7.2 - The T-Junction Power Divider

Reading Assignment: *pp. 315-318*

Three-port couplers are also known as **T-Junction** Couplers, or T-Junction Dividers.

HO: THE T-JUNCTION COUPLER

We will study **three** standard T-Junction couplers:

HO: THE RESISTIVE DIVIDER

HO: THE LOSSLESS DIVIDER

HO: CIRCULATORS

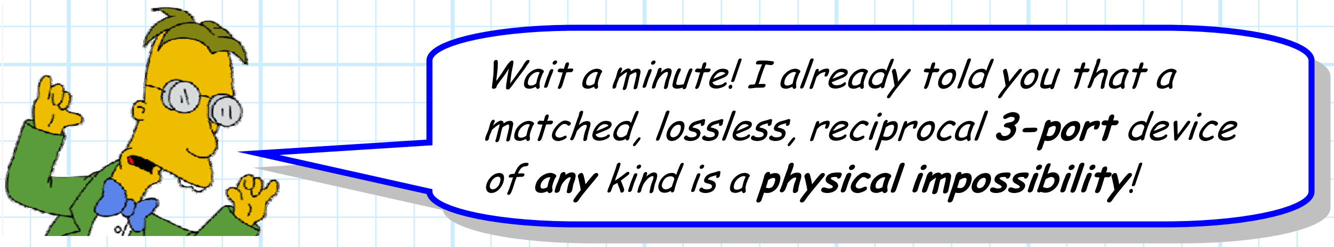
Now let's consider a 3dB power divider from another viewpoint; let's consider the **scattering matrix** of a (nearly) **ideal** 3dB power divider.

HO: THE (NEARLY) IDEAL POWER DIVIDER

This ideal 3dB power divider **can** be constructed! It is the **Wilkinson Power Divider**—the subject of the next section.

The T-Junction Coupler

Say we desire a **matched** and **lossless** 3-port coupler.



Absolutely true! Our desire in this case will be **unfulfilled**. There are, however, a few designs that come **close**.

1. **The Lossless Divider** – As the name states, this coupler is lossless. It is likewise reciprocal, and thus is **not matched**.

2. **The Resistive Divider** – As the name implies, this coupler **is lossy**. However, it is both matched and reciprocal.

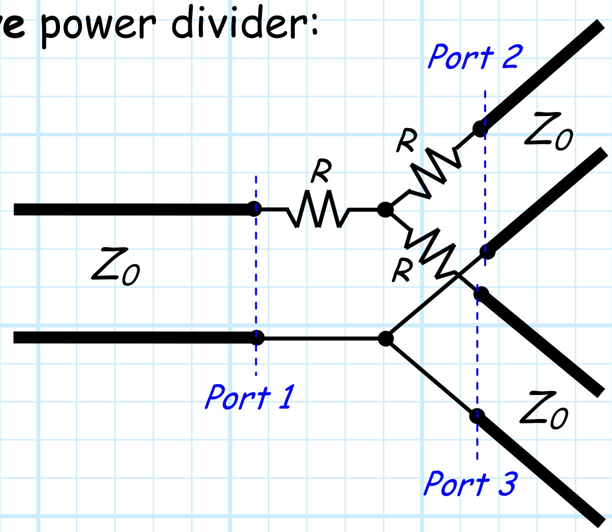
3. **The Circulator** – This three-port coupler is both matched and (ideally) lossy. This of course means that it is **not reciprocal**!

4. **The Wilkinson Divider** - Like the resistive divider, it is matched and reciprocal, and thus **is lossy**. However, it is lossy in a way that is not apparent when power is **divided** (i.e., power can be divided **without loss**).

As a result, the Wilkinson Power Divider is in most ways as **ideal** a T-junction as there is. Accordingly, it has its very **own section** in your textbook!

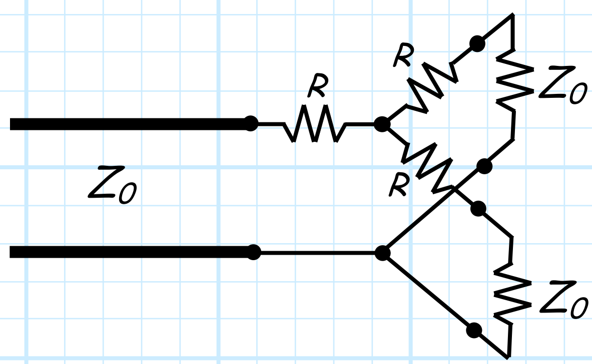
The Resistive Divider

Consider the **resistive** power divider:



This symmetric power divider will be matched at port 1 **if** R is selected as:

$$
\begin{aligned}
Z_0 &= R + (R + Z_0) \| (R + Z_0) \\
&= R + \frac{R + Z_0}{2} \\
&= 1.5R + \frac{Z_0}{2}
\end{aligned}
$$



Solving this equation, we find that port 1 is matched **if**:

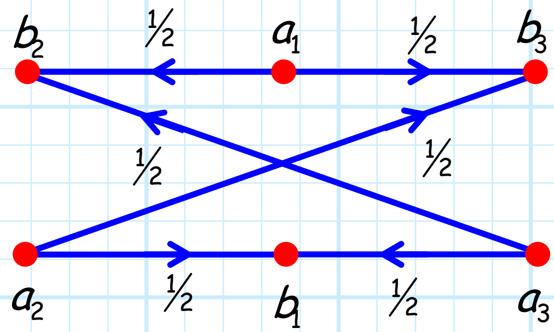
$$R = \frac{Z_0}{3}$$

From the **symmetry** of the circuit, we find that all the **other** ports will be matched as well (i.e., $S_{11} = S_{22} = S_{33} = 0$). Moreover, it can be shown that:

$$S_{12} = S_{21} = S_{31} = S_{31} = S_{23} = S_{32} = \frac{1}{2}$$

So:

$$\mathcal{S} = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$



Note the magnitude of each column is less than one. E.G.,:

$$|S_{21}|^2 + |S_{31}|^2 = \frac{1}{2} < 1$$

Therefore this power divider is **lossy**!

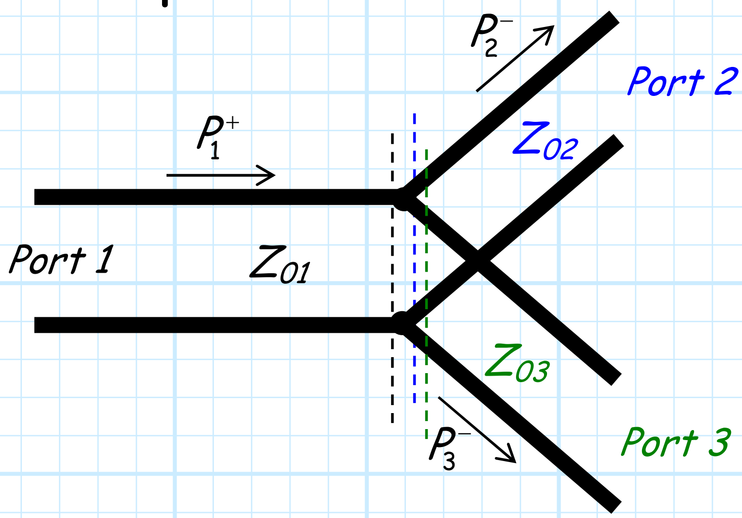
In fact, we find that the power out of each port is just **one-quarter** of the input power:

$$P_2^- = P_3^+ = \frac{P_1^+}{4}$$

In other words, **half** the input power is **absorbed** by the divider!

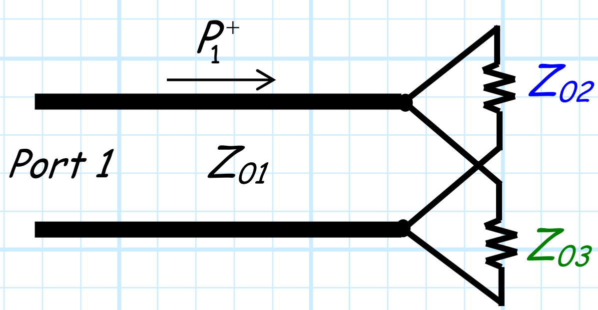
The Lossless Divider

Consider the **lossless power divider**:



To be ideal, we want $S_{11} = 0$. Thus, when ports 2 and port 3 are **terminated** in matched loads, the input impedance at port 1 must be equal to Z_{01}. This will only be true if the values Z_{02} and Z_{03} are selected such that:

$$Z_{01} = \left(\frac{1}{Z_{02}} + \frac{1}{Z_{03}} \right)^{-1} = \frac{Z_{02}\, Z_{03}}{Z_{02} + Z_{03}}$$



Note however that this circuit is **not** symmetric, thus we find that $S_{22} \neq 0$ and $S_{33} \neq 0$!

It is evident that this divider is **lossless** (no resistive components), so that:

$$P_1^+ = P_2^- + P_3^-$$

where P_1^+ is the power incident (and absorbed if $S_{11} = 0$) on port 1, and P_2^- and P_3^- is the power absorbed by the matched loads of ports 2 and 3.

Unless $Z_{02} = Z_{03}$, the power will not be divide equally between P_2^- and P_3^-. With a little microwave circuit analysis, it can be shown that the **division ratio** α is :

$$\alpha = \frac{P_2^-}{P_3^-} = \frac{Z_{03}}{Z_{02}}$$

Thus, if we desire an **ideal** ($S_{11} = 0$) divider with a specific division ratio α, we will find that:

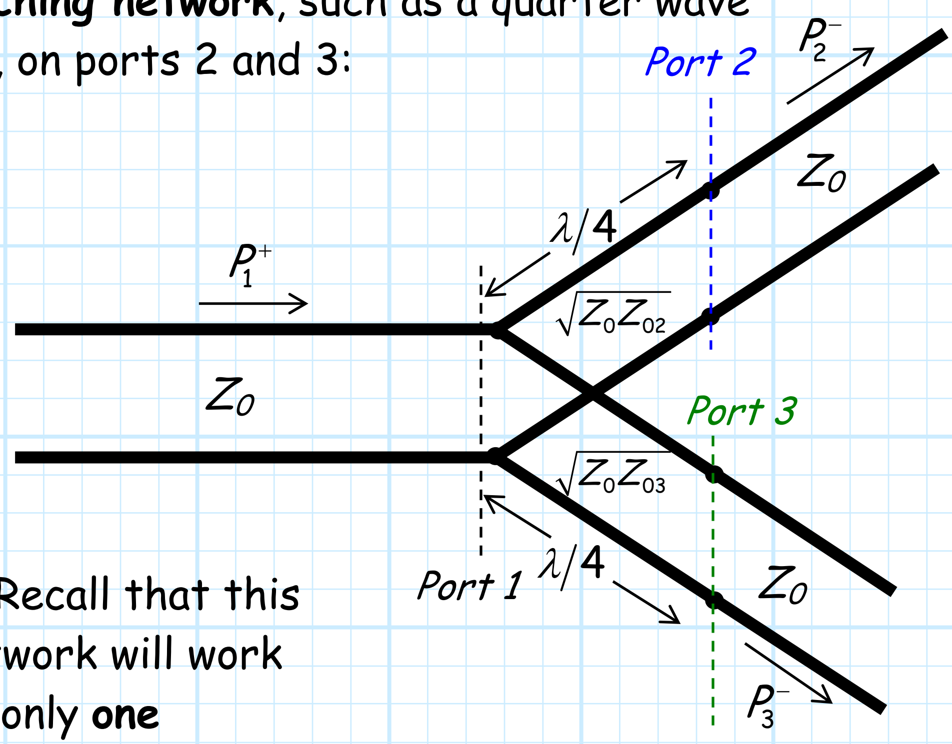
$$Z_{02} = Z_{01}\left(1 + \frac{1}{\alpha}\right)$$

and:

$$Z_{03} = Z_{01}\left(1 + \alpha\right)$$

Q: *I don't understand how this is helpful. Don't we typically want the characteristic impedance of all three ports to be equal to the* ***same*** *value (e.g., $Z_{01} = Z_{02} = Z_{03} = Z_0$)?*

A: True ! A more practical way to implement this divider is to use a **matching network**, such as a quarter wave transformer, on ports 2 and 3:



But beware! Recall that this matching network will work perfectly at only **one** frequency.

This lossless divider has a scattering matrix (at the design frequency) of this form:

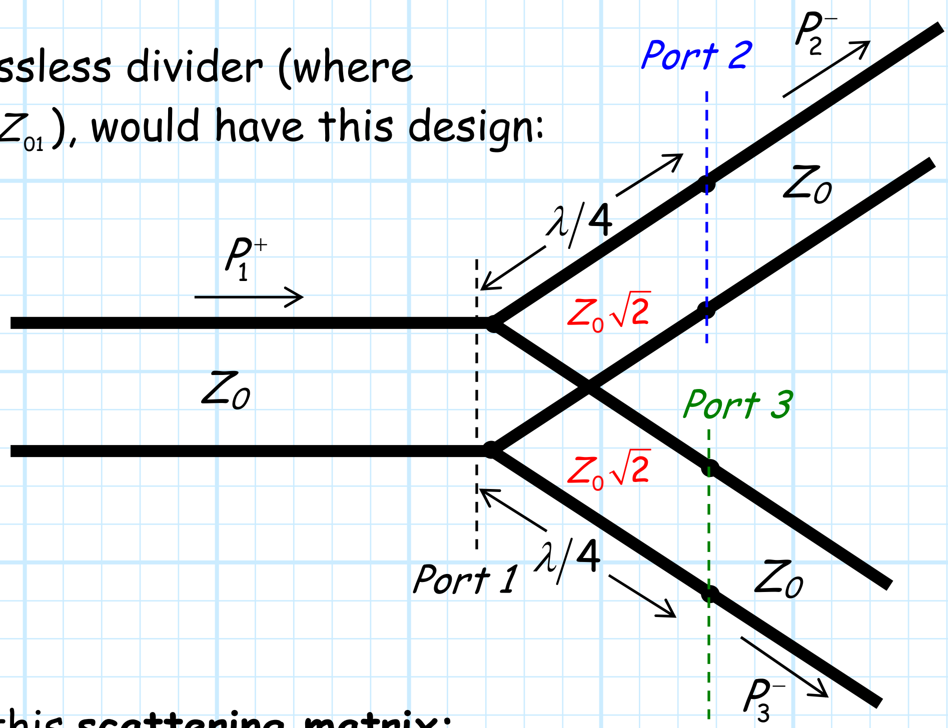
$$\mathcal{S} = \begin{bmatrix} 0 & {}^{-j}/_{\sqrt{2}} & {}^{-j}/_{\sqrt{2}} \\ {}^{-j}/_{\sqrt{2}} & S_{22} & S_{23} \\ {}^{-j}/_{\sqrt{2}} & S_{32} & S_{33} \end{bmatrix}$$

where the (non-zero!) values of S_{22}, S_{23}, S_{32}, and S_{33} depend on the division ratio α.

Note that if we desire a **3 dB** divider (i.e., $\alpha = 1$), then:

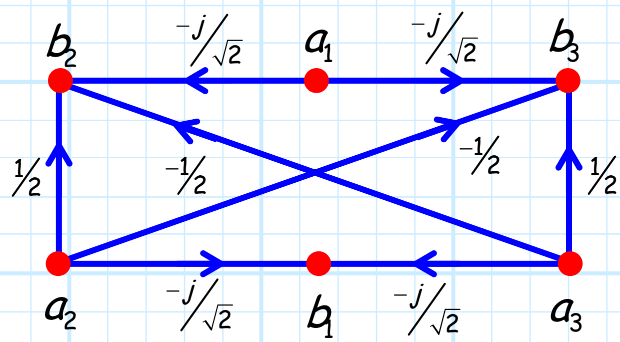
$$Z_{02} = Z_{03} = 2\, Z_{01}$$

This **3dB** lossless divider (where $Z_{02} = Z_{03} = 2\,Z_{01}$), would have this design:



Along with this **scattering matrix**:

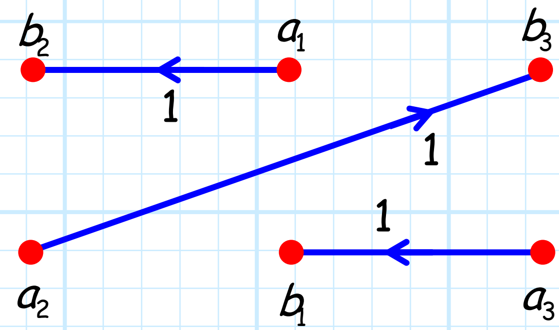
$$\mathcal{S} = \begin{bmatrix} 0 & -j/\sqrt{2} & -j/\sqrt{2} \\ -j/\sqrt{2} & 1/2 & -1/2 \\ -j/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}$$



Circulators

A circulator is a matched, lossless but **non-reciprocal** 3-port device, whose scattering matrix is **ideally**:

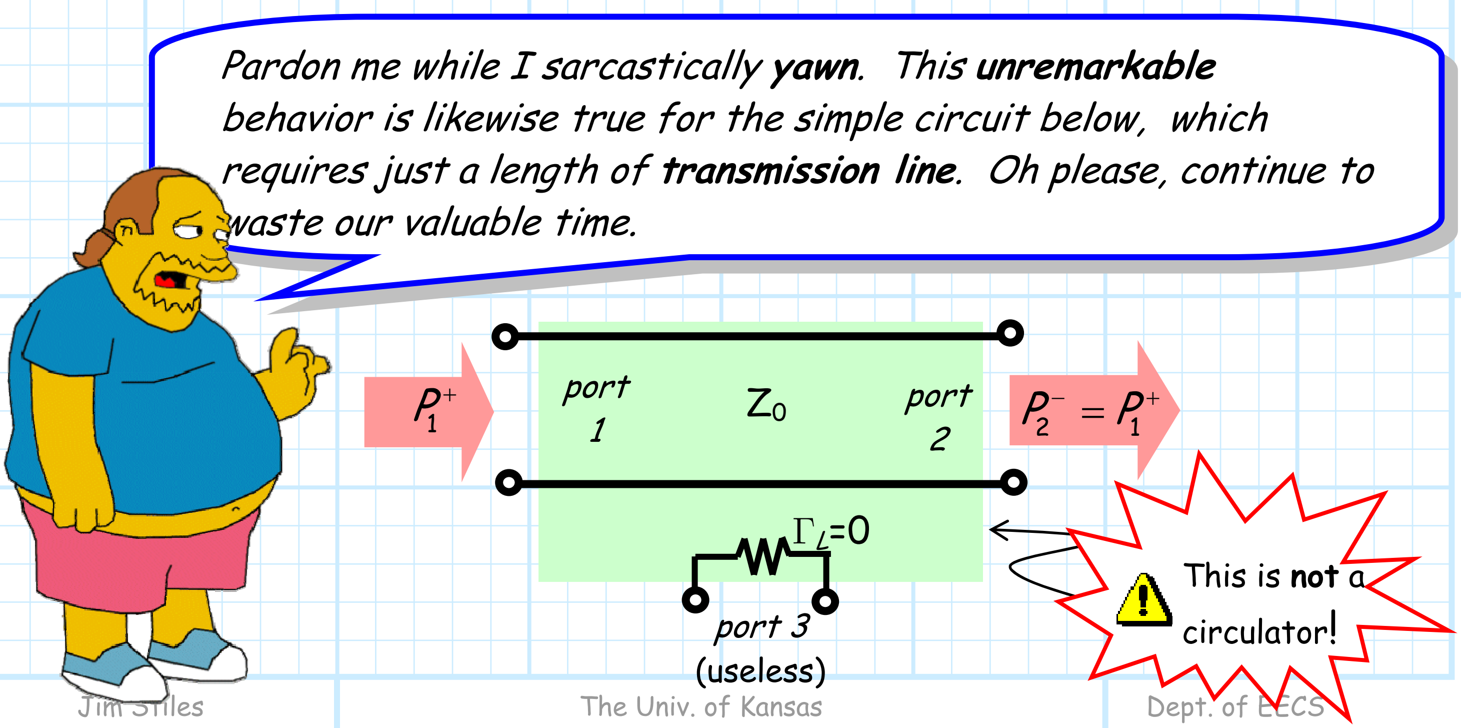
$$\mathcal{S} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



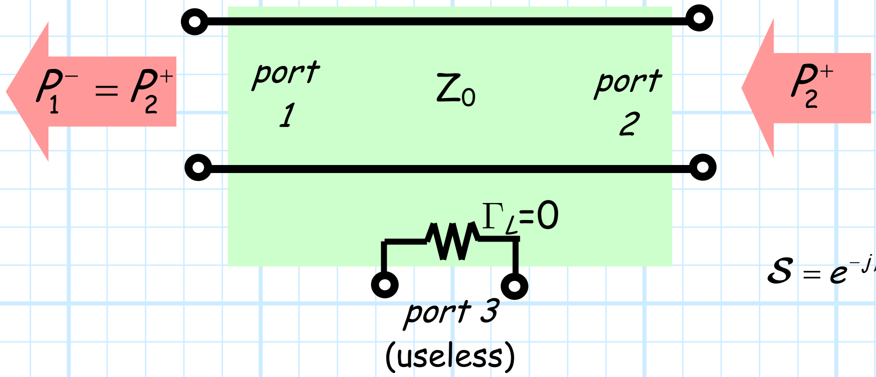
Circulators use anisotropic **ferrite** materials, which are often "biased" by a permanent magnet! → The result is a **non-reciprocal** device!

First, we note that for a circulator, the power incident on port 1 will exit **completely** from port 2:

$$P_2^- = P_1^+$$

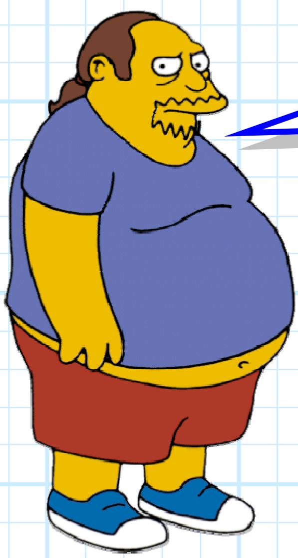


True! But a transmission line, being a **reciprocal** device, will likewise result in the power **incident** on **port 2** of your simple circuit to **exit** completely from **port 1** ($P_1^- = P_2^+$):



$$\mathcal{S} = e^{-j\beta\ell}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

But, this is **not** true for a circulator! If power is incident on port 2, then **no power** will exit port 1 !

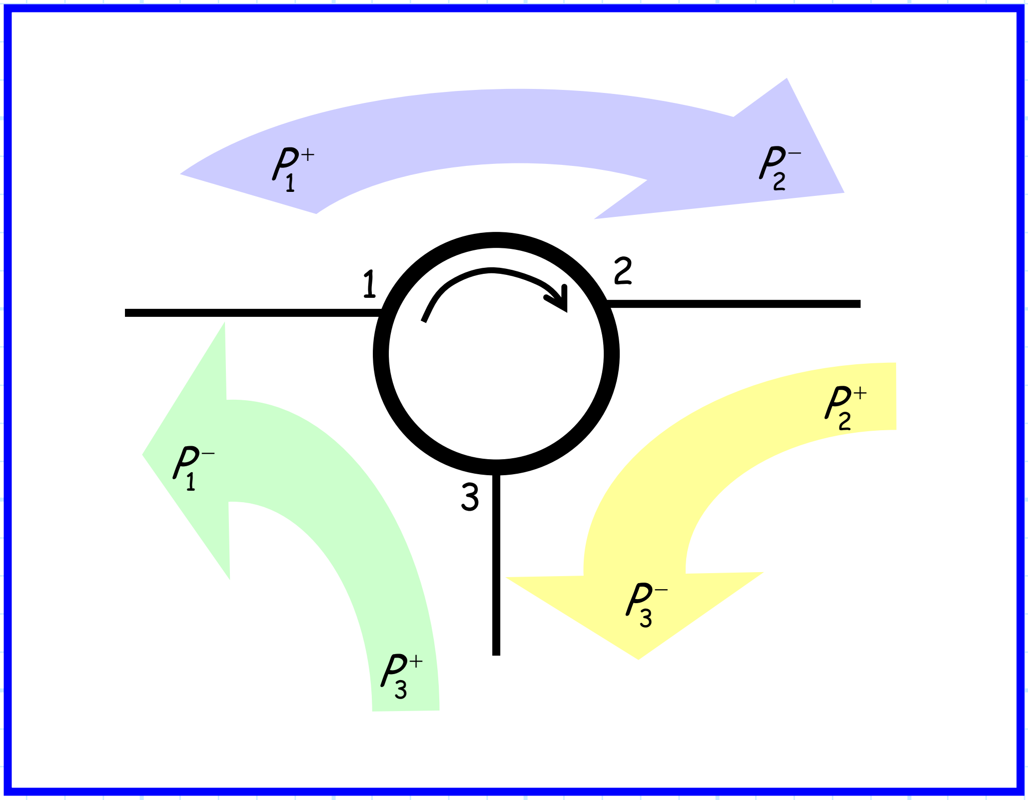


Q: *You have been* ***surprisingly*** *successful in regaining my interest. Please tell us then, just* ***where*** *does the power incident on port 2* ***go****?*

A: It will exit from port **3** !

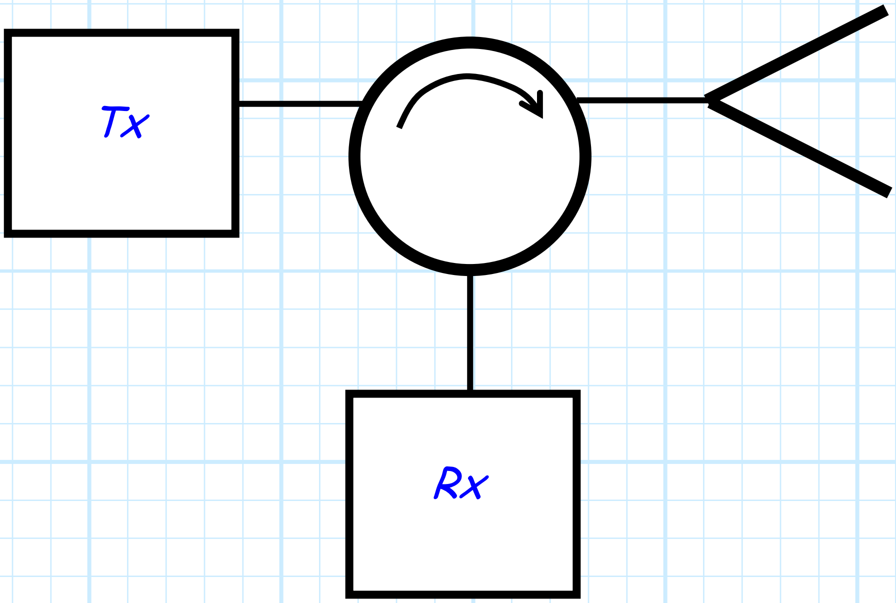
Likewise, power flowing into **port 3** will exit—**port 1**!

It is evident, then how the circulator gets its **name**: power appears to **circulate** around the device, a behavior that is emphasized by its device **symbol**:



We can see that, for example, a **source** at port **2** "thinks" it is attached to a **load** at port **3**, while a **load** at port **2** "thinks" it is attached to a **source** at port **1**!

This behavior is useful when we want to use **one** antenna as **both** the transmitter and receiver antenna. The transmit antenna (i.e., the load) at port 2 **gets** its power from the transmitter at **port 1**. However, the receive antenna (i.e., the source) at port 2 **delivers** its power to the receiver at **port 3**!



It is **particularly** important to keep the transmitter power from getting to the receiver. To accomplish this, the **antenna** must be **matched** to the transmission line. Do **you** see why?

Finally, we should note some major **drawbacks** with a circulator:

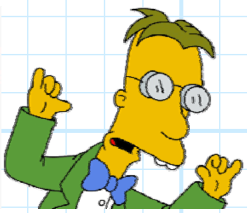
1. They're **expensive**.

2. They're **heavy**.

3. The generally produce a large, static **magnetic field**.

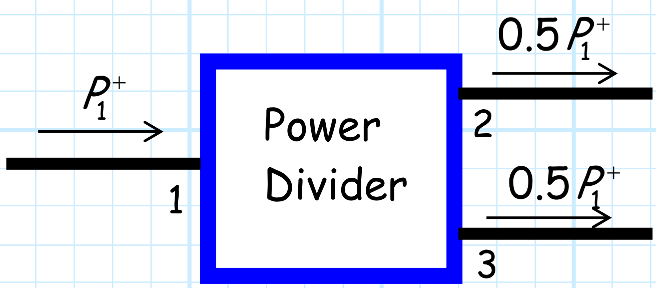
4. They typically exhibit a large **insertion loss** (e.g., $|S_{21}|^2 = |S_{32}|^2 = |S_{13}|^2 \approx 0.75$).

The (nearly) Ideal T-Junction Power Divider



Recall that we **cannot** build a matched, lossless reciprocal **three**-port device.

So, let's **mathematically** try and determine the scattering matrix of the best possible T-junction 3 dB **power divider**.



To **efficiently** divide the power **incident** on the input port, the port (port 1) must first be **matched** (i.e., all incident power should be delivered to port 1):

$$S_{11} = 0$$

Likewise, this delivered power to port 1 must be divided efficiently (i.e., **without loss**) between ports 2 and 3.

Mathematically, this means that the first column of the scattering matrix must have **magnitude of 1.0:**

$$|S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 = 1$$

Since we have already determined that $S_{11} = 0$, this simply means :

$$|S_{21}|^2 + |S_{31}|^2 = 1$$

Provided that we wish to evenly divide the input power, we can conclude from the expression above that:

$$|S_{21}|^2 = |S_{31}|^2 = 1/2 \qquad \therefore \; |S_{21}| = |S_{31}| = 1/\sqrt{2}$$

Note that **this** device would take the power into port 1 and divide into **two equal parts**—half exiting **port 2**, and half exiting **port3** (provided ports 2 and 3 are terminated in matched loads!).

$$P_2^- = |S_{21}|^2 P_1^+ = 0.5\, P_1^+ \qquad P_3^- = |S_{31}|^2 P_1^+ = 0.5\, P_1^+$$

In addition, it is **desirable** that ports 2 and 3 be **matched** (the whole device is thus matched):

$$S_{22} = S_{33} = 0$$

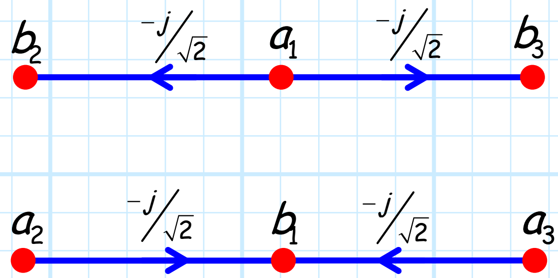
And also **desirable** that ports 2 and 3 be **isolated**:

$$S_{23} = S_{32} = 0$$

This last requirement ensures that no signal incident on port 2 (e.g., reflected from a load) will **"leak"** into port 3—and vice versa.

This ideal 3 dB power divider **could** therefore have the form:

$$\mathcal{S} = \begin{bmatrix} 0 & -j/\sqrt{2} & -j/\sqrt{2} \\ -j/\sqrt{2} & 0 & 0 \\ -j/\sqrt{2} & 0 & 0 \end{bmatrix}$$



Since we can describe this ideal power divider **mathematically**, we can potentially build it **physically**!

Q: *Huh!? I thought you said that a matched, lossless, reciprocal three-port device is **impossible**?*

A: It is! This divider **is** clearly a **lossy** device. The magnitudes of both column 2 and 3 are less than one:

$$|S_{12}|^2 + |S_{22}|^2 + |S_{32}|^2 = \left|-j/\sqrt{2}\right|^2 + 0 + 0 = 0.5 < 1.0$$

$$|S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 = \left|-j/\sqrt{2}\right|^2 + 0 + 0 = 0.5 < 1.0$$

Note then that **half** the power incident on port 2 (or port 3) of this device would **exit** port 1 (i.e., reciprocity), but no power would exit port 3 (port2), since ports 2 and 3 are **isolated**. I.E.,:

$$P_1^- = |S_{12}|^2 P_2^+ = 0.5\, P_2^+ \qquad P_3^- = |S_{32}|^2 P_2^+ = 0$$

$$P_1^- = |S_{13}|^2 P_3^+ = 0.5\, P_3^+ \qquad P_2^- = |S_{23}|^2 P_3^+ = 0$$

Q: *Any ideas on how to **build** this thing?*

A: Note that the **first column** of the scattering matrix is precisely the same as that of the **lossless 3 dB divider**.

Also note that since the device is **lossy**, the design must include some **resistors**.

Lossless Divider + resistors = The Wilkinson Power Divider

Q: *What is the Wilkinson Power Divider?*

A: It's the **subject** of our next section!