Quantum Theory of Many-Body systems in Condensed Matter (4302112) 2020

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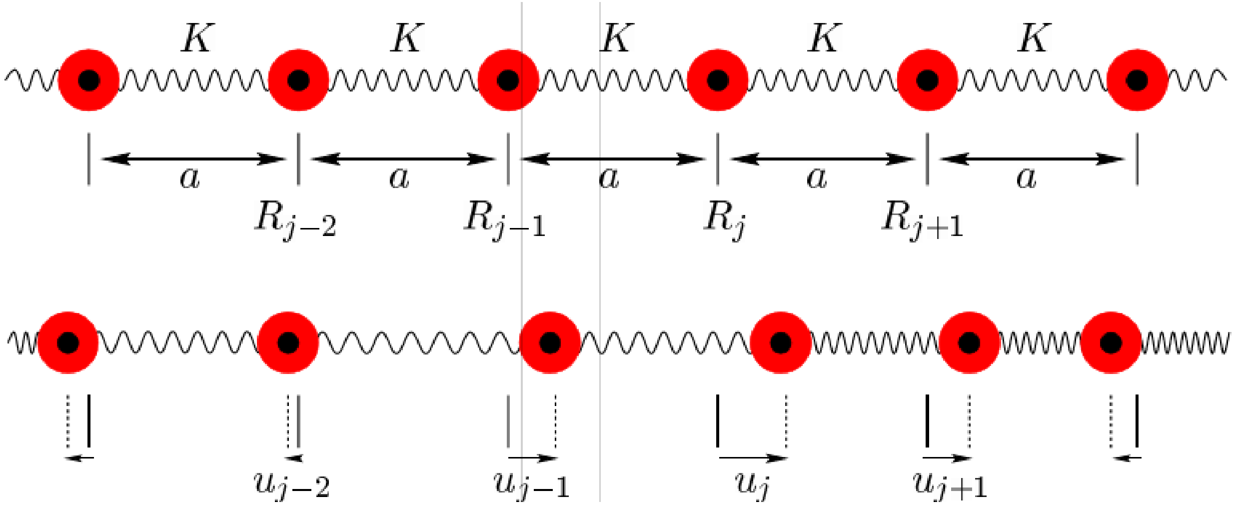
Today's class: *Phonons*

- Acoustic phonons in 1D.
- Phonons in 3D: Debye model.
- Electron-phonon interaction (e-ph).
- e-ph interaction in the jellium model.

Acoustic phonons in 1D



$$\hat{H}_{\rm ph} = \sum_{j=1}^{N} \frac{\hat{p}_j^2}{2M} + \frac{K}{2}\left(\hat{u}_j - \hat{u}_{j-1}\right)^2$$

$$\hat{H}_{\rm ph} = \sum_{q} \frac{\hat{p}_q \hat{p}_{-q}}{2M} + \frac{M}{2}\omega_q^2 \hat{u}_q \hat{u}_{-q}$$

$$\begin{cases} \hat{u}_q = \dfrac{1}{\sqrt{N}} \displaystyle\sum_{j=1}^{N} \hat{u}_j e^{-iqR_j^{(0)}} \\ \hat{p}_q = \dfrac{1}{\sqrt{N}} \displaystyle\sum_{j=1}^{N} \hat{p}_j e^{-iqR_j^{(0)}} \end{cases}$$

$$\begin{cases} \hat{b}_q = \dfrac{1}{\sqrt{2}} \left(\dfrac{\hat{u}_q}{\ell_q} + i \dfrac{\hat{p}_q}{\hbar/\ell_q} \right) \\ \hat{b}_q^\dagger = \dfrac{1}{\sqrt{2}} \left(\dfrac{\hat{u}_{-q}}{\ell_q} - i \dfrac{\hat{p}_{-q}}{\hbar/\ell_q} \right) \end{cases}$$

$$\omega_q = 2\sqrt{\frac{K}{M}} \left| \sin \frac{qa}{2} \right| \approx v_s q$$

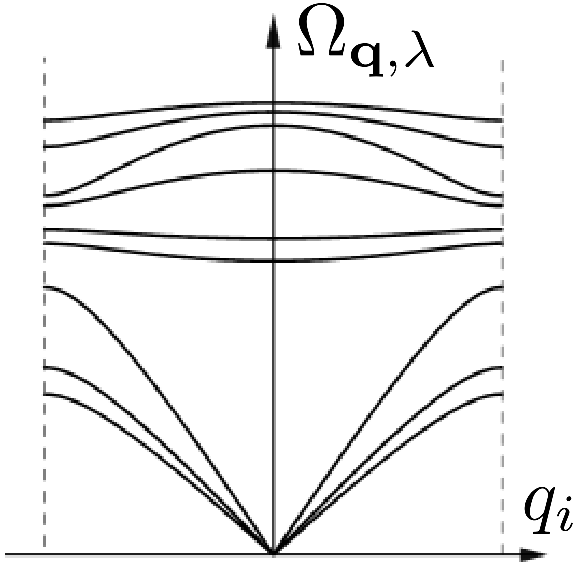
$$\hat{H}_{\rm ph} = \sum_q \hbar\omega_q \left(\hat{b}_q^\dagger \hat{b}_q + 1/2 \right)$$

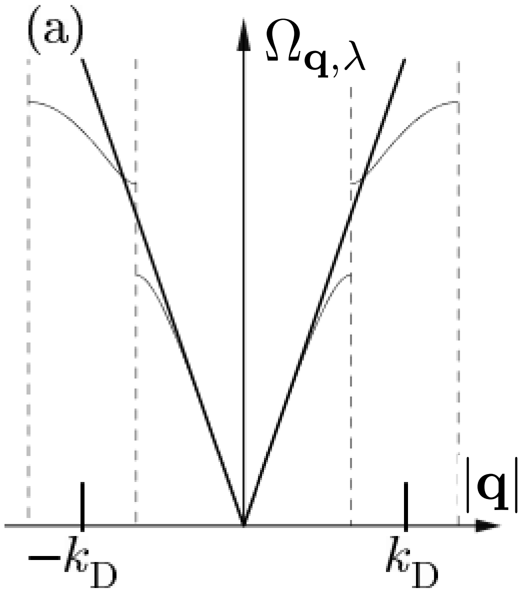
$$\ell_q = \sqrt{\frac{\hbar}{M\omega_q}} \qquad \left[\hat{b}_q, \hat{b}_{q'}^\dagger \right] = \delta_{qq'}$$

Phonons in 3D

$$\hat{H}_{\text{ph}} = \sum_{\mathbf{q},\lambda} \hbar\Omega_{\mathbf{q},\lambda}\left(\hat{b}^\dagger_{\mathbf{q},\lambda}\hat{b}_{\mathbf{q},\lambda} + 1/2\right)$$

Debye model: $\begin{cases} \Omega_{\mathbf{q},\lambda} \approx v_D|\mathbf{q}| \\ -k_D \leq |\mathbf{q}| \leq +k_D \end{cases}$

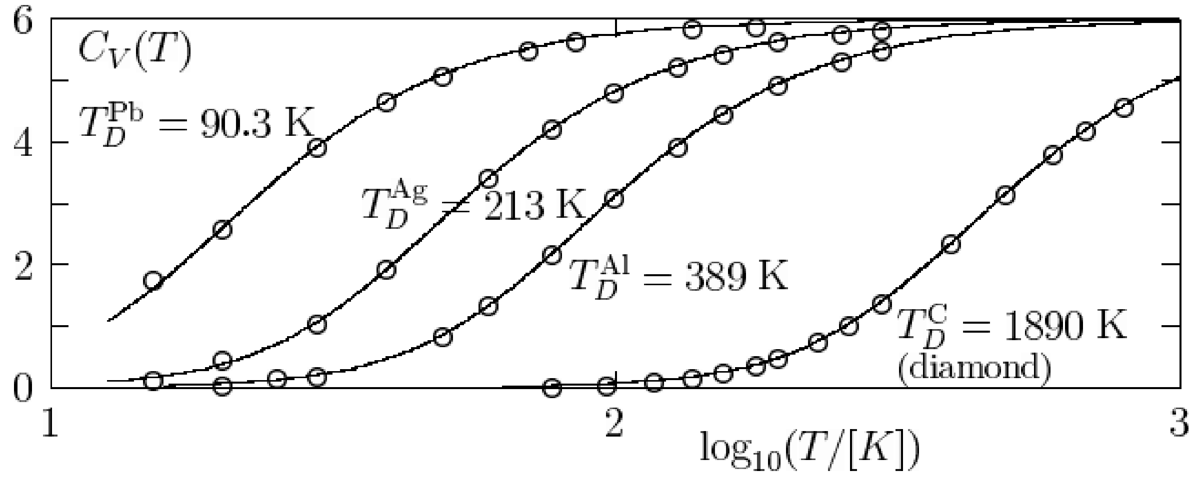




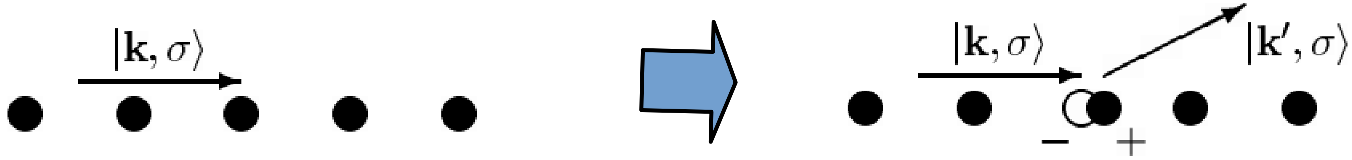
$$N_{\text{ion}} = \frac{V_{\mathbf{r}}}{(2\pi)^3}\frac{4}{3}\pi(k_D)^3$$

$$\hbar\omega_D = \hbar v_D k_D = k_B T_D$$

$$D_{\text{ph}}(\varepsilon) = \frac{dN_{\text{ph}}(\varepsilon)}{d\varepsilon} = \frac{3V_{\mathbf{r}}}{2\pi^2}\frac{\varepsilon^2}{\hbar v_D}$$



Electron-phonon interaction



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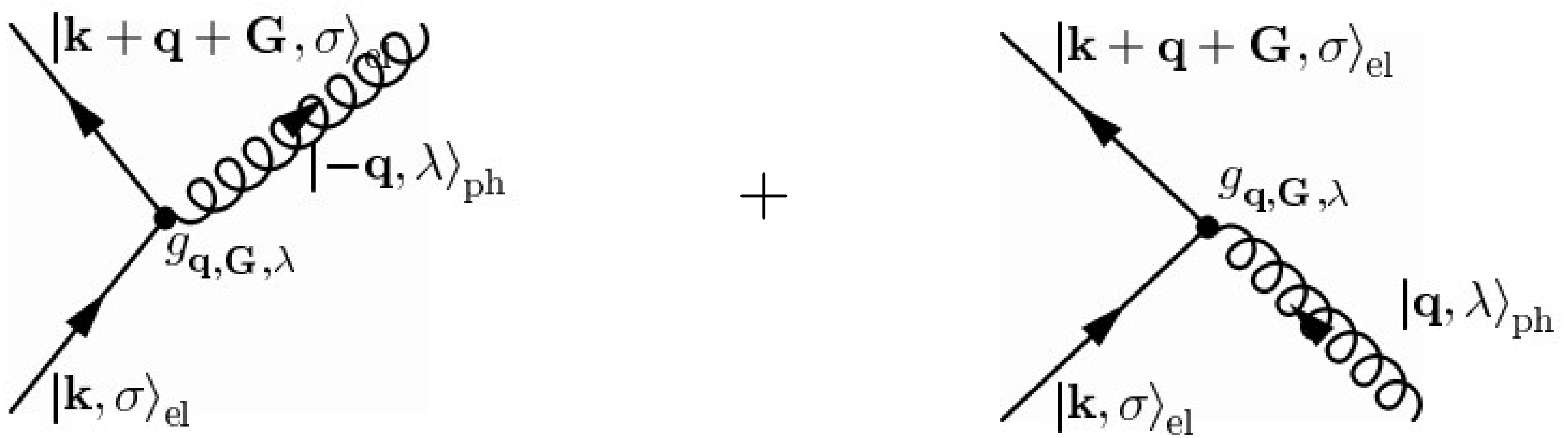
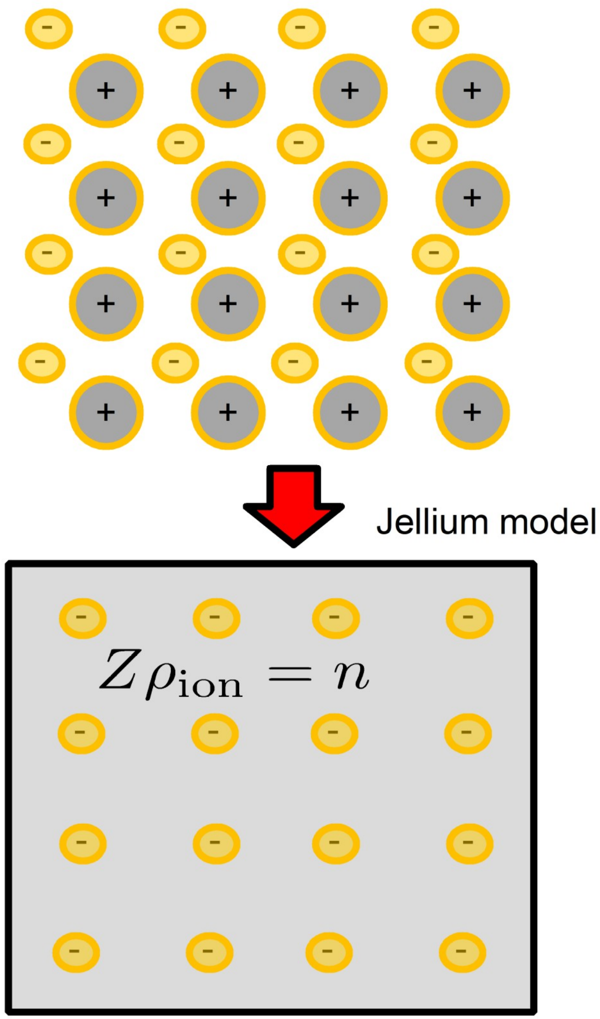
$$\hat{U}_{\mathrm{el-ph}} = \frac{1}{V_{\mathbf{r}}} \sum_{\mathbf{k},\mathbf{q}} g_{\mathbf{q}} \hat{c}^{\dagger}_{\mathbf{k}+\mathbf{q}} \hat{c}_{\mathbf{k}} \left(\hat{b}^{\dagger}_{-\mathbf{q}} + \hat{b}_{\mathbf{q}} \right)$$

$$g_{\mathbf{q}} = ie\sqrt{\frac{N\hbar}{2M\omega_q}}\, qV(\mathbf{q})$$



Electron-phonon in the jellium model



Plasma frequency: $$\Omega^2 = \frac{Ze^2 n}{M\epsilon_0}$$

$$\hat{U}_{\text{el-ph}} = \frac{1}{V_{\mathbf{r}}} \sum_{\mathbf{k},\mathbf{q}} g_{\mathbf{q}}^{\text{jell}} \hat{c}_{\mathbf{k}+\mathbf{q}}^{\dagger} \hat{c}_{\mathbf{k}} \left(\hat{b}_{-\mathbf{q}}^{\dagger} + \hat{b}_{\mathbf{q}} \right)$$

$$g_{\mathbf{q}}^{\text{jell}} = ieqV(\mathbf{q}) \sqrt{\frac{n\hbar}{2M\Omega}} \sqrt{V_{\mathbf{r}}}$$

Useful: $$\frac{1}{V_{\mathbf{r}}} |g_{\mathbf{q}}^{\text{jell}}|^2 = \frac{1}{2} V(\mathbf{q}) (\hbar\Omega)$$