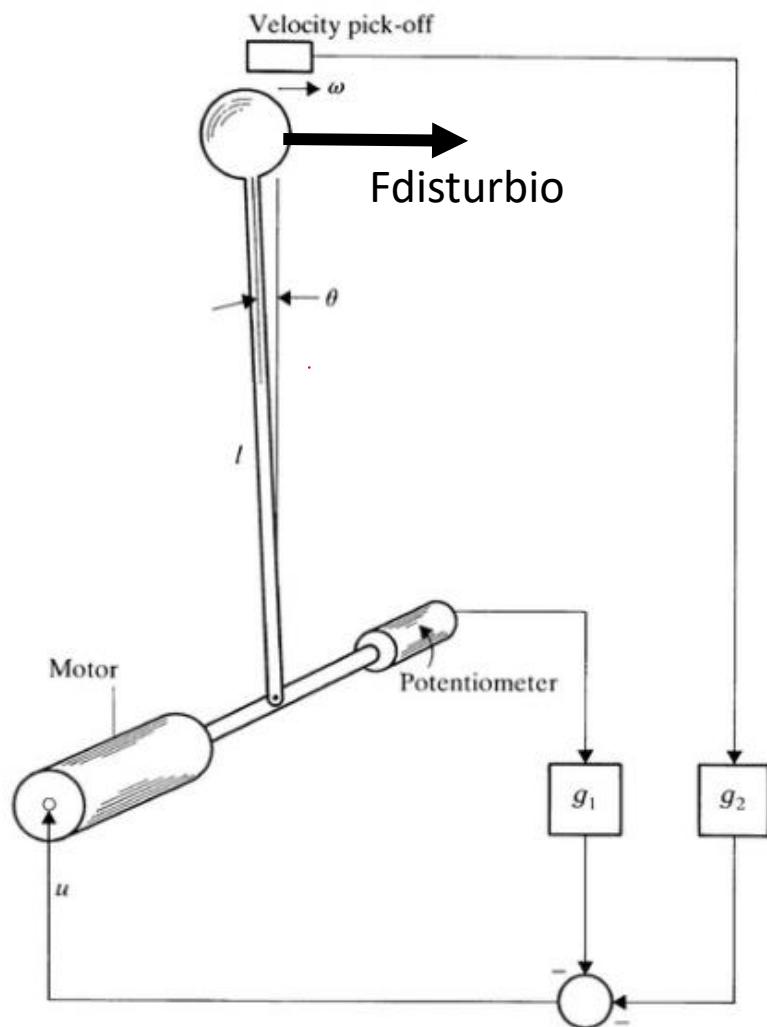


OBS 3) VALE P/ y_d

CONSTANTE, MAS PODE SER
USADA SE y_d VARIAR LENTA-
MENTE

OBS 4) SE A e B NÃO FOREM
CONECTADOS COM EXATIDÃO

$$y \rightarrow y_d \neq y_d$$



$$\dot{\theta} = \omega$$

$$\dot{\omega} = \Omega^2 \theta - \alpha \omega + \beta u$$

$$\alpha = -K^2/JR \quad \beta = K/JR$$

$$J = J_m + ml^2$$

$$\Omega^2 = \frac{mgl}{J + ml^2} = \frac{g}{l + J/ml}$$

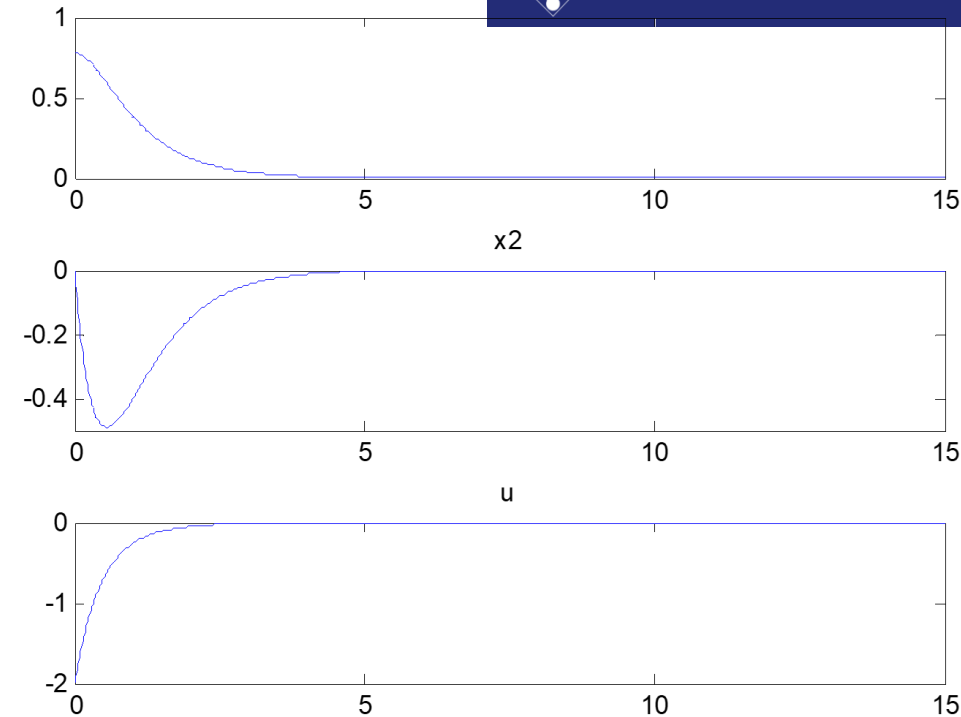
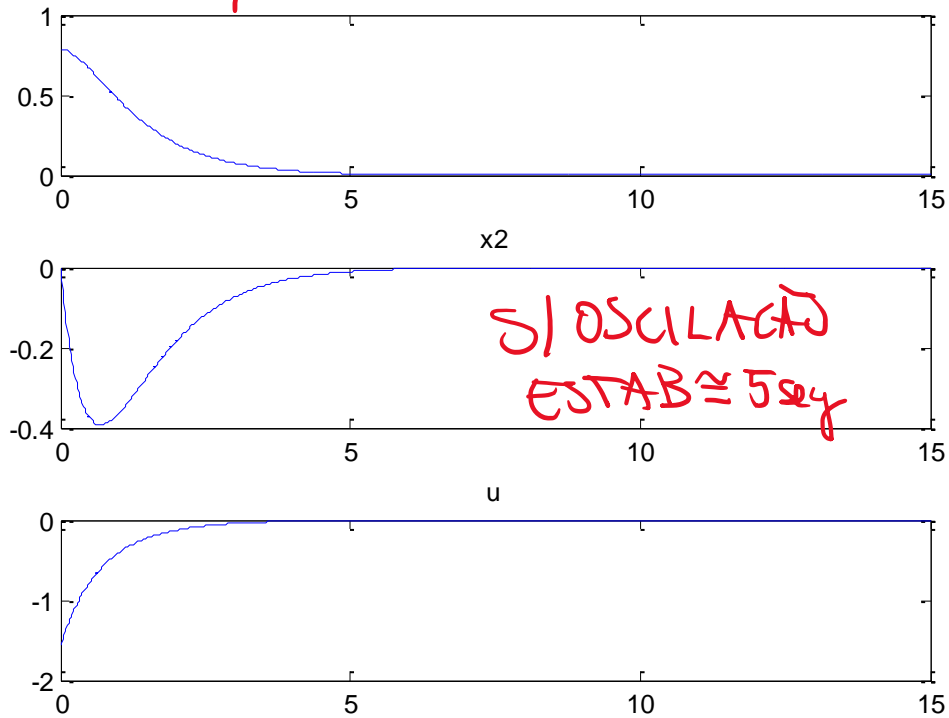
$$A = \begin{bmatrix} 0 & 1 \\ \Omega^2 & -\alpha \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ \beta \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta} \\ \dot{\omega} \end{bmatrix} = \begin{pmatrix} 0 & 1 \\ 2 & -0,1 \end{pmatrix} \begin{pmatrix} \theta \\ \omega \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \end{pmatrix} \cdot u + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot w$$

DISTURBIO \swarrow

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} \theta \\ \omega \end{pmatrix}$$

SIMULAÇÃO COM CONDIÇÕES INICIAIS ≠ 0 (SEM SET-POINT)



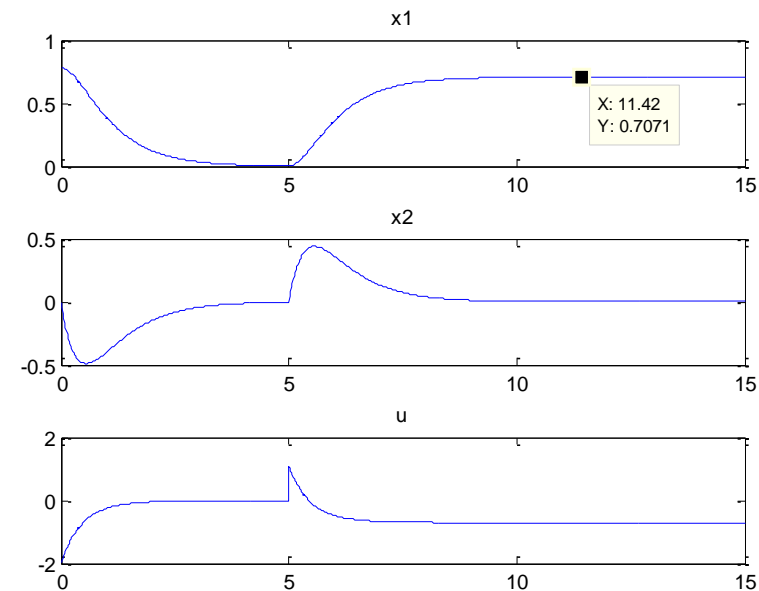
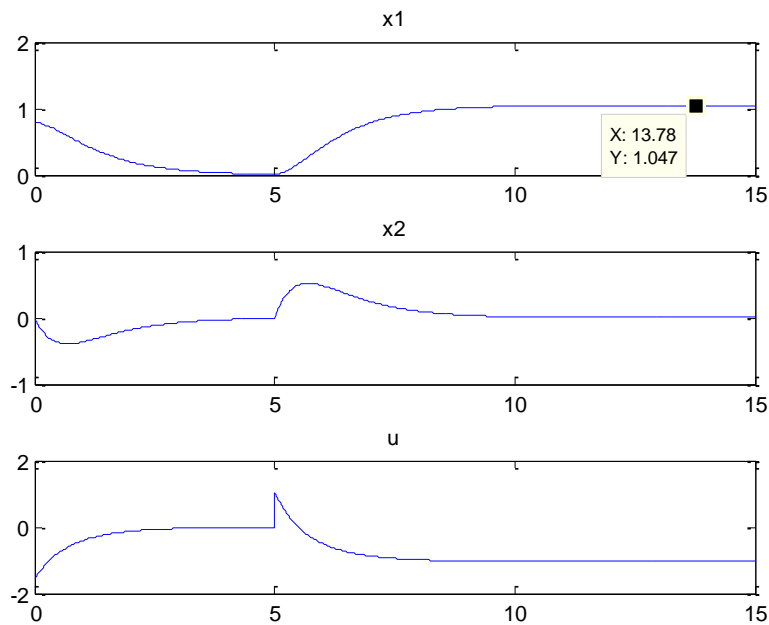
```
A = [0 1  
     2 -0.1];  
B = [0; 2];  
C = [1 0];  
sys = ss(A, B, eye(2), 0);  
g = place(A, B, [-1 -2])
```

2 POLOS REAIS

PROJETADO G
CONSIDERANDO 20% ERRO A
E -20% ERRO B
↓
COMPORTAMENTO "ROBUSTO"

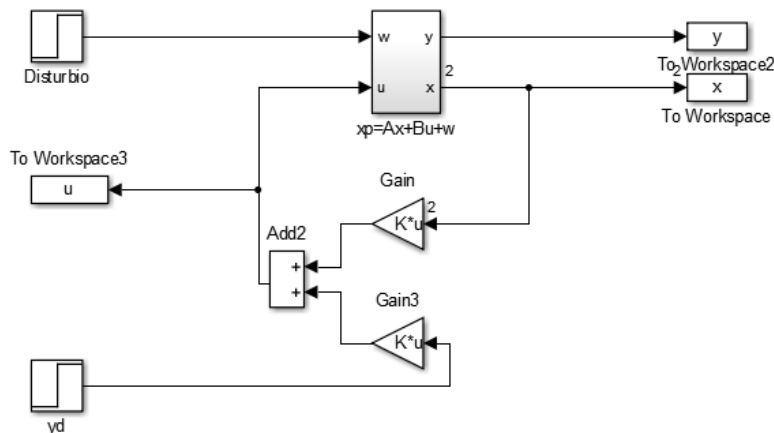


Simulação com SetPoint de 60deg aplicado em t=5s



$$G_{yd} = \text{inv}(C * \text{inv}(A - B * g) * B);$$

$$G_{yd} = \text{inv}(C * \text{inv}(1.2A - 0.8B * g) * 0.8B);$$



COM ERROS 20%, O PUNTO ESTABILIZOU LONGE DOS 60° DO SET-POINT
⇒ PONTO "ROBUSTO"

CONTROLE INTEGRAL

DISTURBIO E TRACKING

$$\dot{x} = Ax + Bu + w$$

$$y = C \cdot x$$

NOVA VARIÁVEL $z = \int y - y_d dt$

$$\dot{z} = y - y_d$$

SIST. ESTENDIDO

$$\begin{pmatrix} \dot{x} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} Ax + Bu + w \\ y - y_d \end{pmatrix} = \begin{pmatrix} Ax + Bu + w \\ Cx - y_d \end{pmatrix}$$

$$\begin{pmatrix} \dot{x} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} A & | & 0 \\ C & | & 0 \end{pmatrix} \begin{pmatrix} x \\ z \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} u + \begin{pmatrix} w \\ -y_d \end{pmatrix}$$



$$u = -G_{ext} \cdot x_{ext} = -Gx - G_i z$$

COM G, G_i OBTIDO POR ALOCAÇÃO DE
POCOS DE $\begin{pmatrix} * & | & 0 \\ \hline C & | & 0 \end{pmatrix} \begin{pmatrix} B \\ \hline 0 \end{pmatrix}$

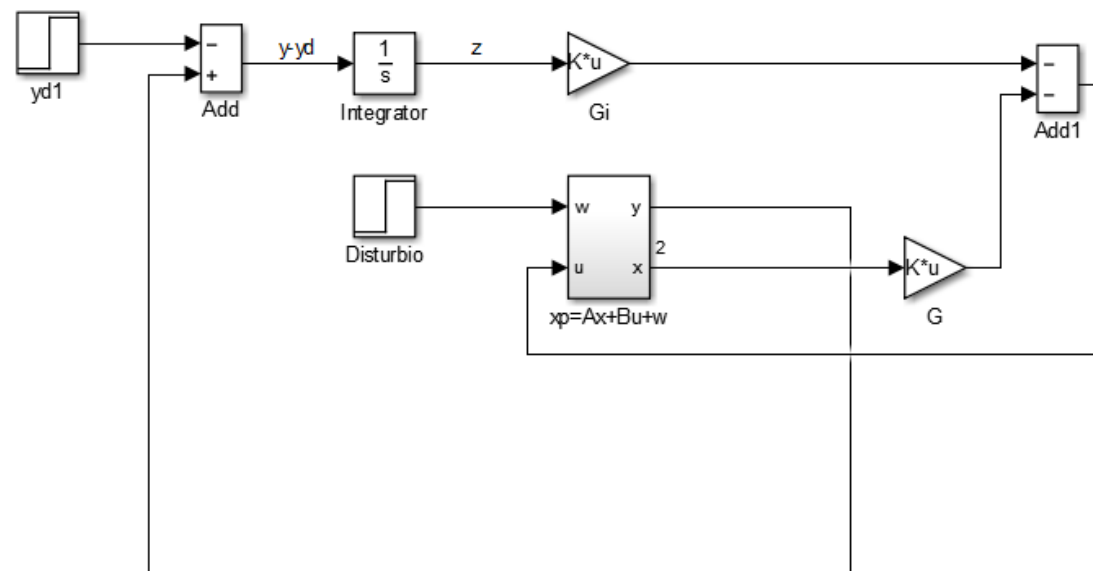
COM ISSO $\Rightarrow \dot{x}_{ext} \rightarrow 0 \Rightarrow$

$$\begin{pmatrix} \dot{x} \\ \dot{z} \end{pmatrix} = 0 \Rightarrow \dot{z} = 0 \Rightarrow y \rightarrow y_d$$

VAMOS GARANTIR QUE

$$\begin{pmatrix} \dot{x} \\ \dot{z} \end{pmatrix} = 0 \Rightarrow \dot{z} = y - y_d \Rightarrow \boxed{y \rightarrow y_d}$$

↳ INDEPENDENTE DO DISTÚRBO w

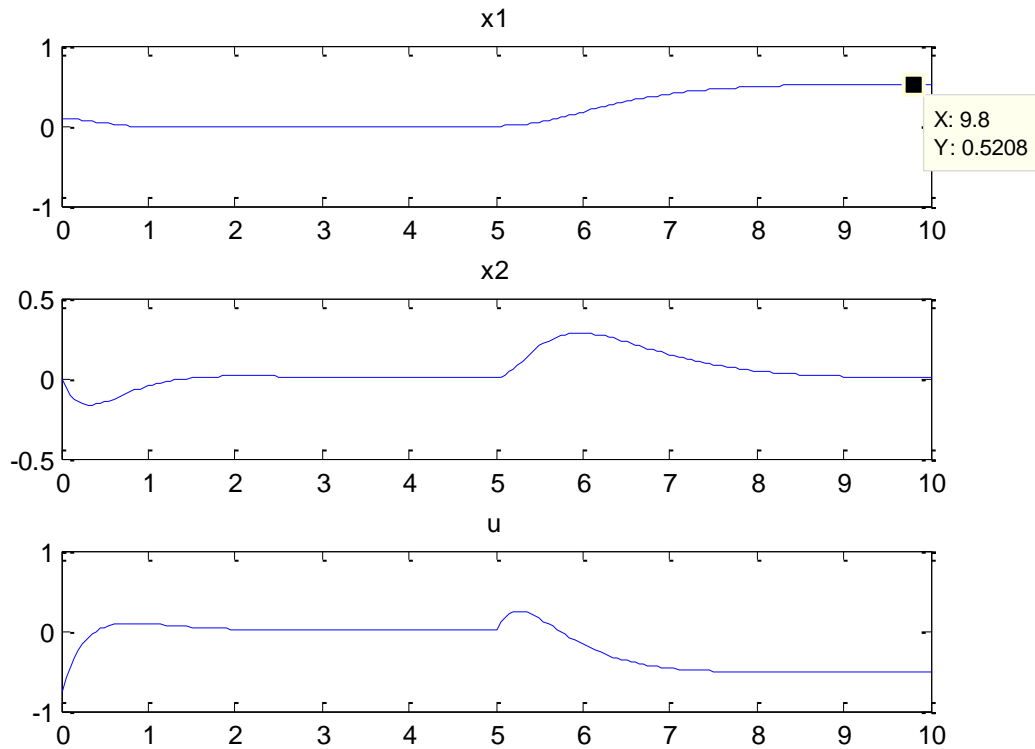


`% com integral`

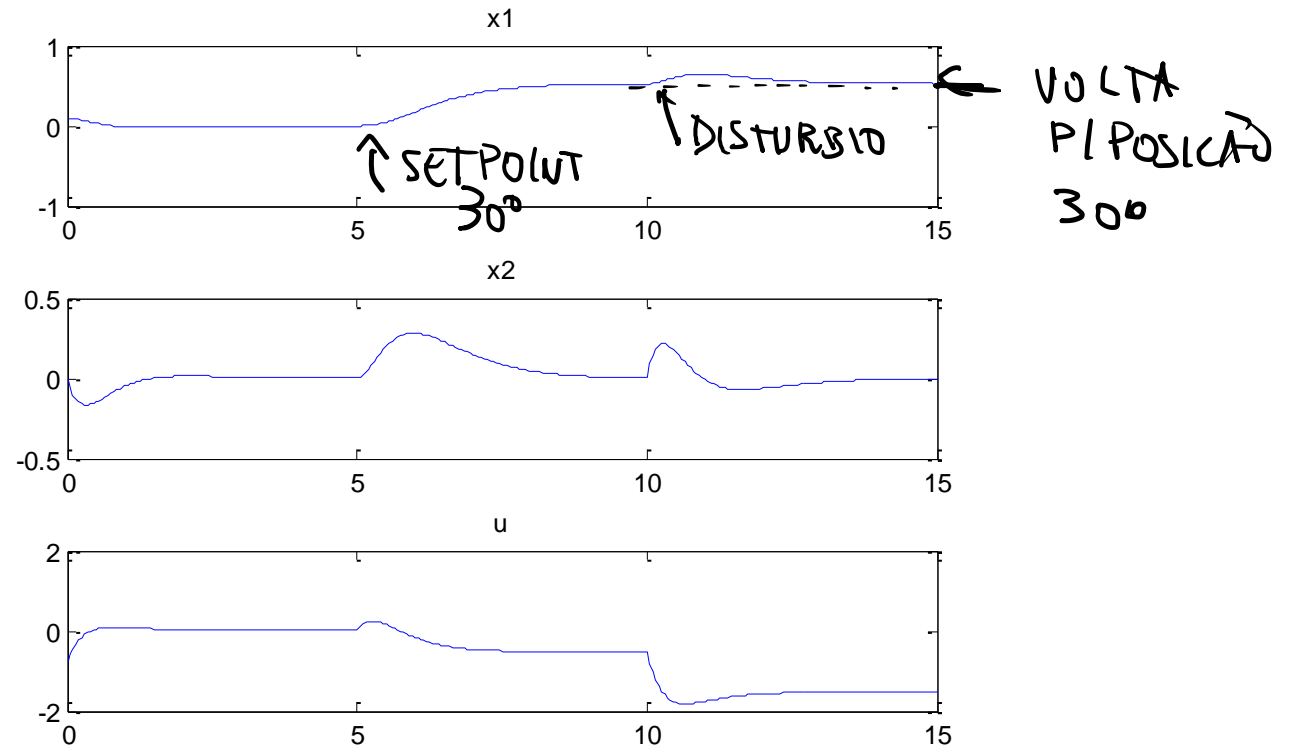
```
Ae = [A zeros(2,1)
      C 0
      ];
```

```
Be = [B
      0];
```

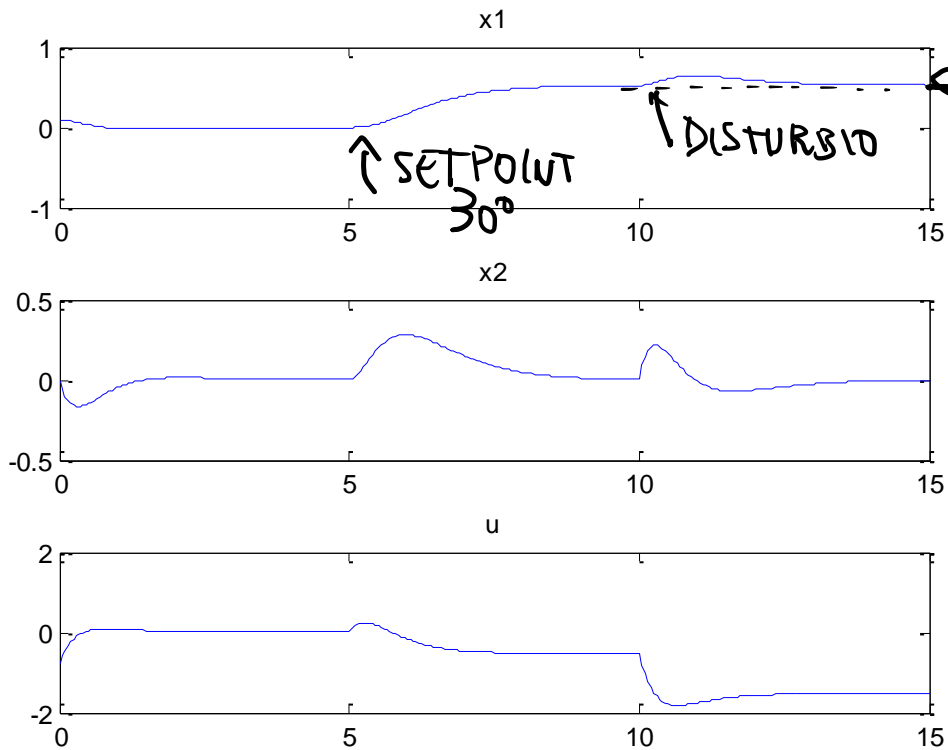
```
g2 = place(Ae, Be, [-3 -2 -1.5]);
```

SET POINT 30°

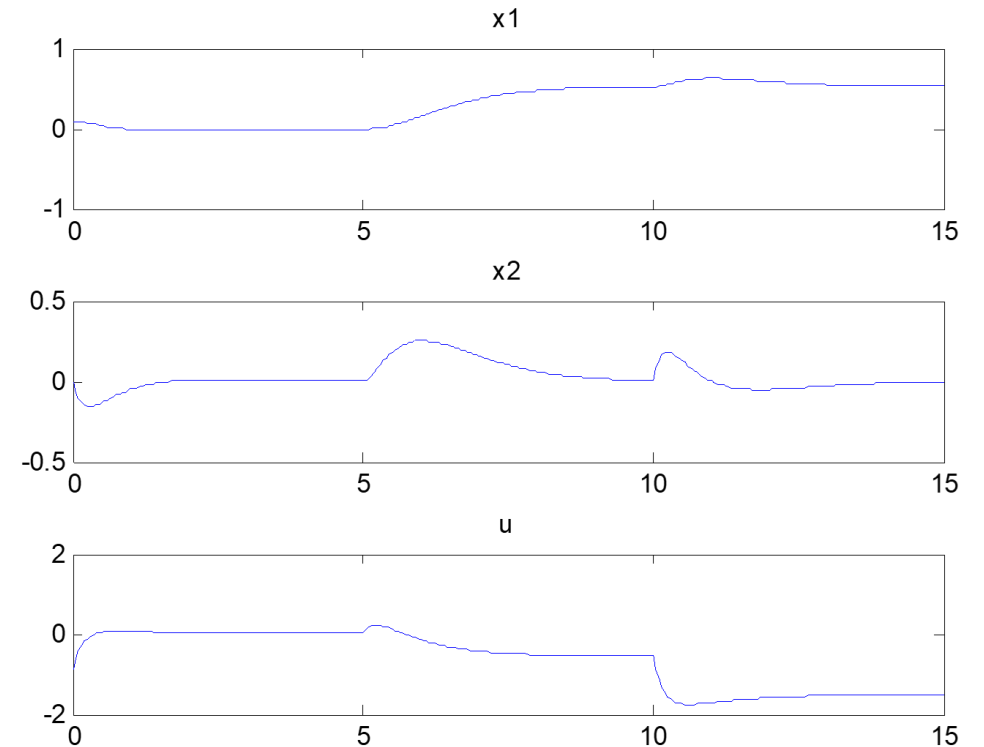


ROBUSTEZ DISTURBIO

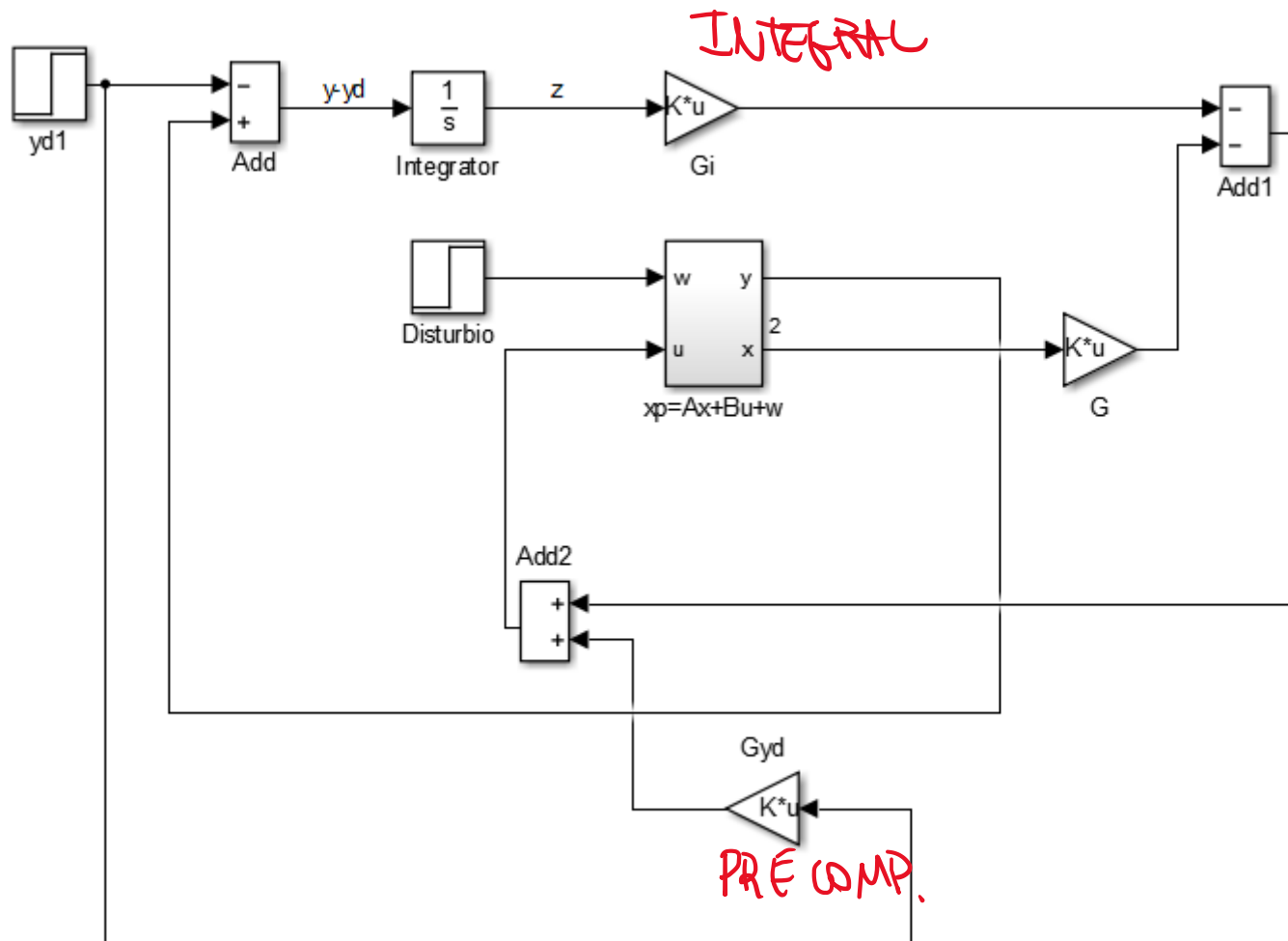


VOLTA
PI POSIÇÃO
300

ROBUSTEZ DISTÚRBO



PROJETO COM 20% ERRO
MODELO → PODE-SE ATESTAR
BOA ROBUSTEZ CONTROLE
INTEGRAL.



ASSOCIANDO

- CONTROLE INTEGRAL
+
- CONTROLE PRÉCOMPENSADO
SET POINT

→ ROBUSTEZ

↓
RAPIDO



