Escola Politécnica da Universidade de São Paulo
Departamento de Engenharia Mecatrônica e de Sistemas Mecânicos - PMR

Aula 9

Projeto de Controle – PID Discreto e Projeto Direto

Prof. Eduardo A. Tannuri

PMR 3409 – Controle II

PID DIGITAL

$$PID(s) = K_P\left(1 + \frac{1}{T_i s} + T_D s\right) \Rightarrow PID(z) = K_P\left(1 + \frac{T}{T_i}\frac{z+1}{z-1} + \frac{T_D}{T}\cdot\frac{z-1}{z}\right)$$

K_P = GANHO PROPORCIONAL

T_i = TEMPO INTEGRAL

T_D = TEMPO DERIVATIVO

$$\boxed{PID(z) = \bar{K}_P + \bar{K}_I \frac{1}{1-z^{-1}} + \bar{K}_D\left(1-z^{-1}\right)}$$

↓ APROXIMAR DERIVADA POR DERIVAÇÃO P/ TRÁS

$$s \to \frac{z-1}{Tz}$$

DERIVATIVO

INTEGRAL POR TUSTIN

$$s \to \frac{2}{T}\frac{z-1}{(z+1)}$$

$$\bar{K}_P = K_P\left(1 - \frac{T}{2T_i}\right)$$

$$\bar{K}_I = \frac{K_P T}{T_i}$$

$$\bar{K}_D = \frac{K_P T_D}{T}$$

$$\boxed{PID(z) \simeq \bar{K}_P + \bar{K}_I \frac{1}{1-z^{-1}} + \bar{K}_D \left(1-z^{-1}\right)}$$

$$PID(z) = \frac{U(z)}{E(z)} = \frac{\bar{K}_P + \bar{K}_I + \bar{K}_D - \left(\bar{K}_P - 2\bar{K}_D\right) z^{-1} + K_D z^{-2}}{1-z^{-1}}$$

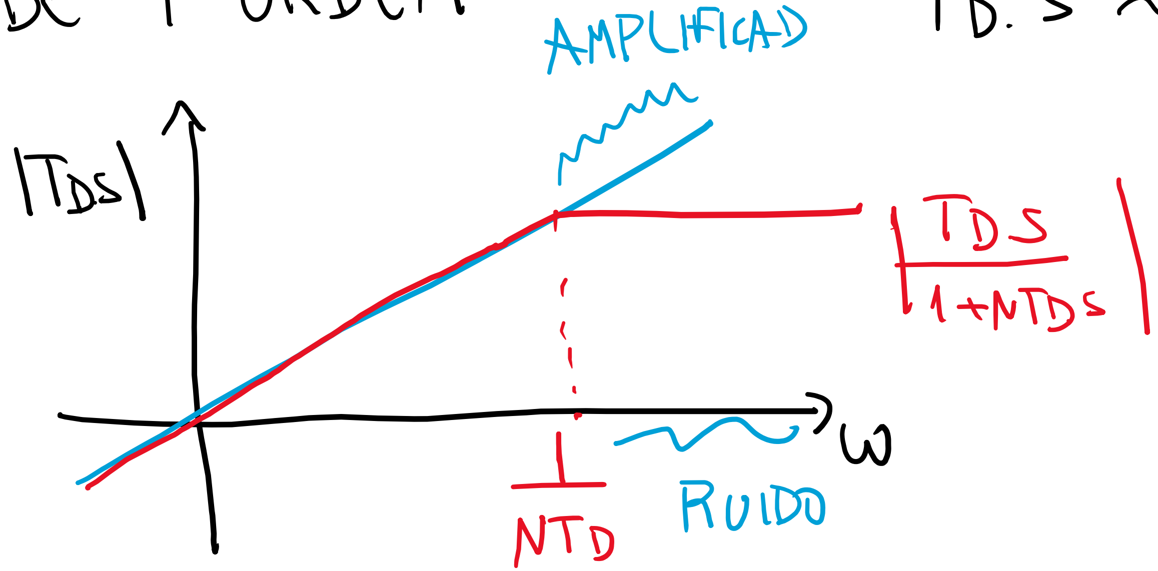
$$\frac{U(z)}{E(z)} = \frac{a_1 + a_2 z^{-1} + a_3 z^{-2}}{1-z^{-1}} \Rightarrow \boxed{u[k] = a_1 \cdot e[k] + a_2 e[k-1] + a_3 e[k-2] + u[k-1]}$$

OBS1) O TERMO $T_D S$ → AMPLIFICA RUIDO, LOGO É SEMPRE ACOPLADO A UM FILTRO DE 1º ORDEM

$$T_D . S \rightsquigarrow \frac{T_D . S}{1 + N T_D . S} \qquad N \simeq 0{,}05 \sim 0{,}2$$

OBS1) O TERMO $T_D S$ → AMPLIFICA RUIDO, LOGO É SEMPRE ACOPLADO A UM FILTRO DE 1º ORDEM

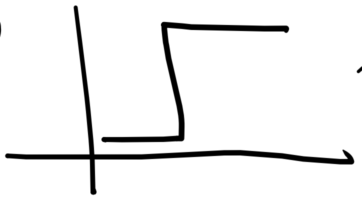
$$T_D \cdot S \rightsquigarrow \frac{T_D \cdot S}{1 + N T_D \cdot S} \qquad N \approx 0{,}05 \sim 0{,}2$$



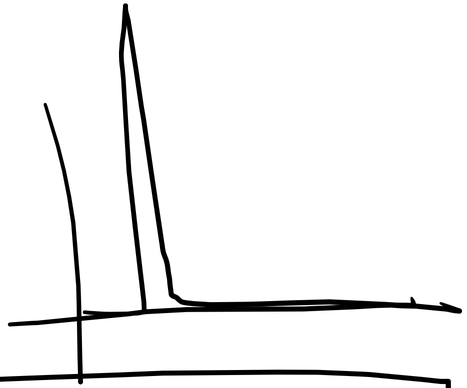
$$PID(z) = K_P \left(1 + \frac{T}{2T_I} \frac{z+1}{z-1} - \frac{z-1}{N\left(z - e^{-T/NT_d}\right)} \right)$$

OBS2) PARA EVITAR SAIDAS ELEVADAS P/ VARIAÇÃO DEGRAU NO SET POINT

$$E(s) = R(s) - Y(s)$$



$\rightsquigarrow \dot{e}(t) =$



$$\Rightarrow \boxed{U(s) = K_p \cdot \left(1 + \frac{1}{T_i s}\right) \cdot E(s) - T_D \cdot s \cdot Y(s)}$$

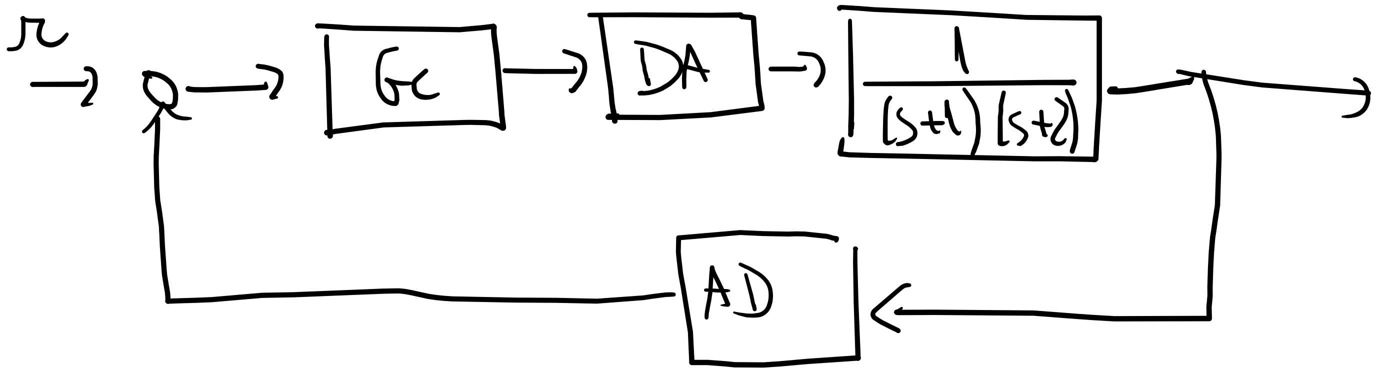
hipótese $\boxed{\dot{r}(t) \cong 0}$

$$\boxed{U(z) = \left[\bar{K}_p + \bar{K}_I \cdot \frac{1}{1-z^{-1}}\right] \cdot E(z) - \bar{K}_D \cdot (1-z^{-1}) \cdot Y(z)}$$

erro

MEDIDA SENSOR

Ex)



- G_c é um PID
- erro nulo p/ $r(KT)$ degrau
- polos dominantes $\zeta = 0,5$; $\omega_N = 2\,rad/s$

SOLUÇÃO POR ALOCAÇÃO ALGEBRICA DE POLOS EM Z (MÉTODO DIRETO)

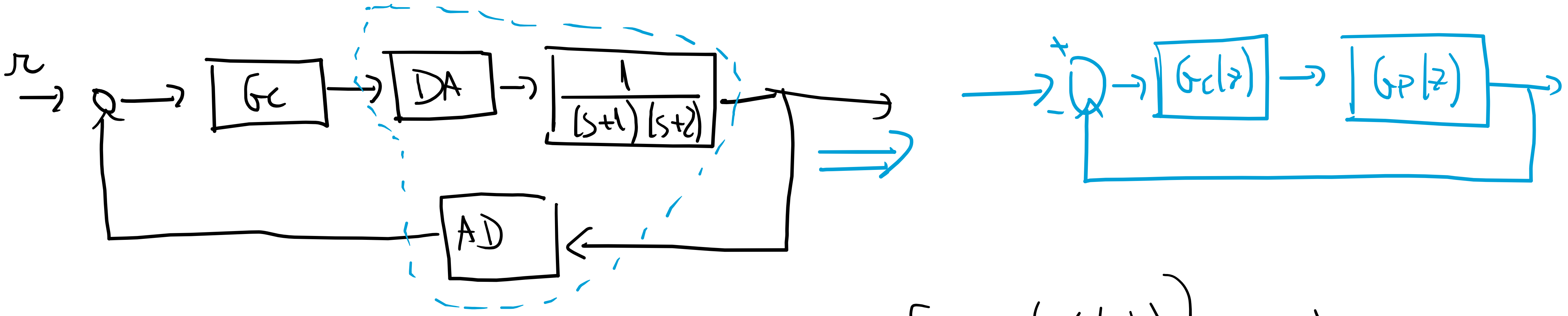
$$s_{1,2} = -\zeta\omega_N \pm j\omega_N\sqrt{1-\zeta^2} = -1 \pm \sqrt{3}\,j \; \dots \; \left\{ \begin{array}{l} M_p = 16\% \\ t_s^{2\%} = 4s \end{array} \right.$$

DEFINIR PERÍODO AMOSTRAGEM

$$T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\sqrt{3}} = 3,63s \Rightarrow \text{ESCOLHER } T \cong 10\text{x MENOR}$$

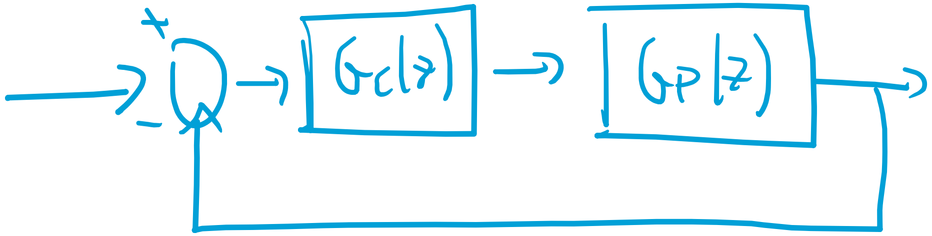
$$T_d \Rightarrow \boxed{0,4s}$$

$$z_{1,2} \Rightarrow e^{T\cdot s} = e^{0,4\cdot\left(-1 \pm \sqrt{3}j\right)} = \boxed{0,52 \pm 0,43j}$$



$$G_P(z) = \text{equivalente discreto planta} \Rightarrow \mathcal{Z}\left\{\mathcal{L}^{-1}\left(\frac{G(z)}{z}\right)\right\}\cdot\left(1-z^{-1}\right)$$

$$G_p = \frac{0,054z + 0,036}{z^2 - 1,12z + 0,3} = \frac{0,054\cdot(z+0,67)}{(z-0,67)(z-0,45)}$$



$$G_c(z) = \frac{(\bar{K}_P + \bar{K}_I + \bar{K}_D)z^2 - (\bar{K}_P + 2\bar{K}_D)z + \bar{K}_D}{z(z-1)} = \frac{K(z+C_1)\cancel{(z+C_2)}}{z(z-1)}$$

ZEROS $-C_1, -C_2$

POLOS $0, 1$

$$G_p(z) = \frac{0{,}054\,(z+0{,}67)}{(z-0{,}45)\cancel{(z-0{,}67)}}$$

ZERO $-0{,}67$

POLOS $0{,}45;\ 0{,}67$

ARBITRÁRIO $\Rightarrow$ $C_2 = -0{,}67$ (CANCELA UM POLO PLANTA)

$$G(z) = G_c(z) \cdot G_p(z) = \frac{0{,}054K(z+C_1)(z+0{,}67)}{z(z-1)(z-0{,}45)}$$

$$\frac{Y}{R} = \frac{G}{1+G} = \frac{0{,}054\,K(z+C_1)(z+0{,}67)}{z^3 + (0{,}054\,K - 1{,}45)z^2 + (0{,}45 + 0{,}054\,KC_1 + 0{,}036\,K)z + 0{,}036\,KC_1}$$

POLINOMIO CARACTERISTICO M.F. → 3 POLOS

- 2 DESEJADOS $(0{,}52 \pm 0{,}43j)$

- 3º POLO p

$$F(z) = (z - 0{,}52 + 0{,}43j)(z - 0{,}52 - 0{,}43j)(z - p)$$

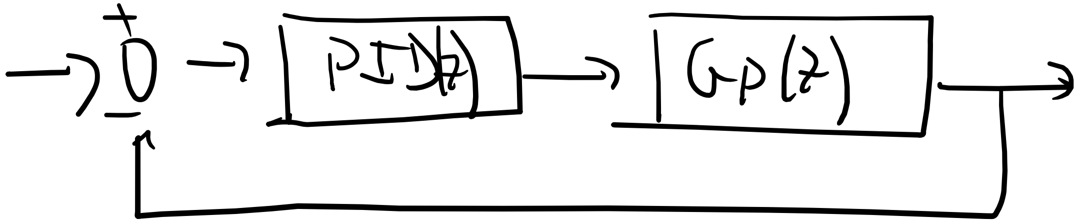
$$F(z) = z^3 - (p + 1{,}032)z^2 + (1{,}032\,p + 0{,}45)z - 0{,}45\,p$$

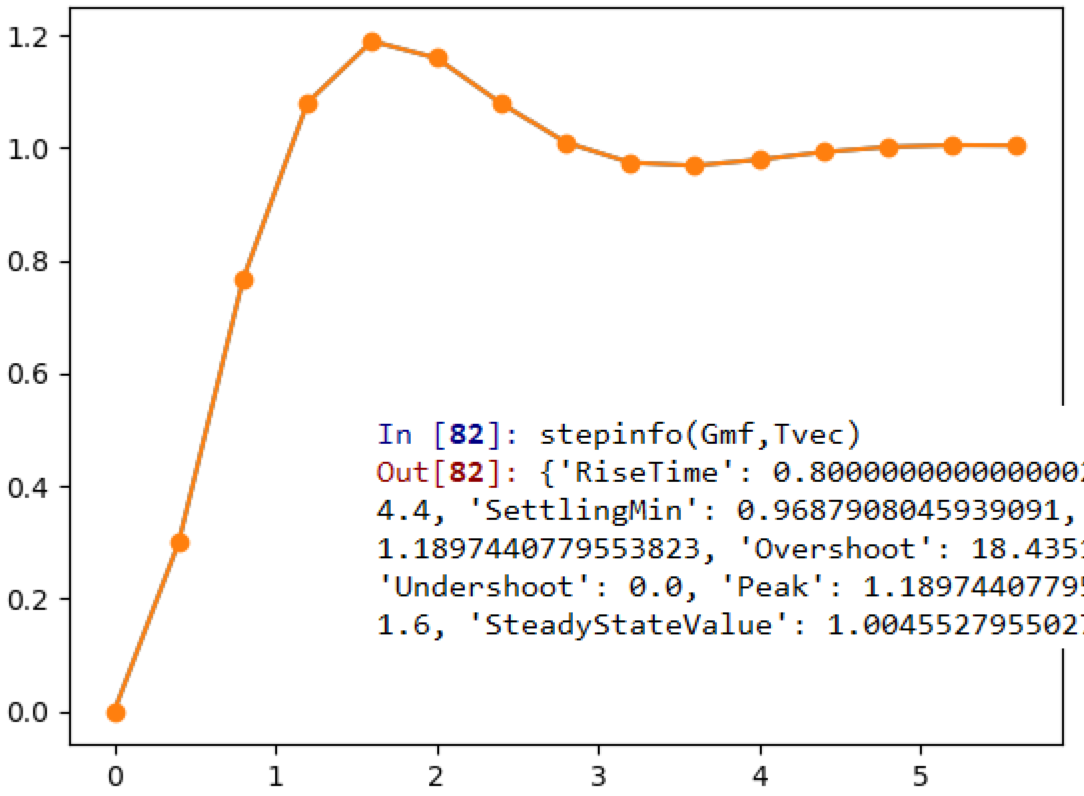
$$\begin{cases} C_1 = -0{,}26 \\ K = 5{,}51 \\ P = 0{,}11 \end{cases} \Rightarrow$$

VERIFICAR 3º POLO → $z_3 = 0{,}11$

$s_3 = \frac{1}{T} \ln z \Rightarrow -5{,}51 \to 5\times$ MAIS LONGE QUE $s_{1,2} \Rightarrow$ NÃO DOMINANTE

PID) $5{,}51 \cdot \dfrac{(z-0{,}26)(z-0{,}67)}{z(z-1)}$






```
In [82]: stepinfo(Gmf,Tvec)
Out[82]: {'RiseTime': 0.8000000000000002, 'SettlingTime':
4.4, 'SettlingMin': 0.9687908045939091, 'SettlingMax':
1.1897440779553823, 'Overshoot': 18.43519656525174,
'Undershoot': 0.0, 'Peak': 1.1897440779553823, 'PeakTime':
1.6, 'SteadyStateValue': 1.0045527955027238}
```

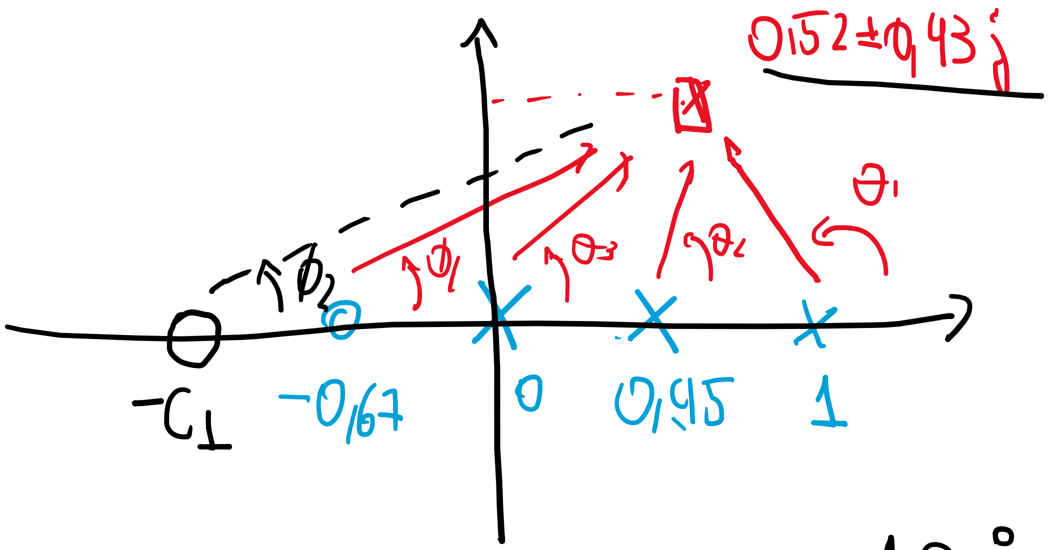
FORMA 2) LUGAR DAS RAIZES

$$G_c(z) = \frac{K(z+c_1)(\cancel{z+c_2})}{z(z-1)}$$

$$G_p(z) = \frac{0{,}054\,(z+0{,}67)}{(z-0{,}45)(\cancel{z-0{,}67})}$$

$C_2 = -0{,}67$ (ZERO DO CONTROLE CANCELA 1 DOS POLOS PLANTA)

$\Rightarrow C_1, K \rightarrow$ CALCULADO POR L.R.



$$-\theta_1 - \theta_2 - \theta_3 + \phi_1 + \phi_2 = -180^\circ$$

$\Rightarrow C_1 = -0{,}26 \quad (\phi_2 = 59^\circ)$

$$\boxed{G_c(z) = \frac{K.(z-0{,}26)(z-0{,}67)}{z(z-1)}}$$

E O GANHO?

COND. MÓDULO

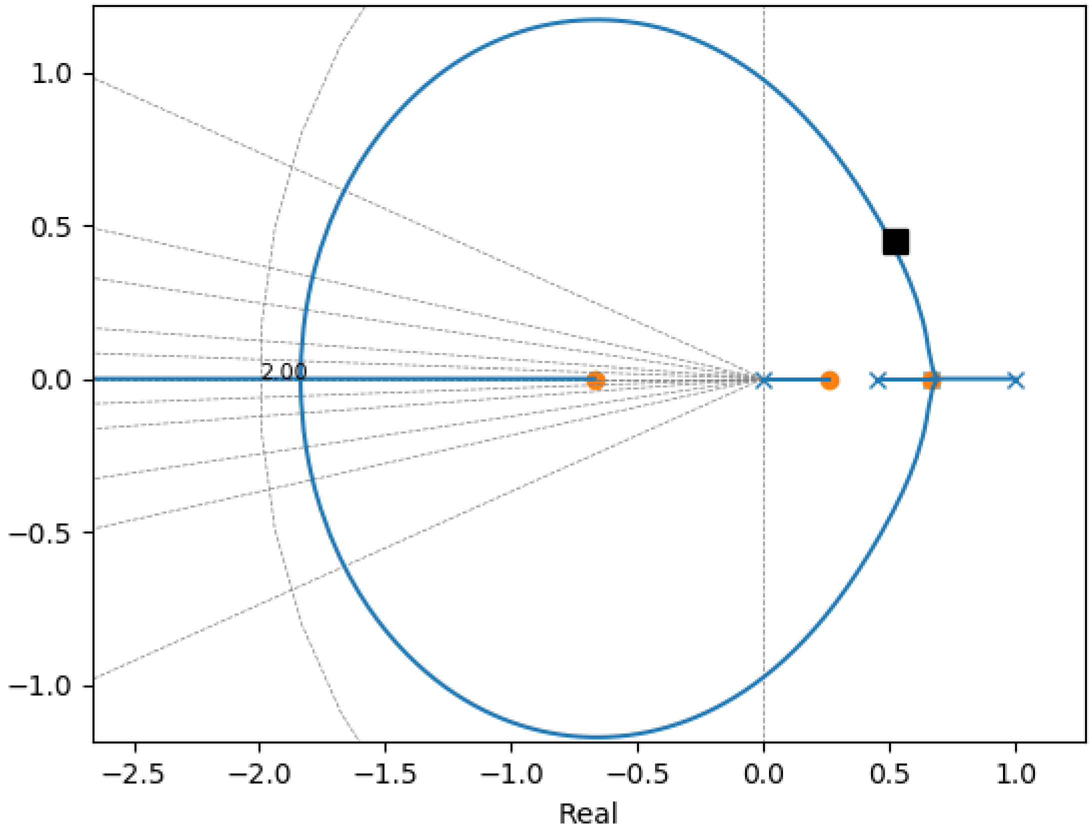
$$\left| G_p(z) \cdot G_c(z) \right|_{z=-0{,}52 \pm 0{,}43j} = 1$$

$$\left| \frac{0{,}059K \cdot (z-0{,}26)(z+0{,}67)}{z(z-1)(z-0{,}45)} \right| = 1 \Rightarrow \boxed{K = 5{,}51}$$

CORRETO

NÃO CORRETO

Clicked at: 0.5235 +0.4485j gain: 5.685 damp: -0.7595




```
# rlocus
z = [0.26,0.67]
p = [0,1]
k = 1
Gc = signal.zpk2tf(z , p , k)
Gcd = tf(Gc[0],Gc[1],Ts)
```

EM MATLAB

OK