12 a)

Reduzir $\lim\limits_{n} \left(1+\frac{1}{n}\right)^{n} = e$ $\qquad \left(1+\frac{1}{kn}\right)^{kn} \to e$

$$\lim \left(1+\frac{1}{n}\right)^{n+3} = \lim \underbrace{\left(1+\frac{1}{n}\right)^{3}}_{\to ?} \underbrace{\left(1+\frac{1}{n}\right)^{n}}_{\to e}$$

$$\lim \left(\underbrace{1}_{\to 1} + \underbrace{\frac{1}{n}}_{\to 0}\right) = \lim 1 + \lim \frac{1}{n} = 1+0 = 1$$

$f(x) = x^3$ é contínua $\Rightarrow \lim \left(1+\frac{1}{n}\right)^{3} = \lim f\left(1+\frac{1}{n}\right)$

$$= f\left(\lim \left(1+\frac{1}{n}\right)\right)$$

$$= f(1) = 1^3 = 1$$

10 b)

$a_n < 2 \quad \forall n$

Base $\Rightarrow$ $a_1 = \sqrt{2} < 2$ ✓

P.I. $\Rightarrow$ $a_n < 2$ p/ $n = n$ (H.I.)

$a_{n+1} < 2 \iff \underbrace{a_{n+1}^2 < 4}_{}$

$\sqrt{2}$

$n \Rightarrow n+1$

$n-1 \Rightarrow n$

or $a_n = \sqrt{2 + a_{n-1}}$ $\quad a_{n+1} = \sqrt{2 + a_n}$

$a_n < a_{n+1}$

$m < n \Rightarrow a_m < a_n$

$a_{n-1} < 2$ $\quad a_{n+1} > a_n$

$\boxed{a_{n+1}^2 > a_n^2}$ (*) $\iff$ $a_{n+1} > a_n$ $\quad 2 > a_n$ $\quad \underbrace{\sqrt{2 + a_n} > \sqrt{a_n + a_n}}_{}$

$a_{n+1}^2 = 2 + a_n > a_n + a_n = 2a_n > a_n \cdot a_n = a_n^2$

$$\lim \left(1+\frac{1}{n}\right)^3 = 1$$

$$\lim \left(1+\frac{1}{n}\right)^n = e \quad \Rightarrow \lim \left(1+\frac{1}{n}\right)^{n+3} = \lim \left(1+\frac{1}{n}\right)^3 \cdot \lim \left(1+\frac{1}{n}\right)^n$$

$$= 1 \cdot e$$

$$= e$$

$$n \geq 1$$

$$e^{x \cdot \ln\left(1+\frac{1}{2x}\right)}$$

$$\left(\left(1+\frac{1}{10n}\right)^{10n}\right)^{\frac{1}{10}}$$

$\hookrightarrow e$

$\hookrightarrow e^{\frac{1}{10}}$

$$x^{\frac{1}{10}} = \sqrt[10]{x}$$

$$K = n+1$$

$$K-1 = n \geq 1$$

$$\frac{n+2}{n+1} = \frac{n+1+1}{n+1}$$

$$= \frac{n+1}{n+1} + \frac{1}{n+1} = 1 + \frac{1}{n+1}$$

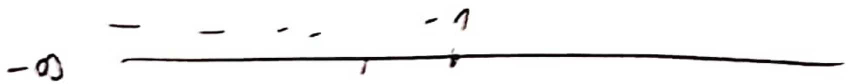
$$\left(1+\frac{1}{n+1}\right)^n =$$

$$= g(n) \cdot \left(1+\frac{1}{K}\right)^K \Rightarrow L \cdot e$$

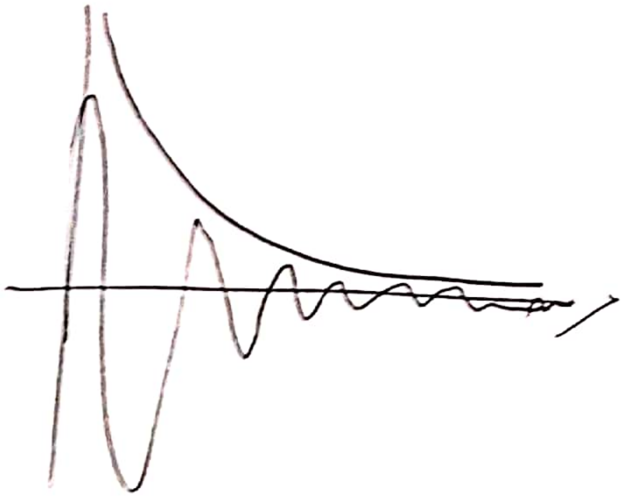
$g(n) \hookrightarrow L$ $\quad \left(1+\frac{1}{K}\right)^K \hookrightarrow e$

$\sqrt{2} = a_1 < a_2$

~~a_n~~ $a_n = -n$



$\frac{1}{2^n}$



$$\sum_{n=0}^{\infty} a_n$$