$$PV^{\gamma} = C$$

$$\log(PV^{\gamma}) = \log(C)$$

$$\log(P) + \log(V^{\gamma}) = \log(C)$$

$$\log(P) + \gamma \log(V) = \log(C)$$

$$\log P = \log C - \gamma \log V$$

<u>NOVO PROBLEMA</u>

APROXIME $F(V) = \log(P(V))$

POR

$$G(V) = a_0 + a_1 \log V$$

PELO MMQ UMA VEZ

OBTIDA A RESPOSTA, TEMOS

$c = e^{a_0}$, $\gamma = -a_1$

V	10	20	30	40
P	6 0	2 2	1 3	0 8

<u>OBS</u> ESCOLHI O LOGARITMO NATURAL. PODERIA USAR OUTRA BASE

PARA RESOLVERMOS O PROBLEMA, PRECISAMOS DOS LOGARITMOS DE V E P

$\log V$	2 30259	2 99573	3 4012	3 6888
$\log P$	1 7918	0 78846	0.26223	-0 223

$g_0(v) = 1$, $g_1(v) = \log V$

MATRIZ DO SISTEMA NORMAL

$$\langle g_0, g_0 \rangle = \sum_{i=1}^{4} g_0(v_i)^2 = 4$$

$$\langle g_0, g_1 \rangle = \sum_{i=1}^{4} g_0(v_i)\, g_1(v_i) =$$

$$\sum_{i=1}^{4} \log v_i = 12\;3884 = \langle g_1, g_0 \rangle$$

$$\langle g_1, g_1 \rangle = \sum_{i=1}^{4} g_1(v_i)^2$$

$$= \sum_{i=1}^{4} (\log v_i)^2 = 394523$$

$$\langle g_0, F \rangle = \sum_{i=1}^{4} g_1(v_i)\, F(v_i) =$$

$$\sum_{i=1}^{4} \log P_i = 2\;61944$$

$$\langle g_1, F \rangle = \sum_{i=1}^{4} g_1(v_i) F(v_i)$$

$$= \sum_{i=1}^{4} (\log v_i)(\log P_i)$$

$$= 6\ 55689$$

$$\begin{bmatrix} 4 & 12\ 3884 \\ 12\ 3884 & 39\ 4523 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 2\ 61944 \\ 6\ 55689 \end{bmatrix}$$

SOLUÇÃO

$a_0 = 5\ 09899$

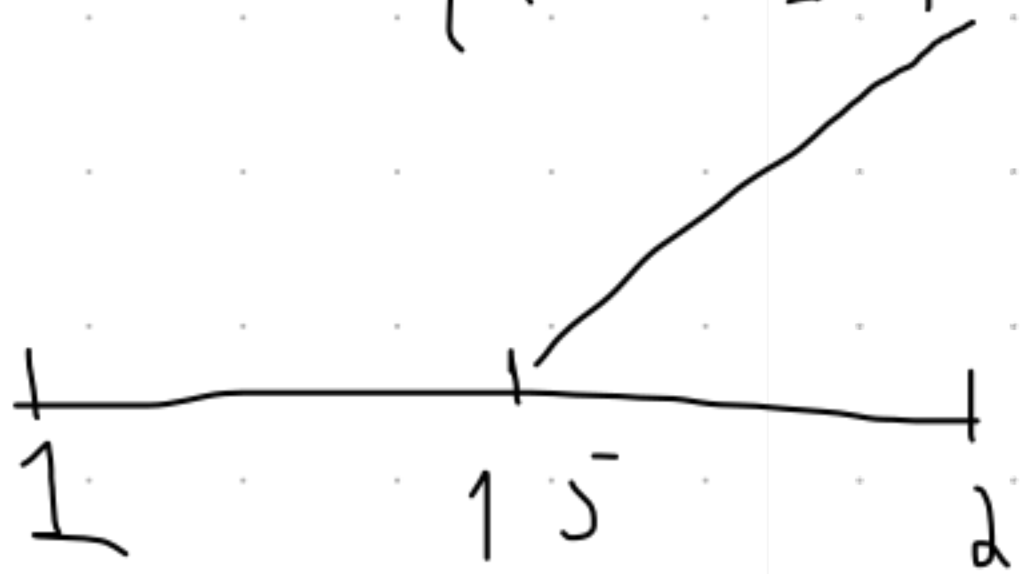
$a_1 = -1\ 43494$

$C = e^{a_0} = 163857$

$\gamma = -a_1 = 1\ 43494$

QUESTÃO 2

$$f(x) = \begin{cases} 0, & 1 \leq x \leq 1{,}5 \\ 2x-3, & 1{,}5 \leq x \leq 2 \end{cases}$$



OBS

$f(2) = 1$

OS POLINÔMIOS DE LEGENDRE SÃO ORT. EM $[-1, 1]$

MUDANÇA DE VARIÁVEL PARA FICARMOS COM UMA FUNÇÃO DEFINIDA EM $[-1,1]$

$$\ell : [-1,1] \longrightarrow [1,2]$$

$$\ell(t) = \alpha t + \beta$$

$$\ell(-1) = 1 \quad -\alpha + \beta = 1$$

$$\ell(1) = 2 \quad \alpha + \beta = 2$$

$$\Longrightarrow \alpha = \frac{1}{2}, \ \beta = \frac{3}{2}$$

$$\ell(t) = \frac{t+3}{2}$$

$$F(t) = f(\ell(t))$$

$$= f\left(\frac{t+3}{2}\right)$$

$$-1 \leq t \leq 0, \ 1 \leq \ell(t) \leq 1.5$$

$$\Longrightarrow f(\ell(t)) = 0$$

$$0 \leq t \leq 1 \Longrightarrow 1.5 \leq \ell(t) \leq 2$$

$$\Longrightarrow f(\ell(t)) = 2\,\ell(t) - 3$$

$$= 2\,\frac{t+3}{2} - 3 = t$$

$$F : [-1,1] \longrightarrow \mathbb{R}$$

$$F(t) = \begin{cases} 0, & -1 \leq t \leq 0 \\ t, & 0 \leq 1 \leq 1 \end{cases}$$

$$G(t) = c_0 P_0(t) + c_1 P_1(t) + c_2 P_2(t)$$

ORTOGONALIDADE $\Longrightarrow$

$$c_K = \frac{\langle P_K, F \rangle}{\langle P_K, P_K \rangle}$$ ONDE

$$\langle u, v \rangle = \int_{-1}^{1} u(t)\, v(t)\, dt$$

OS P_K SÃO ORT E

$$\langle P_K, P_K \rangle = \frac{2}{2K+1}$$

$$P_0(t) = 1$$

$$c_0 = \frac{\langle P_0, F\rangle}{\langle P_0, P_0\rangle} = \frac{1}{2}\int_{-1}^{1} F(t)\,dt =$$

$$\frac{1}{2}\int_{0}^{1} t\,dt = \frac{1}{2}\,\frac{1}{2} = \frac{1}{4}$$

— / —

$$P_1(t) = t$$

$$c_1 = \frac{\langle P_1, F\rangle}{\langle P_1, P_1\rangle} = \frac{1}{2/3}\int_{-1}^{1} t\,F(t)\,dt$$

$$= \frac{3}{2}\int_{0}^{1} t\,t\,dt = \frac{3}{2}\,\frac{1}{3} = \frac{1}{2}$$

$$P_2(t) = \frac{3}{2}t^2 - \frac{1}{2}$$

$$C_2 = \frac{\langle P_2, F \rangle}{\langle P_2, P_2 \rangle} = \frac{1}{2/5} \int_{-1}^{1} \left(\frac{3}{2}t^2 - \frac{1}{2}\right) F(t)\, dt$$

$$= \frac{5}{2} \int_{0}^{1} \left(\frac{3}{2}t^2 - \frac{1}{2}\right) \cdot t\, dt$$

$$= \frac{5}{2} \int_{0}^{1} \left(\frac{3}{2}t^3 - \frac{1}{2}t\right) dt$$

$$= \frac{5}{2}\left(\frac{3}{2}\,\frac{1}{4} - \frac{1}{2}\,\frac{1}{2}\right)$$

$$= \frac{5}{8}\left(\frac{3}{2} - 1\right) = \frac{5}{16} \,//$$

$$G(t) = \frac{1}{4} P_0(t) + \frac{1}{2} P_1(t) + \frac{5}{16} P_2(t) =$$

$$\frac{1}{4} + \frac{1}{2} t + \frac{5}{16} \left(\frac{3}{2} t^2 - \frac{1}{2} \right)$$

$$= \frac{1}{4} + \frac{1}{2} t + \frac{15}{32} t^2 - \frac{5}{32}$$

$$= \frac{3}{32} + \frac{1}{2} t + \frac{15}{32} t^2 \quad //$$

PRECISAMOS FAZER A MUDANÇA INVERSA PARA TERMOS O DOMÍNIO EM [1, 2]

$$\psi : [1, 2] \longrightarrow [-1, 1]$$

$$\psi(x) = \gamma x + \delta$$

$\psi(1) = -1 \qquad \gamma + \delta = -1$

$\psi(2) = 1 \qquad 2\gamma + \delta = 1$

$$\Longrightarrow \gamma = 2$$
$$\delta = -3$$

$$\psi(x) = 2x - 3$$

SOLUÇÃO,

$$g(x) = G(\psi(x)) =$$

$$\frac{3}{32} + \frac{1}{2}\psi(x) + \frac{5}{32}[\psi(x)]^2$$

$$= \boxed{\frac{3}{32} + \frac{1}{2}(2x-3) + \frac{5}{32}(2x-3)^2}$$

$$= \frac{3}{32} + x - \frac{3}{2} + \frac{5}{32}(4x^2 - 12x + 9)$$

etc.

QUESTÃO 3

(a) NOTE QUE $g_{m+1}(x)$ É A PROGEÇÃO ORTOGONAL DE f, SEGUNDO O PRODUTO INTERNO

$$\langle u, v \rangle = \int_0^{2\pi} u(x)\, v(x)\, dx,$$

NO ESPAÇO G GERADO PELAS FUNÇÕES

$$1,\ \cos(kx),\ \text{sen}(kx),$$

$$1 \leq k \leq m+1$$

OU SEJA, $f - g_{m+1}$ É

ORTOGONAL A QUALQUER ELEMENTO DE G SEGUNDO O PRODUTO INTERNO ACIMA

$$g_{m+1}(x) = a_0 + \sum_{k=1}^{m+1} a_k \cos(kx) + b_k \operatorname{sen}(kx)$$

$$g_m(x) = a_0 + \sum_{k=1}^{m} a_k \cos(kx) + b_k \operatorname{sen}(kx)$$

a_l, b_l COEF DE FOURIER

$$g_{m+1}(x) - g_m(x) = a_{m+1} \cos[(m+1)x] + b_{m+1} \operatorname{sen}[(m+1)x]$$

LOGO,

$g_{m+1} - g_m \in G$

$\Longrightarrow f - g_{m+1} \perp g_{m+1} - g_m$

(b)

$$\underbrace{(f - g_{m+1})}_{} + \underbrace{(g_{m+1} - g_m)}_{} = f - g_m$$

ORTOGONAIS

PITÁGORAS

$$|f - g_{m+1}|^2 + |g_{m+1} - g_m|^2 = |f - g_m|^2$$

$$\langle f - g_{m+1}, f - g_{m+1} \rangle + \langle g_{m+1} - g_m, g_{m+1} - g_m \rangle$$
$$= \langle f - g_m, f - g_m \rangle$$

$$\langle f-g_m, f-g_m\rangle =$$

$$\langle f-g_{m+1}+g_{m+1}-g_m, f-g_{m+1}+g_{m+1}-g_m\rangle$$

$$= \langle f-g_{m+1}, f-g_{m+1}\rangle + \underbrace{\cancel{\langle f-g_{m+1}, g_{m+1}-g_m\rangle}}_{0} \; (a)$$

$$+ \underbrace{\cancel{\langle g_{m+1}-g_m, f-g_{m+1}\rangle}}_{0} \; (a)$$

$$+ \langle g_{m+1}-g_m, g_m-g_m\rangle$$

$$\langle f-g_m, f-g_m\rangle = \int_0^{2\pi} [f(x)-g_m(x)]^2 \, dx$$

$$\langle f-g_{m+1}, f-g_{m+1}\rangle = \int_0^{2\pi} [f(x)-g_{m+1}(x)]^2 \, dx$$

$$\langle g_{m+1}-g_m, g_{m+1}-g_m\rangle =$$

$$= \int_0^{2\pi} \left[a_{m+1}\cos((m+1)x) + b_{m+1}\,\mathrm{sen}((m+1)x)\right]^2 dx$$

$$= a_{m+1}^2 \underbrace{\int_0^{2\pi} \cos^2[(m+1)x]\,dx}_{\pi} +$$

$$2a_{m+1}b_{m+1} \underbrace{\int_0^{2\pi} \cos((m+1)x)\,\mathrm{sen}((m+1)x)\,dx}_{0}$$

$$+ b_{m+1}^2 \underbrace{\int_0^{2\pi} \mathrm{sen}^2[(m+1)x]\,dx}_{\pi}$$

$$= \pi\left(a_{m+1}^2 + b_{m+1}^2\right)$$

USE O ITEM (b)